ALY6050 Introduction to Enterprise Analytics

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Module 4

Inventory Model

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Contents

Introduction

Inventory management and modeling are often among the most influential decisions an organization can make to impact their financial performance. Maintaining adequate inventory to ensure demand can be met, while minimizing the overhead cost associated with purchasing, storing and maintaining excess inventory can make or break a business. The risks are high – too little inventory can lead to missed sales or delayed production time. Too much inventory can quickly lead to excess costs in warehousing, transportation, or even deterioration of items. For this study, a manufacturing company is seeking advice on ordering decisions based around inventory of a key engine component. Through internal research, they have determined that the annual demand for this component is 15,000 units, and that demand remains constant throughout the year. Each unit costs the company $80 and the overall opportunity cost for storing a unit for a year is 18%. Every order placed costs the company $220 in addition to the $80 per unit charge. The company policy is to order whenever the inventory level reaches a predetermined reorder point that provides sufficient stock to meet demand until the supplier's order can be shipped and received; and then to order twice as many units. This study is to help the company determine the optimal ordering strategy to minimize costs associated with the inventory of the engine component through the development of a decision model.

Part One

The parameters for this decision model are the unit cost, opportunity (or holding) rate and the ordering cost. The uncontrollable factor is the annual demand for the component, and the decision variable is the quantity to be ordered.

The mathematical functions needed for the model are as follows:

- Unit holding cost (per unit, per year) = unit cost * holding rate
- Total orders per year = annual demand / order quantity
- Average inventory = order quantity / 2
- Annual ordering cost = orders per year * ordering cost
- Annual holding cost = average inventory * unit holding cost

- Total cost = annual ordering cost + annual holding costs

The objective of the model is to minimize the total cost.

In Excel, the model is built by establishing the parameters and setting a seed value in the order quantity values. Then, using the What-If tool, a data table of values for potential order values can be generated.

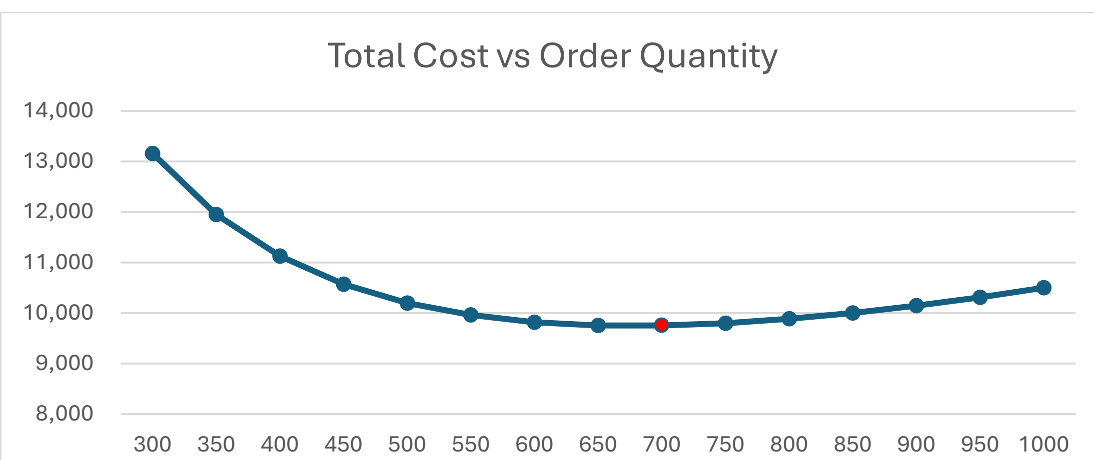
Data Table	$ 9,749
300	13,160
350	11,949
400	11,130
450	10,573
500	10,200
550	9,960
600	9,820
650	9,757
700	9,754
750	9,800
800	9,885
850	10,002
900	10,147
950	10,314
1000	10,500

As seen in the resulting table, the optimum order value from this table is 700 units. If there was a constraint from the vendor that orders needed to be placed in sets of 10 or 50, this would be the answer.

