# ROB-GY 6323
# reinforcement learning and optimal control for robotics

## Lecture 9
## Q-learning

## Course material

All necessary material will be posted on Brightspace
Code will be posted on the Github site of the class

https://github.com/righetti/optlearningcontrol

## Discussions/Forum with Slack

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## Tentative schedule (subject to change)

| Week | Lecture | | Homework | Project |
|---|---|---|---|---|
| 1 | Intro | Lecture 1: introduction | | |
| 2 | Trajectory optimization | Lecture 2: Basics of optimization | HW 1 | |
| 3 | | Lecture 3: QPs | | |
| 4 | | Lecture 4: Nonlinear optimal control | | |
| 5 | | Lecture 5: Model-predictive control | | |
| 6 | | Lecture 6: Sampling-based optimal control | HW 2 | |
| 7 | Policy optimization | Lecture 7: Bellman's principle | | |
| 8 | | Lecture 8: Value iteration / policy iteration | | Project 1 |
| 9 | | Lecture 9: Q-learning | HW 3 | |
| 10 | | Lecture 10: Deep Q learning | | |
| 11 | | Lecture 11: Actor-critic algorithms | | |
| 12 | | Lecture 12: Learning by demonstration | HW 4 | Project 2 |
| 13 | | Lecture 13: Monte-Carlo Tree Search | | |
| 14 | | Lecture 14: Beyond the class | | |
| 15 | Finals week | | | |

## Infinite horizon problems

In general the sum of costs might diverge

We introduce <u>discounted costs</u> $\lim_{N\to\infty} \min_{\pi(x_n)} \sum_{n=0}^{N} \alpha^n l(x_n, \pi(x_n))$

$0 < \alpha < 1$ Discount factor

This will work as long as $l(x_n, \pi(x_n))$ is <u>bounded:</u> $|l(x, u)| < M$

# Value iteration

For any bounded function $J(x)$, the iteration

$$J^{n+1}(x) = \min_u g(x,u) + \alpha J^n(f(x,u))$$

with $J^0(x) = J(x)$ converges to the optimal value function, i.e.

$$\lim_{n\to\infty} J^{n+1}(x) = J^*(x)$$

<u>Value iteration algorithm</u>: start from an arbitrary J(x) and iterate $J^{n+1}(x)$

Policy <u>evaluation</u>: how good is a policy?

# Policy evaluation algorithm I

For any bounded function $J(x)$, the iteration

$$J_\mu^{n+1}(x) = g(x, \mu(x)) + \alpha J_\mu^n(f(x, \mu(x)))$$

converges to the total cost of the stationary policy $\mu(x)$, i.e.

$$\lim_{n\to\infty} J_\mu^n \quad (x) = J_\mu(x)$$

<u>Policy evaluation algorithm</u>: start from an arbitrary J(x) and iterate $J_\mu^{n+1}(x)$

## Policy Evaluation Algorithm II

Given a policy $\mu$ :

Solve $(I - \alpha A)J_\mu = \bar{g}$

$J_\mu$ can also be written as a vector $\begin{pmatrix} J_\mu(x_1) \\ J_\mu(x_2) \\ \vdots \\ J_\mu(x_N) \end{pmatrix}$

Let $\bar{g}$ the vector of costs for all the states, i.e. $\bar{g} = \begin{pmatrix} g(x_1, \mu(x_1)) \\ g(x_2, \mu(x_2)) \\ \vdots \\ g(x_N, \mu(x_N)) \end{pmatrix}$

$J_\mu(f(x, \mu(x)))$ can be written using the matrix $A$ of transitions such that we get the following linear equation

$$J_\mu = \bar{g} + \alpha A J_\mu$$

## Policy <u>iteration</u>: finding the optimal policy

## Policy Iteration algorithm: optimal policy through policy evaluation

Start with an initial guess for the policy $\mu_0$

1. Policy evaluation step:
   Compute $J_{\mu_n}(x)$ using the policy evaluation algorithm

2. Policy update step:
   Update the policy using

$$\mu_{k+1} = \arg\min_u g(x,u) + \alpha J_{\mu_k}(f(x,u))$$

Iterate until convergence
(guaranteed to happen in a finite number of iteration!)

Checking all the possible x is not always possible
Can we do something less “perfect” but more practical?

=> reinforcement learning!

## A historical digression on reinforcement learning

## What is reinforcement learning?

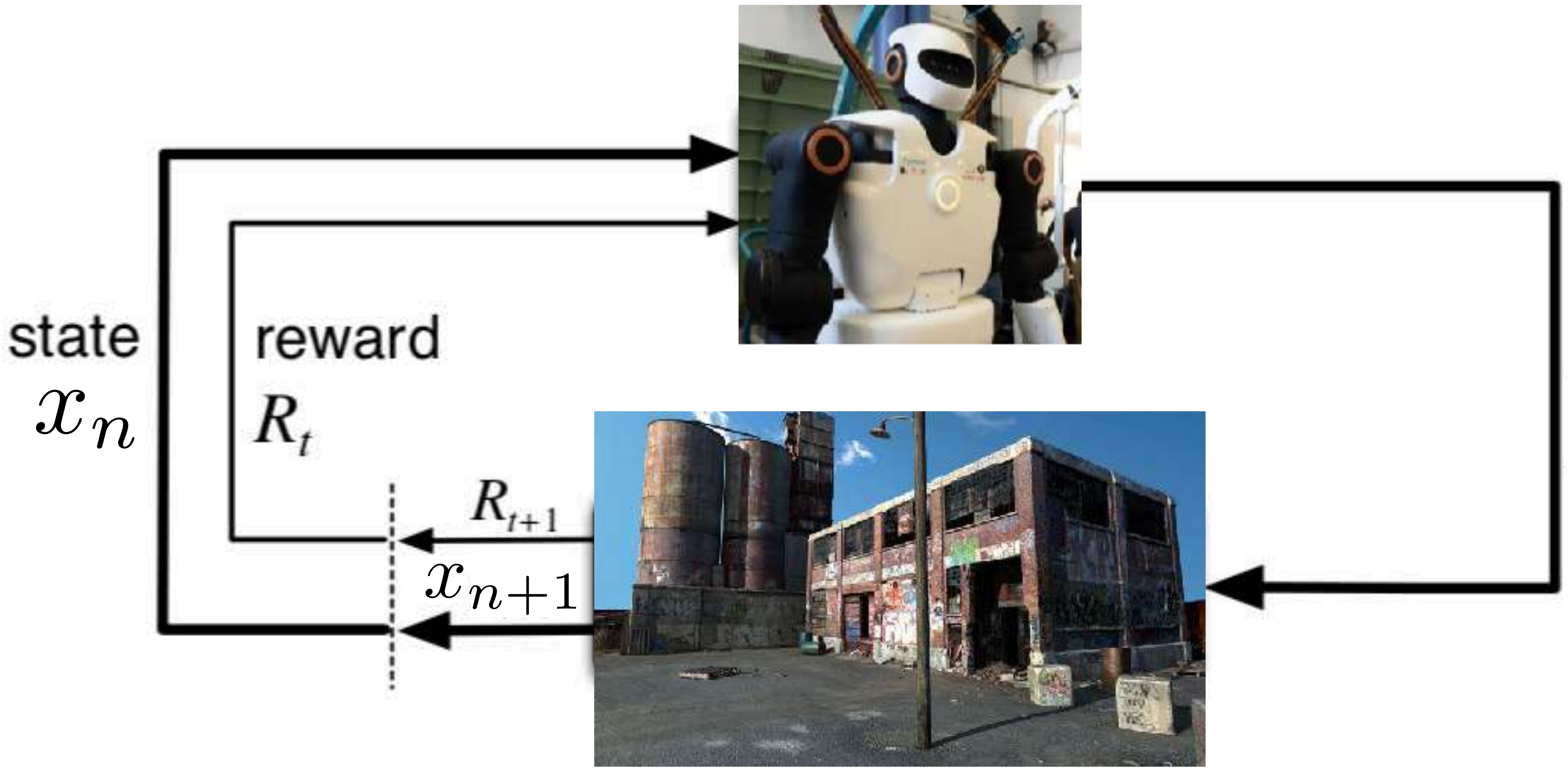
Learning “how to act” from direct interaction with the environment with the goal to “maximize” a reward

=> aka optimal control without a model

=> original RL was supposed to work on real robots

Not a new field!
Early versions of TD-learning from the 1950s
Q-learning in the 1980s
RL with neural networks in the 1990s

$$\max_{u_n} \sum_{n=0}^{N} R_n(x_n, u_n)$$



$= \pi_n(x_n)$

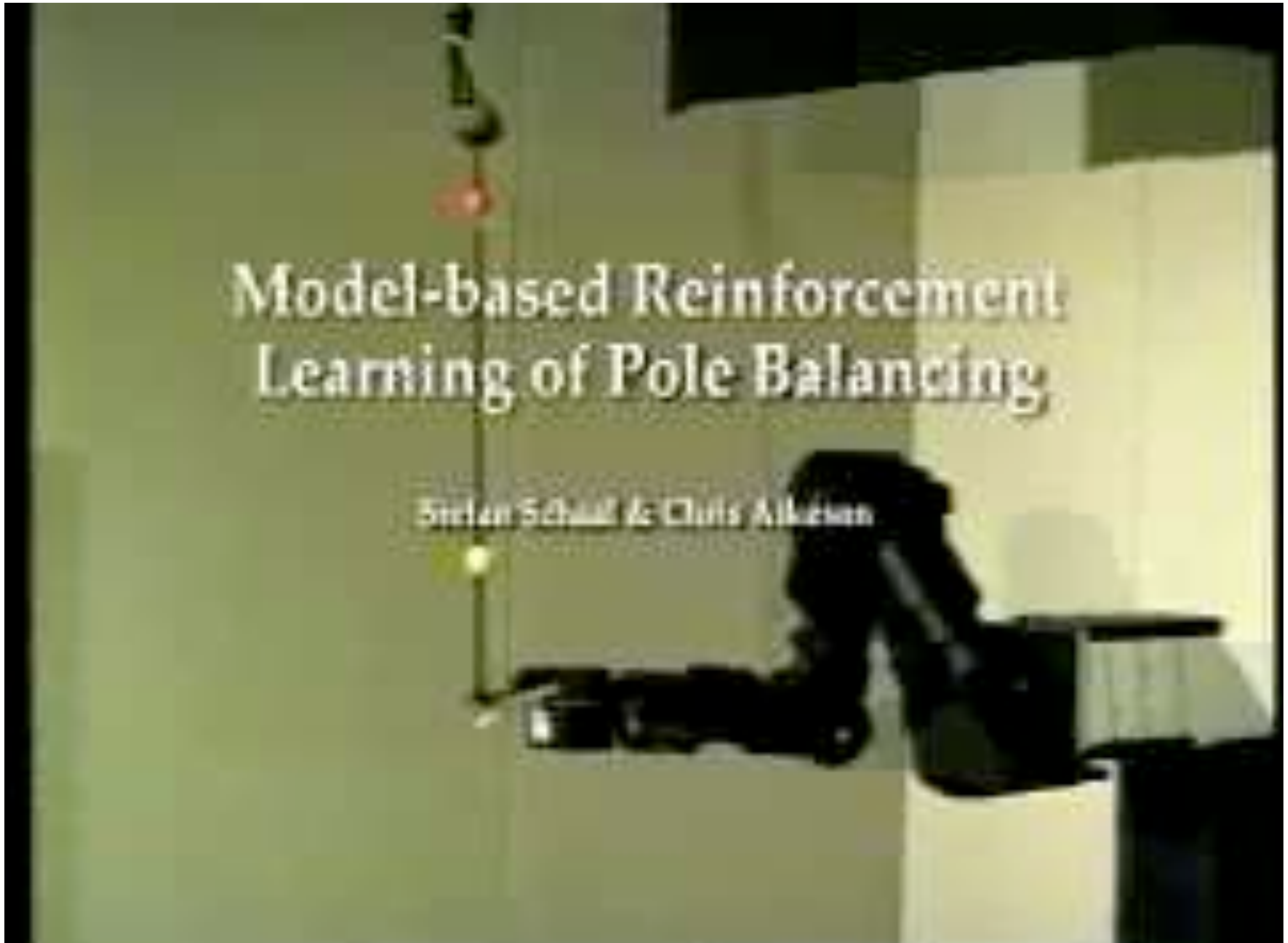
Optimal Policy ?

$$x_{n+1} = f(x_n, u_n)$$

Markov Decision Process

# Early model-based reinforcement learning in robotics

[Schaal and Atkeson ~1995]


Model-based Reinforcement
Learning of Pole Balancing
Stefan Schaal & Chris Atkeson

# Early model-based reinforcement learning in robotics

[Schaal and Atkeson ~1995]

Model-based Reinforcement
Learning of Devilsticking

Stefan Schaal & Chris Atkeson

## Apprenticeship learning

[Abbeel, 2010]

# Apprenticeship learning

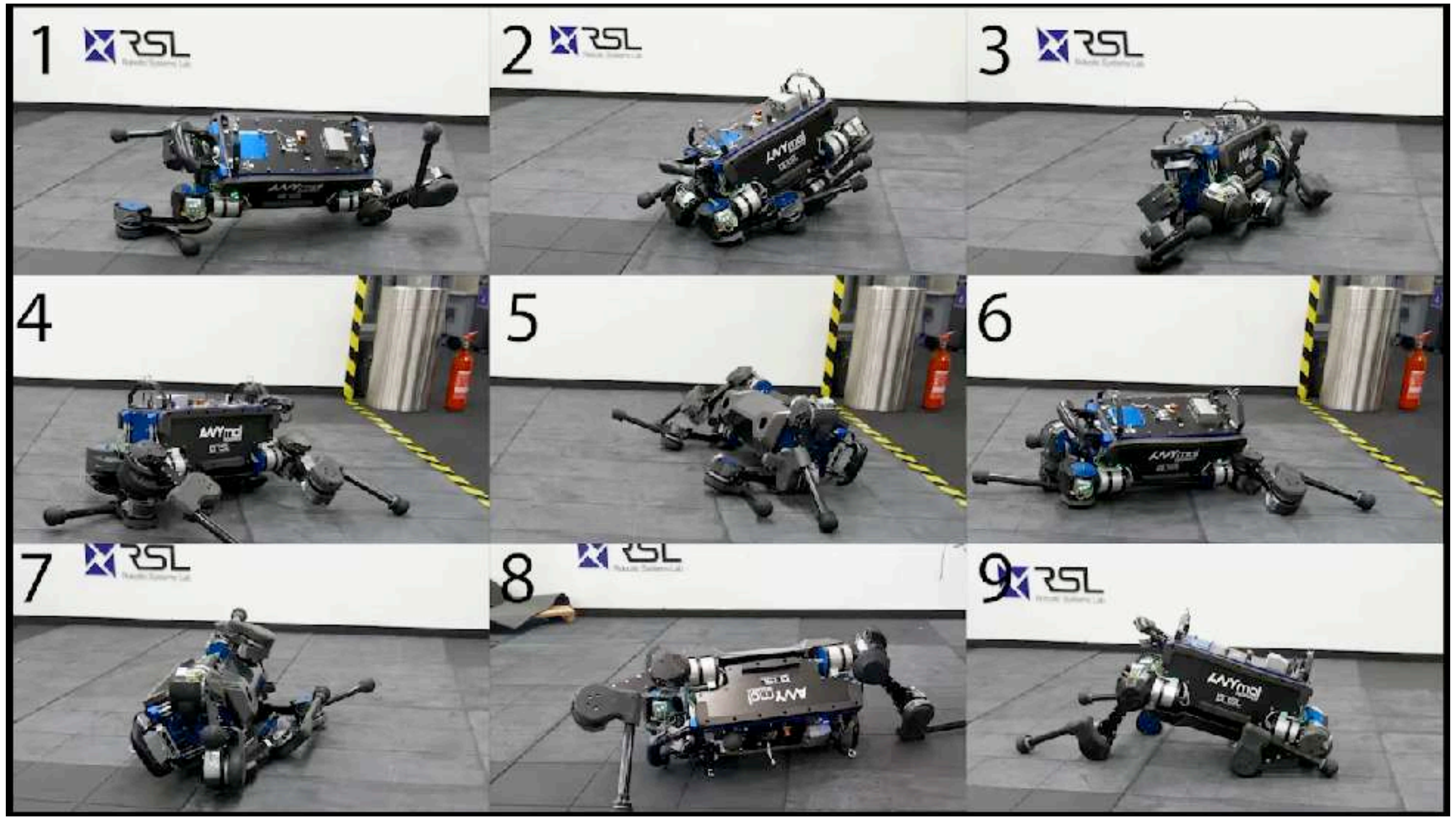
[Abbeel, 2010]

## What is reinforcement learning?

Today RL algorithms mostly uses simulators (i.e. models) and are data intensive
RL algorithms are really just optimization algorithms to find <u>policies</u>

RL algorithms are really just optimization algorithms to find <u>policies</u>
Deep learning has unlocked the ability to approximate complex policies
=> RL can be applied to very complex problems

[Silver et al., 2016]



[Hwangbo et al. 2019]

## Typical RL problems

Set of actions is discrete

State is discrete (countable)

## Robotics RL problems

State is continuous

Action space is continuous

Most methods designed for discrete state/action models do not carry over to continuous state/action models

## Some notations

In the RL literature

States $S_n$

Control inputs are called actions $A_n$

Non-deterministic systems are considered

$$x_{n+1} = f(x_n, u_n, \omega_n)$$

So the optimization criteria is often

$$\min_{u_n} \mathbb{E}_{\omega_n} \left( \sum_{n=0}^{N-1} g_n(x_n, u_n) \right)$$

RL algorithms are often “sampling based”

## Running an <u>episode</u>

Idea: try some policy and record the states and instantaneous costs

execute a policy guess on the robot (or on a simulator) and collect data

update the value/policy guess based on the data

Remember: trajectory optimization can also be sampling based!

