

# Supervised Learning: Classification

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All material in these lecture notes is based on our textbook:

Aurélien Géron, *Hands-On Machine Learning with Scikit-Learn and TensorFlow*, 3rd ed., O'Reilly, 2022

# Outline

- Classification Types
  - Binary Classification
  - Multiclass Classification
  - Multilabel Classification
  - Multioutput Classification (*outside the scope of this module*)
- Performance Measures
  - Accuracy
  - Confusion Matrices
  - Precision and Recall
  - $F_1$  Score

# Example Case Study

- Handwritten Digit classification
- The MNIST dataset - The “Hello World” of ML
- 70,000 images
- Each image has 28 x 28 pixels = 784 features
- Each feature is a value between 0 (white) and 255 (black)
- Visit our textbook’s Jupyter notebooks collection here: [homl.info/colab3](https://homl.info/colab3) and select *03\_classification*

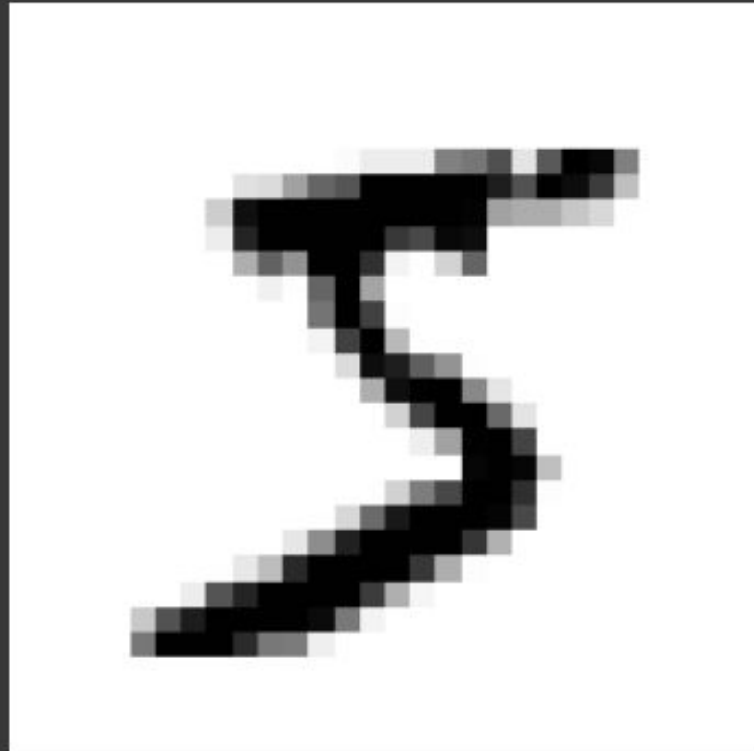
# The MNIST dataset

## Plotting one digit

```
import matplotlib.pyplot as plt

def plot_digit(image_data):
    image = image_data.reshape(28, 28)
    plt.imshow(image, cmap="binary")
    plt.axis("off")

some_digit = x[0]
plot_digit(some_digit)
save_fig("some_digit_plot") # extra code
plt.show()
```



# The MNIST dataset

Plotting an indicative number of digits

```
# extra code - this cell generates and saves Figure 3-2
plt.figure(figsize=(9, 9))
for idx, image_data in enumerate(X[:100]):
    plt.subplot(10, 10, idx + 1)
    plot_digit(image_data)
plt.subplots_adjust(wspace=0, hspace=0)
save_fig("more_digits_plot", tight_layout=False)
plt.show()
```



# Binary Classification

- Binary classification is about identifying instances that belong to **one target class  $\Rightarrow$  positive class**
- In other words, the aim is to **distinguish between two classes**
  - An instance either belongs to the target class or it doesn't  **$\Rightarrow$  negative class**
- In relation to MNIST: identify one digit
  - i.e. 5 and non-5

# Performance Measure: Accuracy

- **Accuracy** measures the **ratio** (percentage) **of correct predictions**
- Evaluating the performance of such a classifier can be more challenging than evaluating a regressor!
  - If 10% of images are 5s and an algorithm classifies everything as a non-5, it will be correct 90% of the times!
- Accuracy is **not** the best performance measure for classifiers
  - Especially when dealing with **skewed datasets**, i.e. when some classes are more frequent than others, i.e. when your instances are far from equally distributed among the classes

