



Kendall's Tau

Kendall's Tau is a correlation coefficient and is thus a measure of the relationship between two variables.

Difference Kendall's Tau and Pearson Correlation

In contrast to the Pearson correlation, the Kendall's rank correlation is a non-parametric test procedure. For the calculation of Kendall's tau, the data must not be normally distributed and the two variables must only have an ordinal scale level.



Difference Kendall's Tau and Spearman Rank Correlation

Exactly the same is true for Spearman rank correlation, right? That's right! The Kendall's tau is very similar to Spearman's rank correlation coefficient.

However, Kendall's Tau should be preferred over Spearman's correlation when there is very little data and many rank ties!

Calculate Kendall's Tau

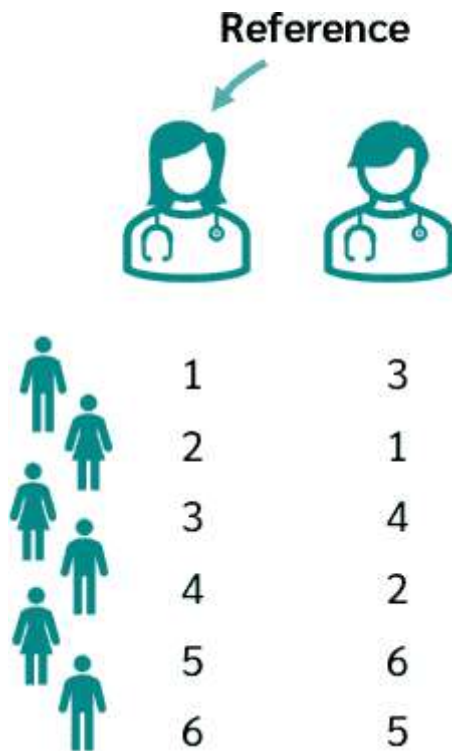
We can calculate the Kendall's Tau with this formula:

$$\tau = \frac{C - D}{C + D}$$

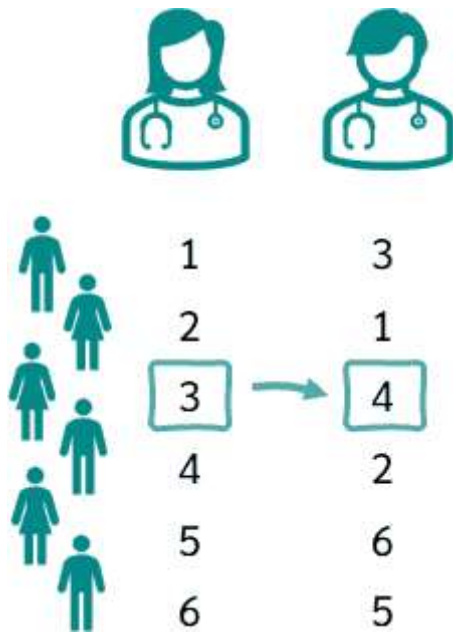
Where C is the number of concordant pairs and D is the number of discordant pairs. But, what are concordant and discordant pairs?

Example Kendall's Tau

Suppose two doctors rank 6 patients by descending physical health. One of the two doctors, in this case the female, is now defined as the reference and the patients are sorted from 1 to 6.

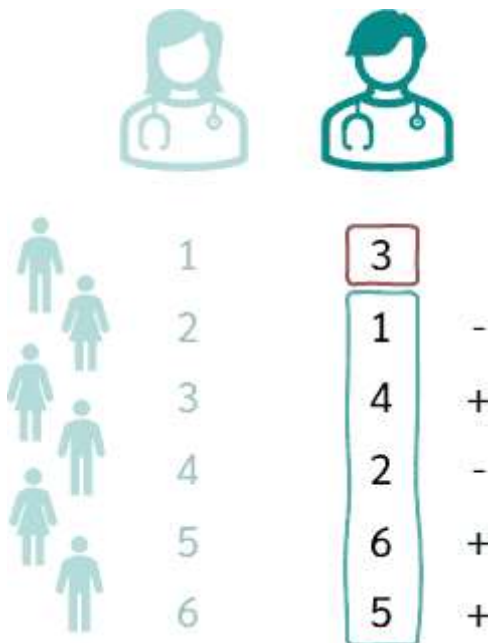


Now it is possible to compare the sorted ranks with the ranks of the second doctor, e.g. the patient who is ranked 3 by the female doctor is ranked 4 by the male doctor.