# Single shooting with sampling based gradient descent

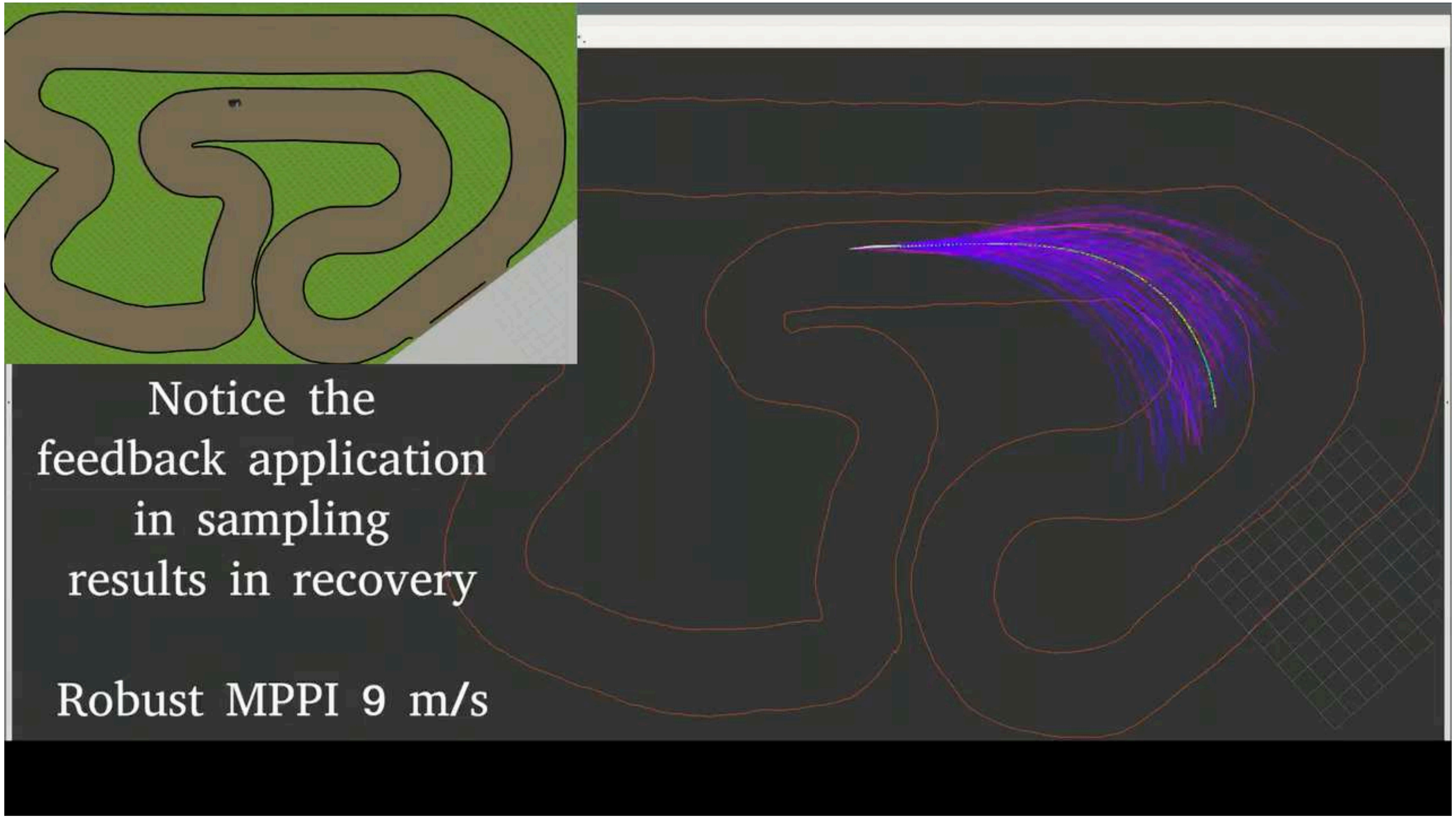
$$\min_{u_0,\cdots,u_{N-1}} \sum_{n=0}^{N-1} l_n(f^n(x_0,u_0,\cdots,u_{n-1}),u_n) + l_N(f^N(x_0,u_0,\cdots,u_{N-1}))$$

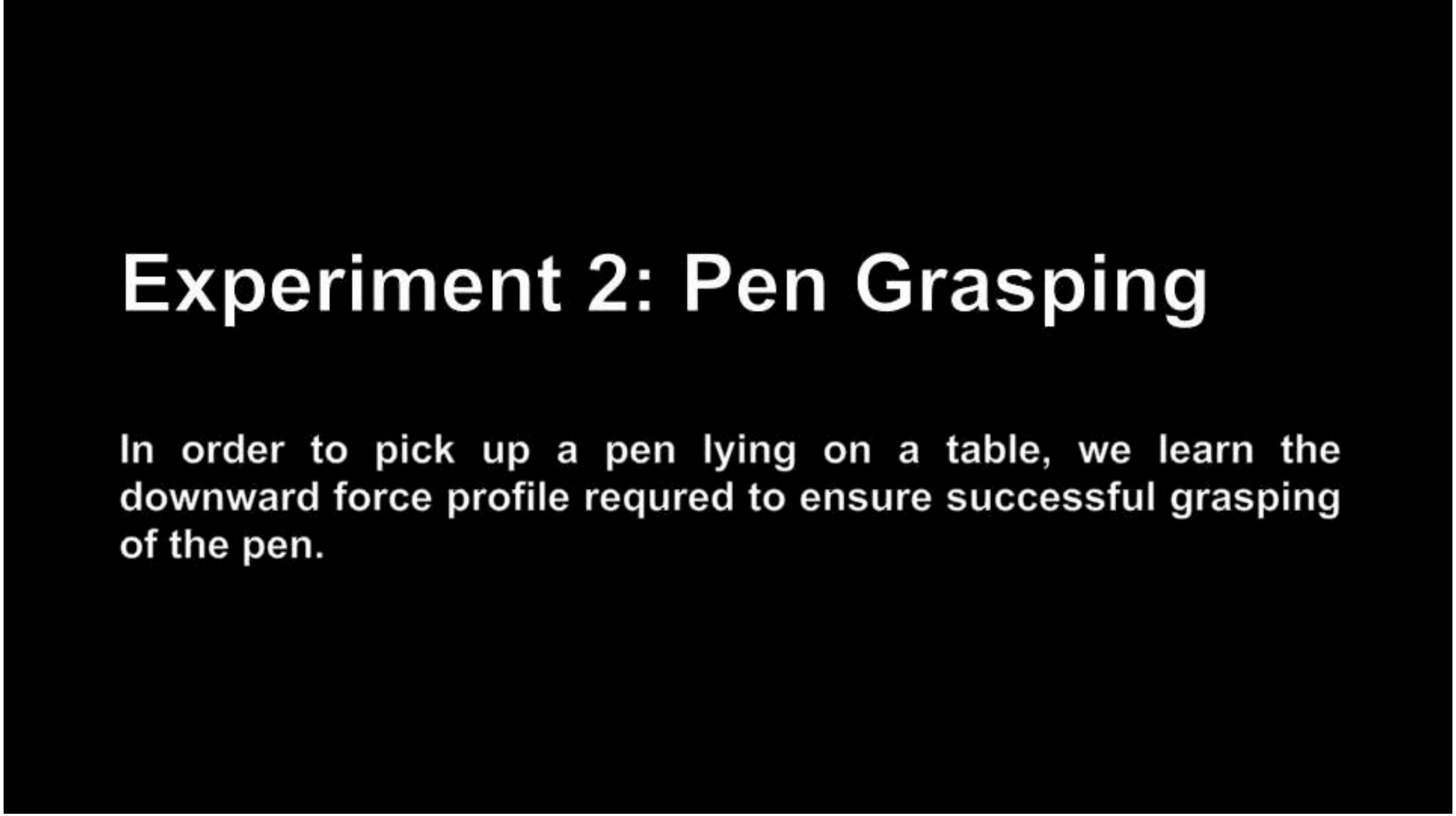
Start with a control guess $\bar{u} = [u_0, \cdots, u_{N-1}]$, then repeat until convergence

- Sample $N$ trajectories $\bar{u} + \epsilon_n$ where $\epsilon\ \mathcal{N}(0, I)$
- Estimate the gradient of the cost $\nabla l(\bar{u}) \simeq \frac{1}{N\sigma} \sum (f(\bar{u} + \epsilon_n) - f(\bar{u}))\, \epsilon_n$
- Update $\bar{u} \leftarrow \bar{u} + \alpha \nabla l(\bar{u})$

Very convenient as we can replace the dynamics f() by a simulator!

# MPPI (Model-predictive path integral control)





Experiment 2: Pen Grasping

In order to pick up a pen lying on a table, we learn the downward force profile requred to ensure successful grasping of the pen.

[Kalakrishnan et al. 2011]

## Trajectory optimization with real-world data

Remember: trajectory optimization can also be sampling based!

Most modern RL algorithms optimize a policy

## On-policy vs. off-policy methods

<u>On-policy methods</u> evaluate or improve the policy that is used to generate episodes (i.e. learn while you act)

<u>Off-policy methods</u> evaluate or improve a policy different from the method used to generate the data / episodes.

## Before optimizing a policy… we evaluate the value of a policy

## Policy evaluation with sampling 1
## Monte-Carlo methods

## Policy evaluation with Monte-Carlo methods

## Policy evaluation with sampling II
## TD-learning

## Optimal Value function

$$J = \min_u g(x,u) + \alpha J(f(x,u))$$

## Value Function for Policy $\mu(x) : x \to u$

$$J_\mu(x) = g(x, \mu(x)) + \alpha J_\mu(f(x, \mu(x)))$$

## Action-value function $Q(x_t, u_t)$

$$Q(x_t, u_t) = g(x_t, u_t) + \alpha \min_u Q(x_{t+1}, u)$$

$$\underbrace{J_\mu(x_t)}_{\substack{\text{current guess} \\ \text{of cost for state } x_t}} \overset{?}{=} \underbrace{g(x_t, \mu(x_t)) + \alpha J_\mu(x_{t+1})}_{\substack{\text{actual return} \\ \text{following time t}}}$$

Temporal difference error (TD error)

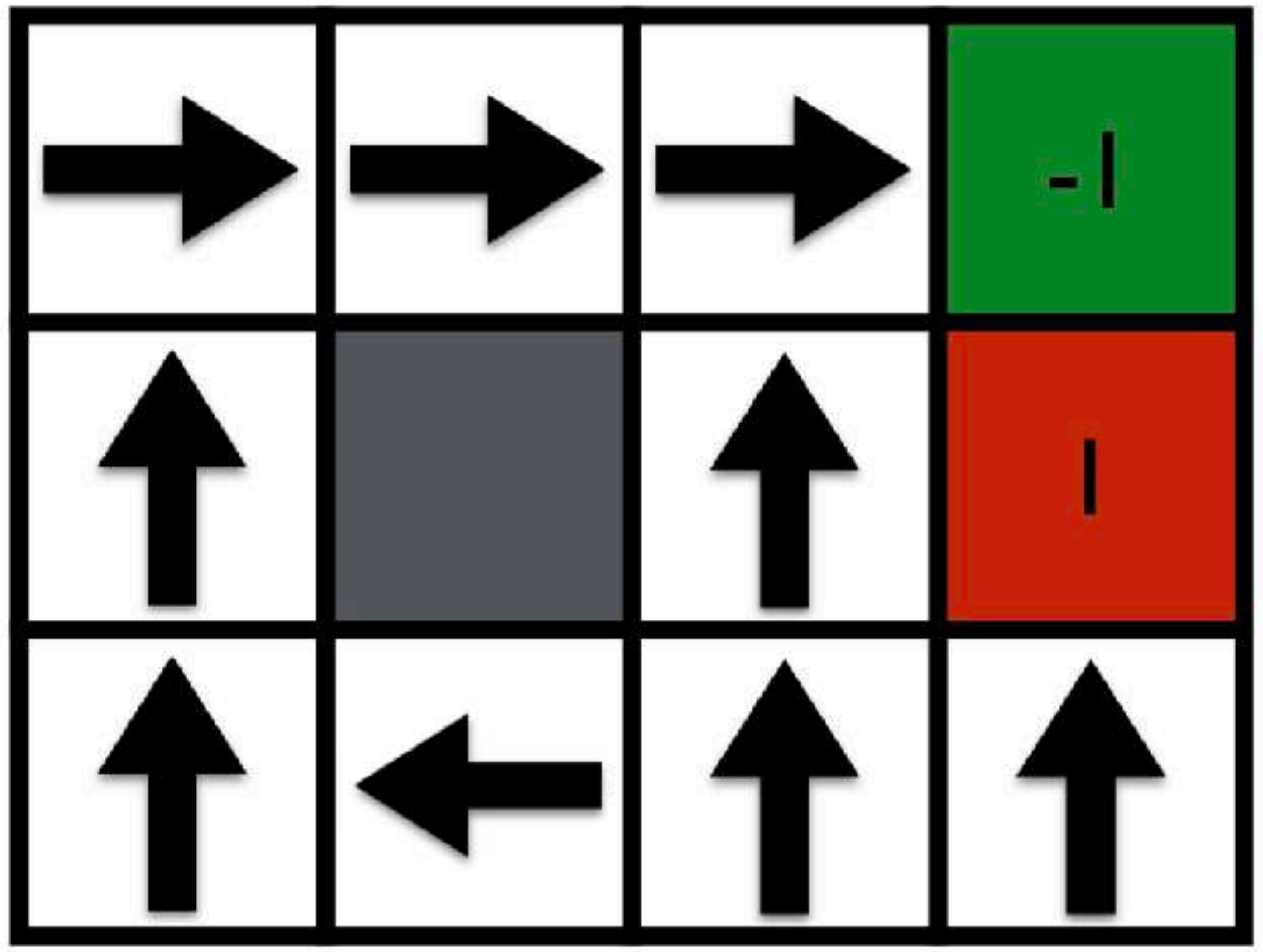
$$\delta_t = g(x_t, \mu(x_t)) + \alpha J_\mu(x_{t+1}) - J_\mu(x_t)$$

# Temporal difference learning

[Sutton 1988]
[Samuel 1959]

TD(0) learning for estimating $J_\mu$

```
Input: policy to be evaluated $\mu$
Choose a step size $\gamma \in [0, 1]$
Initialize $J_\mu$ for all states x
For each episode of length N:
    Choose an initial state $x_0$
    Loop for each step of the episode:
        Do $\mu(x_t)$
        Observe $x_{t+1}$ Compute $g(x_t, \mu(x_t))$
        Update $J_\mu(x_t) \leftarrow J_\mu(x_t) + \gamma\delta_t$
        using $\delta_t = g(x_t, \mu(x_t)) + \alpha J_\mu(x_{t+1}) - J_\mu(x_t)$
```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## How can we improve the policy?

# Q-learning: off-policy TD control

[Watkins 1989]

Action-value function $Q(x_t, u_t)$

$$Q(x_t, u_t) = g(x_t, u_t) + \alpha J^*(x_{t+1})$$

$Q(x_t, u_t)$: cost of doing $u_t$ at state $x_t$ and behaving optimally after

$J^*(x_t) = \min_u Q(x_{t+1}, u)$ **Optimal value function**

$\mu^*(x) = \arg\min_u Q(x_t, u)$ **Optimal policy**

## Q-learning: off-policy TD control

[Watkins 1989]

Action-value function $Q(x_t, u_t)$

$$Q(x_t, u_t) = g(x_t, u_t) + \alpha \min_u Q(x_{t+1}, u)$$

$$J^*(x_t) = \min_u Q(x_{t+1}, u)$$ Optimal value function

TD-error $\delta_t = g(x_t, u_t) + \alpha \min_u Q(x_{t+1}, u) - Q(x_t, u_t)$

Update $Q(x_t, u_t) \leftarrow Q(x_t, u_t) + \gamma \delta_t$

## Q-learning

$$\delta_t = g(x_t, u_t) + \alpha \min_u Q(x_{t+1}, u) - Q(x_t, u_t)$$

$Q(x,u)$ is stored as a table

| | $x=0$ | $x=0.1$ | $x=0.2$ | | $x=6.2$ |
|---|---|---|---|---|---|
| $u=-5$ | $Q(x_1,u_1)$ | | | $\cdots$ | |
| $u=-4.9$ | | | | $\cdots$ | |
| | | | | $\cdots$ | |
| | $\vdots$ | $\vdots$ | | $\ddots$ | |
| $u=0$ | | | | | |
| | $\vdots$ | $\vdots$ | | $\ddots$ | |
| $u=5$ | | | | $\cdots$ | |

# The exploration/exploitation trade-off

How do we choose the policy in an episode?

