Optimal strategies in mental rotation tasks

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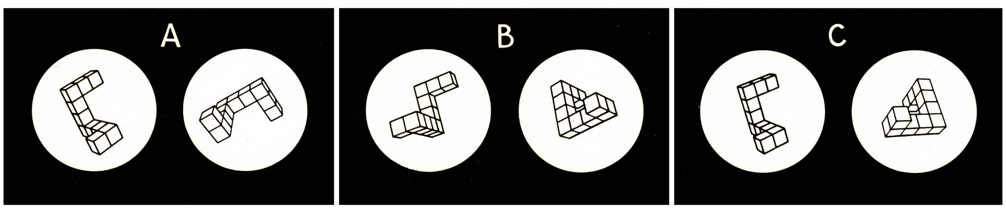

Figure 1: Classic mental rotation task from [1].

1 Background

Consider the objects in Figure 1. In each panel, are the two depicted objects identical (except for a rotation), or distinct? When presented with this mental rotation task, people default to a strategy in which they visualize one object rotating until it is congruent with the other [1]. There is strong evidence for such *mental imagery* or *mental simulation*: we can imagine three-dimensional objects in our minds and manipulate them, to a certain extent, as if they were real [2].

However, the use of mental simulation is predicated on determining appropriate parameters to give the simulation, and people's cognitive constraints may furthermore place limitations on the duration or precision of simulation. One hypothesis for how these issues are handled argues that people use a "rational" solution, meaning that it is optimal under the given constraints [3, 4, 5].

2 Question

In the case of the classic mental rotation task, we might ask: in what direction should the object be rotated? What are the requirements for "congruency"? When should one stop rotating and accept the hypothesis that the objects are different? In this project, I will investigate the optimal computational solution to this task. Thus, the specific question this project aims to answer is: what is the rational computational solution to the mental rotation task, and does it predict the people's behavior on the task?

3 Method

We can formalize the mental rotation task as an instance of function learning. Given two images I_a and I_b, we wish to determine the probability of seeing both images given the hypothesis (h_1) that one is merely a rotation of the other: $\Pr(I_b, I_a|h_1)$. We can generate some mental image I_M using mental rotation, but this is a costly operation and each I_M must be computed sequentially (that is, each I_t must be generated from $I_{t-\delta}$, where δ is a small angle). Let us define the pixel-space similarity between I_b and I_M as $S(I_b|I_M)$. Then:

$$\Pr(I_b, I_a|h_1) \approx \int_{I_M} S(I_b|I_M) \Pr(I_M|I_a) \Pr(I_a) \, \mathrm{d}I_M \propto \int_{I_M} S(I_b|I_M) \, \mathrm{d}I_M \tag{1}$$

We do not know the form of S, and so cannot compute this integral analytically. We cannot rely on a simple Monte Carlo simulation to estimate it, either: because mental rotation is costly in terms of cognitive resources and must be performed in a sequence, we cannot evaluate $S(I_b|I_M)$ at arbitrary I_M. Instead, we must choose a relatively small, sequential set of I_M to estimate the integral.

Bayesian quadrature [6] provides an alternate method for evaluating the integral by placing a prior distribution on functions and then computing a posterior over values of the integral. While Bayesian quadrature itself does not address the issue of choosing points to evaluate the function at, recent work in statistics and machine learning has examined how to optimally select these samples [7].

In this project, I will apply these methods to the classic mental rotation task in both two and three dimensions. From the extensive literature on mental imagery, I will determine appropriate cognitive limitations and compute the optimal tradeoff between computation time (precision) and certainty of answer (accuracy). Finally, I will compare the results of this model with real behavioral data to test whether people are, in fact, using a rational solution in the mental rotation task.

References

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