Study Guide 2

Feb 2 - Lecture 7 } Hw 3
4 - Lecture 8 }
9 - Lecture 9 } Hw 4.
11 Lecture 10 }

Textbooks
Ash 4.3, 3.4

G+S Chapter 2
Chapter 5
Chapter 6
Chapter 8
Chapter 9

Dists: Geometric
Poisson
Uniform Continuous

density $= f(x)$ $\quad f(x) = \frac{\partial F}{\partial x}$, $F(x) = \int_{-\infty}^{x} f(t)\,dt$

cdf = cumulative distribution fn $= F(x)$

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correspondences

Poisson $\leftrightarrow$ geometric Memoryless.
Bernoulli $\leftrightarrow$ Gaussian

Normal density, shift and rescale $N_x(\mu = \mu_0, \sigma = \sigma_0)$
$\to N(0, 1)$
$= \frac{1}{\sqrt{2\pi}} \int e^{-x^2/2}$
if x in first dist. above transformed
$x \to \left(\frac{x - \mu}{\sigma}\right)$

~~Rayler~~ Maxwell-Rayleigh Dist
$$\int r^n e^{-r^2} dr$$

Chi-squared, $\sum \frac{(O_i - E_i)^2}{E_i}$

Expectation rules $\langle x \cdot y \rangle = \langle x \rangle \cdot \langle y \rangle$ x, y, indep
$\langle x + y \rangle = \langle x \rangle + \langle y \rangle$, indep. not required

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Def. of Varience

$$V(X) = E\left((X-\mu)\right)^2 = \langle X^2\rangle - \langle X\rangle^2$$

$$D(X) = \sqrt{V(X)}$$

$$V = \sum_x (x-\mu)^2 m(x) \Rightarrow \int_\Omega (X-\mu)^2 f(x)\,dx$$

properties of μ and V.

$$\langle cX\rangle = c\langle X\rangle$$

$$\langle X + c\rangle = \langle X\rangle + c$$

$$V(cX) = c^2 V(X)$$

$$V(X+c) = V(X)$$

$$V(X+Y) = \langle (X+Y)^2\rangle - \left(\langle X\rangle + \langle Y\rangle\right)^2$$

$$= V(X) + V(Y) \quad X, Y \text{ indep}$$

Def. i.i.d. r.v = "Identical Independently Distributed Random Variables"

S_n = sum of n iidrv trials

$$V(S_n) = n\sigma^2 \qquad \sigma^2 = \text{Var. of 1 trial}$$

$$\sigma\,\cancel{D}(S_n) = \sigma\sqrt{n} = \sqrt{V} = D.$$

Continuum version $\mu = \langle X\rangle = \int x\, f(x)\,dx$

$$\langle XY\rangle = \iint xy\, f_x(x) f_y(y)\,dx\,dy = \langle X\rangle\langle Y\rangle \text{ if indep.}$$

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Conditional Expectation

$$E(X|F) = \sum_j x_j \, P(X=x_j|F)$$

(take away x_j weighting and you have $P(X|F) = \sum_j P(X=x_j|F)$)

TOTAL PROB. $P(X) = \sum_j P(X|F_j)P(F_j)$

expectation: $E(X) = \sum_j E(X|F_j)P(F_j)$

fair game: expected total winnings do not change at each step.

Pascal's Wager as instance of expectation value

Moments: $E(X^n) = \int x^n f(x)\,dx$ or $\sum_{j=1}^{n} x_j^n \, P(x_j)$

first moment: $\int x\, f(x)\,dx$ or $\sum x_j \, P(x_j) = \mu$ (mean)

2nd moment: $\int x^2 f(x)\,dx$ or $\sum x_j^2 \, P(x_j^2)$

Use both in variance: $V(X) = E(X^2) - (E(x))^2$

Covariance like variance, but with two random variables:

$\beta = E((R_1-\mu_1)(R_2-\mu_2)) \rightarrow$ covariance

instead of $E((R_1-\mu_1)^2) =$ Variance

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Covariance, con't

$$\mathrm{Cov}(R_1, R_2) = \langle R_1 R_2 \rangle - \langle \underset{\mu_1}{R_1} \rangle \langle \underset{\mu_2}{R_2} \rangle$$

Variance: $\langle R_1^2 \rangle - \langle \underset{\mu_1}{R_1} \rangle^2$

Test for independence:

2 vars indep. iff $\mathrm{Cov}(R_1, R_2) = 0$.

Completely dependent iff $\mathrm{Cov}(R_1, R_2) = \pm 1$.

Correlation: $\rho(R_1, R_2) = \dfrac{\mathrm{Cov}(R_1, R_2)}{\sigma_1 \sigma_2}$

if $R_1 = R_2$, Cov $\to$ Variance $\quad \rho(R_1, R_1) \to \dfrac{\langle R_1^2 \rangle - \langle R_1 \rangle^2}{\sigma_1^2}$

$= \dfrac{\sigma_1^2}{\sigma_1^2} = 1.$ (Cov. of variable w/ self is 1.)

= complete dependence

Law of large numbers

for any $\epsilon \leq 1$, $P(|X - \mu| \geq \epsilon) \leq \dfrac{V(X)}{\epsilon^2}$

Central Limit theorem

If S_n = sum of iidrv's, $S_n \to$ normal density function

$$f_{\mu,\sigma}(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2}$$

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Standard form of normal dist

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

(prev. formula w/ $\mu = 0, \sigma = 1$)