# Performance Measure: Confusion Matrix

- A Confusion Matrix counts how many times instances of a Class A are classified as Class B for all A-B pairs of classes
  - Rows represent the actual classes
  - Columns represent the predicted classes

```
array([[53892, 687],  
       [ 1891, 3530]])
```

- Assuming a perfect performance, only the main diagonal has non-zero values

```
array([[54579, 0],  
       [ 0, 5421]])
```



# Confusion Matrix

For the digit 5 of the  
MNIST dataset

```
from sklearn.metrics import confusion_matrix
```

```
cm = confusion_matrix(y_train_5, y_train_pred)  
cm
```

```
array([[53892,   687],  
       [ 1891,  3530]])
```

```
y_train_perfect_predictions = y_train_5 # pretend we reached perfection  
confusion_matrix(y_train_5, y_train_perfect_predictions)
```

```
array([[54579,    0],  
       [    0,  5421]])
```

# Performance Measure: Confusion Matrix

**True Negatives:**  
non-5s that were  
**correctly** classified  
as non-5s

		Predicted	
		non-5s	5s
A c t u a l	non-5s	53892	687
	5s	1891	3530

**False Positives:**  
non-5s that were  
**incorrectly** classified  
as 5s

**True Positives:**  
5s that were  
**correctly** classified  
as 5s

**False Negatives:**  
5s that were  
**incorrectly** classified  
as non-5s

# Performance Measures: Confusion Matrix, Recall, Precision

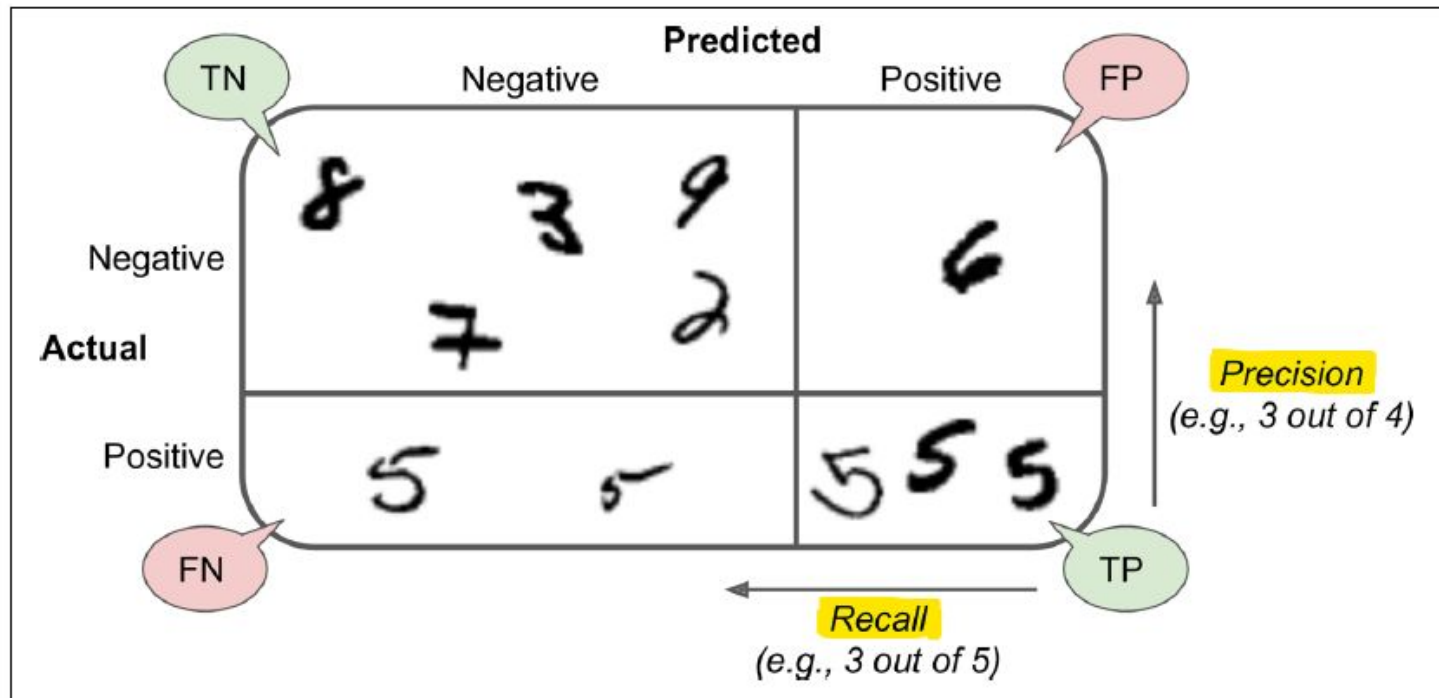


Figure 3-2. An illustrated confusion matrix shows examples of true negatives (top left), false positives (top right), false negatives (lower left), and true positives (lower right)