Current optimal guess: $u_t = \arg\min_u Q(x_t, u)$

If we always choose the optimal guess, we might miss better actions/states that we would never try

If we only choose random actions, we might be not be able to get a good guess for Q

## $\epsilon$ -greedy policy

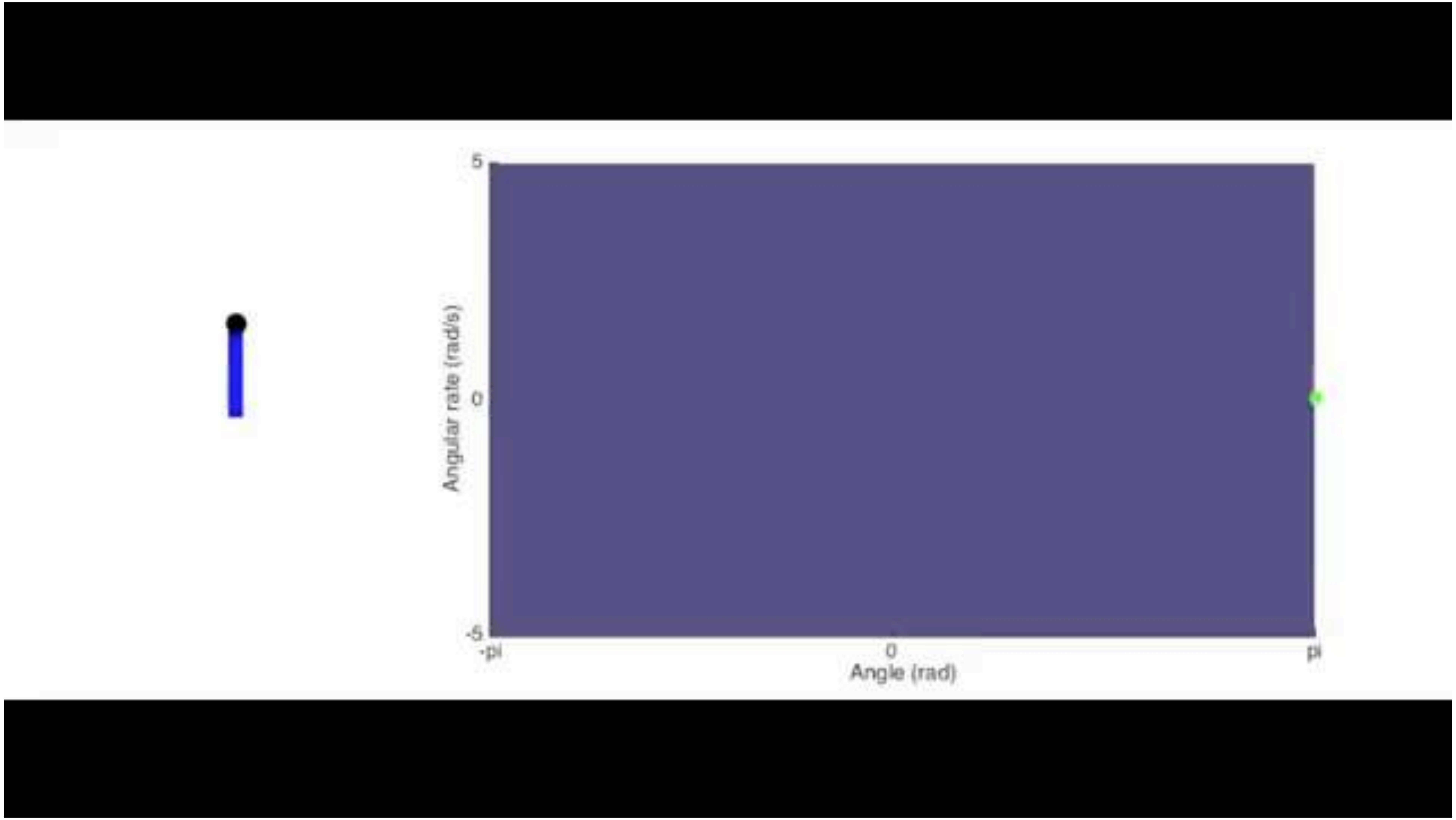
$$u_t = \begin{cases} \arg\min_u Q(x_t, u) & \text{with probability } 1 - \epsilon \\ \text{random action} & \text{with probability } \epsilon \end{cases}$$

# Q-learning

[Watkins 1989]

```
Choose a step size $\gamma \in [0, 1]$ and small $\epsilon$
Initialize $Q(x, u)$ for all states x and actions u

For each episode:
    Choose an initial state $x_0$
    Loop for each step of the episode:
        Choose an action using an $\epsilon$-greedy policy from Q
        Observe $x_{t+1}$   Compute $g(x_t, \mu(x_t))$
        Update $Q(x_t, u_t) \leftarrow Q(x_t, u_t) + \gamma \delta_t$
          using $\delta_t = g(x_t, u_t) + \alpha \min_u Q(x_{t+1}, u) - Q(x_t, u_t)$
```

## Q-learning



[source: https://www.youtube.com/watch?v=YLAWnYAsai8]

# Q-learning

Model-free approach to learn optimal policies
(value iteration with a twist)

$$Q(x_t, u_t) \leftarrow Q(x_t, u_t) + \gamma \delta_t$$

$$\delta_t = g(x_t, u_t) + \alpha \min_u Q(x_{t+1}, u) - Q(x_t, u_t)$$

Guaranteed to converge at infinity BUT can take a long time!

Need to store Q / discrete actions and states

Need to compute the min of Q (expensive!)

## Typical RL problems

Set of actions is discrete

State is discrete (countable)

## Robotics RL problems

State is continuous

Action space is continuous

Most methods designed for discrete state/action models do not carry over to continuous state/action models

# Q-learning

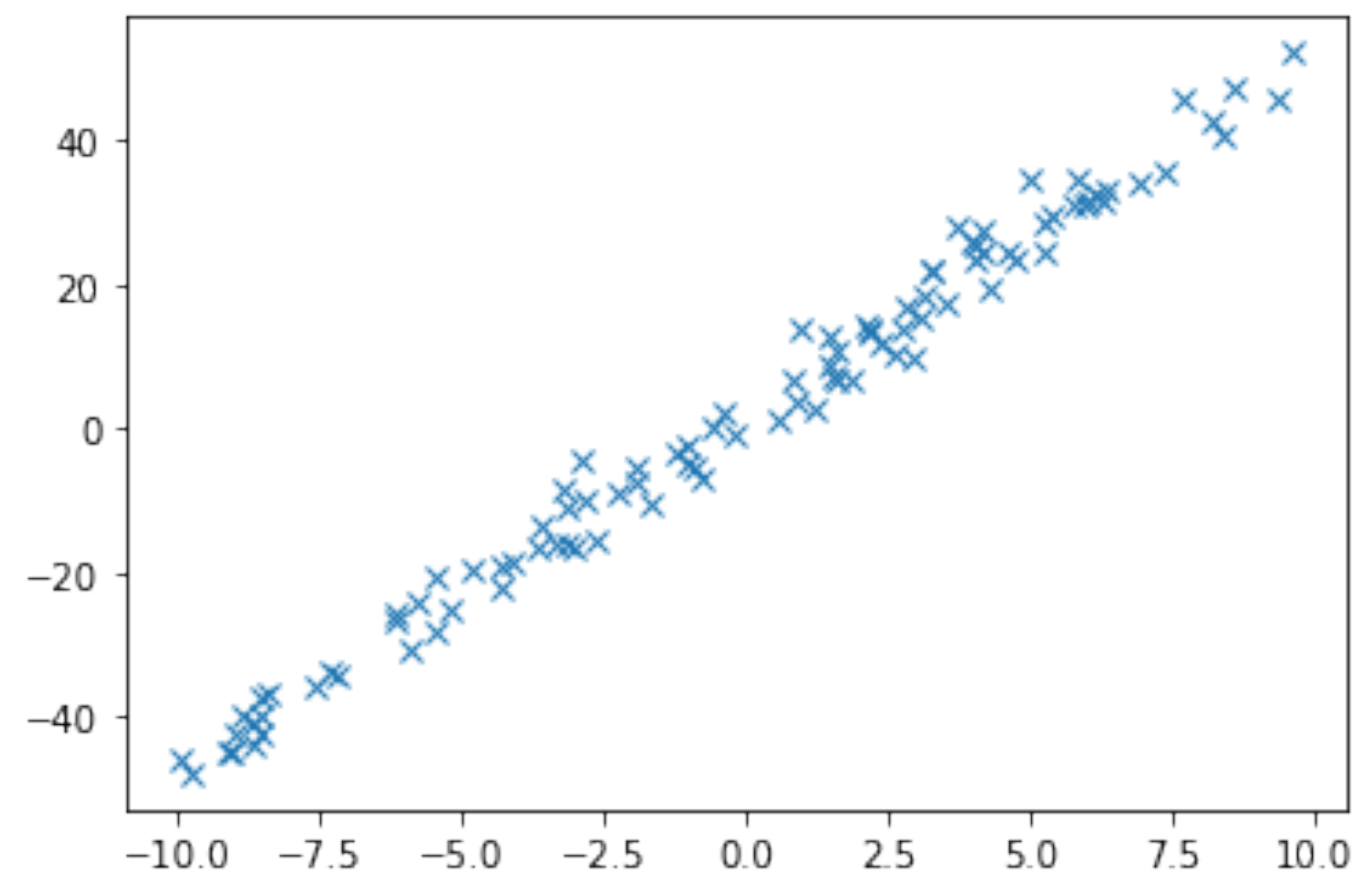
$$\delta_t = g(x_t, u_t) + \alpha \min_u Q(x_{t+1}, u) - Q(x_t, u_t)$$

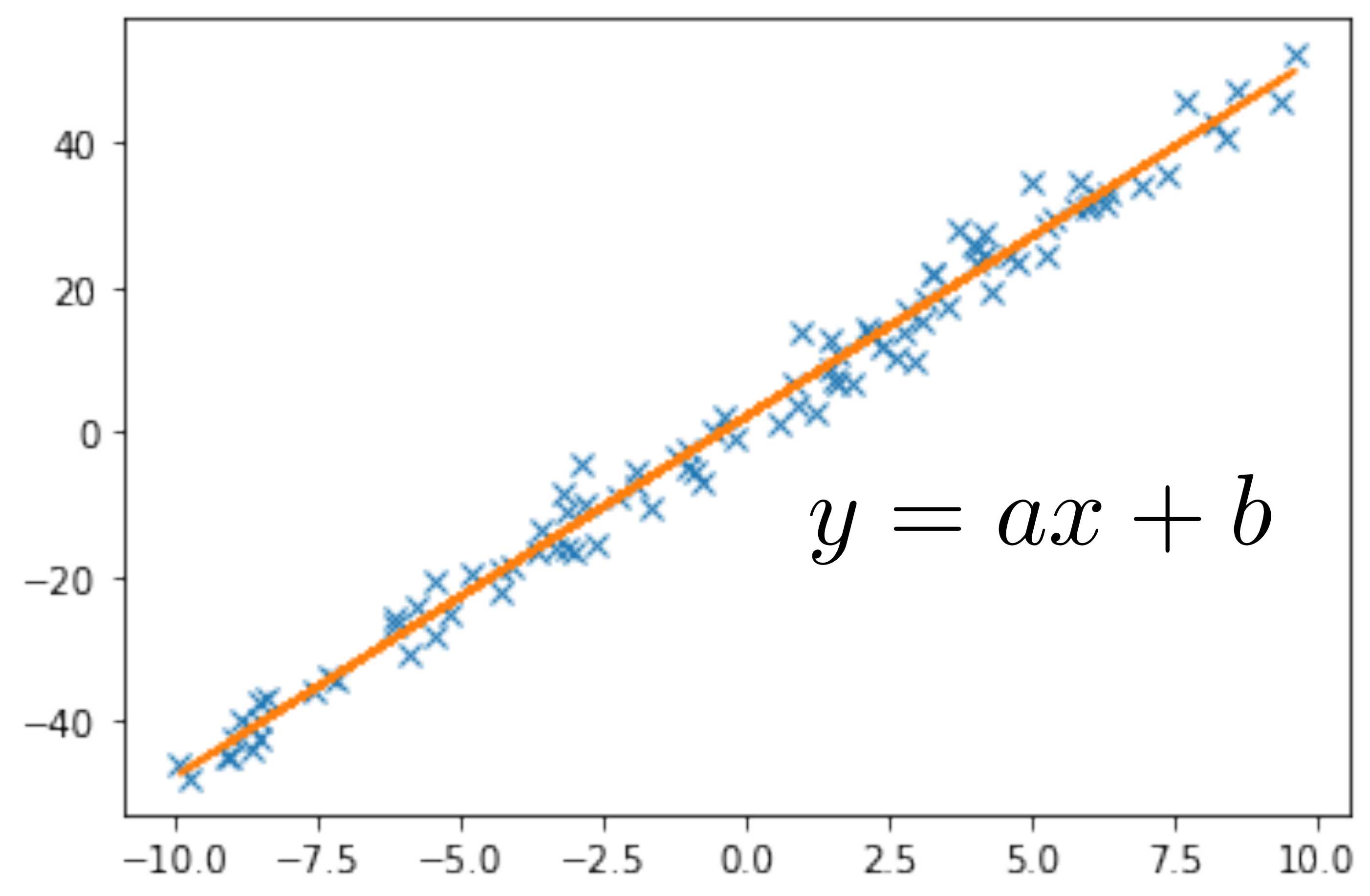
Can we get rid of the discrete states and actions and the table?

$Q(x, u)$ is a function - can we approximate it?

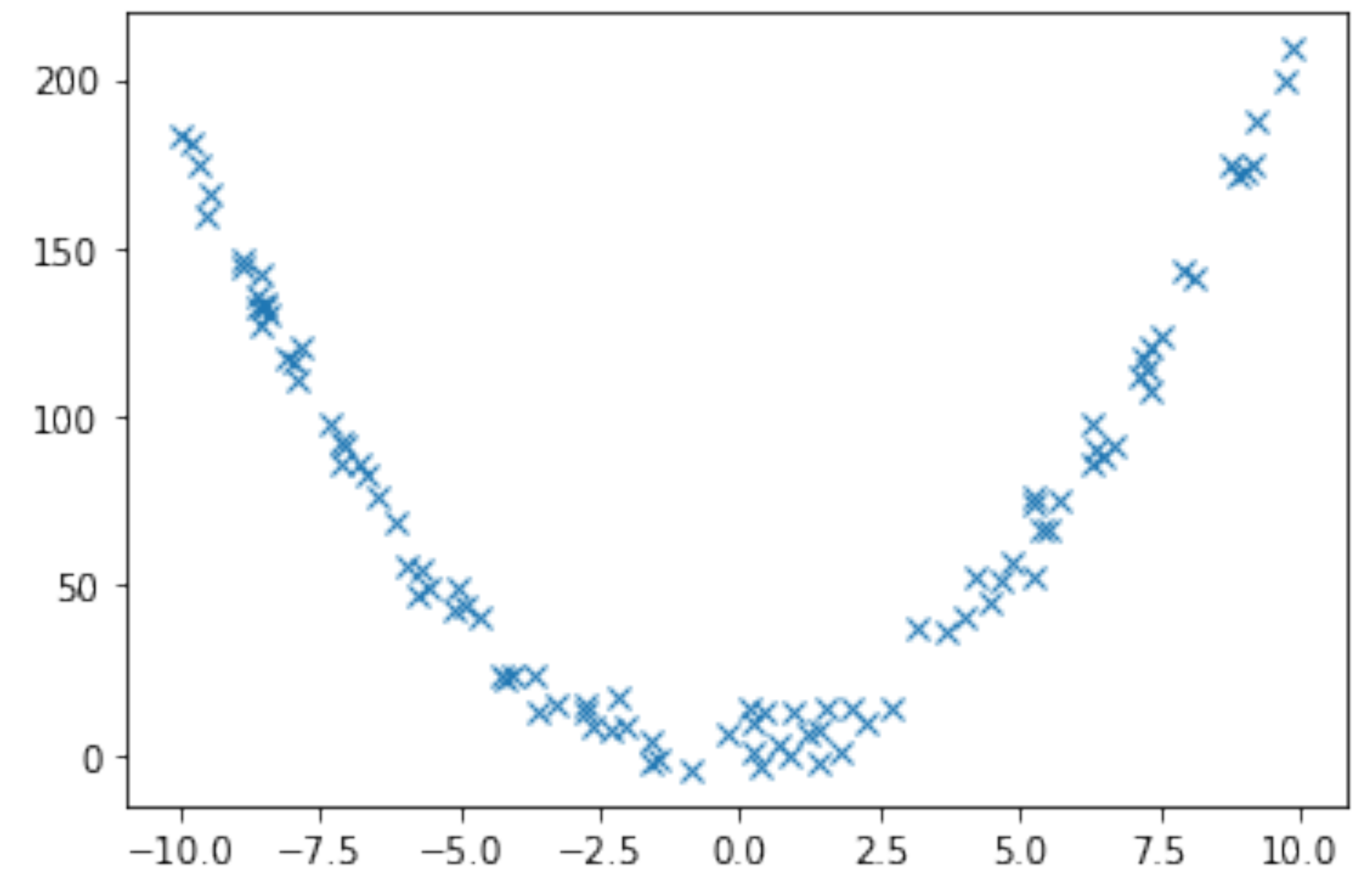
| | $x = 0$ | $x = 0.1$ | $x = 0.2$ | | $x = 6.2$ |
|---|---|---|---|---|---|
| $u = -5$ | $Q(x_1, u_1)$ | | | $\cdots$ | |
| $u = -4.9$ | | | | $\cdots$ | |
| | | | | $\cdots$ | |
| | $\vdots$ | $\vdots$ | | $\ddots$ | |
| $u = 0$ | | | | | |
| | $\vdots$ | $\vdots$ | | $\ddots$ | |
| $u = 5$ | | | | $\cdots$ | |

