APPENDIX

A. MEASURING LINE VELOCITIES VIA THE TEMPLATE-FITTING METHOD IN SNE IC-BL

The blueshift and width of specific spectral absorption features, such as Fe II λ 5169, are used to track the photospheric velocity of SNe. For SN Ic spectra, it is straight-forward to identify absorption due to Fe II λ5169, while Fe II $\lambda\lambda$ 4924, 5018 lines are usually blended into a single feature (see Fig. 9). For SNe Ic-bl, all three Fe II lines are heavily blended due to the high expansion velocities in the ejecta (see Fig. 9 and 10). Thus, it is difficult to accurately identify Fe II λ 5169 and measure its absorption velocity. There have been several attempts in the literature to solve this problem. Lyman et al. (2014), for example, used Si II λ6355 instead, whenever they could not identify Fe II λ5169. However, we find that the Fe II λ5169 absorption velocity is systematically higher than the Si II λ6355 measured using the same method (see Section 4.4 and Bianco et al, in prep). Thus, the two velocities cannot be used interchangeably to track photospheric velocity in order to calculate physical parameters for SNe. What makes it worse is that Si II λ6355 is sometimes blended with the Na I D lines into one absorption feature (e.g., PTF 10qts). Thus, it is at times difficult to identify Si II λ6355 as well. Schulze et al. (2014) analyzed spectra of SN -GRBs (many of the same spectra as in our SN-GRBs sample) and attributed the blended and broad feature due to the three Fe II lines to only Fe II λ5169. Thus, they derive systematically high velocities, up to $-50{,}000$ km s^{-1}. Pignata et al. (2011) used SYNOW, a spectral synthesis code, to obtain velocities of Fe II $\lambda\lambda\lambda$4924, 5018, 5169 for SN 2009bb. Different spectral synthesis codes, however, yield different velocities for the same SN using different methods, as shown in Section 4.3.

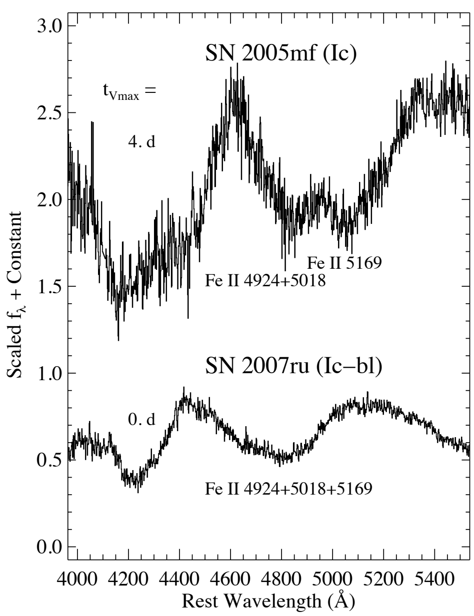

FIG. 9.— SN spectra of a SN Ic and a SN Ic-bl showing the wavelength region around the Fe II lines.

We developed a data-driven and consistent method to identify Fe II λ5169 and measure its velocity in the blended spectra of SNe Ic-bl: we used a template fitting approach in which we found the best fit between a convolved and blueshifted SN Ic template and the SN Ic-bl spectrum under consideration, at similar phases, for the wavelength region of the feature under consideration. This is similar to what we did in Section 3.2 for the whole spectrum, but now it is a formal fit, which we implement via Monte Carlo Markov Chain (MCMC) methods. The fits that this procedure yields are generally good, as indicated by the $\sim$ unity values for the reduced χ^2 values.

Our SN Ic template spectra are the mean SN Ic spectra of Section 3, but with a different time spacing: now we construct them for every two-day-phase interval (as opposed to five-day-intervals for the mean spectra of Section 3.1), in the range between $t = -10$ and $t = +72$ days with respect to maximum light (i.e, $t = -10, -8, -6, -4$, etc. days). Each mean spectrum includes SN Ic spectra in 5-day bins centered around the middle, i.e. "target" phase. For example: in order to measure the photospheric velocity for a SN Ic-bl spectrum at phase t =0 days, we use the mean SN Ic spectrum constructed using SN Ic spectra between phases -2.5 and 2.5 days and centered at phase $t = 0$ days. Thus, we note that template spectra at adjacent phases may not be independent from each other, but this aspect should not influence our results. For measuring the velocity of a SN Ic-bl spectrum at phase $t = -2$ days, we use the mean SN Ic spectrum centered at $t = -2$ days (constructed from SN Ic spectra between -4 and 0 days) for template fitting. Each mean spectrum includes no more than one spectrum per SN Ic, in order to avoid biasing the mean spectrum towards better-observed SNe. If two or more spectra of the same SN Ic are available within the phase range (within ±2.5 days of the target phase), we choose the one nearest to the target phase. If there is still more than one spectrum

