Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Nonlinear Vehicle Dynamics Control – A Flatness Based Approach

Stefan Fuchshumer, Kurt Schlacher and Thomas Rittenschober

***Abstract*— The central issue of this contribution is the discussion of the differential flatness of the planar holonomic bicycle model. The components of a flat output are given as the lateral and the longitudinal velocity component of a distinguished point located on the longitudinal axis of the vehicle. This property is shown for the front-, rear- and all-wheel driven vehicle, without referring to particular representatives of the functions modelling the lateral tire forces. The clear physical meaning of the flat output is regarded as particularly useful for the control design task. The vehicle dynamics control design is accomplished following the flatness based control theory.**

I. INTRODUCTION

This contribution proposes a novel approach for the nonlinear vehicle dynamics control which is essentially based on the observation that the dynamics of the planar holonomic bicycle model depicted in Fig. 1 are differentially flat. The contact between the tires and the road is modelled in terms of contact forces, which implies that the tires are enabled to slip and slide on the road. The steering angle and the longitudinal tire forces are regarded as control inputs.

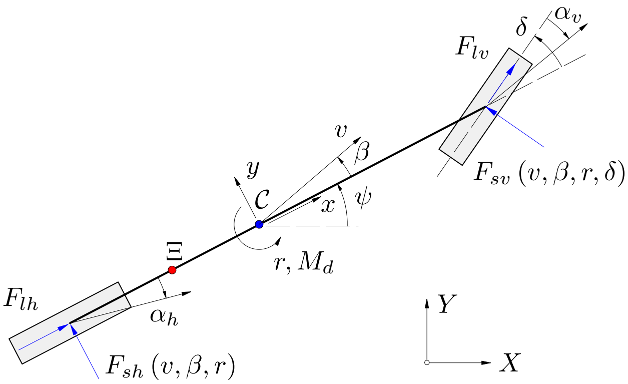

Fig. 1. The planar (holonomic) bicycle model.

While the flatness property of the non-holonomic kinematic vehicle, which is based on the arrangement of ideal rolling contact of the wheels, is well-known in the literature and numerously exploited for tracking applications of vehicles (with trailers), cf. e.g. [4], the differential flatness of the (holonomic) bicycle model of Fig. 1 has not been reported yet.

The bicycle model as introduced in [9] emerges from the four-wheel vehicle by gluing together the front and the rear wheels to a single (mass-less) front and rear wheel, respectively, located at the longitudinal axis of the car. This planar model, known as a well-established basis for the design of vehicle dynamics control systems, see, e.g., [1], [3], is capable of rendering the longitudinal, lateral and yaw dynamics of the vehicle. The pitch and roll dynamics of a vehicle are clearly not involved in the scope of this model.

Vehicle dynamics control systems as e.g. ESP (electronic stability program) acting on the brakes and the traction control system are implemtented in series-production vehicles. These systems support the driver in emergency situations by producing a (counter-) yaw torque due to individually controlled braking of all four wheel in the case of exceedance of a certain yaw rate. Accordingly, the potential of involving the steering system to handle emergency situations is very rich. The automotive industry aims at utilizing these possibilities e.g. by means of supporting the driver for the difficult task of counter-steering by application of a respective artificial torque to the steering wheel, i.e., as a haptic recommendation for the driver. As a very recent advance, active front steering (AFS) [5] has been implemented in series-production vehicles. The basic function of AFS is to mechanically add an additional steering angle (adjusted e.g. by an electric drive) to the steering angle given by the driver. Besides the objective to enable a velocity-dependent gain of the steering mechanism, the AFS system can be used for vehicle dynamics control, too.

This paper is organized as follows: After a brief revisit of the bicycle dynamics in Section II, the discussion is concerned with the system analysis in paragraph III yielding a physically relevant representative for the flat output of the bicycle model as the main result. This representative of the flat output does not depend on the particular choice of the vehicle's actuation, i.e., it holds for the rear-, front- and all-wheel driven car equivalently. Additionally, the flat output is attached with a clear physical meaning. Up to a certain family of functions which have to be excluded, the system analysis does not refer to particular representatives for the functions describing the lateral tire forces.