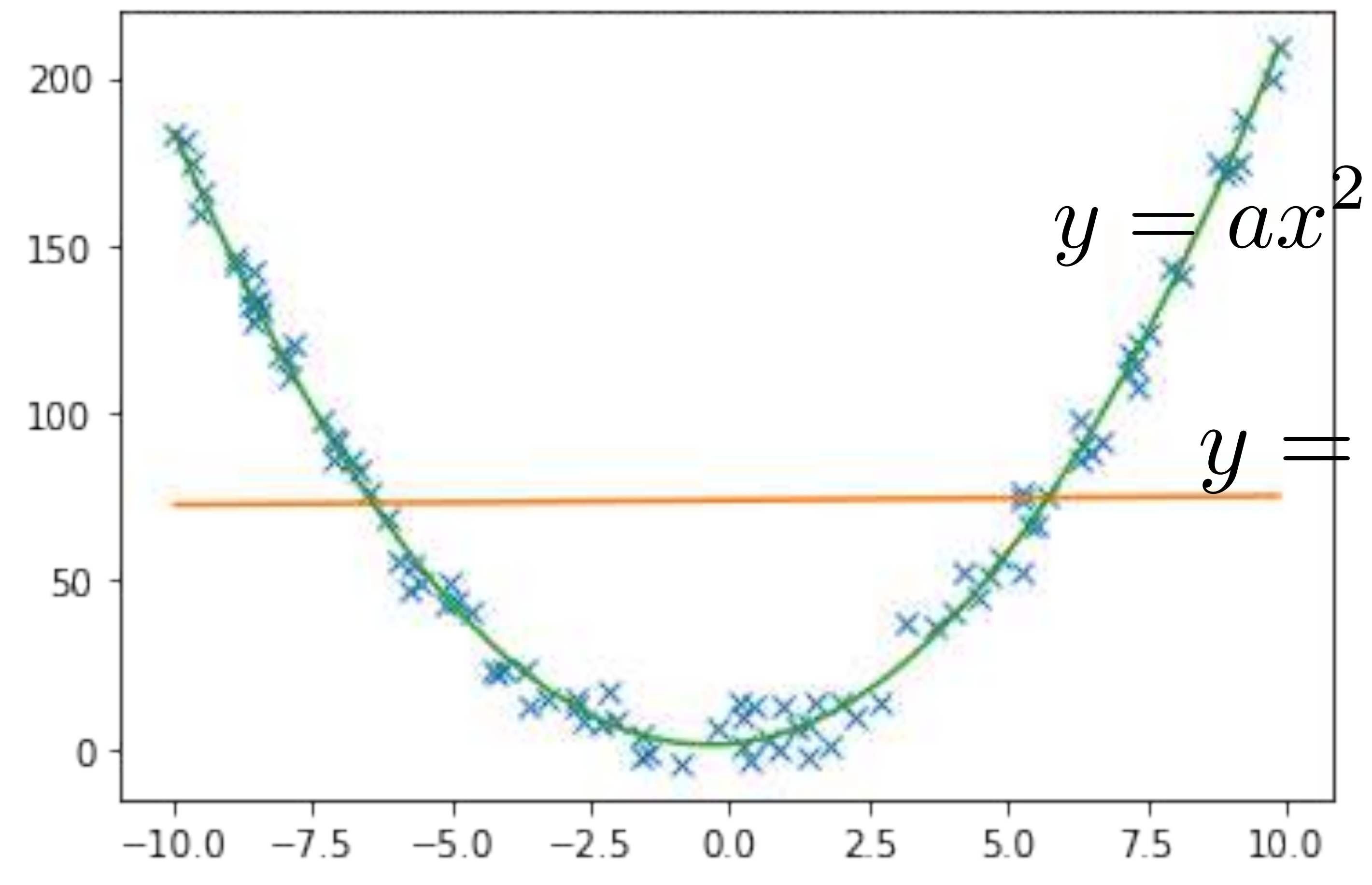
# Function approximation





$y = ax + b$





$y = ax^2 + bx + c$

$y = ax + b$

# Linear least squares

Given N data point $(x_1, y_1)\ (x_2, y_2) \cdots (x_N, y_N)$

$$y = \sum_{k=0}^{K} a_k x^k$$

Find a function that is a linear combination of polynomials of the input x

Minimize the least square error between the output data and the function

$$\min_{a_0 \cdots a_K} \sum_{i=0}^{N-1} \left(\sum_{k=0}^{K} a_k x_i^k - y_i\right)^2$$

# Linear least squares

Minimize the least square error between the output data and the function

$$\min_{a_0 \cdots a_K} \sum_{i=0}^{N-1} (\sum_{k=0}^{K} a_k x_i^k - y_i)^2$$

We can write these relations in matrix form by noticing that

$$\sum_{k=0}^{K} a_k x_i^k = \begin{bmatrix} 1 & x_i & x_i^2 & \cdots & x_i^K \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_K \end{bmatrix}$$

# Linear least squares

We can write these relations in matrix form by noticing that

$$\sum_{k=0}^{K} a_k x_i^k = \begin{bmatrix} 1 & x_i & x_i^2 & \cdots & x_i^K \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_K \end{bmatrix}$$

Using all the data points and the knowledge of the degree K and we can then construct the $N \times K$ matrix

$$X = \begin{bmatrix} 1 & x_0 & x_0^2 & \cdots & x_0^K \\ 1 & x_1 & x_1^2 & \cdots & x_1^K \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{N-1} & x_{N-1}^2 & \cdots & x_{N-1}^K \end{bmatrix}$$

and the vector

$$Y = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-1} \end{bmatrix} \qquad a = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_K \end{bmatrix}$$

where each row $i$ of $X$ and $Y$ is defined by the sample $i$ from the dataset.

# Linear least squares

We now have

$$Xa - Y = \begin{bmatrix} \sum_{k=0}^{K} a_k x_0^k - y_0 \\ \sum_{k=0}^{K} a_k x_1^k - y_1 \\ \vdots \\ \sum_{k=0}^{K} a_k x_{N-1}^k - y_{N-1} \end{bmatrix}$$

and the original problem can be written as

$$\min_a (Xa - Y)^T (Xa - Y)$$

which is equal to

$$\min_a a^T X^T X a - 2Y^T X a + Y^2$$

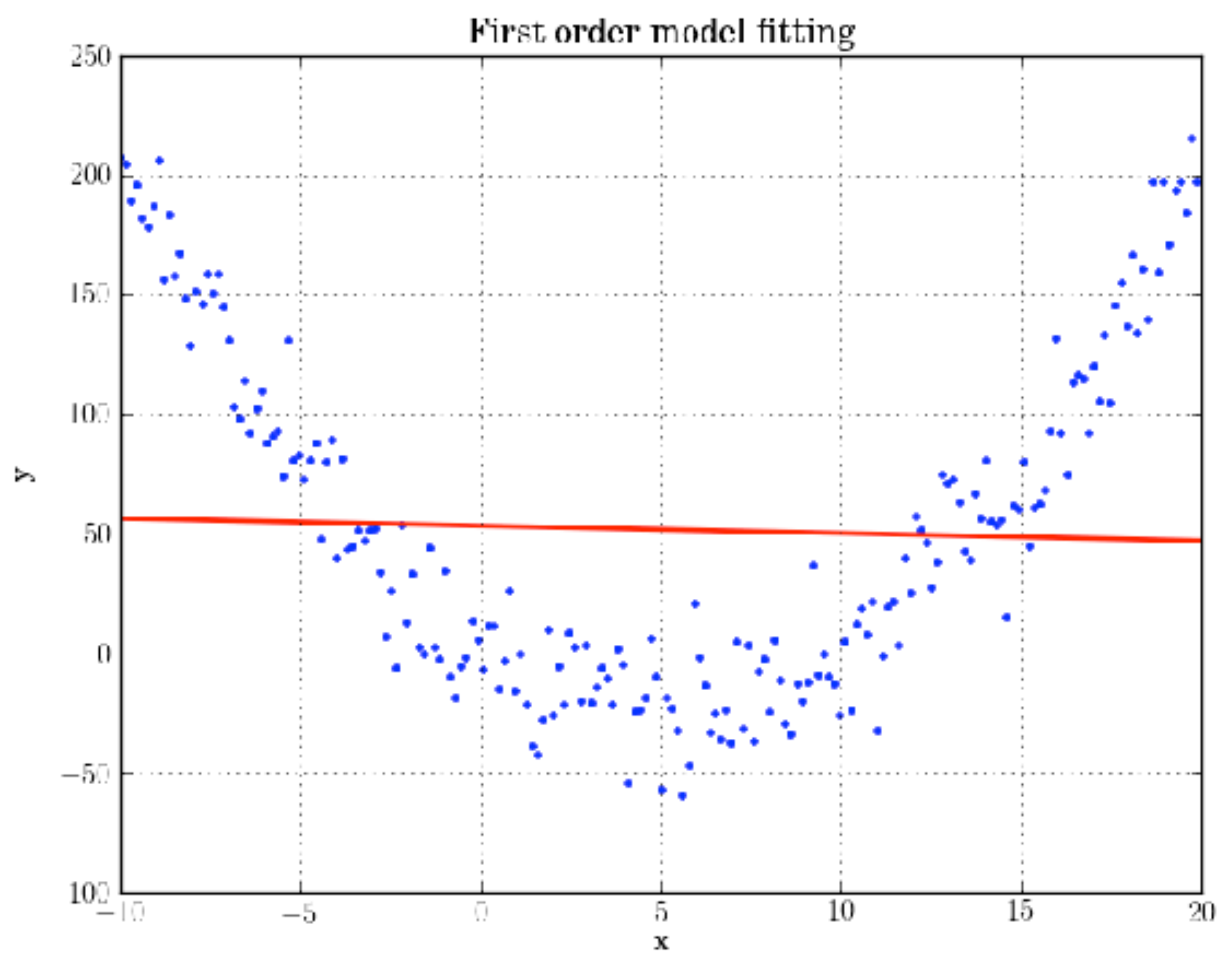
$$\frac{\partial}{\partial a}\left(a^T X^T X a - 2Y^T X a + Y^2\right) = 2X^T X a - 2X^T Y = 0$$

$$a = (X^T X)^{-1} X^T Y$$

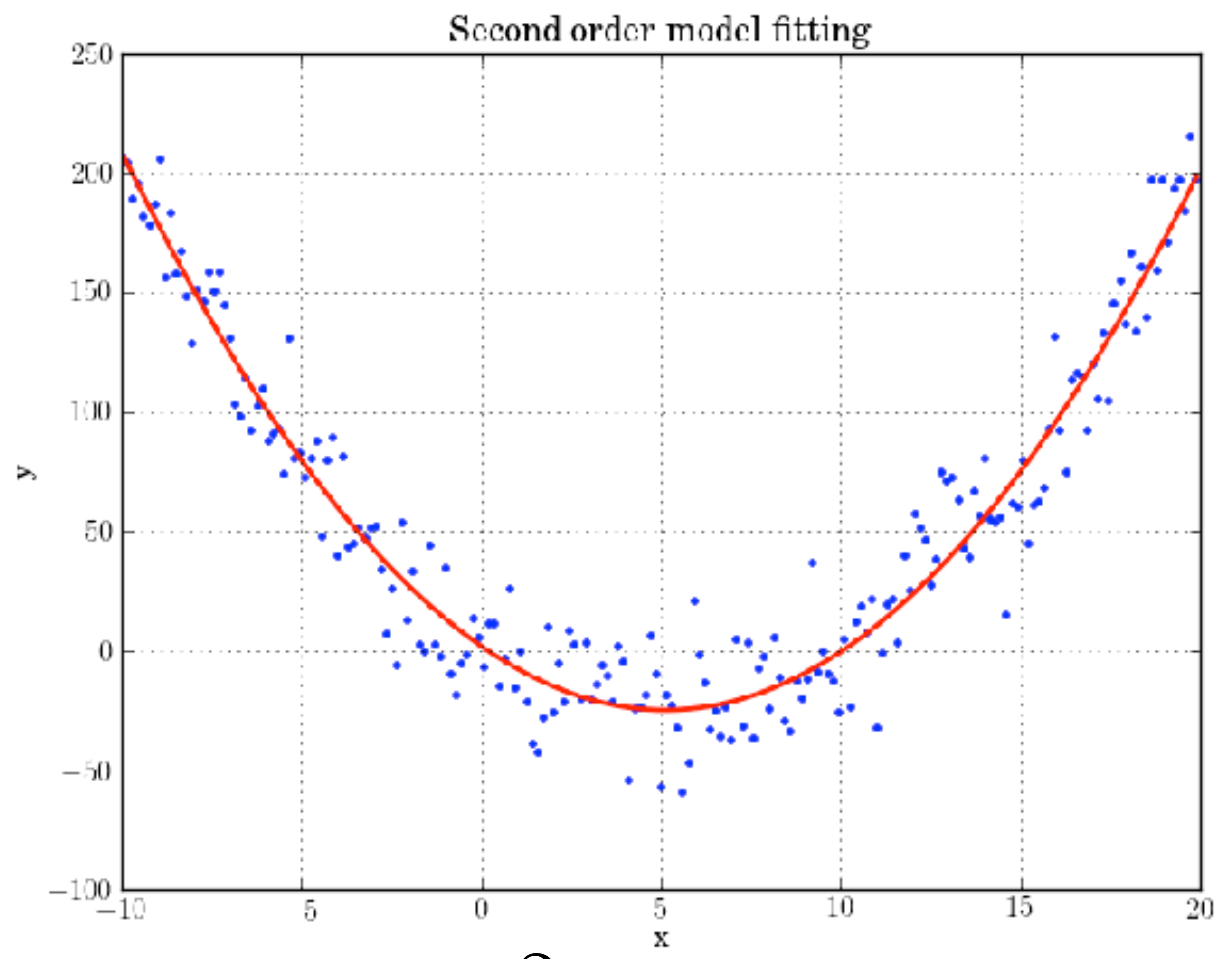
# Function approximation

500 noisy samples $\mathcal{D} = \{(x_0, y_0), (x_1, y_1), \cdots, (x_N, y_N)\}$

from a quadratic function



$$y = a_1 x + a_0$$



$$y = a_2 x^2 + a_1 x_1 + a_0$$

## Nonlinear least-square

$$y = f(x, w)$$

$$\min \sum_{i=0}^{N} (y_i - f(x_i, \omega))^2$$

we cannot compute w explicitly as before because the function is not linear in w anymore

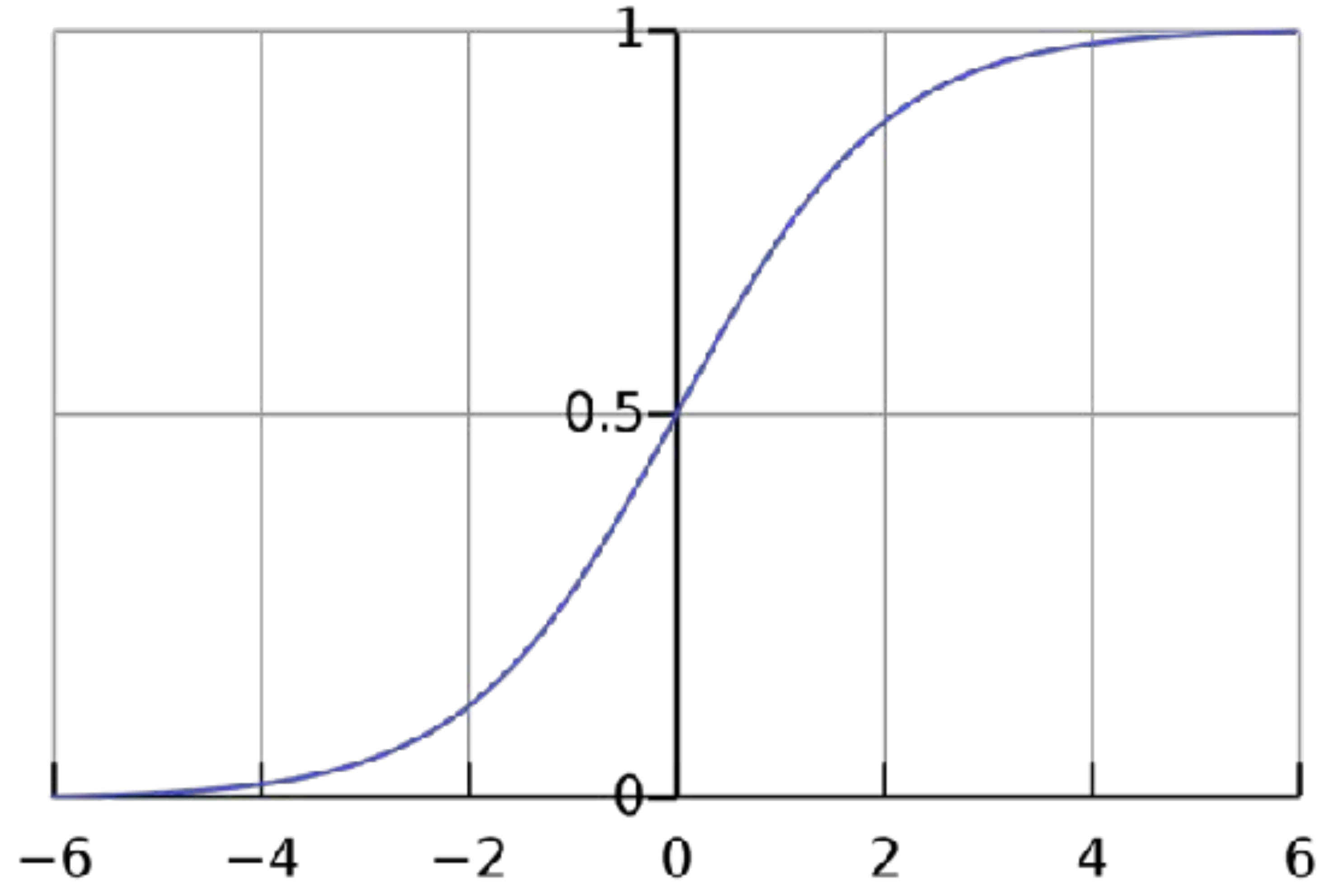
=> do gradient descent to minimize the function

