FX Model Computer Assignment Report

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Please refer to ComputerAssignment.ipynb for code details

1 SARB Calibration

1.1 Restore strike K

Using the market instruments given, I first restored strikes for these instruments using:

$$\Delta = P^f \frac{K}{F} N(d2), \text{ for call option}$$

and

$$\Delta = -P^f \frac{K}{F} N(-d2), \text{ for put option}$$

where $P^f = e^{-r_f T}$ is the zero coupon bond and $F = e^{(r_d - r_f)T}$ is the forward rate.

I used the fsolve package from scipy.optimize to solve for K

1.2 Hagan et. al. approximation

The implied volatility of the strikes obtained above can be approximated by using Hagan et. al. approximation to the SARB model.

$$\begin{aligned}\sigma_{IV}(K, t_e) = & \frac{\alpha}{(FK)^{(1-\beta)/2}(1 + \frac{(1-\beta)^2 log(\frac{F}{K})^2}{24} + \frac{(1-\beta)^4)(log(\frac{F}{K})^4}{1920}} * \frac{z}{X(z)} \\ & *(1 + [\frac{(1-\beta)^2\alpha^2}{24(FK)^{1-\beta}} + \frac{\rho\beta\nu\alpha}{4(FK)^{(1-\beta)/2}} + \frac{(2-3\rho^2)\nu^2}{24}]T_e)\end{aligned} \tag{1}$$

The obtained implied volatility are expressed in terms of unknown model parameters α, ρ, and ν

1.3 Calibration

As we have obtained implied volatility for all strikes and market instruments volatility, we can use the following linear system to solve for 3 SARB model parameters.

$$\sigma_{RR} = \sigma_{RRCall} - \sigma_{RRPut}$$

$$\sigma_{BF} = (\sigma_{BFCall} + \sigma_{BFPut})/2 + \sigma_{ATMIV}$$

$$\sigma_{ATM} = \sigma_{ATMIV}$$

I used the least_square package from scipy.optimize to solve the linear system, with parameter constrain $\alpha >= 0$, $\rho \in (-1, 1)$, and $\nu > 0$

1.4 Implied volatility

With SARB model parameters solved for each case, we can calculate the implied volatility for each strike from the previously obtained expressions in α, ρ, and ν

2 Volatility Smile

Using the market data in 1Y case, and the previous procedure, we can obtained the strikes for 0.1d call and -0.1d put. Then we use Hagan et. al. approximation again to build a volatility smile curve for strikes in this range.