The task of (real-time) trajectory planning amounts to mapping the inputs supplied by the driver, i.e., the current position of the throttle/brake pedal and the angle of the steering wheel, to suitable trajectories for the flat output. By virtue of the free parametrization of the nonlinear vehicle dynamics by means of the flat output, these trajectories can

S. Fuchshumer is with the Christian Doppler Laboratory for Automatic Control of Mechatronic Systems in Steel Industries, Institute of Automatic Control and Control Systems Technology, Johannes Kepler University of Linz, Austria. stefan.fuchshumer@jku.at

K. Schlacher is the head of the Christian Doppler Laboratory for Automatic Control of Mechatronic Systems in Steel Industries and the Institute of Automatic Control and Control Systems Technology, Johannes Kepler University of Linz, Austria. kurt.schlacher@jku.at

T. Rittenschober is with Profactor Produktionsforschungs GmbH, Steyr, Austria. thomas.rittenschober@profactor.at

0-7803-9568-9/05/$20.00 ©2005 IEEE

be shaped arbitrarily by the design engineer in order to entail the desired dynamic behavior/handling of the vehicle. The physical meaning of the flat output is regarded to facilitate this objective significantly. Informations gathered by scanning the environment, e.g., regarding the conditions of the road, or the position of a detected obstacle, can also be incorporated for the real-time shaping of the trajectories. The trajectory shaping task, which inherently involves subjective sensations of the driver, is not addressed in this contribution. Finally, to illustrate the proposed control approach, simulation results given in Section IV conclude this contribution.

II. THE BICYCLE MODEL

To start with, let us first revisit the modelling assumptions and the dynamics of the bicycle model depicted in Fig. 1. A detailed discussion on these issues can be found e.g. in [9] or [3]. The global position (X, Y) of the center of gravity $\mathcal{C}$ and the orientation ψ of the longitudinal axis represent the degrees of freedom of the bicycle. The parameters are given as follows: m denotes the vehicle mass, J is the moment of inertia with respect to the yaw axis fixed at $\mathcal{C}$, and l_v, l_h denote the distances between $\mathcal{C}$ and the front and the rear wheel, respectively. The steering angle δ as well as the longitudinal tire forces F_{lv} and F_{lh}, which are due to the motor (or the braking) torque, are regarded as control inputs. The torque M_d denotes a disturbance acting w.r.t. the yaw axis. The inputs are collected to the vector $u^T = [\delta, F_{lv}, F_{lh}, M_d]$.

Let v represent the (magnitude of the) velocity of $\mathcal{C}$, and let β denote the angle between the longitudinal axis and the velocity vector $\dot{X}\partial_X + \dot{Y}\partial_Y$ at $\mathcal{C}$, i.e.,

$$v = \sqrt{\dot{X}^2 + \dot{Y}^2}, \qquad \beta = \arctan\left(\dot{Y}/\dot{X}\right) - \psi. \quad (1)$$

Thus, we have

$$\dot{X} = v\cos(\beta + \psi), \quad \dot{Y} = v\sin(\beta + \psi), \quad \dot{\psi} = r, \quad (2)$$

with r denoting the yaw rate. Notice that in the scope of vehicle dynamics control (to assist the driver in emergency situations), only the case $v > 0$ is considered.

In order to describe the tire/road contact via suitable mathematical models for the lateral tire forces F_{sv} and F_{sh}, the literature offers a wide variety of modelling assumptions. A very common and well-established assumption, cf. e.g. [2], is that the lateral tire forces, which allow for changes of the vehicle's orientation, are regarded as functions of the respective side-slip angles of the wheels. The side-slip angles α_h and α_v represent the angles between the velocity vector at the rear/front wheel and the associated tire plane, see Fig. 1,

$$\begin{aligned} \alpha_h(v, \beta, r) &= -\arctan\left(\frac{v\sin\beta - l_h r}{v\cos\beta}\right), \\ \alpha_v(v, \beta, r, \delta) &= \delta - \arctan\left(\frac{v\sin\beta + l_v r}{v\cos\beta}\right). \end{aligned} \quad (3)$$

However, in the scope of the system analysis to be given in Section III, we do not a-priori select a particular representative for the functions of the lateral tire forces, i.e., we will regard the functions $F_{sh}(v, \beta, r)$, $F_{sv}(v, \beta, r, \delta)$ as arbitrary smooth functions[1]. The dynamics of the bicycle model read

$$\dot{\bar{x}} = f(\bar{x}, u), \qquad \bar{x}^T = \begin{bmatrix} v & \beta & r \end{bmatrix} \quad (4)$$

with the components of the vector field f given as

$$\begin{aligned} f^1 = \frac{1}{m}&\left(F_{sv}(v, \beta, r, \delta)\sin(\beta - \delta) + \right. \\ &\left. + F_{lv}\cos(\beta - \delta) + F_{sh}(v, \beta, r)\sin\beta + F_{lh}\cos\beta\right), \end{aligned} \quad (5)$$

$$\begin{aligned} f^2 = -r + \frac{1}{mv}&\left(F_{sv}(v, \beta, r, \delta)\cos(\beta - \delta) - \right. \\ &\left. - F_{lv}\sin(\beta - \delta) + F_{sh}(v, \beta, r)\cos\beta - F_{lh}\sin\beta\right) \end{aligned} \quad (6)$$

and

$$\begin{aligned} f^3 = \frac{1}{J}&\left(l_v\left(F_{sv}(v, \beta, r, \delta)\cos\delta + F_{lv}\sin\delta\right) - \right. \\ &\left. - l_h F_{sh}(v, \beta, r) + M_d\right). \end{aligned} \quad (7)$$

The reason for selecting the representation (4)-(7) is twofold: First, this particular choice for the coordinates $\bar{x}$ implies that the functions f^α do not depend on the vehicle's orientation ψ (and the global position X, Y as well). Thus, these coordinates are often referred to as vehicle coordinates. Secondly, as the vehicle dynamics control under consideration is not concerned with position control of the car, but with control objectives depending on $\bar{x}$ only, the subsystem (2) has been dropped from the entire bicycle dynamics.