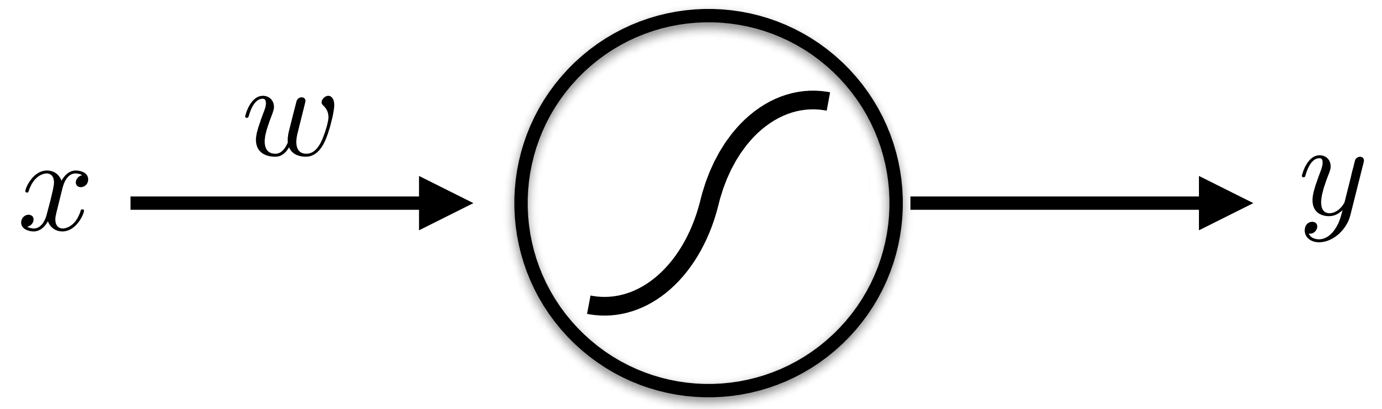
$$\omega \leftarrow \omega - \eta \frac{\partial}{\partial \omega} \left( \sum_{i=0}^{N} (y_i - f(x_i, \omega))^2 \right) = \omega + 2\eta \sum_{i=0}^{N} \left( (y_i - f(x_i, \omega)) \frac{\partial f}{\partial \omega} \right)$$

## Example: a simple nonlinear function

$$y = \frac{1}{1 + \mathrm{e}^{-wx}}$$

a sigmoid function

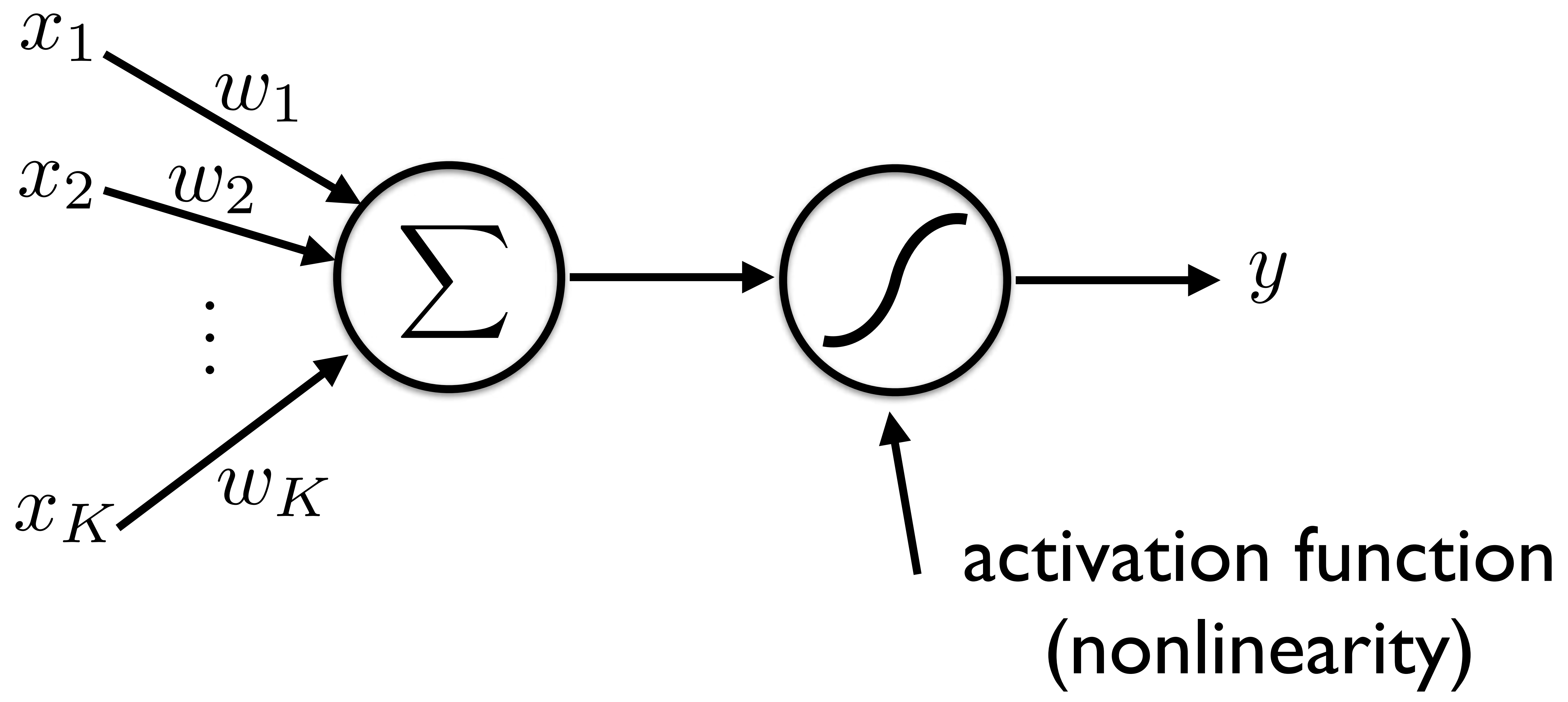




# Example: a simple nonlinear function

multiple inputs and one output

$$y = \frac{1}{1 + \mathrm{e}^{-\sum_k w_k x_k}}$$

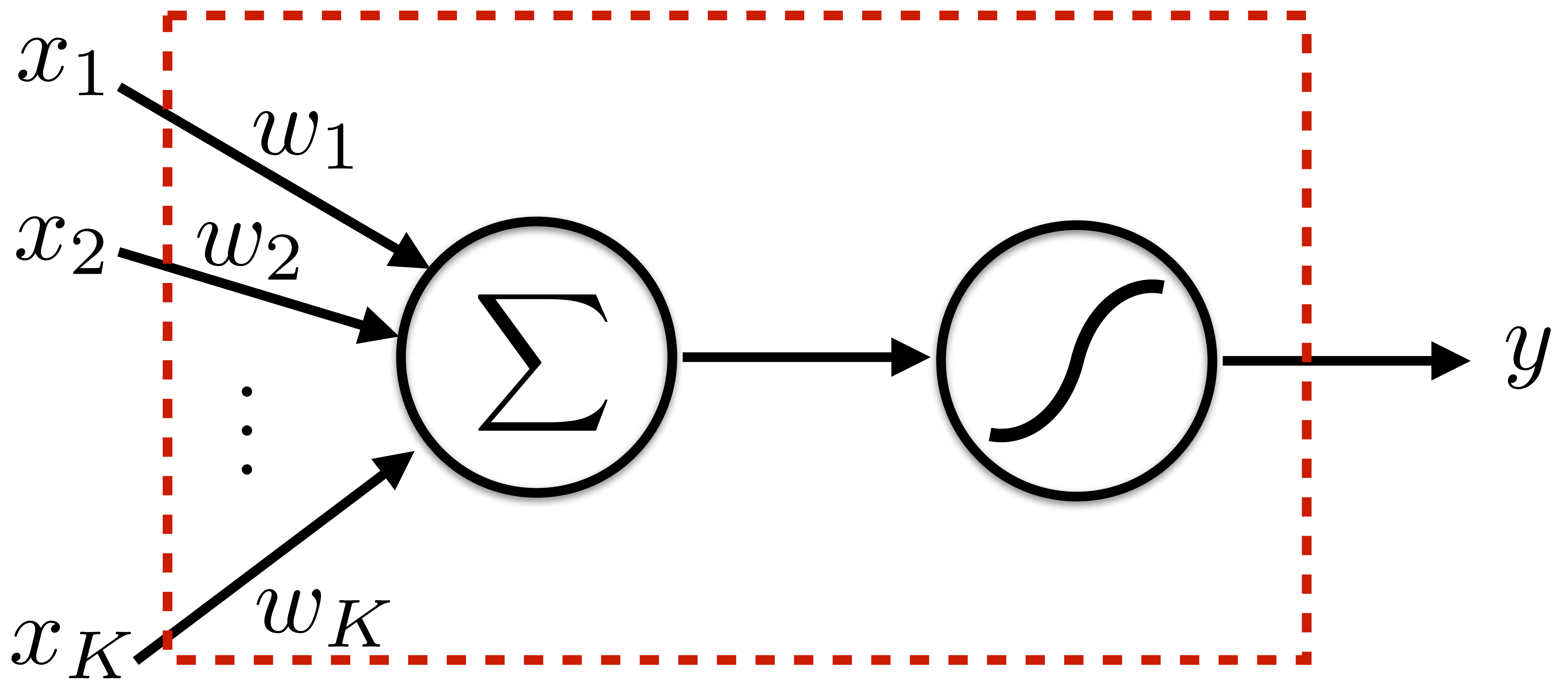


$w_k$ weights (unknown parameters to find or "learn")

# Example: a simple nonlinear function

multiple inputs and one output

$$y = \frac{1}{1 + \mathrm{e}^{-\sum_k w_k x_k}}$$



one "neuron"

$$\min_w \sum_{n=0}^{N} \left( y_i - \frac{1}{1 + \mathrm{e}^{-\sum_k w_k x_{k,i}}} \right)$$

# Example: a simple nonlinear function

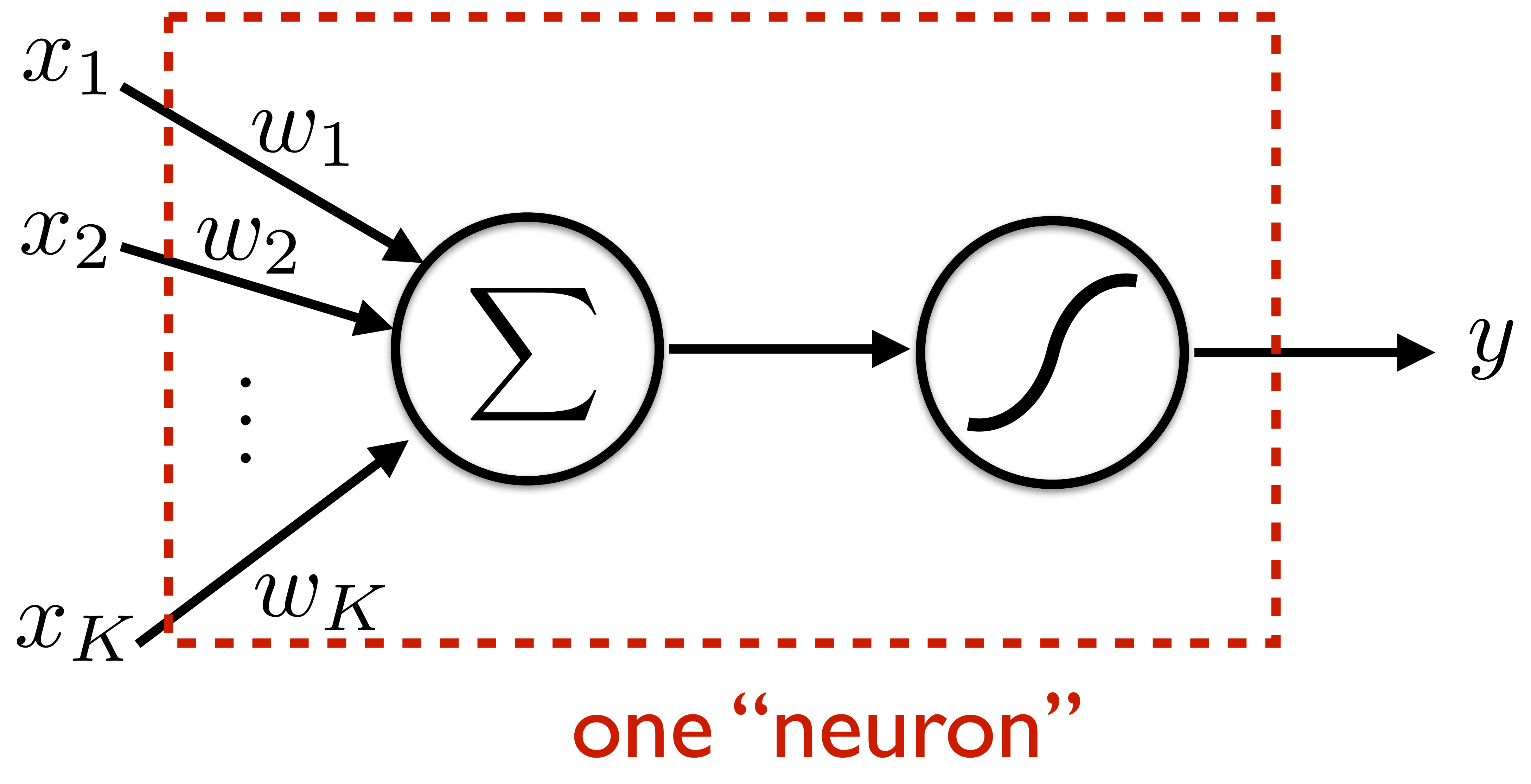
$$\min_{w} \sum_{n=0}^{N} \left( y_i - \frac{1}{1 + \mathrm{e}^{-\sum_k w_k x_{k,i}}} \right)$$

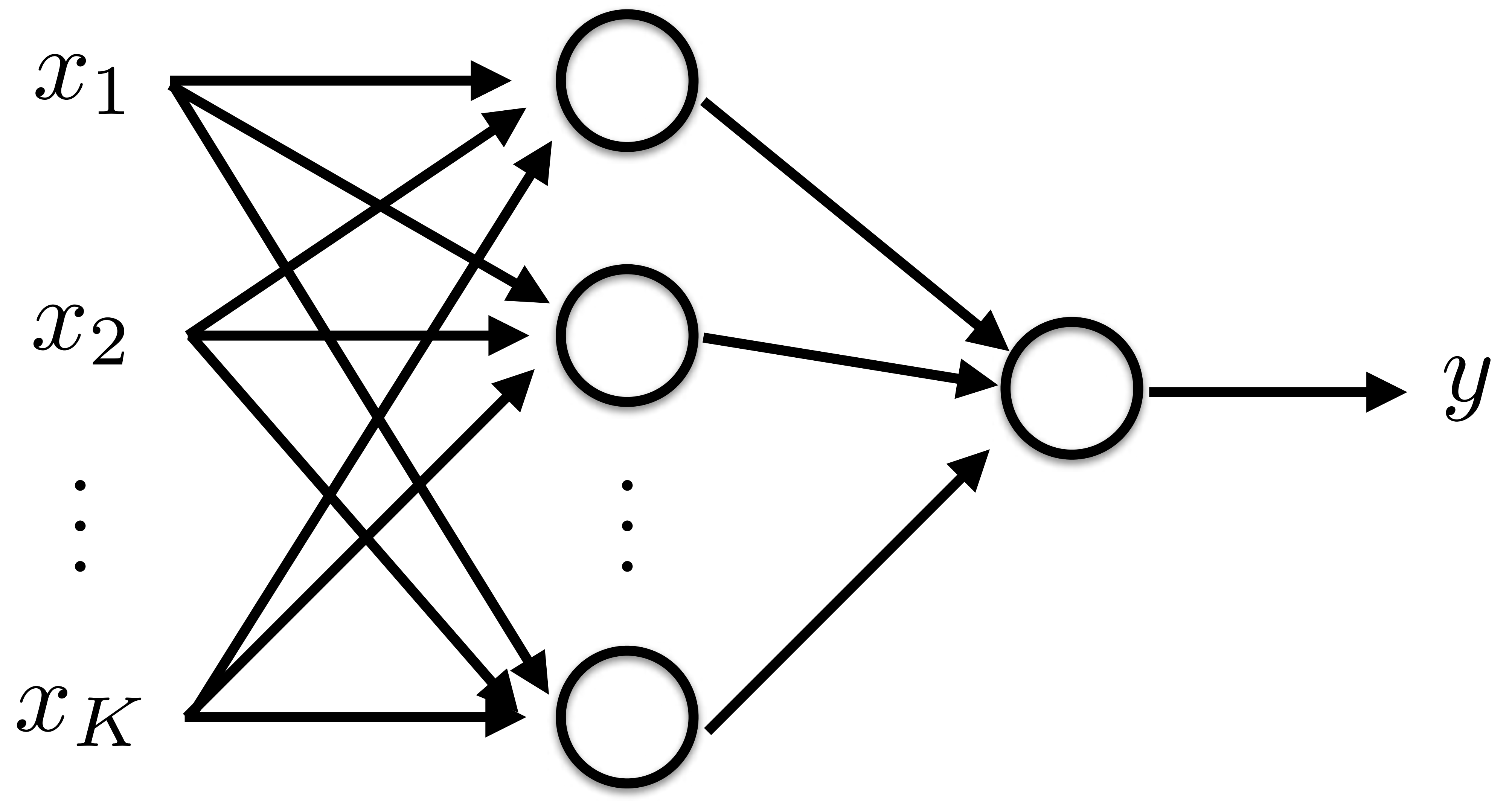
Dataset has multidimensional input

$$\mathcal{D} = \{(x_{0,0}, x_{1,0}, \cdots, x_{M,0}, y_0), (x_{0,1}, x_{1,1}, \cdots, x_{M,1}, y_1), \cdots, (x_{0,N}, x_{1,N}, \cdots, x_{M,N}, y_N)\}$$