# Terminology

## **Accuracy**

The ratio of correct predictions

## **Confusion Matrix**

It counts how many times instances of a Class A are classified as Class B for all A-B pairs of classes  
i.e. all the true positive/negatives,  
and all the false positives/negatives

## **Precision**

The accuracy of the positive predictions

$$\text{precision} = \frac{TP}{TP + FP}$$

## **Recall** or **Sensitivity** or **TPR (True Positive Rate)**

The ratio of positive instances that are correctly detected

$$\text{recall} = \frac{TP}{TP + FN}$$

# Terminology

## **F<sub>1</sub> score**

The **harmonic mean** of **precision** and **recall**

- for a high F<sub>1</sub> score, both precision as well as recall have to be high
- It favours models that have **similar** precision and recall

$$F_1 = \frac{2}{\frac{1}{\text{precision}} + \frac{1}{\text{recall}}} = 2 \times \frac{\text{precision} \times \text{recall}}{\text{precision} + \text{recall}} = \frac{TP}{TP + \frac{FN + FP}{2}}$$

# Performance Measures: The Precision/Recall trade-off

- **Increasing precision reduces recall and vice-versa**
  - If someone says, "Let's reach 99% precision," you should ask, "At what recall?"
- Classifiers typically calculate a score for each instance using a **decision function**
- **If the score is higher than a threshold, the instance is classified in the positive class** (otherwise to the negative)
- The trade-off depends on the value of the **decision threshold**