Following the usual tensor notation, coordinates and component functions of vector fields are indicated with superscript indices, and the Einstein convention for summation on repeated indices is arranged. Additionally, let $\partial_\alpha = \partial/\partial\bar{x}^\alpha$, $\partial_\delta = \partial/\partial\delta$, and let $\mathcal{L}_f c = f^\alpha \partial_\alpha c$ denote the Lie derivative of the function $c(\bar{x})$ along the vector field $f = f^\alpha\partial_\alpha$.

III. THE FLATNESS PROPERTY OF THE BICYCLE DYNAMICS

Let $M_d = 0$ in the scope of the following system analysis. We will commence by investigating the general case of the so-called all-wheel driven vehicle, which means that the motor (or braking) torque can be supplied to the front and the rear wheel with a given transmission ratio. To this end, let us introduce the distribution of the aggregate longitudinal force F_l to the front and the rear wheel as

$$F_{lh} = \gamma F_l, \qquad F_{lv} = (1 - \gamma) F_l, \quad (8)$$

with $\gamma \in [0, 1]$ referred to as the transmission ratio. This ratio γ is regarded either as a function of the time, $\gamma(t) : \mathbb{R} \mapsto [0, 1]$, or as a function of the vehicle's state $\bar{x}$. The first point of view amounts to regarding γ as an input given by the driver, the second viewpoint, however, is understood as an action of a vehicle control system.

The central result for this type of vehicle is given in the following proposition.

[1]This C^∞ assumption is arranged to avoid mathematical subtleties. Thus, it is a sufficient condition in the scope of the following discussions.

Proposition 1: Let the lateral tire forces $F_{sh}(v,\beta,r)$, $F_{sv}(v,\beta,r,\delta)$ be arbitrary (smooth) functions up to the requirements

$$F_{sh}(v,\beta,r) \neq \frac{(ml_v)^2}{J(l_v+l_h)} v^2 \sin\beta\cos\beta + \\ + \varkappa_h\left(v\cos\beta, v\sin\beta - \frac{J}{ml_v}r\right), \tag{9}$$

and

$$F_{sv} \neq \frac{\varkappa_v(v,\beta,r) - (1-\gamma)(\gamma\sin\delta + (1-\gamma)\delta)F_l}{1-\gamma(1-\cos\delta)}, \tag{10}$$

with $\varkappa_h$, $\varkappa_v$ as arbitrary (smooth) functions. Then, the bicycle dynamics (4)-(8), $u^T = [\delta, F_l]$, $M_d = 0$, with $\gamma \in [0,1]$ regarded either as a function of the time or the state $\bar{x}$, are differentially flat (for $v \neq 0$). Moreover, this system is exact input/state linearizable via static state feedback. An output $y = (y^1, y^2)$ entailing the exact linearization with relative degree $(1,2)$, i.e., a flat output, is given as

$$y^1 = c^1(\bar{x}) = v\cos\beta \tag{11}$$

and

$$y^2 = c^2(\bar{x}) = v\sin\beta - \frac{J}{ml_v}r. \tag{12}$$

Proof: (sketch) The exact linearizability is straightforwardly verified with (11), (12). For the (local) coordinates transformation $z = \varphi(\bar{x}) = \left[c^1(\bar{x}), c^2(\bar{x}), \mathcal{L}_f c^2(\bar{x})\right]^T$,

$$\begin{bmatrix} z^1 \\ z^2 \\ z^3 \end{bmatrix} = \begin{bmatrix} v\cos\beta \\ v\sin\beta - Jr/(ml_v) \\ \mathcal{L}_f c^2(\bar{x}) \end{bmatrix}, \tag{13}$$

with

$$\mathcal{L}_f c^2 = \frac{l_v + l_h}{ml_v} F_{sh}(v,\beta,r) - vr\cos\beta, \tag{14}$$

to qualify as a diffeomorphism, the requirement

$$B(v,\beta,r) - v^2\cos\beta/(l_v+l_h) \neq 0 \tag{15}$$

with

$$B = \frac{Jv(\partial_v F_{sh})\sin\beta + J(\partial_\beta F_{sh})\cos\beta + ml_v v(\partial_r F_{sh})}{m^2 l_v^2}$$

has to be fulfilled due to the implicit function theorem. The requirement (15) amounts to excluding the family (9) of functions for the lateral rear tire force F_{sh}. Remark 5 provides a comment on the condition (9) to show that it does not imply a practically relevant restriction on the choice of F_{sh}.