If however, the vendor will accept an order of any amount, the Solver tool in Excel can be used to find the exact optimum values. From this tool, the ideal order value is 677 units. This equates to 22 orders a year and keeps the total annual inventory cost below $9,750.

Decision Variable		
Order Quantity (Q)		677
Cost Calculations		
Annual Ordering Cost	$	4,874
Annual Holding Cost	$	4,874
Total Annual Cost	$	9,749
Metrics		
Orders per year		22
Cycle Time (days)		16.47
Average Inventory		339

Plotting the total costs against the order quanity helps visualize this conclusion.



Further exploration of the relationships between these variables, a two-way What-If analysis is produced to explore the relationship between order quantities and annual demand on costs.

		Annual Demand						
	$ 9,749	10,000	12,000	14,000	16,000	18,000	20,000	22,000
Order Quantity	300	$ 9,493	$ 10,960	$ 12,427	$ 13,893	$ 15,360	$ 16,827	$ 18,293
	400	$ 8,380	$ 9,480	$ 10,580	$ 11,680	$ 12,780	$ 13,880	$ 14,980
	500	$ 8,000	$ 8,880	$ 9,760	$ 10,640	$ 11,520	$ 12,400	$ 13,280
	600	$ 7,987	$ 8,720	$ 9,453	$ 10,187	$ 10,920	$ 11,653	$ 12,387
	700	$ 8,183	$ 8,811	$ 9,440	$ 10,069	$ 10,697	$ 11,326	$ 11,954
	800	$ 8,510	$ 9,060	$ 9,610	$ 10,160	$ 10,710	$ 11,260	$ 11,810
	900	$ 8,924	$ 9,413	$ 9,902	$ 10,391	$ 10,880	$ 11,369	$ 11,858
	1000	$ 9,400	$ 9,840	$ 10,280	$ 10,720	$ 11,160	$ 11,600	$ 12,040
	1100	$ 9,920	$ 10,320	$ 10,720	$ 11,120	$ 11,520	$ 11,920	$ 12,320
	1200	$ 10,473	$ 10,840	$ 11,207	$ 11,573	$ 11,940	$ 12,307	$ 12,673

As well as between demand and holding cost rates

		Holding Cost Rate							
	$ 9,749	10%	12%	14%	16%	18%	20%	22%	24%
Annual Demand	10,000	$ 5,958	$ 6,499	$ 7,041	$ 7,582	$ 8,124	$ 8,666	$ 9,207	$ 9,749
	11,000	$ 6,283	$ 6,824	$ 7,366	$ 7,907	$ 8,449	$ 8,991	$ 9,532	$ 10,074
	12,000	$ 6,608	$ 7,149	$ 7,691	$ 8,232	$ 8,774	$ 9,316	$ 9,857	$ 10,399
	13,000	$ 6,933	$ 7,474	$ 8,016	$ 8,557	$ 9,099	$ 9,641	$ 10,182	$ 10,724
	14,000	$ 7,257	$ 7,799	$ 8,341	$ 8,882	$ 9,424	$ 9,965	$ 10,507	$ 11,049
	15,000	$ 7,582	$ 8,124	$ 8,666	$ 9,207	$ 9,749	$ 10,290	$ 10,832	$ 11,374
	16,000	$ 7,907	$ 8,449	$ 8,991	$ 9,532	$ 10,074	$ 10,615	$ 11,157	$ 11,699
	17,000	$ 8,232	$ 8,774	$ 9,316	$ 9,857	$ 10,399	$ 10,940	$ 11,482	$ 12,024
	18,000	$ 8,557	$ 9,099	$ 9,641	$ 10,182	$ 10,724	$ 11,265	$ 11,807	$ 12,349
	19,000	$ 8,882	$ 9,424	$ 9,965	$ 10,507	$ 11,049	$ 11,590	$ 12,132	$ 12,673
	20,000	$ 9,207	$ 9,749	$ 10,290	$ 10,832	$ 11,374	$ 11,915	$ 12,457	$ 12,998
	21,000	$ 9,532	$ 10,074	$ 10,615	$ 11,157	$ 11,699	$ 12,240	$ 12,782	$ 13,323
	22,000	$ 9,857	$ 10,399	$ 10,940	$ 11,482	$ 12,024	$ 12,565	$ 13,107	$ 13,648
	23,000	$ 10,182	$ 10,724	$ 11,265	$ 11,807	$ 12,349	$ 12,890	$ 13,432	$ 13,973
	24,000	$ 10,507	$ 11,049	$ 11,590	$ 12,132	$ 12,673	$ 13,215	$ 13,757	$ 14,298
	25,000	$ 10,832	$ 11,374	$ 11,915	$ 12,457	$ 12,998	$ 13,540	$ 14,082	$ 14,623