Gradient descent enables the optimization of the unknown parameters (or neuron weights) w

# One layer neural network

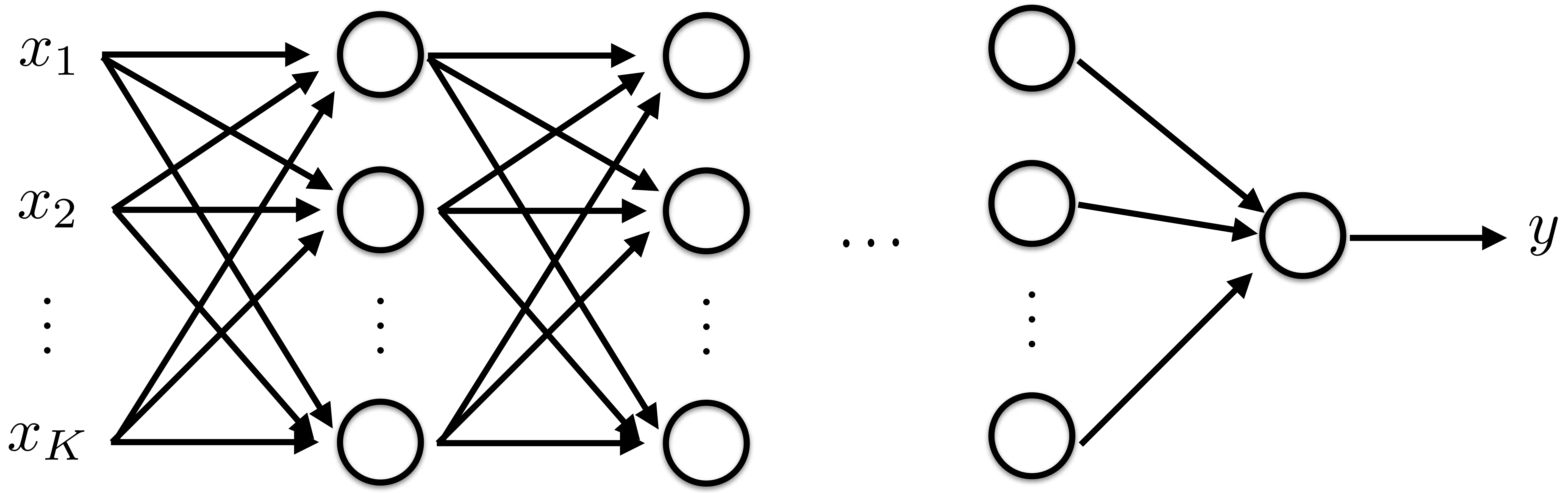




Simple neural network: a combination of neurons to create a (complex) nonlinear function

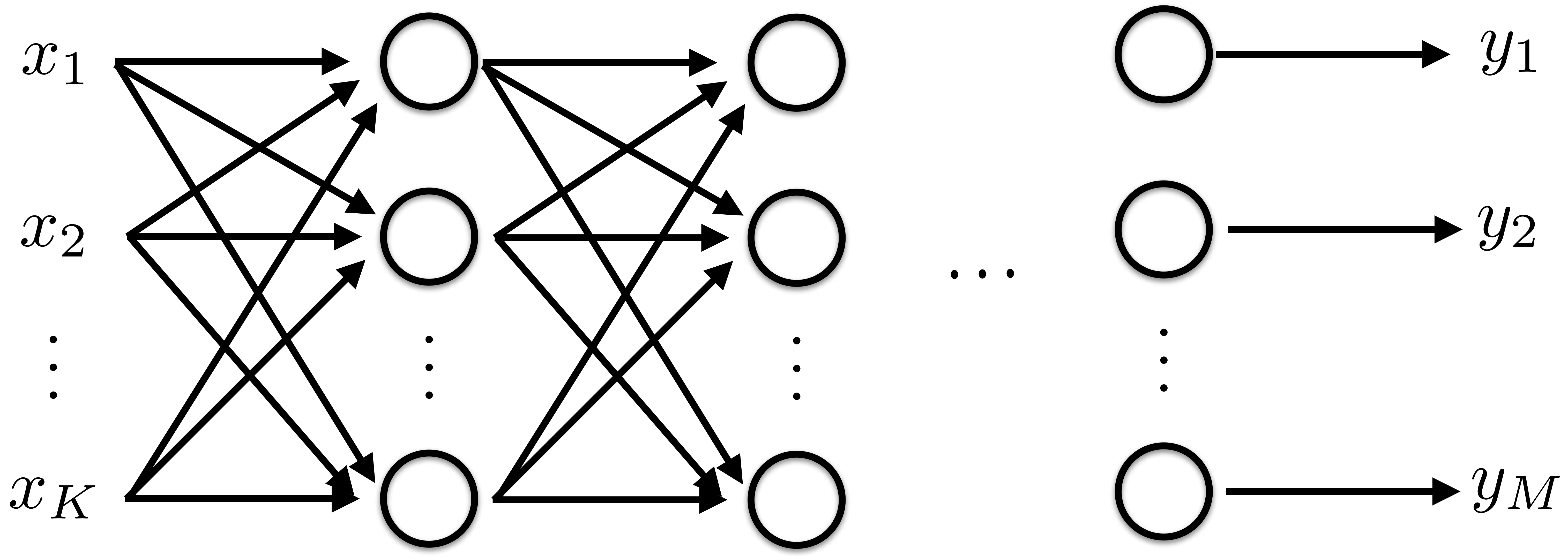
# Deep neural network

A neural network with many layers

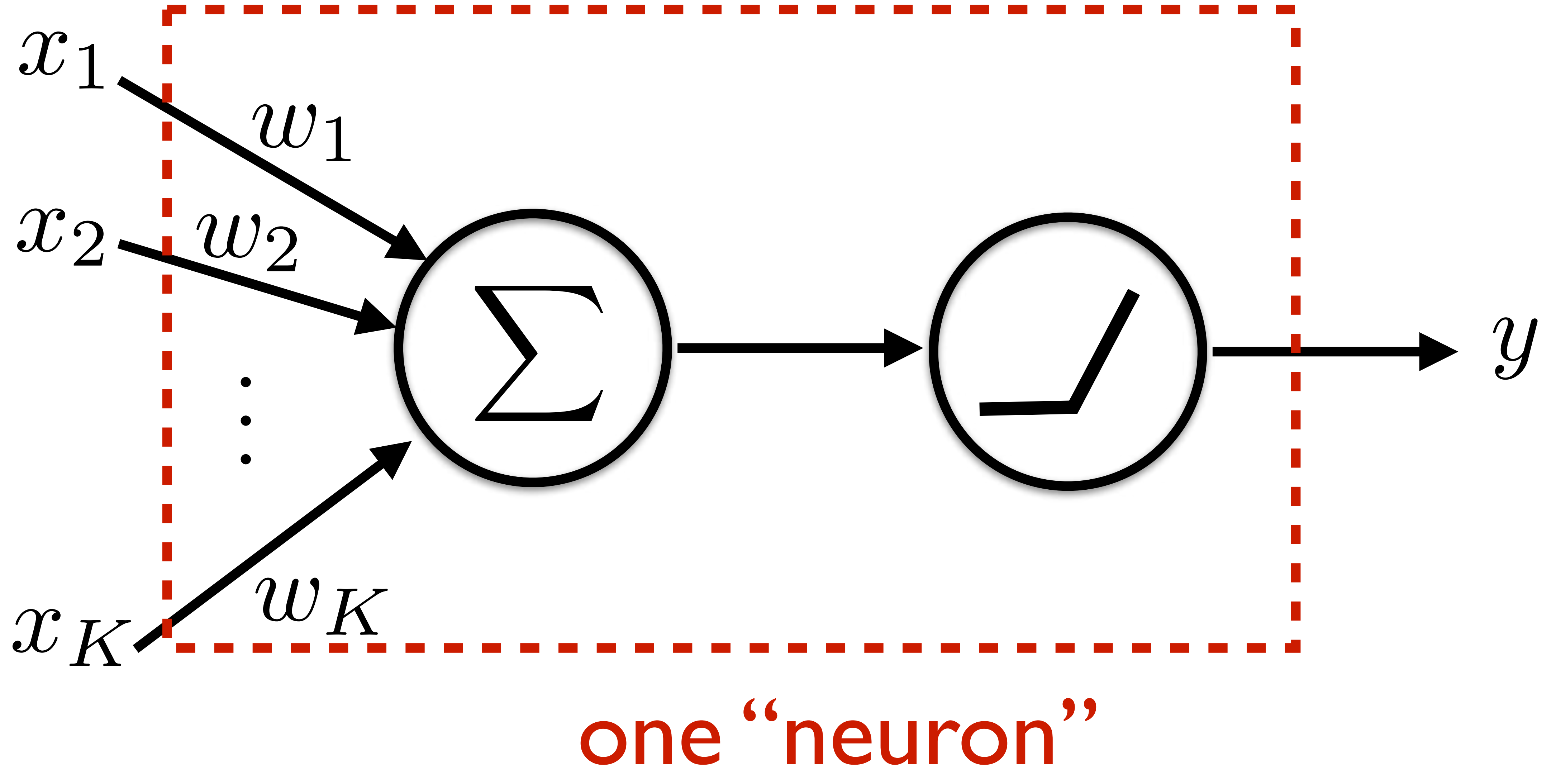


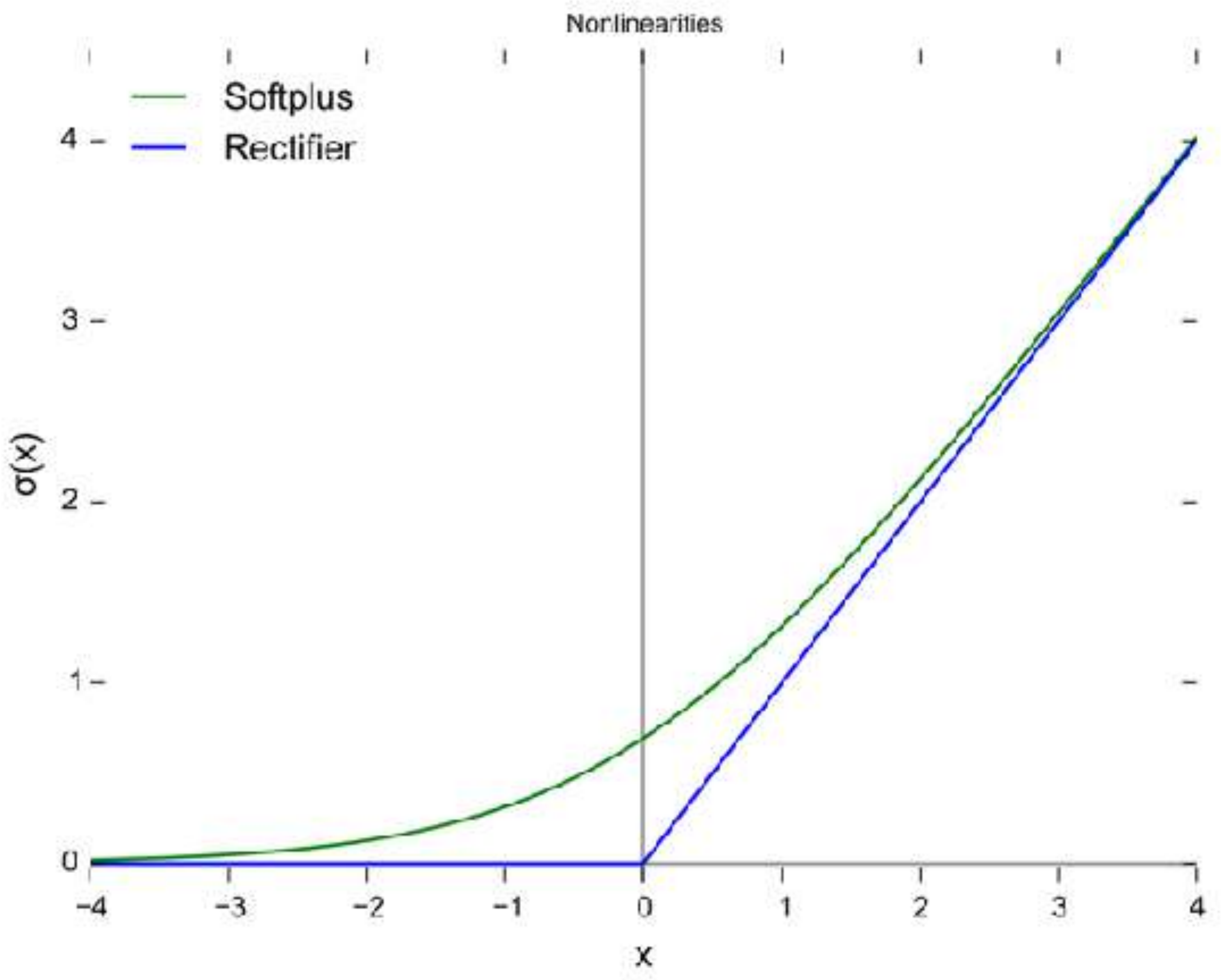
Example of a fully connected deep neural network

## Multi-dimensional output



# Different activation functions





we can replace the sigmoid by other nonlinear functions, popular ones are

- Rectified Linear Units (ReLU) $y = \max\{0, \sum_k w_k x_k\}$
- Softplus $y = \log(1 + e^{\sum_k w_k x_k})$

# Different architectures

We can decide:

- How many layers
- How many neurons per layer
- Which activation functions are used
- How layers connect to each other
- etc

## Stochastic gradient descent

Usually we cannot do gradient descent using all the data point available in our dataset!

Stochastic gradient descent: randomly select a small number of data points (a mini-batch) and do gradient descent using these points