The static state feedback entailing the linear and time-invariant dynamics outlined in the coordinates z, with the new input $w^T = [w^1, w^2]$, is obtained (locally) as the solution of

$$\left(\mathcal{L}_f c^1\right)(\bar{x},u) = w^1, \qquad \left(\mathcal{L}_f^2 c^2\right)(\bar{x},u) = w^2 \tag{16}$$

with respect to $u^T = [\delta, F_l]$. Following the implicit function theorem, (local) solvability is guaranteed iff the condition

$$\gamma F_{sv}\sin\delta - (1-\gamma(1-\cos\delta))(\partial_\delta F_{sv} + (1-\gamma)F_l) \neq 0, \tag{17}$$

as well as (15) is met. Thus, additionally to (9), we have to impose the requirement (10). Again, Remark 5 provides a comment on this restriction. ■

A. Some Key Observations Associated with the Flatness Property of the Bicycle Dynamics

Remark 1: Note that, except for the restrictions (9) and (10), the property of exact linearizability for (11), (12) does not depend on the particular choice of the functions $F_{sv}(v,\beta,r,\delta)$ and $F_{sh}(v,\beta,r)$. Additionally, note that the restrictions (9), (10) are associated with the particular choice for $c^1(\bar{x})$, $c^2(\bar{x})$ as given in (11), (12).

Remark 2: The function $v\sin\beta - J/(ml_v)r$ occurring in (12) depicts the y-component v_y^Ξ of the velocity of the point Ξ located on the vehicle axis with the (vehicle) coordinates $(x,y) = (-J/(ml_v), 0)$, see Fig. 1. Thus, the output c^2 of (12) is attached with a clear physical meaning.

Remark 3: The function $c^1(\bar{x})$ of (11) depicts the x-component of the velocity of points located on the vehicle's longitudinal axis.

Remark 4: The location of the point Ξ as introduced above only depends on the parameters J, m and l_v which are known (rather) accurately in applications.

Remark 5: Typically, as sketched in Section II, the lateral tire forces F_{sh} and F_{sv} are considered as functions of the respective side-slip angles α_h, α_v, cf. e.g. [2] for a very well-established approach. In view of this, the conditions (9) and (10) do not impose practically relevant restrictions. Particularly, to this end, note that the (arbitrary, smooth) function $\varkappa_v$ of the condition (10) on F_{sv} must not depend on the steering angle δ. Additionally, note that the function $\varkappa_h$ involved in the restriction (9) for F_{sh} is a function of the velocity components v_x^Ξ, v_y^Ξ of the point Ξ.

Remark 6: Note that the transmission ratio γ, regarded as $\gamma(t)$ or $\gamma(v,\beta,r)$, does not explicitly appear in $\dot{y}^2 = z^3 = \mathcal{L}_f c^2$, see (13) and (14).

The point Ξ is interesting also from another point of view. To this end, let us calculate the (x,y)-decomposition of the acceleration of a point $\mathcal{Q}$ located at the x-axis at the distance $x = \zeta$ from the center of gravity $\mathcal{C}$. Let $R = SO(2)$, i.e., the group of rotations in the plane,

$$R(\psi) = \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix}, \qquad 0 \leq \psi < 2\pi$$

denote the mapping from the inertial frame (X,Y) to the vehicle coordinates (x,y). Variables augmented with the subscripts x, y are related to the vehicle coordinate system, whereas the subscripts X, Y indicate the respective inertial frame representation. The velocity of $\mathcal{Q}$ is given as

$$\begin{bmatrix} v_x^{\mathcal{Q}} \\ v_y^{\mathcal{Q}} \end{bmatrix} = \begin{bmatrix} v\cos\beta \\ v\sin\beta + \zeta r \end{bmatrix}, \quad \begin{bmatrix} v_X^{\mathcal{Q}} \\ v_Y^{\mathcal{Q}} \end{bmatrix} = R^{-1} \begin{bmatrix} v_x^{\mathcal{Q}} \\ v_y^{\mathcal{Q}} \end{bmatrix},$$

and the acceleration reads

$$\begin{bmatrix} a_x^{\mathcal{Q}} \\ a_y^{\mathcal{Q}} \end{bmatrix} = R\frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} v_X^{\mathcal{Q}} \\ v_Y^{\mathcal{Q}} \end{bmatrix} = \begin{bmatrix} \dot{v}_x^{\mathcal{Q}} \\ \dot{v}_y^{\mathcal{Q}} \end{bmatrix} + \begin{bmatrix} -v_y^{\mathcal{Q}} \\ v_x^{\mathcal{Q}} \end{bmatrix} \dot{\psi}.$$