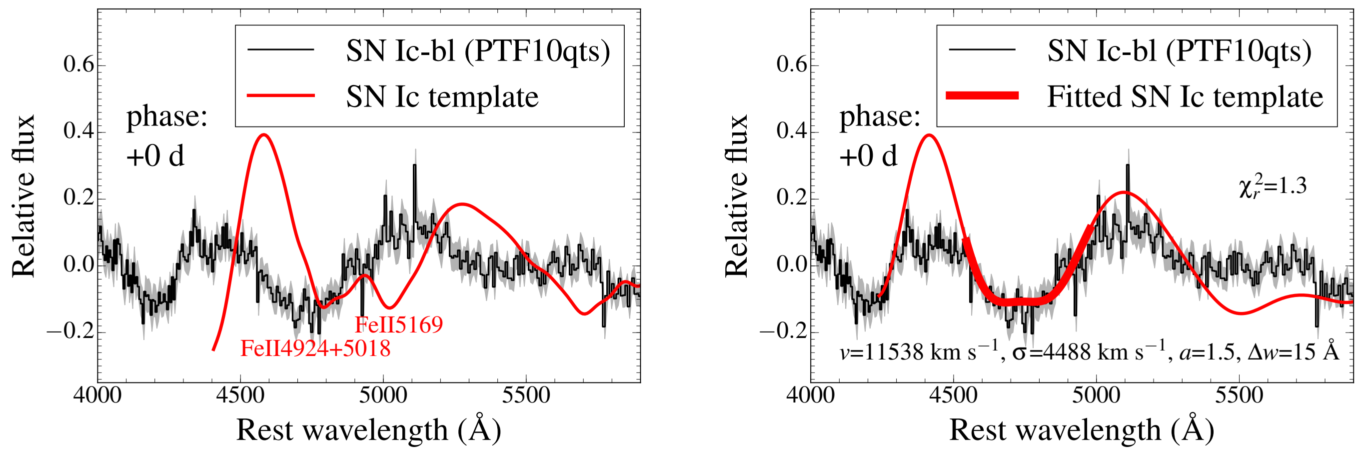

FIG. 10.— Example for convolution fitting of the Fe II feature for the spectrum of a SN Ic-bl (here: PTF10qts) at t=0 days. In both plots, the SN Ic-bl spectrum is in black with its error spectrum overlaid in grey. *Left:* The SN Ic template spectrum (before the fit) is overplotted in red, and shows a "W" feature between 4600 Å and 5300 Å due to the FeII triplet: the red absorption trough is due to Fe II λ5169, while the blue one is a blend of the other two Fe II lines. In the spectrum of the SN Ic-bl PTF10qts, the three Fe II lines are blended into one feature. *Right*: The fitted SN Ic template spectrum is overplotted in red, with the 50th percentile confidence values and their error bars (corresponding to $50^{th} - 16^{th}$ and $84^{th} - 50^{th}$ percentiles respectively) determined by Markov-Chain Monte Carlo (MCMC) methods. The SN Ic template has been dopplershifted by the 50th percentile value of velocity parameter v $(= 11,540^{+240}_{-250}$ km s^{-1}), broadened by the Gaussian kernel with the 50th percentile value of sigma parameter σ $(= 4490^{+300}_{-490}$ km s^{-1}), and stretched along the y-axis with the best value of amplitude parameter a $(= 1.5^{+0.1}_{-0.1})$. The thick red line denotes the spectrum that is used for the fit with the best value of the wavelength-range parameter Δw $(= 15^{+3}_{-2}$ Å), by which the initial wavelength range (between the two maxima) gets subtracted. Note that the final reported value (and its uncertainty) of the Doppler velocity parameter v includes the contribution of the SN Ic template itself for each phase at which the fit is performed. Also shown on the plot is the reduced χ^2 value for this fit.