Understanding these relationships not only help understand the between the variables but also provide guidance to decision makers should some of the current fixed parameters change do to market conditions.

Part Two

For the second half of the analysis, it is assumed that the annual demand is no longer fixed at 15,000 units but rather has a triangular probability distribution between 13,000 and 17,000 units and a mode of 15,000.

Simulation

In order to identify the optimum ordering strategy, a Monte Carlo simulation will be conducted with 1,000 simulations. This will be run in R, utilizing the triangle package with parameters from the assumptions above.

```
set.seed(123)
random_demand <- rtriangle(1000, a = 13000, b = 17000, c = 15000)
min_value <- vector(length = 1000)
objective_value <- vector(length = 1000)
annual_orders <- vector(length = 1000)

for ( i in 1:1000) {
  result <- optimize(Total_cost, c(300,1000), d = random_demand[i], maximum = FALSE)
  min_value[i] <- round(result$minimum, 0)
  objective_value[i] <- result$objective
  annual_orders[i] <- round(random_demand[i]/ min_value[i], 0)
}
```

The for loop above creates 1,000 simulations of the data and utilizes the optimize() function to find the optimal order size. For each scenario, a record is created with the optimum order number (min_value), the total costs associated with that order (objective_value) and the number of orders placed per year (annual_orders) – see the appendix for full results.

1. Total Cost

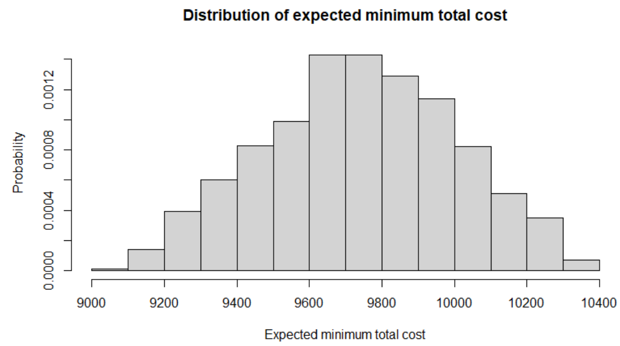
From this simulation, a 95% confidence interval can be constructed for the minimum total cost of between $9,726.37 and $9,759.17.

```
> t.test(df$total_cost, conf.level = 0.95)

        One Sample t-test

data:  df$total_cost
t = 1165.9, df = 999, p-value < 2.2e-16
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 9726.369 9759.167
sample estimates:
mean of x
 9742.768
```

A histogram can also be created to look at the distribution of the costs.



At first, the distribution appears to follow a normal distribution. Further analysis can be conducted by performing a Chi-Squared goodness of fit test. The results produce a p-value of 0.0329, so at a 95% confidence interval, the null hypothesis is rejected, meaning there is a difference between this distribution and the normal distribution.