Thus, we have

$$a_x^{\mathcal{Q}} = \frac{1}{m}\left(F_{lv}\cos\delta + F_{lh} - F_{sv}\left(v,\beta,r,\delta\right)\sin\delta\right) - \zeta r^2$$

for the longitudinal component and

$$a_y^{\mathcal{Q}} = \frac{1}{m}\left(\left(1-\zeta\frac{ml_h}{J}\right)F_{sh}\left(v,\beta,r\right)+\right. \\ \left. +\left(1+\zeta\frac{ml_v}{J}\right)\left(F_{lv}\sin\delta + F_{sv}\left(\cdot\right)\cos\delta\right)\right) + \frac{\zeta}{J}M_d$$

for the lateral acceleration, with $(\cdot) = (v,\beta,r,\delta)$.

Remark 7: The lateral acceleration a_y at the point Ξ does not explicitly depend on the contact forces of the front wheel, i.e, $F_{sv}\left(v,\beta,r,\delta\right)$ and F_{lv}.

Remark 8: For $\zeta = J/\left(ml_h\right)$, the lateral acceleration does not explicitly depend on the lateral force $F_{sh}\left(v,\beta,r\right)$ of the rear wheel. This point, with coordinates $(x,y) = \left(J/\left(ml_h\right),0\right)$, occurs in the analysis of Ackermann leading to the "robustly decoupling control law of DLR" [1], [3].

Remark 9: There is an interesting relation to prior work on flatness, in particular, on configuration flatness of Lagrangian systems underactuated by one control [8]. E.g., the planar rigid body actuated by two body-fixed forces allows for a configuration-flat output (Huyghens oscillation center), see, e.g., [6], with the PVTOL being a well-known representative of this class. However, in contrast to the flatness based PVTOL (position) control, the proposed vehicle dynamics control involves static state feedback only, which is a consequence of the control objective involving functions of $\bar{x}$ instead of configuration coordinates.

B. The Front- and the Rear-Wheel Driven Bicycle

The flatness property of the rear-wheel driven vehicle ($\gamma = 1$) and the front-wheel driven vehicle ($\gamma = 0$) follow as special cases from Proposition 1. For the rear-wheel driven car, the restriction (17) yields $F_{sv}\tan\delta - \partial_\delta F_{sv} \neq 0$, and, thus, the requirement

$$F_{sv}\left(v,\beta,r,\delta\right) \neq \varkappa_v\left(v,\beta,r\right)\cos^{-1}\delta,$$

see (10). In the case of the front-wheel driven bicycle, the restriction

$$F_{sv}\left(v,\beta,r,\delta\right) \neq \varkappa_v\left(v,\beta,r\right) - F_l\delta$$

has to be obeyed, see (10).

IV. SCHEME OF A FLATNESS BASED VEHICLE DYNAMICS CONTROL

Given the trajectory of the flat output y, the associated trajectories of the system variables $\bar{x}^T = [v,\beta,r]$ and the control inputs $u^T = [\delta, F_l]$ are obtained as the solution of

$$\mathcal{F} = \left\{c^1\left(\bar{x}\right) = y^1,\ c^2\left(\bar{x}\right) = y^2,\ \mathcal{L}_f c^2\left(\bar{x}\right) = \dot{y}^2,\right. \\ \left.\mathcal{L}_f c^1\left(\bar{x},u\right) = \dot{y}^1,\ \mathcal{L}_f^2 c^2\left(\bar{x},u\right) = \ddot{y}^2\right\}. \tag{18}$$

By obeying the restrictions regarding the lateral tire forces F_{sv}, F_{sh} given in Proposition 1 and Section III-B, respectively, the implicit function theorem guarantees the local solvabiltiy of (18).

Following [4], [10], [11], Fig. 2 depicts the scheme of a flatness based control, which is applied to the bicycle model. The nonlinear state feedback $u^i = \chi^i\left(\bar{x},w\right)$, $i = 1,2$, derived from (16) entails the exact linearization of the bicycle dynamics, $\dot{z}^1 = w^1$, $\dot{z}^2 = z^3$, $\dot{z}^3 = w^2$, see also (13). This implies the linearity and time-invariance of the tracking error dynamics, with the tracking error $e = y - y_d$. Here, the subscript d is used to indicate the desired values. The asymptotic stabilization of the tracking error dynamics

$$\begin{aligned} \dot{e}^1 &= \dot{y}^1 - \dot{y}_d^1 = \dot{z}^1 - \dot{y}_d^1 = w^1 - \dot{y}_d^1 \\ \ddot{e}^2 &= \ddot{y}^2 - \ddot{y}_d^2 = \dot{z}^3 - \ddot{y}_d^2 = w^2 - \ddot{y}_d^2 \end{aligned} \tag{19}$$