that satisfies the above condition, we use the one that covers the largest optical wavelength range, or the one with the highest signal-to-noise ratio. Each mean spectrum contains spectra of N SNe, where $N \geq 3$, except the ones at phases of $t = +62$ and $+64$ days. Thus, for SNe Ic-bl spectra with phases between $t = +61$ and $+65$ days, we use the mean spectrum with the nearest phase — either the one at phase $t = +60$ days or the one at phase $t = +66$ days — to replace mean spectra at phases 62 and 64. Since SN Ic-bl spectra have broader features and higher velocities than those of SNe Ic, we convolve the template with a Gaussian kernel (see Section 3.2) and doppler blueshift it using the relativistic Doppler formula.

There are four parameters in our fit: a parameter for the width of the convolution Gaussian (σ), a velocity parameter (v) for the doppler shift, an amplitude parameter (a), and a wavelength-range parameter (Δw), by which we add to or subtract, times two, from the initial guess for the wavelength range of the SNe Ic-bl spectrum over which we will fit the template. The initial guess of the wavelength range is based on finding the local maxima on the blue and the red side of the feature and using the range in-between. We choose to use a single parameter instead of two (i.e., one for each end) to control the range because we want to limit the dimensionality of our parameter space, as a higher dimensional parameter space induces higher uncertainty in the parameter distributions, and requires more computational power. We use the following priors on our parameters: uniform prior for absorption velocity with respect to the SN Ic template (0−20,000km s^{-1}, where positive values refer to blueshifts, i.e., negative velocities), σ (0−20,000 km s^{-1}, where positive values refer to wider kernels), and amplitude (0−3). We used a gaussian prior for the wavelength-range parameter Δw, where we centered the Gaussian on zero and used 100 Å for the 99.7th percentile region. We note that these choice are only valid to up to $t_{Vmax} < 60$ days, as afterwards the SNe Ic-bl spectra become too similar to the SN Ic template spectra.

In summary, our likelihood function is:

$$\ln p(f_{Icbl}|f_{Ic}, err_{fIcbl}, \sigma, v, a, \Delta w) = -0.5\times \sum_{x=wbi+\Delta w}^{wri-\Delta w} \left\{ \frac{\{f_{Icbl}(x) - [a \times D(v) \times (G(s) * f_{Ic})](x)\}^2}{err_{fIcbl}(x)^2 \times (N-4)} + \ln[err_{fIcbl}(x)^2] \right\}, \tag{A1}$$

where $f_{Icbl}(x)$ is the flux of a SN Ic-bl spectrum at wavelength x, $D(v)$ is the relativistic doppler shift at velocity v, $G(s)$ is the Gaussian function with width σ, f_{Ic} is the flux of the SN Ic mean spectrum, err_{fIcbl} is the uncertainty of f_{Icbl}, N is the number of data points between wavelength $wbi + \Delta w$ and $wri - \Delta w$, and 4 represents the number of parameters in our model. We sample the posterior distribution function via the **emcee** package (Foreman-Mackey et al. 2013) which implements an affine-invariant ensemble Markov chain Monte Carlo (MCMC) sampler (Goodman & Weare 2010). The velocity parameter v and its error bar are based on its marginalized distribution: the Doppler velocity parameter v corresponds to the 50th percentile in the marginalized v distribution (i.e., median), while its