We want to know if there is a correlation between the two assessments using *Kendall's Tau*. To calculate it, we only need the ranks on the right-hand side, i.e. the ones from the male doctor.

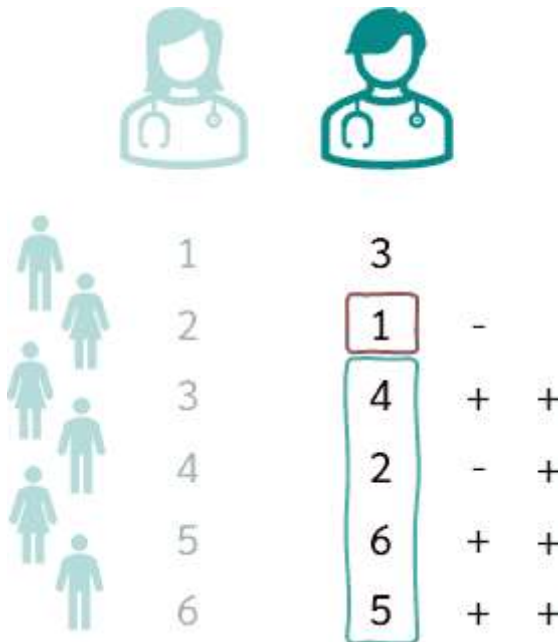
We now look at each rank and note whether the values below it are smaller or larger than itself.



As can be seen in the figure above, we start with the first rank, corresponding to the value 3. 1 is smaller than 3, so it gets a minus, 4 is larger, so it gets a plus, 2 is

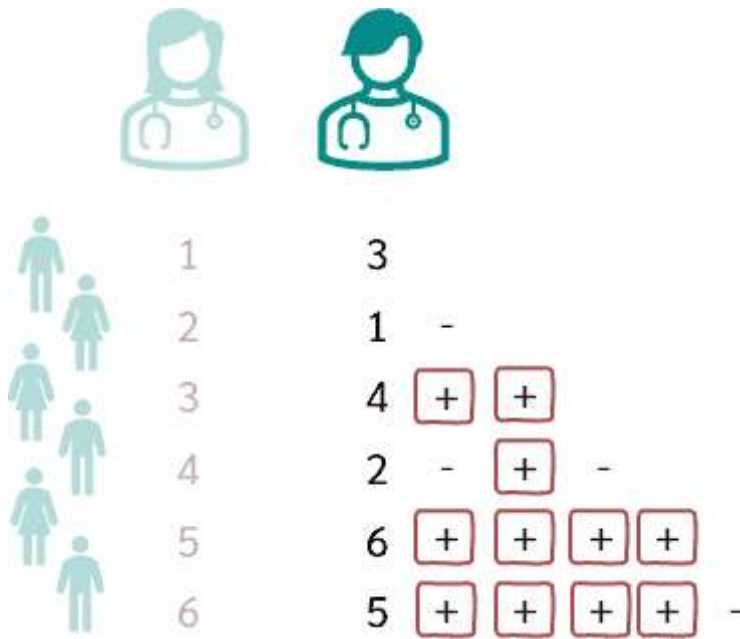
smaller, so it gets a minus, 6 is larger, so it gets a plus, and 5 is also larger, so it also gets a plus.

We now do the same for the second rank, corresponding to the value 1. Of course, each subsequent rank has a greater value than 1, so we have a plus everywhere.