can be obtained by means of the control law

$$\begin{aligned} w^1 &= \dot{y}_d^1 - \mu e^1 - \bar{\mu}\xi^1 = \dot{y}_d^1 - \mu\left(z^1 - y_d^1\right) - \bar{\mu}\xi^1 \\ w^2 &= \ddot{y}_d^2 - \nu_1 e^2 - \nu_2\dot{e}^2 - \bar{\nu}\xi^2 = \\ &= \ddot{y}_d^2 - \nu_1\left(z^2 - y_d^2\right) - \nu_2\left(z^3 - \dot{y}_d^2\right) - \bar{\nu}\xi^2 \end{aligned} \tag{20}$$

with the integral part

$$\dot{\xi}^1 = e^1, \qquad \dot{\xi}^2 = e^2 \tag{21}$$

and $\mu > 0$, $\bar{\mu} > 0$, and ν_1, ν_2, $\bar{\nu}$ such that the characteristic polynomials are Hurwitz. Thus, the error dynamics read $\dot{e}^1 = -\mu e^1 - \bar{\mu}\xi^1$, $\ddot{e}^2 = -\nu_1 e^2 - \nu_2\dot{e}^2 - \bar{\nu}\xi^2$ with (21).

In order to illustrate the proposed vehicle dynamics control approach, a close-to-reality modelled sports car which is provided by the multi-body simulation program MSC.ADAMS [7], is used as a testrig. This demo vehicle comprises fully-detailed models of the suspension, the powertrain and the steering system, while the bodywork is considered as a rigid body, spring-mounted on the chassis. The total number of degrees of freedom amounts to 96.

The parameters of the corresponding bicycle model, which have been extracted from the MSC.ADAMS vehicle, are given by $m = 1529$ kg, $J = 1344$ kgm^2, $l_h = 1.08$ m and $l_v = 1.481$ m. The point Ξ, which is attached with particular meaning regarding the flatness property, see Remark 2, is located at $(x,y) = (-0.594\text{ m}, 0)$.

The lateral forces F_{sh}, F_{sv} acting on the tires are modelled following the widely-used model of Pacejka [2],

$$F_{sk}\left(\alpha_k\right) = 2D_s\sin\left(C_s\arctan\left(B_s\alpha_k\ -\right.\right. \\ \left.\left. -\ E_s\left(B_s\alpha_k - \arctan\left(B_s\alpha_k\right)\right)\right)\right),$$

$k \in \{v,h\}$, often referred to as "Pacejka's magic formula" in the literature. Thus, the lateral forces are given as functions

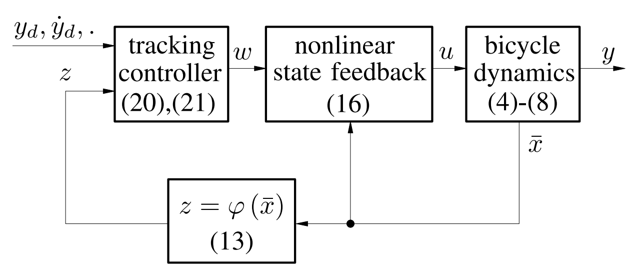

Fig. 2. Scheme of a flatness based vehicle dynamics control.

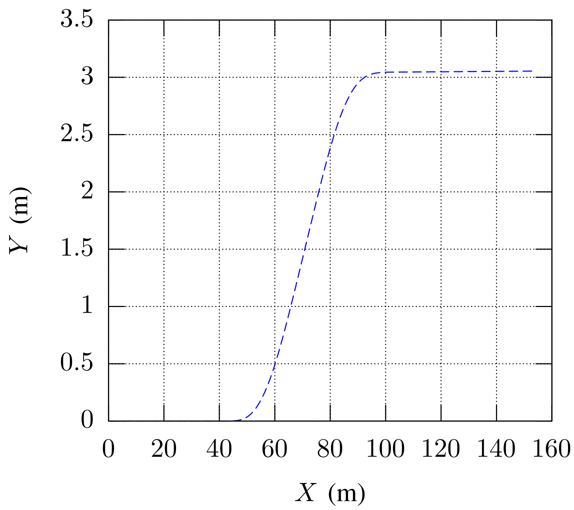

Fig. 3. Single lane change maneuver due to (22), (23): Trajectory of the vehicle's center of gravity $\mathcal{C}$.

of the respective side-slip angles, see also Remark 5. For instance, the parameters of the rear tire model are chosen to be $B_s = 13\ \mathrm{rad}^{-1}$, $C_s = 1.65$, $D_s = 4789$ N, $E_s = 0.68$.

Clearly, for an application, the shaping of the trajectories is to be done in real-time, based on the inputs supplied by the driver. To this end, as the position of the throttle/brake pedal is thought of as to reflect the driver's demands on the longitudinal dynamics, this input (supplied at the current time t) might be used to define the desired trajectory for v_x^Ξ for a future time horizon. Accordingly, the angle of the steering wheel may be regarded as the driver's demand on the lateral dynamics, so, this input might be used to adjust a desired trajectory for v_y^Ξ.

