ALY-6050 MOD 5 – Lab 1

(i) Linear Algebraic Modeling of LP Problems
(ii) R & Excel Solutions of LP Formulations

In this lab, we will first formulate a given linear programming problem using linear algebra. Furthermore, we will obtain solutions by using both Excel and R.

Matrix Forms:

(i) Maximization Problems:
Maximize $\boldsymbol{Z} = \mathbf{c}^{\mathbf{T}}\mathbf{x}$
Subject to: $\mathbf{Ax} \le \mathbf{b}$

(ii) Minimization Problems:
Minimize $\boldsymbol{Z} = \mathbf{c}^{\mathbf{T}}\mathbf{x}$
Subject to: $\mathbf{Ax} \ge \mathbf{b}$

Problem:
Suppose that a manufacturer makes 3 products. Let x_1, x_2, and x_3 denote the number of units of each type respectiuvely. The model is described by the following formulation:

$Maximize \quad \boldsymbol{Z = 20x_1 + 30x_2 + 40x_3}$

$Subject\ to$:
Constraint 1: $\boldsymbol{x_1 + x_2 + x_3 \le 1000}$
Constraint 2: $\boldsymbol{3x_1 + 5x_2 + 8x_3 \le 5000}$
Constraint 3: $x_1 \le 0.4(x_1 + x_2 + x_3) \quad \rightarrow \quad \boldsymbol{0.6x_1 - 0.4x_2 - 0.4x_3 \le 0}$
Constraint 4: $x_3 \ge 250 \quad \rightarrow \quad \boldsymbol{-x_3 \le -250} \quad \rightarrow \quad : \quad \boldsymbol{0x_1 + 0x_2 - x_3 \le -250}$
Constraint 5: $x_1 \ge 0 \quad \rightarrow \quad \boldsymbol{-x_1 \le 0} \quad \rightarrow \quad \boldsymbol{-x_1 + 0x_2 + 0x_3 \le 0}$
Constraint 6: $x_2 \ge 0 \quad \rightarrow \quad \boldsymbol{-x_2 \le 0} \quad \rightarrow \quad \boldsymbol{0x_1 - x_2 + 0x_3 \le 0}$

Therefore, the decision variables vector $\mathbf{x}$, the objective vector $\mathbf{c}$, the constraints' matrix $\mathbf{A}$, and the vector $\mathbf{b}$ of the constraints' right-hand sides are given by:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \qquad \mathbf{c} = \begin{bmatrix} 20 \\ 30 \\ 40 \end{bmatrix}, \qquad \mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 8 \\ 0.6 & -0.4 & -0.4 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1000 \\ 5000 \\ 0 \\ -250 \\ 0 \\ 0 \end{bmatrix}$$

Task:
1. Formulate the above LP problem in Excel by using linear algebra and solve it by using the Solver.
2. Formulate and solve the above problem in R.