

**Homework Assignment 4**

**1.** How would you test for overflow, the result of an addition of two 8-bit operands if the operands were (i) unsigned (ii) signed with 2s complement representation.

Add the following 8-bit strings assuming they are (i) *unsigned* (ii) *signed and represented using 2's complement*. Indicate which of these additions overflow.

- A. 0110 1110 + 1001 1111
- B. 1111 1111 + 0000 0001
- C. 1000 0000 + 0111 1111
- D. 0111 0001 + 0000 1111

**2.** One possible performance enhancement is to do a shift and add instead of an actual multiplication. Since  $9 \times 6$ , for example, can be written  $(2 \times 2 \times 2 + 1) \times 6$ , we can calculate  $9 \times 6$  by shifting 6 to the left three times and then adding 6 to that result. Show the best way to calculate  $0xAB_{hex} \times 0xEF_{hex}$  using shifts and adds/subtracts. Assume both inputs are 8-bit unsigned integers.

**3.** What decimal number does the 32-bit pattern  $0xDEADBEEF$  represent if it is a floating-point number? Use the IEEE 754 standard

**4.** Write down the binary representation of the decimal number 78.75 assuming the IEEE 754 *single precision* format. Write down the binary representation of the decimal number 78.75 assuming the IEEE 754 *double precision* format

**5.** Write down the binary representation of the decimal number 78.75 assuming it was stored using the single precision **IBM format** (base 16, instead of base 2, with 7 bits of exponent).

**6.** IEEE 754-2008 contains a half precision that is only 16 bits wide. The leftmost bit is still the sign bit, the exponent is 5 bits wide and has a bias of 15, and the mantissa (fractional field) is 10 bits long. A hidden 1 is assumed.

(a) Write down the bit pattern to represent  $-1.3625 \times 10^{-1}$ . Comment on how the range and accuracy of this 16-bit floating point format compares to the single precision IEEE 754 standard.

(b) Calculate the sum of  $1.6125 \times 10^1$  (A) and  $3.150390625 \times 10^{-1}$  (B) by hand, assuming operands A and B are stored in the 16-bit half precision described in problem a. above. Assume 1 guard, 1 round bit, and 1 sticky bit, and round to the nearest even. Show all the steps.

**7.** What is the range of representation and relative accuracy of positive numbers for the following 3 formats:

- (i) IEEE 754 Single Precision
- (ii) IEEE 754 – 2008 (described in Problem 6 above)
- (iii) 'bfloat16' shown in the figure below

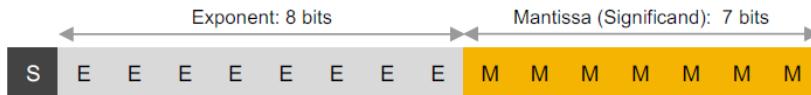
## IEEE 754, Single Precision:



## fp16: Half-precision IEEE Floating Point Format



## **bfloat16: Brain Floating Point Format**



**8.** NVIDIA has a “half” format, which is similar to IEEE 754 except that it is only 16 bits wide. The leftmost bit is still the sign bit, the exponent is 5 bits wide and the Fraction field is 10 bits long. A hidden 1 is assumed.

## REPRESENTATION RANGE OF THE NVIDIA ‘HALF FORMAT’



For each of the following, write the *binary value* and the *corresponding decimal value* of the 16-bit floating point number that is the closest available representation of the requested number. If rounding is necessary use round-to-nearest. Give the decimal values either as whole numbers or fractions. Show your work.

Number	Binary	Decimal
0	00000 0000000000	0
Mass of a neutron: $1.674 \times 10^{-27}$ (Kg)		
Smallest positive normalized number		
Smallest positive <b>denormalized</b> number $> 0$		
Largest positive <b>denormalized</b> number $> 0$		
Largest positive number < infinity		
Average distance b/w proton and neutron in Hydrogen atom = $0.8751 \times 10^{-15}$ m		
Number of Years since ancient 'supercontinent Gondwana' broke up 180,000,000		