However, for the current sake of illustration, the desired trajectory $y_d(t) : [0, T] \to \mathbb{R}^2$ of the flat output is chosen as

$$y_d^1(t) = v_0 + \frac{3t^2T - 2t^3}{T^3}(v_T - v_0) \qquad (22)$$

for the longitudinal component of the velocity at Ξ, and

$$y_d^2(t) = \begin{cases} p(t - t_1, a_1, t_2 - t_1), & t \in [t_1, t_2) \\ p(t - t_2, a_2, t_3 - t_2), & t \in [t_2, t_3) \\ 0, & else \end{cases} \qquad (23)$$

for the lateral component, with

$$p(t, a, \tau) = -a\frac{t^3(\tau - t)^3}{\tau^6}.$$

Here, v_0 and v_T denote the x-component of the velocity at the time $t = 0$ and $t = T$, respectively. Let $t_1 = 1.5$ s, $t_2 = 2.5$ s, $t_3 = 3.5$ s, $T = 5$ s, $v_0 = 27.7$ m/s, $v_T = 33.3$ m/s, $a_1 = 50$ m/s, $a_2 = -57$ m/s, then this trajectory $y_d(t)$ of (22), (23) implies a single lane change maneuver, associated with an acceleration of the vehicle, see Fig. 3. Fig. 4 finally depicts the simulation results of the proposed flatness based control. The coefficients of the tracking controller (20) are chosen as $\mu = 10\ \mathrm{s}^{-1}$, $\bar{\mu} = 10\ \mathrm{s}^{-2}$, and $\nu_1 = 1200\ \mathrm{s}^{-2}$, $\nu_2 = 60\ \mathrm{s}^{-1}$, $\bar{\nu} = 8000\ \mathrm{s}^{-3}$.

Remark 10: The difference between the actual rear tire longitudinal force F_{lh} and the predicted value (dashed line) is due to the fact that the MSC.ADAMS model involves aerodynamic forces and the road resistance, which have to be overcome to follow the desired trajectory.

Remark 11: The deviation of the yaw rate and the steering input results from unequal loading of the inner and the outer track during cornering. This unequal loading clearly affects the lateral and longitudinal tire forces via the normal force.

V. CONCLUSIONS AND FUTURE WORKS

Individually controlled braking of all four wheels (ESP) to maintain the vehicle's stability and steering response has proven as very valuable to increase safety. Comparably, making available the steering system to the vehicle dynamics control offers additional potential to support the driver in emergency situations. To this end, the differential flatness property of the bicycle model might offer new perspectives to cope with the vehicle dynamics control problem involving the longitudinal forces of the tires and the steering angle as control inputs. The flat output revealed in this contribution could be identified as the longitudinal and the lateral component of the velocity of a certain point located on the vehicle's longitudinal axis. The location of this distinguished point is determined in terms of the mass, the moment of inertia and the distance between the front wheel and the center of gravity, which can be regarded as well-known parameters in practical applications. Additionally, this flat output does not depend on the particular actuation of the vehicle, i.e., it holds for the rear-, front- and all-wheel driven car equivalently.

Besides the objective of real-time trajectory shaping for the flat output, our future research will be concerned with observer design and parameter identification of the tire characteristics. While the first issue deals with driver's experience during demanding maneuvers, the second point focuses on the fact that the automotive industry, in general, is strongly reluctant to implement side-slip angle sensors. Finally, vehicle dynamics control is required to work seamlessly under various road conditions.

The authors wish to thank the reviewers for the useful comments.

REFERENCES

[1] J. Ackermann, T. Bünte and D. Odenthal, "Advantages of active steering for vehicle dynamics control", *in 32nd Int. Symp. on Automotive Technology and Automation*, Vienna, Austria, pp. 263-270, 1999.

[2] E. Bakker, H. Pacejka and L. Lidner, A new tire model with an application in vehicle dynamics studies, *SAE Paper No. 890087*, pp. 101-113, 1989.

[3] T. Bünte, Beiträge zur robusten Lenkregelung von Personenkraftwagen, PhD thesis, Technische Hochschule Aachen, 1998.

[4] M. Fliess, J. Lévine, P. Martin and P. Rouchon, Flatness and defect of non-linear systems: Introductory theory and examples, *Int. J. Control*, 61:1327-1361, 1995.

[5] P. Köhn, G. Baumgarten, "Die Aktivlenkung – das neue fahrdynamische Lenksystem von BMW", *in 11. Aachener Kolloquium Fahrzeug- und Motorentechnik*, 2002.

[6] P. Martin, R.M. Murray, P. Rouchon, Flat systems, equivalence and trajectory generation, *CDS Technical Report, CDS 2003-008*, 2003.

[7] MSC.ADAMS, www.mscsoftware.com.

