

Assignment #3 – James Hamski – IS605 Computational Mathematics

1) $\left[\begin{array}{cccc} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 5 & 4 & -2 & -3 \end{array} \right] \rightarrow \left[\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 7 \\ 0 & 1 & -2 & 1 \\ 5 & 4 & -2 & -3 \end{array} \right] \rightarrow \left[\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 7 \\ 0 & 1 & -2 & 1 \\ 5 & 4 & -2 & -3 \end{array} \right] \rightarrow \left[\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 7 \\ 0 & 1 & -2 & 1 \\ 0 & -6 & -17 & -23 \end{array} \right]$

\downarrow

$\left[\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 7 \\ 0 & 1 & -2 & 1 \\ 0 & -6 & -17 & -23 \end{array} \right] \rightarrow \left[\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 7 \\ 0 & -2 & 4 & -2 \\ 0 & -6 & -17 & -23 \end{array} \right] \rightarrow \left[\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 7 \\ 0 & 0 & 8 & 5 \\ 0 & -6 & -17 & -23 \end{array} \right] \rightarrow \left[\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 0 & 6 & 12 & 21 \\ 0 & 0 & 8 & 5 \\ 0 & 0 & -5 & -16 \end{array} \right]$

\downarrow

$\left[\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 0 & 6 & 12 & 21 \\ 0 & 0 & 8 & 5 \\ 0 & 0 & -5 & -16 \end{array} \right] \rightarrow \left[\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 0 & 6 & 12 & 21 \\ 0 & 0 & 40 & 25 \\ 0 & 0 & -40 & -125 \end{array} \right] \rightarrow \left[\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 0 & 6 & 12 & 21 \\ 0 & 0 & 25 & 0 \\ 0 & 0 & 0 & 125 \end{array} \right] \quad \text{Matrix Rank} = 4$

2) MAXIMUM rank = m (no. of rows)

minimum rank = 1 \Rightarrow All other rows could be linearly dependent

3) $\left[\begin{array}{ccc} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{array} \right] * \quad \begin{array}{l} \text{Row 3 is a multiple of row 1} \\ \text{Row 2 is a multiple of row 1} \\ \text{They are linearly dependent} \end{array}$

\downarrow

$\left[\begin{array}{ccc} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{array} \right] \rightarrow \left[\begin{array}{ccc} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 2 & 4 & 2 \end{array} \right] \rightarrow \left[\begin{array}{ccc} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right] \quad \text{Matrix rank} = 1$

Assignment #3 – James Hamski – IS605 Computational Mathematics

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} \quad \det(A - \lambda I) = 0$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = 0$$

$$\det \left(\begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 5 \\ 0 & 0 & 6-\lambda \end{bmatrix} \right) = 0$$

$$(1-\lambda)(4-\lambda)(6-\lambda) + 2 \cdot 5 \cdot 0 + 3 \cdot 0 \cdot 0 - 0 \cdot (4-\lambda)3 - 0 \cdot 5(1-\lambda) - (6-\lambda) \cdot 0 \cdot 2 = 0$$

$$(1-\lambda)(4-\lambda)(6-\lambda) = 0$$

$$\lambda = 1 \quad \lambda = 1, 4, \text{ or } 6$$

$$\begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 5 \\ 0 & 0 & 6-\lambda \end{bmatrix} \quad \lambda = 4 \quad \lambda = 6$$

Assignment #3 – James Hamski – IS605 Computational Mathematics

$$\lambda = -1$$

$$\left[\begin{array}{ccc|c} 1-\lambda & 2 & 3 & 0 \\ 0 & 4-\lambda & 5 & 0 \\ 0 & 0 & 6-\lambda & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 0 & 2 & 3 & 0 \\ 0 & 3 & 5 & 0 \\ 0 & 0 & 5 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 0 & -6 & -9 & 0 \\ 0 & 3 & 10 & 0 \\ 0 & 0 & 5 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 0 & 2 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 5 & 0 \end{array} \right] \rightarrow$$

$$\left[\begin{array}{ccc|c} 0 & 2 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 5 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 5 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad x_1 = \text{free}, x_2 = 0, x_3 = 0$$

$$X_{\lambda=-1} = x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda = 4$$

$$\left[\begin{array}{ccc|c} 1-\lambda & 2 & 3 & 0 \\ 0 & 4-\lambda & 5 & 0 \\ 0 & 0 & 6-\lambda & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} -3 & 2 & 3 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 2 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2/3 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} -1 & 2/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} -1 & 2/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$x_1 = \frac{2}{3}x_2$$

$$x_2 = \text{free}$$

$$x_3 = 0$$

$$X_{\lambda=4} = x_2 \begin{bmatrix} 2/3 \\ 1/3 \\ 0 \end{bmatrix}$$

$$\lambda = 6$$

$$\left[\begin{array}{ccc|c} -5 & 2 & 3 & 0 \\ 0 & -2 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} -5 & 0 & 8 & 0 \\ 0 & -2 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -8/5 & 0 \\ 0 & 1 & -5/2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$x_1 = 8/5x_3$$

$$x_2 = 5/2x_3$$

$$x_3 = \text{free}$$

$$X_{\lambda=6} = x_3 \begin{bmatrix} 8/5 \\ 5/2 \\ 1 \end{bmatrix}$$