Does oil price volatility scale with oil price?

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May 2017

1 Introduction

In a commodity trading market the price level is expected to be tied to commodity system dynamics, namely supply, demand, and delivery of the commodity being traded. Volatility, the variation in price over time, reflects uncertainty in the balance of these system factors. This research paper aims to answer a simply posed question: "does oil price volatility scale with price?"; i.e., can we expect to observe larger price swings when the price is near $100 per barrel vs $20 per barrel?

If oil price volatility reflects uncertainty about supply and demand dynamics, it isn't immediately clear whether we should expect volatility to depend on price level. Higher oil prices are associated with "tightness" in the supply market, meaning there is little excess capacity to increase production, therefore we may expect swings higher but some base price support that results in lower measured volatility. Likewise, low prices may suggest excess capacity that can buffer shocks to the oil delivery system, dampening volatility. Despite these "just so" arguments, a multitude of factors such as storage dynamics, supply chain disruptions, the ability of producers to increase production to bring more oil to market or shut in production capacity in response to prices ("rebalancing"), and market speculation complicate this picture and suggest it must be studied empirically.

If it is found that oil price volatility is dependent on price level, the relationship may follow a scaling formula. For instance, if we can expect volatility of $1/barrel when oil is at $20/barrel, can we expect volatility of $5/barrel at $100/barrel price levels via a simple linear scaling rule? Three methods are presented in this research paper to answer this question: (1) regression modeling of price and volatility, (2) viewing volatility within oil price regimes, and (3) using multivariate Generalized Autoregressive Conditional Heteroskedasticity (GARCH) modeling.

Note that in this research paper *oil price* is used to specifically mean spot-traded crude oil. This represents only one component of the oil markets, and most of the actual oil price is determined by futures and long term delivery contracts (need cite). This research paper is concerned with understanding the energy system using pricing information. In this way, it differs from much of the published research in that it is not concerned with forecasting prices or volatility. Nor is it addressing exogeneous system elements such as equity markets or interest rates, though the literature shows that the crude oil market and larger economic indicators are intertwined (add cite). Instead, it contributes to our understanding of the system dynamics of an essential energy commodity.

2 Exploratory Data Analysis

2.1 Data Source

The data source is the West Texas Intermediate (WTI) nominal (i.e. not inflation adjusted) daily spot price record from the U.S. Energy Information Administration. Using nominal prices maintains the ability to assess volatility for a given time period without introducing an exogenious factor via inflation adjustment. The WTI series was filtered to the date range January 2, 1986 through December 30, 2016 (Table 1).

Table 1: Oil price series date and price ranges.

Date Range	Price Range
Min. :1986-01-03	Min. : 10.25
1st Qu.:1993-09-01	1st Qu.: 19.38
Median :2001-06-11	Median : 28.01
Mean :2001-06-19	Mean : 42.87
3rd Qu.:2009-03-31	3rd Qu.: 63.47
Max. :2016-12-30	Max. :145.31

2.2 Returns and Volatility

In this research paper, volatility is characterized two ways: (1) 5-day historic volatility and (2) 30-day historic volatility. Historic volatility is defined as:

$$V_{H,t} = \sqrt{N} sd(P)$$

In addition, the relationship between the returns themselves and price level is investigated. Daily returns were calculated as:

$$R_t = \ln(P_t - P_{t-1})$$

Note that the standard return formula takes the log of the daily price differences, meaning the day to day price differences are scaled using a natural logarithm. In some sections, the absolute value of "unscaled returns" (i.e. $|P_t - P_{t-1}|$) are used in order to investigate the scaling of the return series.