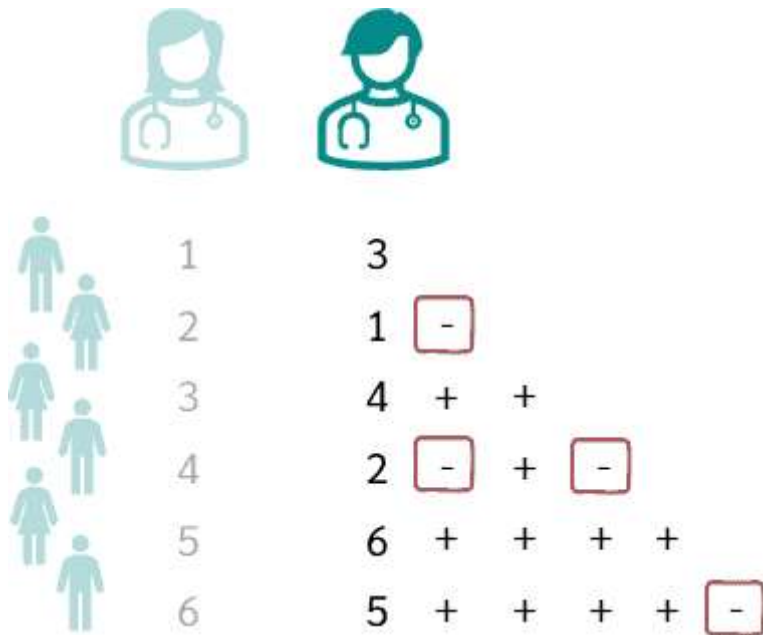


For rank with value 4, 2 is smaller and 6 and 5 are both larger. We proceed in equal manner with the ranks with value 2 and 6.

Now we can easily calculate the number of concordant and discordant pairs. We get the number of concordant pairs by counting all "+". In our example we have a total of 11.



We get the number of discordant pairs by counting through all the "-". In our example we have a total of 4.



C is 11 and D is 4, so the *Kendall's Tau* is $11 - 4$ divided by $11 + 4$, resulting a value of 0.47.

$$\tau = \frac{C - D}{C + D} = \frac{11 - 4}{11 + 4} = \frac{7}{15} = 0.47$$

An alternate formula for the *Kendall's Tau* is obtained herewith: S is $C-D$, so, 7, whereas n is the number of cases, so, 6.

S is $C-D$ so 7

$$\tau = \frac{S}{\frac{n(n-1)}{2}} = \frac{7}{15} = 0.47$$

n is the number of cases, i.e. 6.

If we insert everything, we also get 7 by 15.

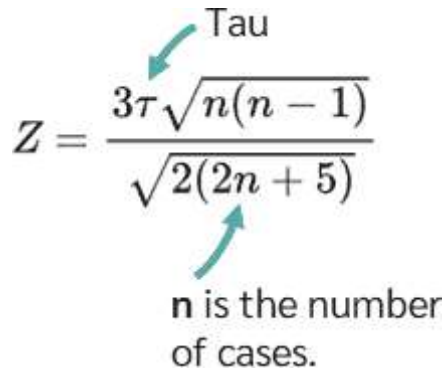
Kendall's Tau Significance

In the case of Kendall's Tau, the null and alternative hypotheses result in:

- **Null hypothesis:** the correlation coefficient $\text{Tau} = 0$ (There is no correlation.)
- **Alternative hypothesis:** the correlation coefficient $\text{Tau} \neq 0$ (There is a correlation.)

Now we want to know if the correlation coefficient, is significantly different from zero. You can determine this either by hand or with software like DATAtab.

For the hand calculation, we can use the z-distribution as an approximation. However, for this we should have more than 40 cases. So the 6 cases from our example are actually too few! We get the z-value using this formula:



The diagram shows the formula for the Z-score of Kendall's Tau:
$$Z = \frac{3\tau\sqrt{n(n-1)}}{\sqrt{2(2n+5)}}$$
 A green arrow points from the word "Tau" to the symbol τ in the numerator. Another green arrow points from the text " n is the number of cases." to the variable n in the denominator.

n is the number of cases.

Calculate Kendall's tau with DATAtab

If you want to calculate the Kendalls tau online with DATAtab, just copy your own data into the table in the Kendalls tau calculator and click on *correlation*.