# Performance Measures: The Precision/Recall trade-off

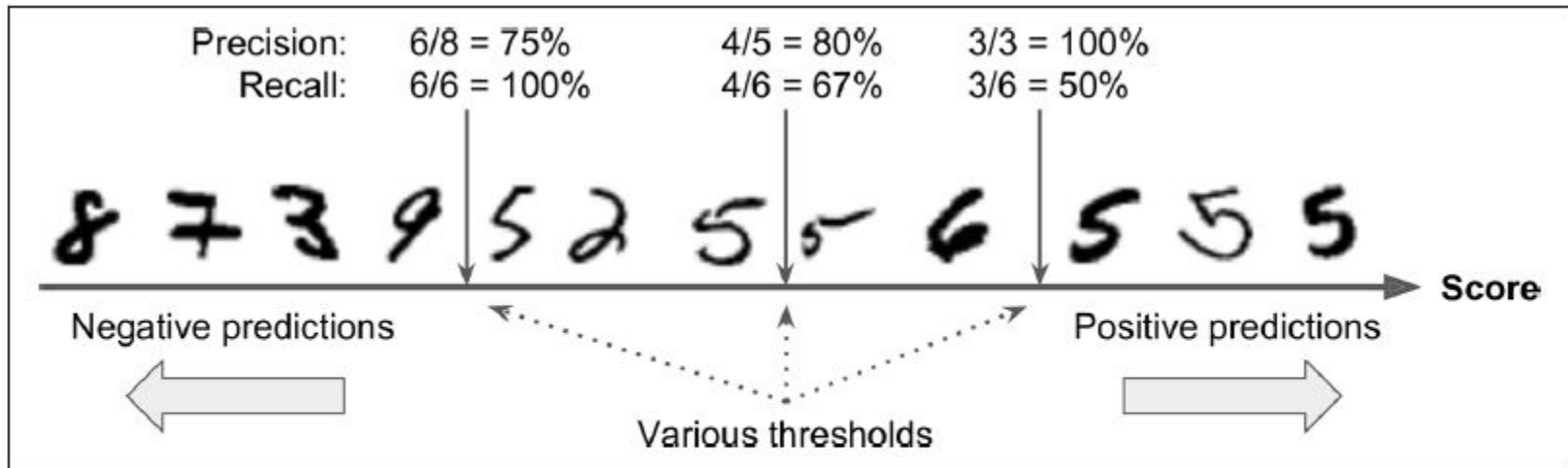
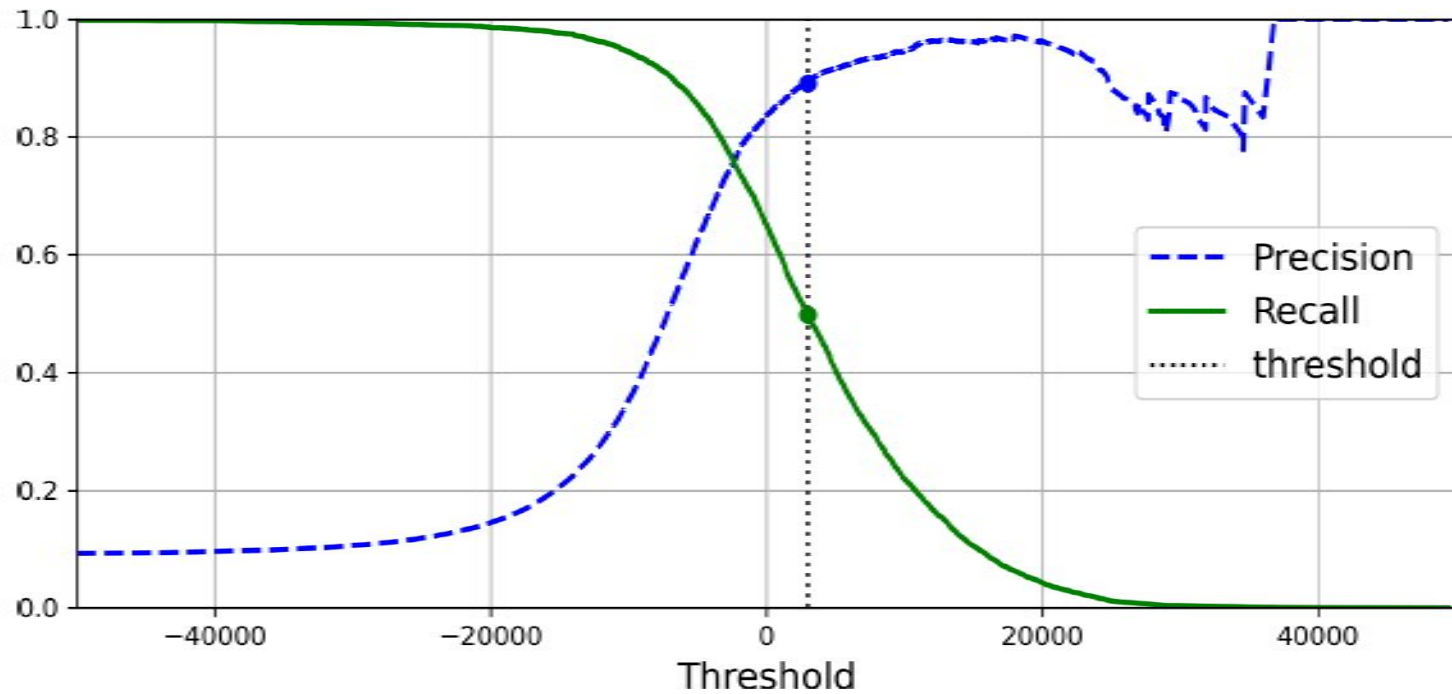


Figure 3-3. In this precision/recall trade-off, images are ranked by their classifier score, and those above the chosen decision threshold are considered positive; the higher the threshold, the lower the recall, but (in general) the higher the precision

# Performance Measures: The Precision/Recall trade-off





# Precision Recall $F_1$ Score

```
from sklearn.metrics import precision_score, recall_score
```

```
precision_score(y_train_5, y_train_pred) # == 3530 / (687 + 3530)
```

```
0.8370879772350012
```

```
# extra code - this cell also computes the precision: TP / (FP + TP)  
cm[1, 1] / (cm[0, 1] + cm[1, 1])
```

```
0.8370879772350012
```

```
recall_score(y_train_5, y_train_pred) # == 3530 / (1891 + 3530)
```

```
0.6511713705958311
```

```
# extra code - this cell also computes the recall: TP / (FN + TP)  
cm[1, 1] / (cm[1, 0] + cm[1, 1])
```

```
0.6511713705958311
```

```
from sklearn.metrics import f1_score
```

```
f1_score(y_train_5, y_train_pred)
```

```
0.7325171197343846
```

```
# extra code - this cell also computes the f1 score  
cm[1, 1] / (cm[1, 1] + (cm[1, 0] + cm[0, 1]) / 2)
```

```
0.7325171197343847
```