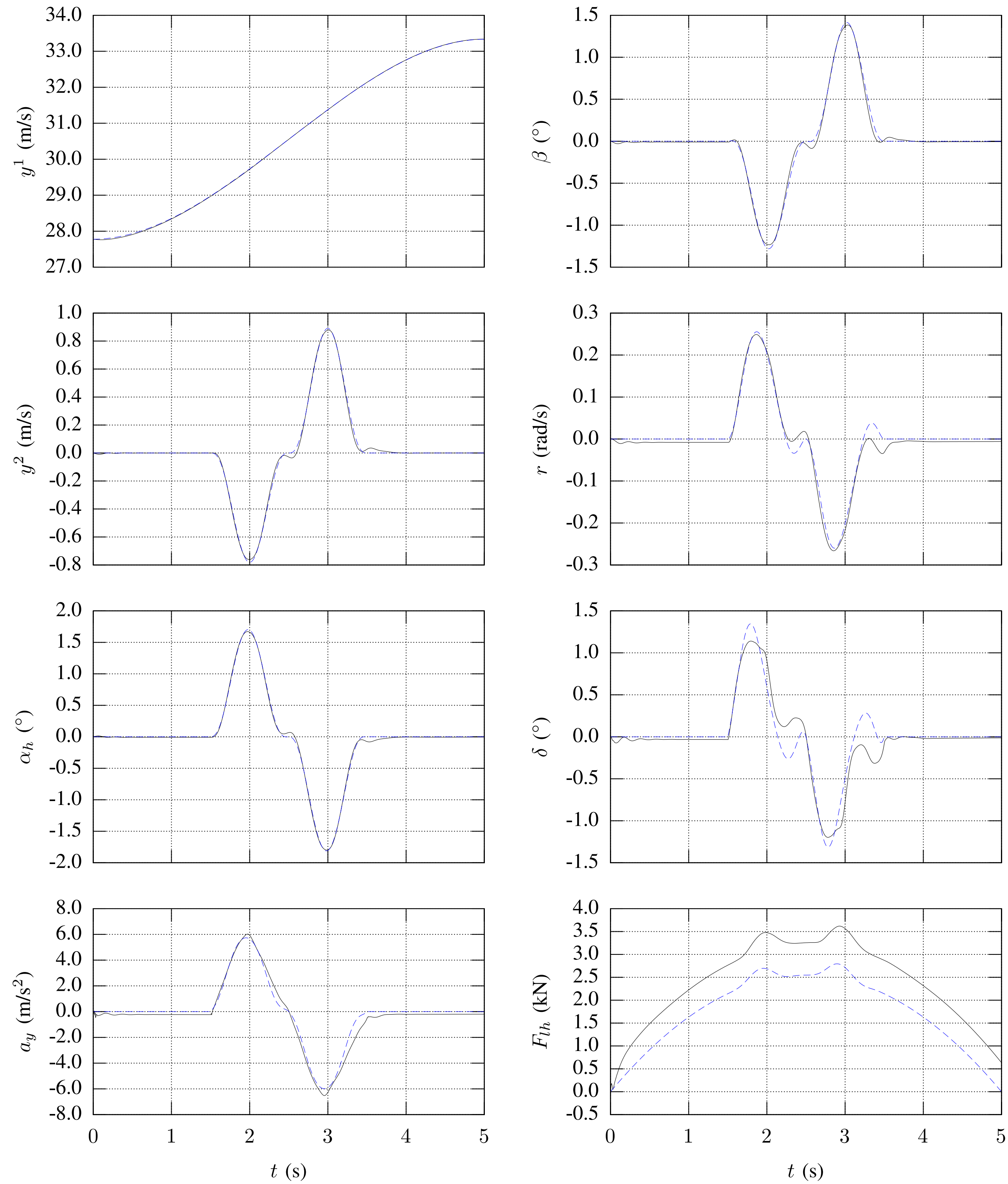

Fig. 4. Simulation results of the proposed flatness based vehicle dynamics control approach, using a close-to-reality modelled sports car provided by the multi-body simulation program MSC.ADAMS. The desired trajectories are indicated with dashed lines.

[8] M. Rathinam, R.M. Murray, Configuration flatness of Lagrangian systems underactuated by one control, *SIAM J. Control Optim.*, 36(1):164-179, 1998.

[9] P. Riekert, T. Schunck, Zur Fahrmechanik des gummibereiften Kraftfahrzeugs, *Ingenieur Archiv*, 11:210-224, 1940.

[10] R. Rothfuß, *Anwendung der flachheitsbasierten Analyse und Regelung nichtlinearer Mehrgrößensysteme*, VDI Verlag, 1997.

[11] J. Rudolph, *Flatness based control of distributed parameter systems*, Shaker, Aachen, 2003.

[12] S. Sastry, *Nonlinear systems, analysis, stability and control*, Springer, New York, 1999.