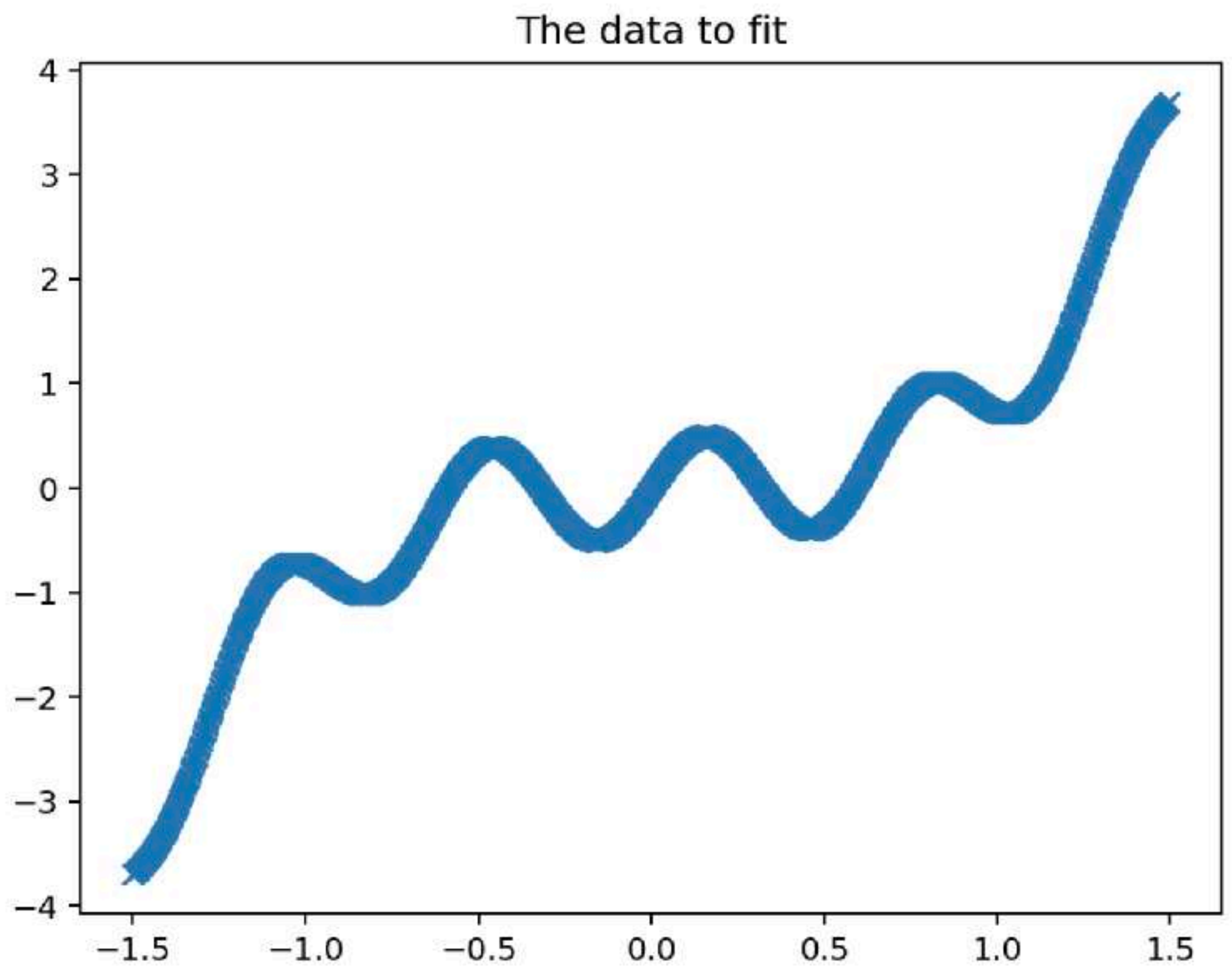
## Libraries to “train” neural networks

PyTorch

# Example

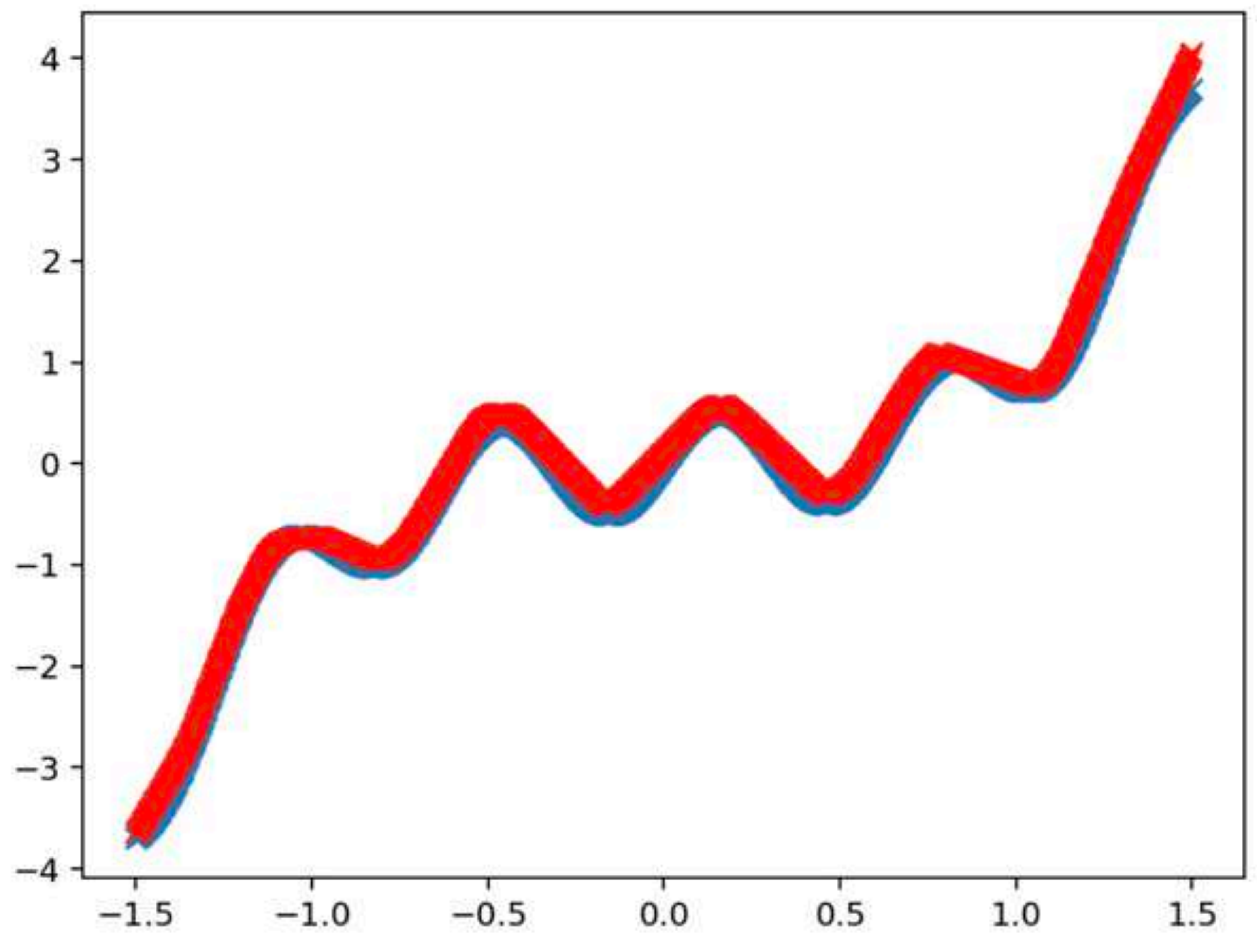
In [1]:
```
%matplotlib notebook

import torch

import numpy as np
import matplotlib as mp
import matplotlib.pylab as plt
```

In [2]:
```
## we create the data  to learn from
N = 1000 # number of data points
x = torch.linspace(-1.5,1.5,steps=N).reshape(N, 1)
y = x**3 + 0.5*torch.sin(10*x)

plt.figure()
plt.plot(x.numpy(),y.numpy(), 'x')
plt.title('The data to fit')
```



```
In [6]: # # we create another model with 3 hidden layers
        model = torch.nn.Sequential(
            torch.nn.Linear(D_in, H),
            torch.nn.ReLU(),
            torch.nn.Linear(H, H),
            torch.nn.ReLU(),
            torch.nn.Linear(H, H),
            torch.nn.ReLU(),
            torch.nn.Linear(H, D_out),
        )

        # we define the learning rate and select an optimizer
        learning_rate = 1e-3
        optimizer = torch.optim.SGD(model.parameters(), lr=learning_rate)

        # we learn doing 5000 iterations
        for t in range(10000):
            # sample a mini batch
            sample_index = torch.tensor(np.random.choice(N, batch_size))

            # Forward pass: compute predicted y by passing x to the model.
            y_pred = model(x[sample_index])

            # Compute the least-square loss.
            loss = loss_fn(y_pred, y[sample_index])

            # use the optimizer object to zero all of the gradients for the variables it will update,
            # i.e. the weights of the model. Checkout docs of torch.autograd.backward for more details.
            optimizer.zero_grad()

            # compute gradient of the loss with respect to model parameters (backward autodiff)
            loss.backward()

            # call the step function of the optimizer to make one update of the parameters
            optimizer.step()

        y_pred = model(x)
        plt.figure()
        plt.plot(x.numpy(),y.numpy(), 'x')
        plt.plot(x.numpy(),y_pred.detach().numpy(), 'rx')
```

```
# D_in is input dimension;
# H is dimension of the hidden layers; D_out is output dimension.
batch_size = 32
D_in, H, D_out = 1, 64, 1
```



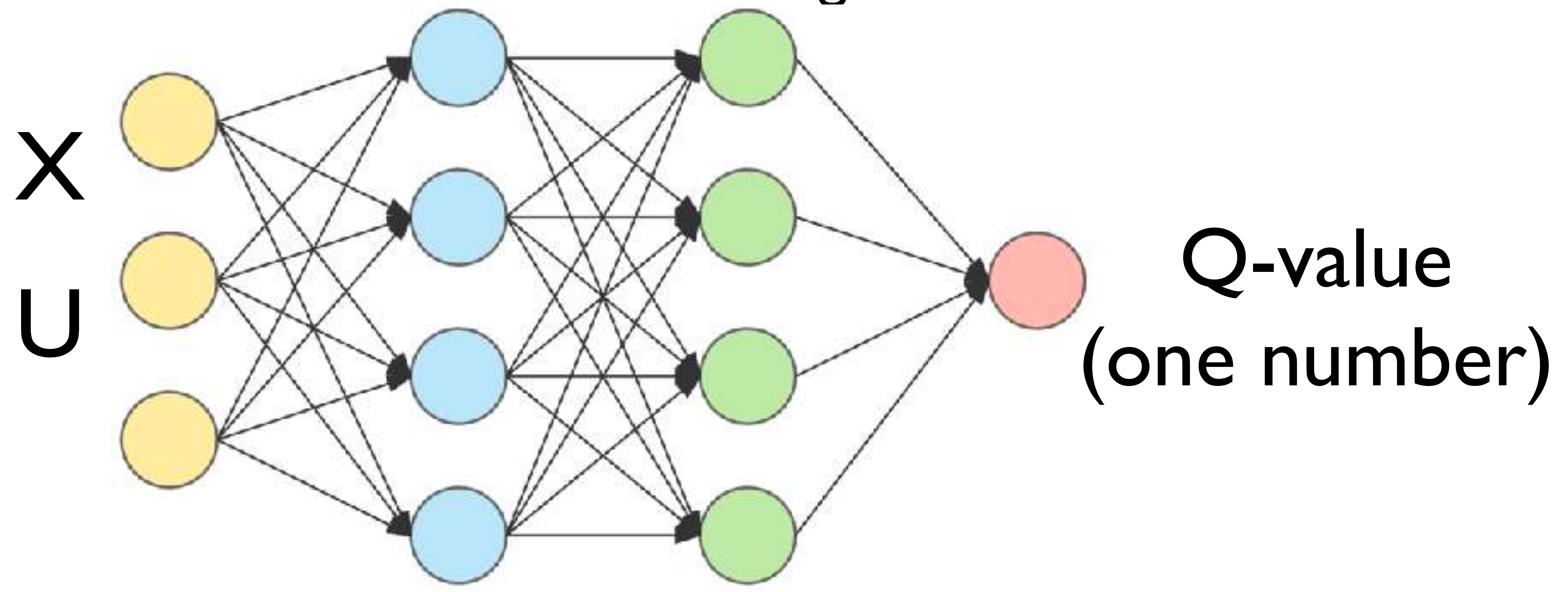
# Back to Q-learning

Q-learning with a table cannot work for high-dimensional spaces nor for continuous state/action spaces!

Idea: replace the table with a function approximator (e.g. a neural network) - still assume discrete number of actions

$$Q(x,u) \simeq Q(x,u,w)$$

Neural network weights



## Back to Q-learning

The problem can be written as a least square problem

We can compute the right side of Bellman equation
from data collected during one episode

$$y_t = g(x_t, u_t) + \alpha \min_a Q(x_{t+1}, a, w)$$

and then do one step of gradient descent on the weights
of the neural network to minimize the TD error

$$\min_w ||y_t - Q(x_t, u_t, w)||^2$$

## Q-learning with a neural network

```
Initialize $Q(x, u, w)$ with random weights $w$

For each episode:
    Choose an initial state $x_0$
    Loop for each step of the episode:
        Choose an action $u_t$ using an $\epsilon$-greedy policy from Q
        Observe the next state $x_{t+1}$
        Compute $y_t = g(x_t, u_t) + \alpha \min_a Q(x_{t+1}, a, w)$
        Update the weights of the neural network by doing one
        iteration of stochastic gradient descent
            $\min_w ||y_t - Q(x_t, u_t, w)||^2$
```

# Back to Q-learning

Problem: a direct (naive) approach using solely current episode data tend to be unstable (i.e. it diverges):

- The sequence of observations are correlated
- Small changes in Q can lead to large changes in policy

# Back to Q-learning

[Mnih et al., Nature, 2015]

## Deep Q-network (DQN)

[Mnih et al., Nature, 2015]

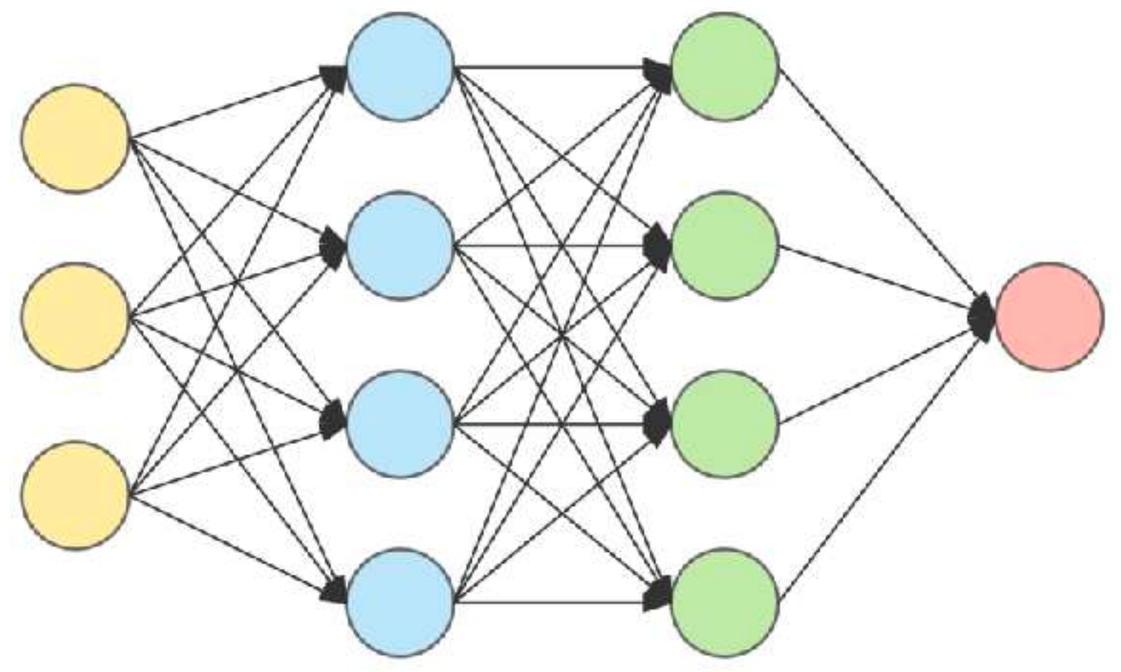
Problem: a direct (naive) approach using solely current episode data tend to be unstable (i.e. it diverges):

- The sequence of observations are correlated
- Small changes in Q can lead to large changes in policy

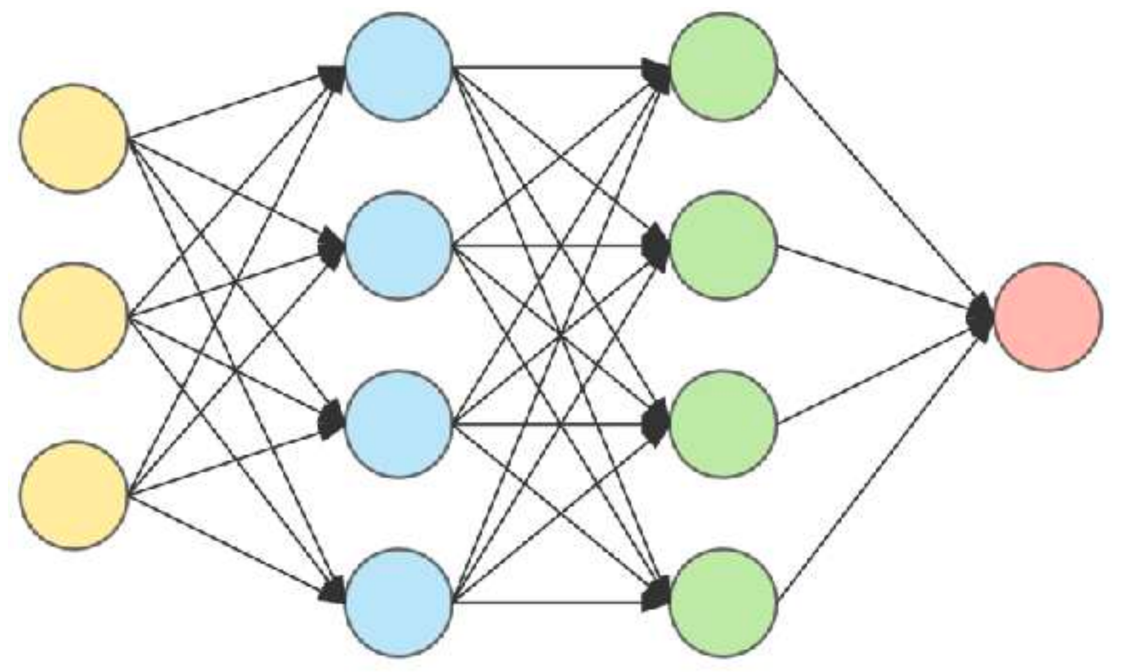
Solution 1)
Use a “replay” memory of a previous samples from which we randomly sample the next training batch (remove correlations)

Solution 2)
Use 2 Q-networks to avoid correlations due to updates

Q-network



Target $\hat{Q}$-network



# Deep Q-network (DQN)

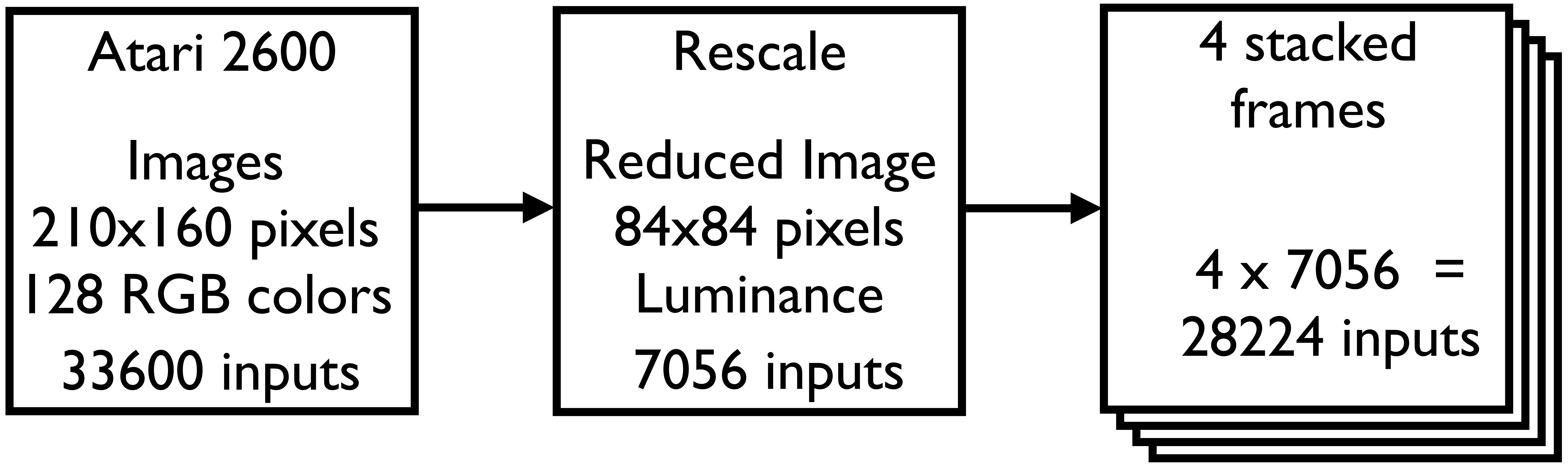
[Mnih et al., Nature, 2015]

```
Initialize replay memory D of size $N$
Initialize Q-network with random weights $\theta$
Initialize target $\hat{Q}$ function with weights $\theta^- = \theta$

For each episode:
    Start from an initial state $x_0$
    Loop for each step $t$ of the episode:
        Choose a control action $u_t$ using $Q$ (e.g. $\epsilon$-greedy policy)
        Do $u_t$ and observe the next state $x_{t+1}$
        Compute $y_t = g(x_t, u_t) + \alpha \min_a \hat{Q}(x_{t+1}, a, \theta^-)$ ← ! here we use the target network
        Store $(x_t, u_t, y_t, x_{t+1})$ in memory $D$
        Sample minibatch $K$ of transitions $(x_k, u_k, y_k, x_{k+1})$ from $D$
        Gradient descent on $\theta$ to minimize $\sum_K ||Q(x_k, u_k, \theta) - y_k||^2$
        Every $C$ steps reset the target network by setting $\theta^- = \theta$
```