# Multiclass Classification

- **Multiclass** (or **multinomial**) classifiers distinguish between more than two classes
- Some classifiers are natively multiclass
  - Logistic Regression, RandomForest
- while others are strictly binary
  - SGDClassifier and SVC (C-Support Vector)
- BUT there are strategies for **using multiple binary classifiers to perform multiclass classification!**

# Multiclass Classification using Binary Classifiers

## OvA (or OvR) Strategy

- Train as many Binary Classifiers as your classes
  - For the Handwritten Digits problem you need **10**, one for each digit: a 0-detector, a 1-detector, ..., a 9-detector
- To classify one instance:
  - Get the decision score from each classifier
  - Select the class with the highest score

⇒ **one-versus-all (OvA)**

⇒ or **one-versus-the-rest (OvR)**

# Multiclass Classification using Binary Classifiers

## OvO Strategy

For  **$N$**  classes you need  
 **$N \times (N - 1) / 2$**   
classifiers

- Train a Binary Classifier for every pair of classes
  - For the Handwritten Digits problem you need **45**,
    - one to distinguish between 0s and 1s
    - one to distinguish between 0s and 2s, ...,
    - one to distinguish between 8s and 9s
- To classify one instance:
  - Select the class that wins more duels!  
 **$\Rightarrow$  one-versus-one (OvO)**
- **Advantage:** each classifier is trained only on part of the Training Set

# Multiclass Classification using Binary Classifiers

## OvA versus OvO

- OvA is generally preferred
  - OvO is preferred in cases when an algorithm scales poorly with the size of the Training Set
    - e.g. Support Vector Machine classifiers
- ⇒ Easier to train **many classifiers on small sets** rather than one/few classifiers on large sets

# Multilabel Classification

- Multilabel classification is about identifying **multiple classes for each instance**
- The output of the classifier is an array of boolean tags
  - each position represents a particular class;
  - the value (true or false) whether the instance belongs to this class

# Multilabel Classification

The MNIST dataset

Learning to identify whether:

- A number is greater or equal to 7
- A number is odd

`some_digit` is a 5:

- Less than 7
- Odd

```
import numpy as np
from sklearn.neighbors import KNeighborsClassifier

y_train_large = (y_train >= '7')
y_train_odd = (y_train.astype('int8') % 2 == 1)
y_multilabel = np.c_[y_train_large, y_train_odd]

knn_clf = KNeighborsClassifier()
knn_clf.fit(X_train, y_multilabel)
```

K-nearest  
Neighbors  
supports  
multilabel  
classification

```
knn_clf.predict([some_digit])

array([[False,  True]])
```

# Multilabel Classification Performance Measure

## $F_1$ Score

- Compute  $F_1$  score per class and average the scores
- Even better: Weigh each score in the average depending on how many instances belonging to each class exist in the set