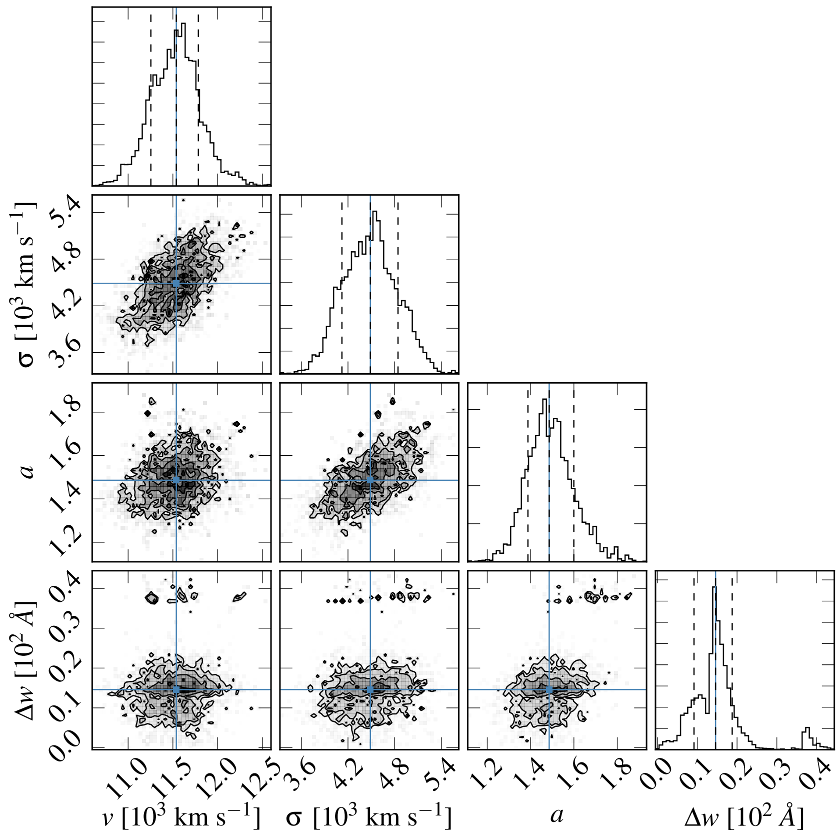

FIG. 11.— One- and two-dimensional projections of the posterior MCMC probability distributions for the spectral fit parameters for PTF 10qts at t_{Vmax}=0, shown in Fig. 10.

error bars correspond to the $50^{th} - 16^{th}$ and $84^{th} - 50^{th}$ percentiles of the marginalized distribution of v (Fig. 11). We compute the final Fe II λ5169 velocity for SNe Ic-bl by adding the Fe II λ5169 absorption velocity of the SN Ic template to the velocity parameter value from the fit. We compute the final uncertainty in the Doppler velocity parameter v by adding in quadrature the MCMC-derived uncertainty to that based on the velocity error in the corresponding SN Ic template. The velocity error in the SN Ic template is based on the fact the template spectrum is generated by spectra of different SNe Ic and at slightly different phases - thus the template velocity error was determined by taking the standard deviation of velocities measured in the individual SN spectra that were used to generate the SN template spectrum. Note that a potential mismatch between the phase of the SN Ic-bl spectrum and that of the mean SN Ic spectrum that was used for the fit does not impact the final absorption velocity measured for the SN Ic-bl spectrum.

In Figure 10, we demonstrate our template-fitting method for the SN Ic-bl PTF10qts at phase $t_{Vmax} = 0$ days, using the SN Ic mean spectrum at phase $t_{Vmax} = 0$ days as a template. Since the SNe Ic mean spectra are constructed using flattened spectra, we use the flattened version of the SNe Ic-bl spectra as well. We note that for SN 1998bw, its blue Fe trough is stronger than the one in the SN Ic template, at all phases. The same applies to SN 2002ap, but only for up to $t_{Vmax} = 10$ days.

We tested our template-fitting method by applying it to SNe Ic spectra and comparing the resultant Fe II 5169 velocities with those from line identifications. We find that the Fe II 5169 velocities from the two methods are consistent within their error bars. Thus, we can be confident that using the template-fitting method, which we use for SNe Ic-bl does not introduce a systematic bias in the velocity measurements. We note that for SN Ic 2013dk, the three iron lines were blended into one feature, such that we used the template-fitting method to measure its absorption velocity.

B. ABSORPTION VELOCITIES OF INDIVIDUAL SNE

While we presented the bulk properties of SNe in the above sections, readers may be interested in the velocity evolution of individual SNe. Thus, in FIg. 12, we present the Fe II λ5169 absorption velocities of individual SNe in three panels for the different SN subtypes. For the plot that shows SN-GRBs, we also distinguish between SN Ic-bl connected with LLGRBs vs. SNe Ic-bl connected with HLGRBs, as well as the radio-relativistic SNe 2009bb and 2012ap.