2. Order quantity

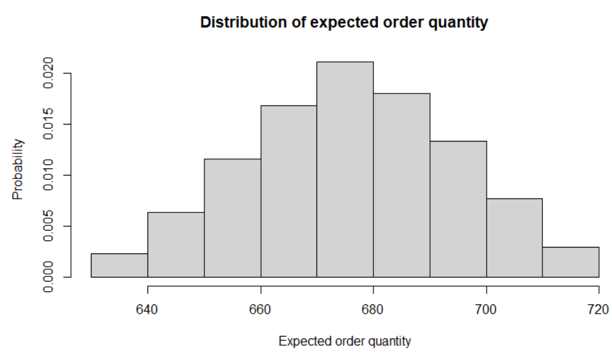
Similarly to the analysis of total cost, a 95% confidence interval is constructed for the order quantity, which returns and interval between 675 and 677 items per order.

```
> t.test(df$order_quantity, conf.level = 0.95)

        One Sample t-test

data:  df$order_quantity
t = 1166.1, df = 999, p-value < 2.2e-16
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 675.4405 677.7175
sample estimates:
mean of x
  676.579
```

And a histogram is again produced. As order quantity is a discrete variable, the most likely distribution to test would be the Poisson distribution. After running a Chi-Squared goodness of fit test, a p-value of 0.02578 which also leads to a rejection of the null hypothesis, meaning there is not evidence to support that this distribution matches the Poisson distribution.



3. Annual orders

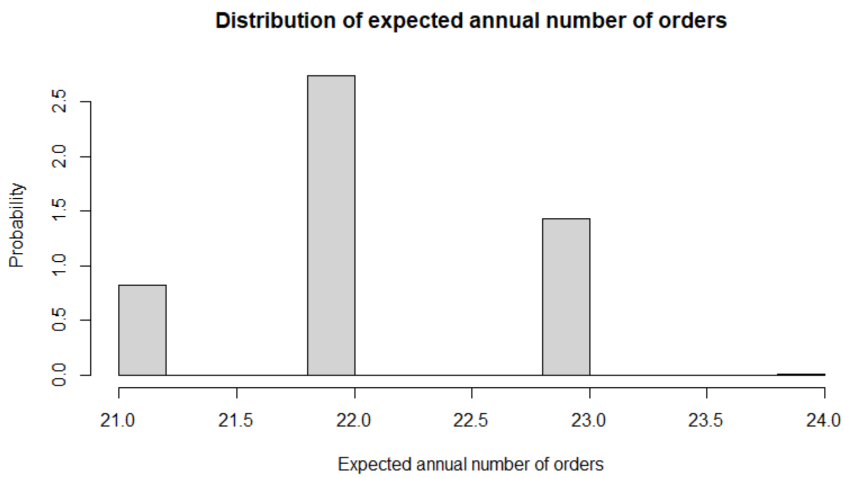
Finally, a 95% confidence interval is constructed for the number of annual orders.

```
> t.test(df$annual_orders, conf.level = 0.95)

        One Sample t-test

data:  df$annual_orders
t = 1054.3, df = 999, p-value < 2.2e-16
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 22.08182 22.16418
sample estimates:
mean of x
   22.123
```

The upper and lower ranges of the confidence interval both round to 22 orders placed per year. The histogram of results tells a similar story.



Again, with a discreet variable in orders, the distribution is tested against the Poisson distribution. The Chi-squared goodness of fit test results in a p-value of 0, so again the null hypothesis is rejected.

Conclusion and Recommendation

After utilizing optimization tools in both R and Excel, the optimum order quantity was determined to be 677 units. This strategy results in 22 orders per year at an annual cost of $9,749. What-if analysis has also been conducted to demonstrate results if changes are detected in either demand or the underlying costs of inventory management. To further the analysis, 1,000 simulations were conducted for a range of potential demand levels, at a 95% confidence interval, the 677 units per order strategy was confirmed to minimize costs to the organization.

References

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