# Multilabel Classification

## Chain Classification

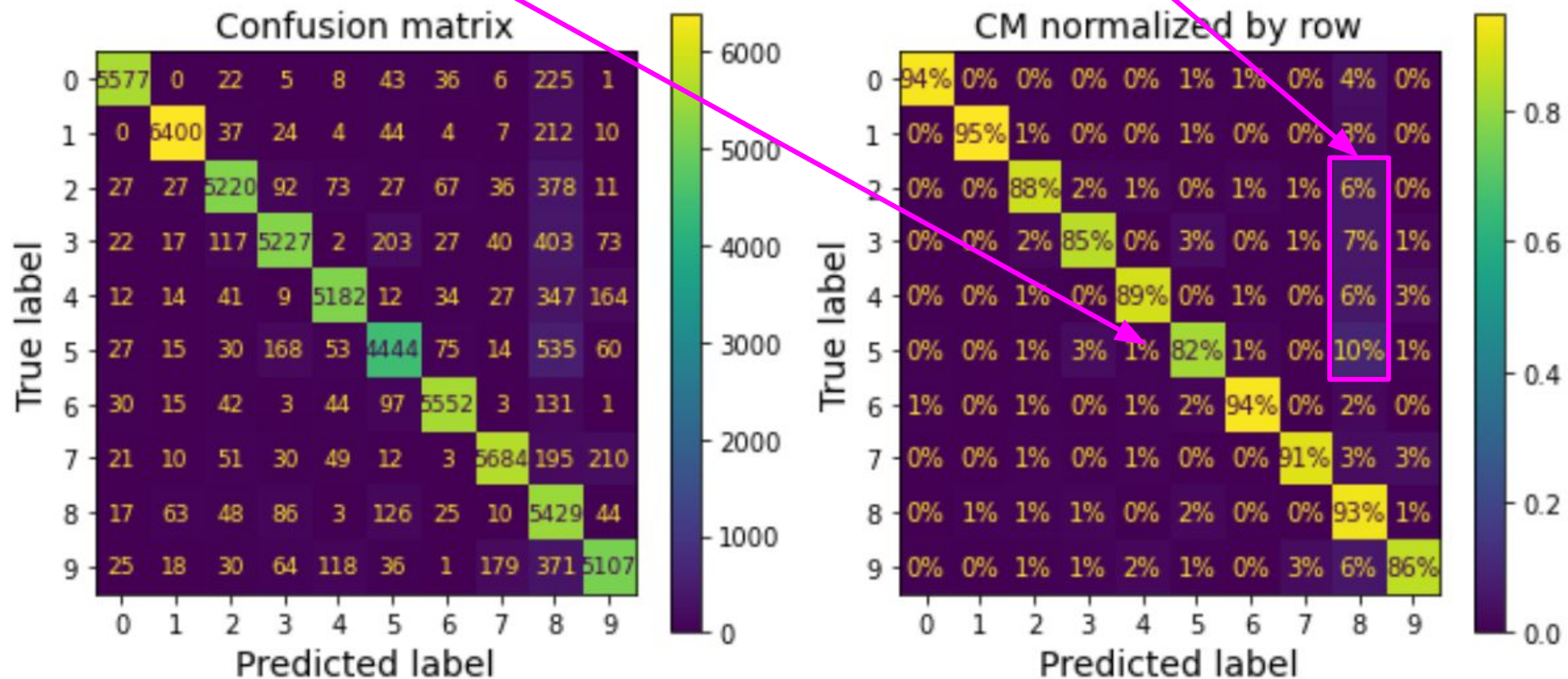
- You may wish to use a classifier that does not support multilabel classification:
- Organise the models in a **chain**:
- Each model in the chain uses
  - The input features of the instance, and
  - The predictions of all the models that come before it in the chain