## Deep Q-network (DQN)

[Mnih et al., Nature, 2015]

Pre-processing states of the system



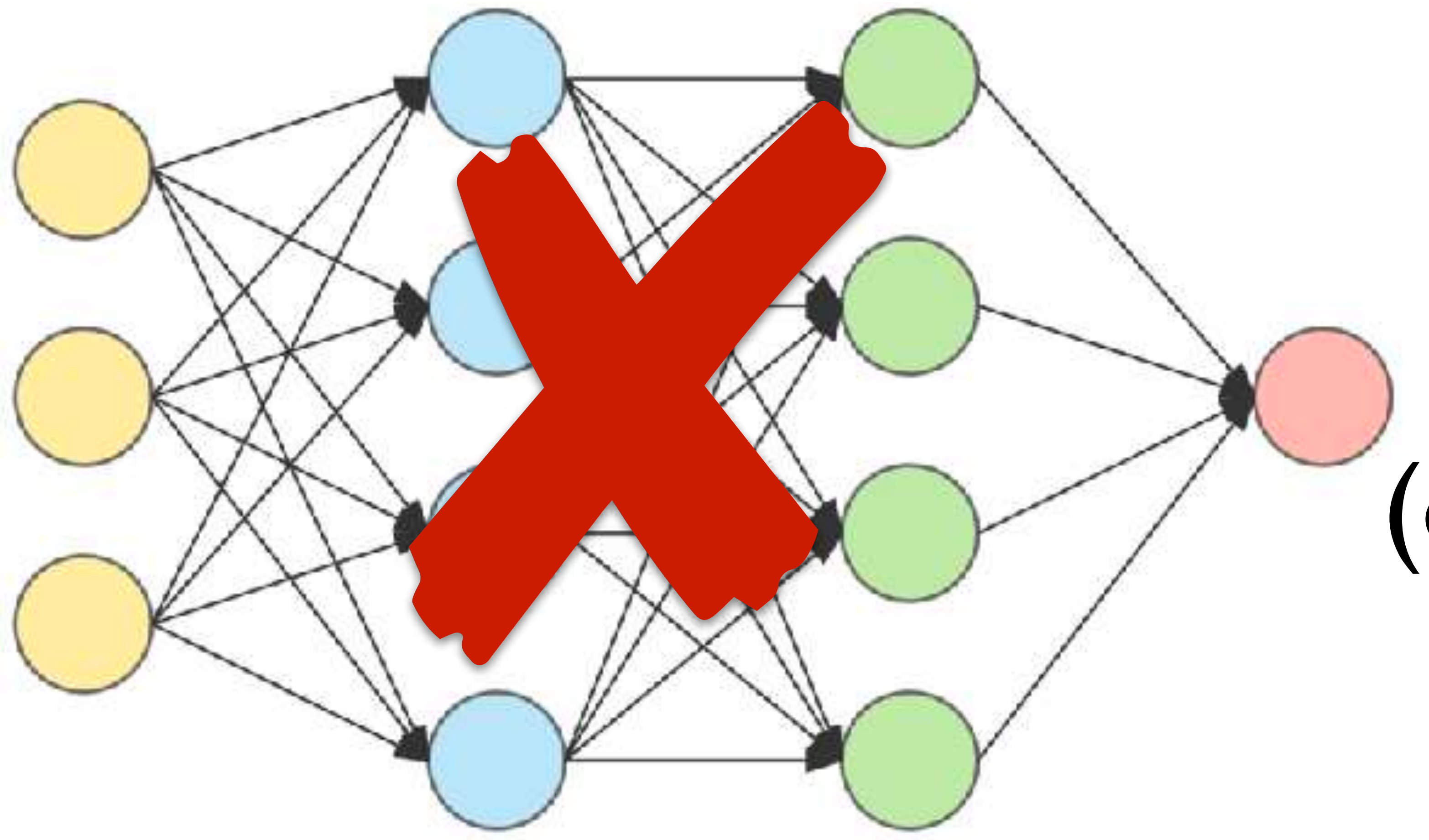
# Deep Q-network (DQN)

[Mnih et al., Nature, 2015]

Network Architecture

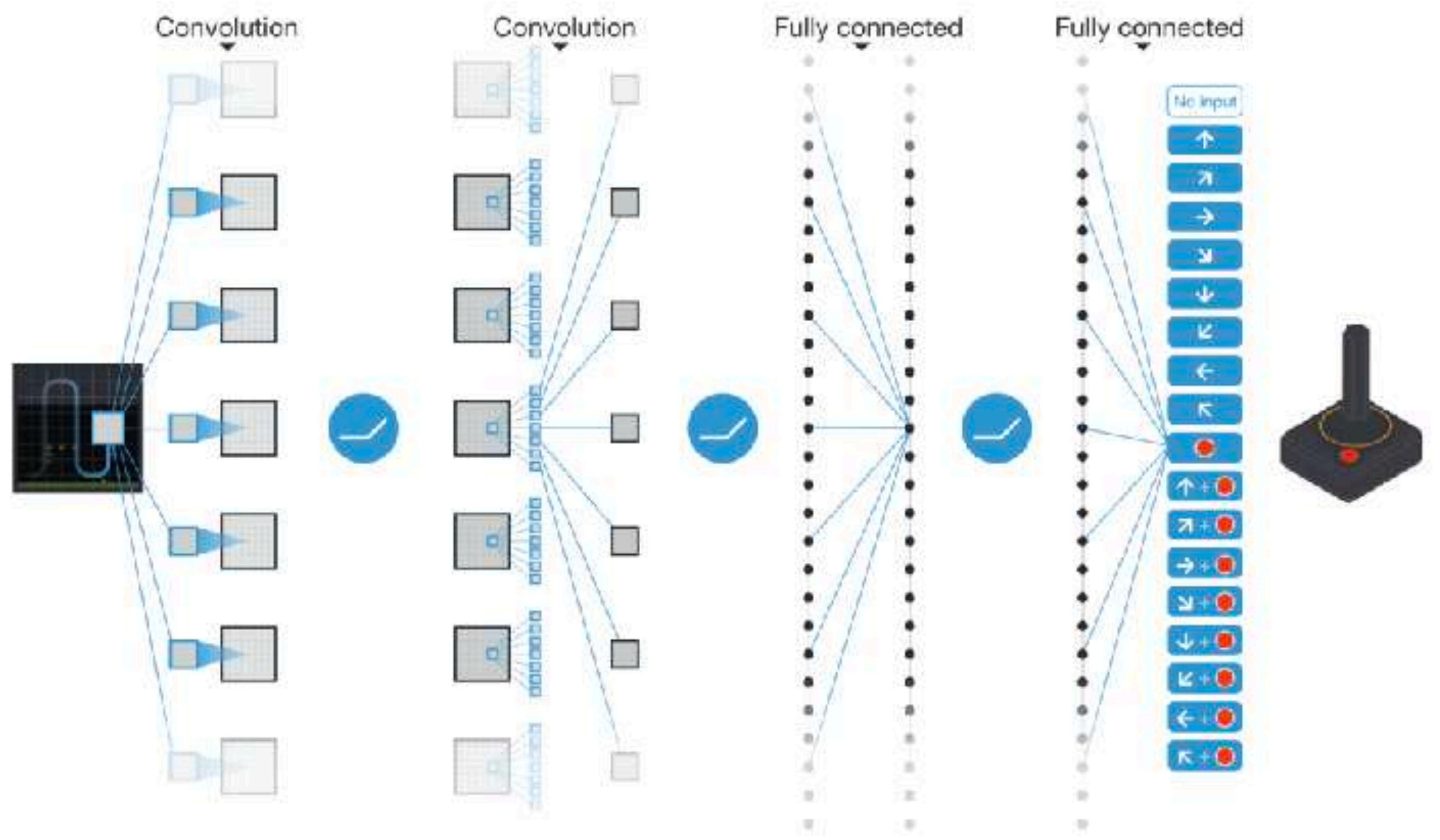
X

U



Q-value
(one number)

X



One Q-value per action
(multiple outputs)

# Deep Q-network (DQN)

[Mnih et al., Nature, 2015]

## Training

49 games:

- a different Q network is used for each game
- same parameters for learning each game

mini-batches of size 32

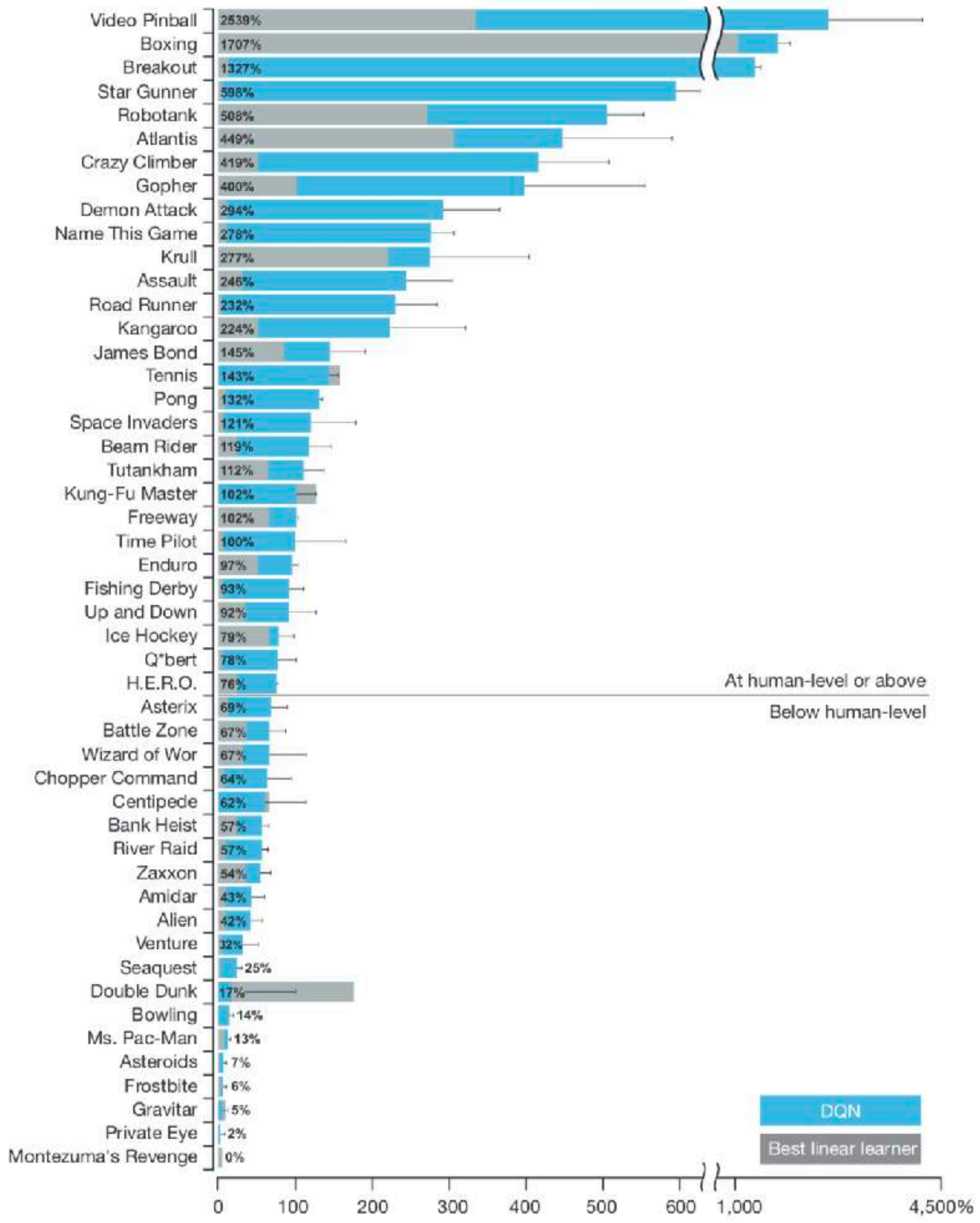
$\epsilon$-greedy with $\epsilon = 1$ at the beginning of learning and linearly decreases until $\epsilon = 0.1$ after first 1 million frames

trained on 50 million frames
=> 32 days of game experience in total!
(the human player was allowed only 2h of training)

replay memory size: 1 million samples (FIFO)

# Deep Q-network (DQN)

[Mnih et al., Nature, 2015]

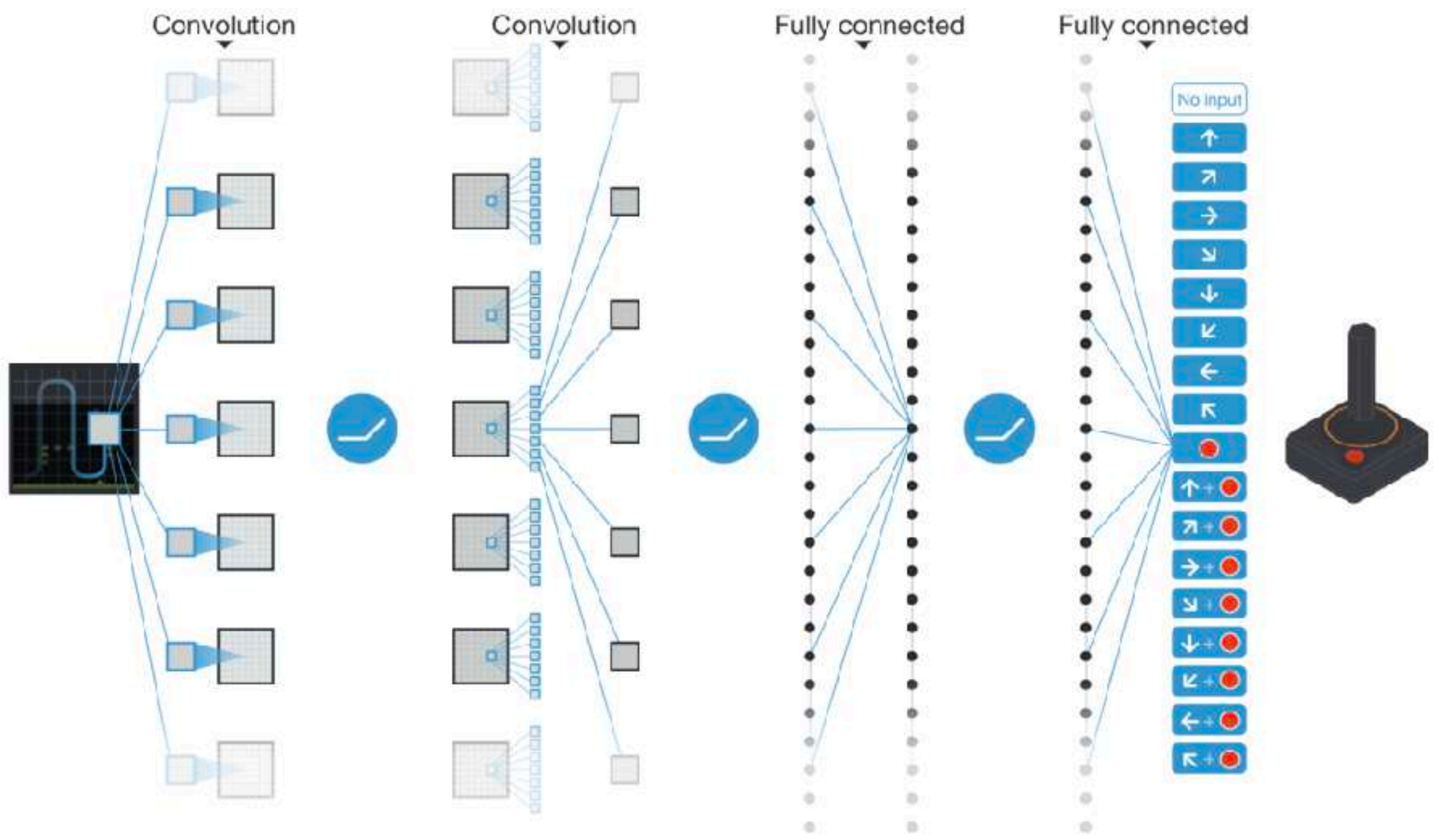


# Deep Q-network (DQN)

[Mnih et al., Nature, 2015]

Now we can do Q-learning using continuous states and high dimensional inputs!

What about a continuous action space?



# What about continuous action space?

Problem: we need to evaluate the min to be able to do Q-learning with a function approximator

$$||Q(x_t, u_t, \theta) - g(x_t, u_t) - \alpha \min_a \hat{Q}(x_{t+1}, a, \theta^-)||2$$

Solution: use another neural network to approximate the min operator (i.e. to approximate the optimal policy)