# Error Analysis using Confusion Matrices

# Normalising the Confusion Matrix by Row (percentages instead of absolute instance numbers)

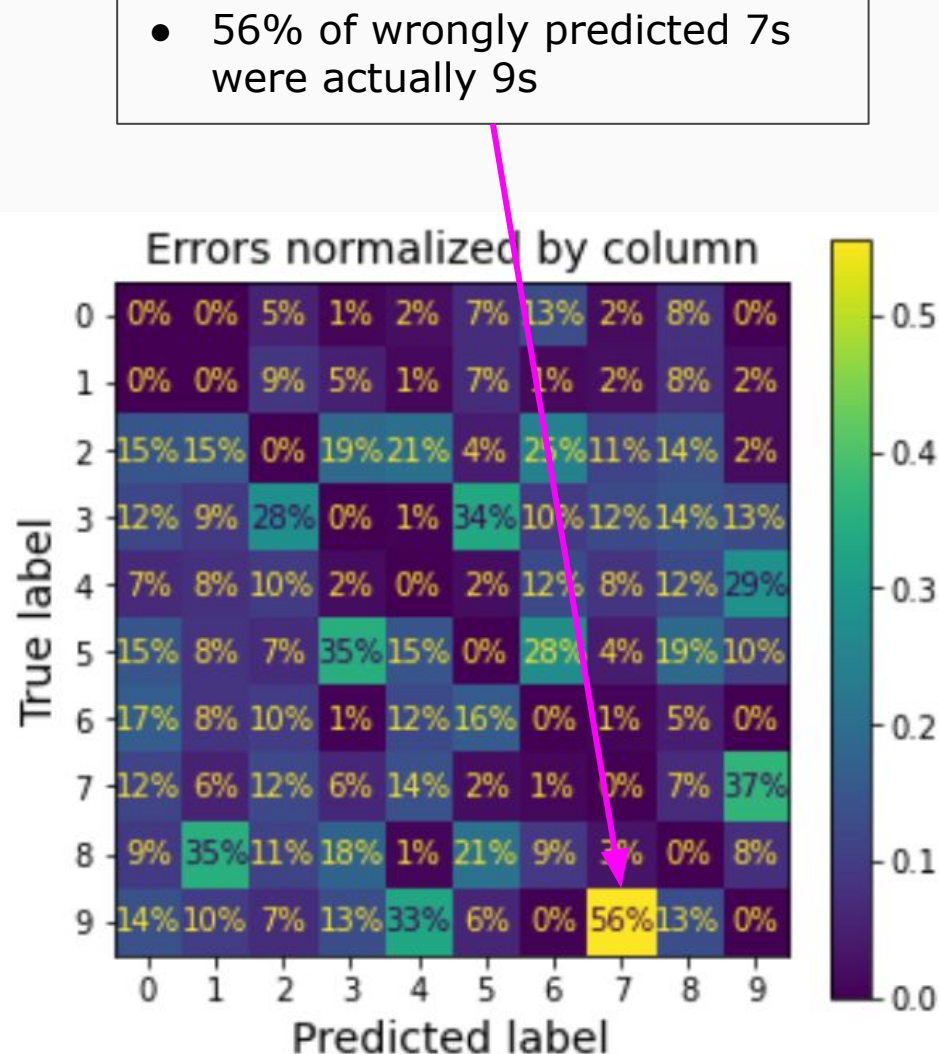
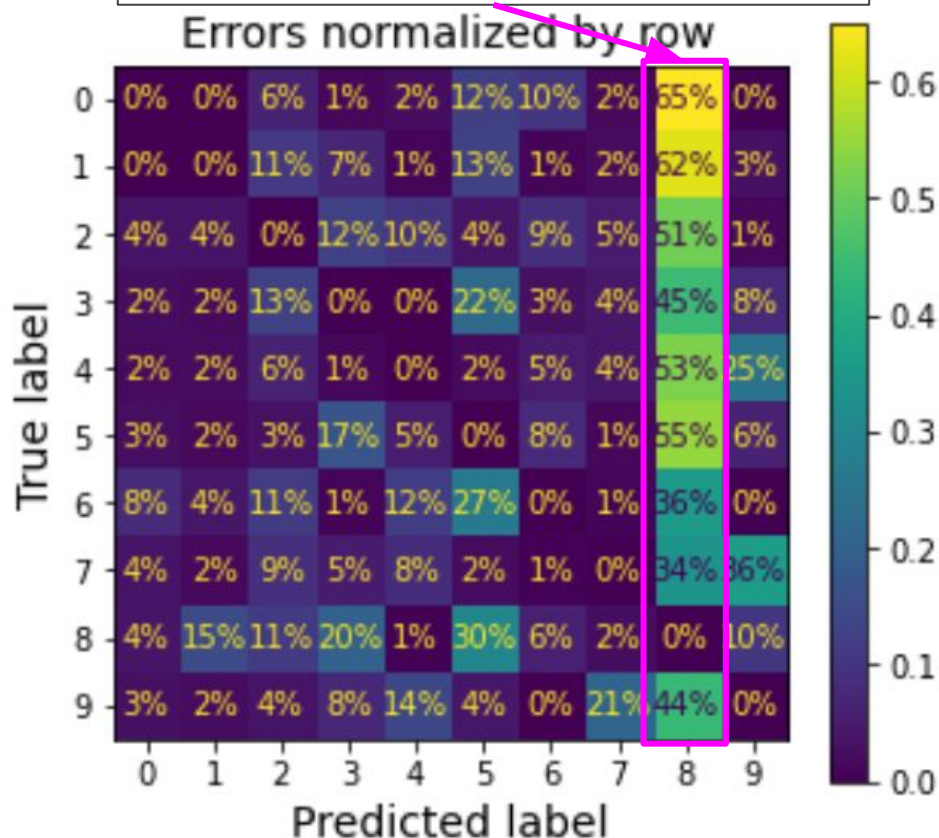
5s are the ones that are most often misclassified

A lot of numbers are misclassified as 8s



# Normalising the **Error** by Row and Column (correct predictions are ignored)

- A lot of digits are misclassified as an 8
- 65% of **misclassified** 0s were classified as 8s, etc.



# Using Error Analysis Results

- After you identify the types of errors your model performs, you can:
  - Gather more training data for the particular classes
  - Engineer new features that could help the classifier
  - Preprocess your data

⇒ **Data Augmentation**

# Thank you!

*Coming up next:*

Supervised Learning:  
Classification Models