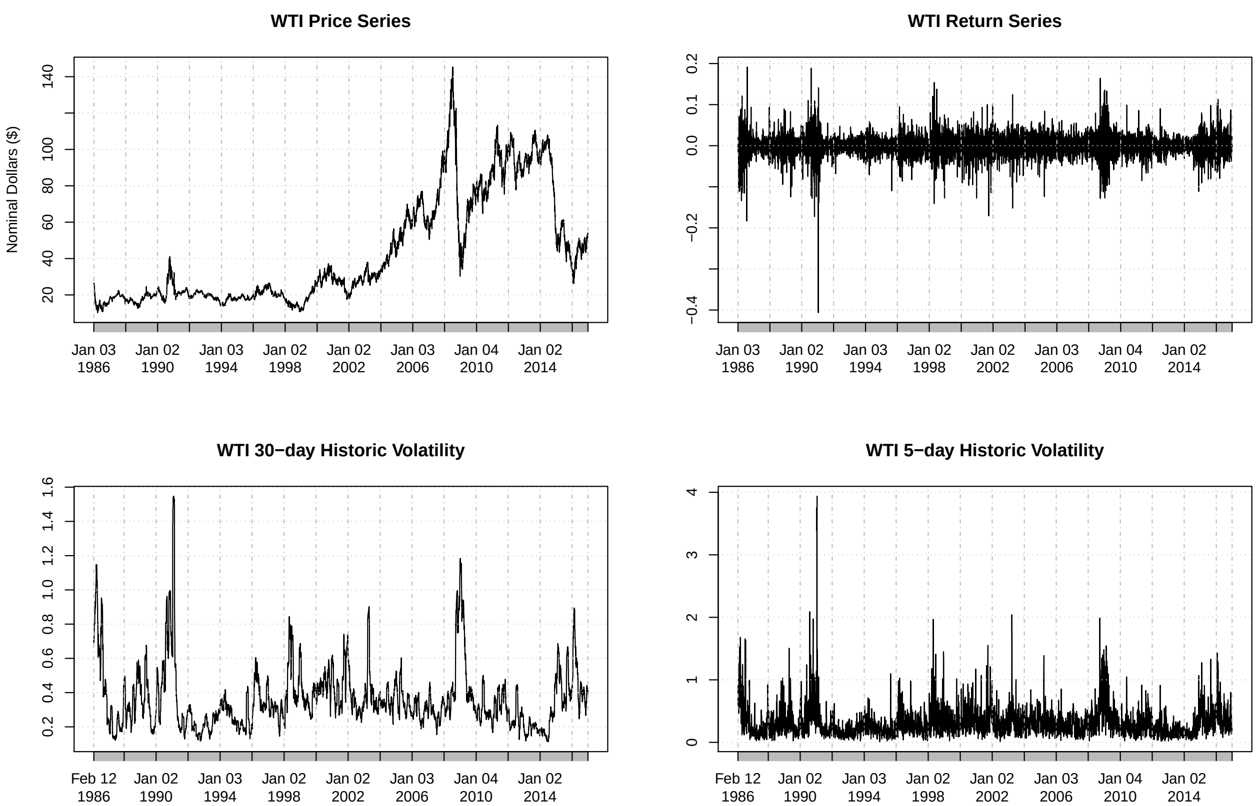
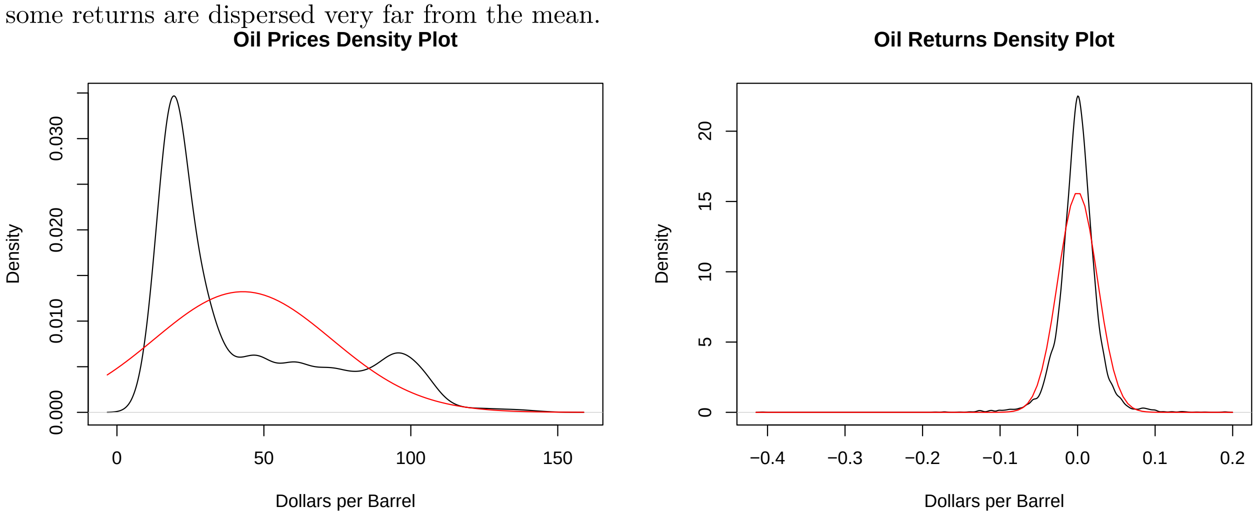


Figure 1: Price level, return, and volatility series plots of WTI spot oil prices.

A seen in Figure 1, most of the series from 1986 through 2004 contains prices between \$10/barrel and \$40/barrel. This results in a price series with a skewness value 0.98 and a long right tail (Figure 2), with most prices being around the series median of \$28 per barrel. The return series exhibits fatter tails and a narrower center than a normal distribution of the same descriptive parameters. The kurtosis value of the return series is 13.52. Returns with excess kurtosis compared to normally distributed returns is a common characteristic in financial time series. This indicates that most returns are very close to the mean return, but some returns are dispersed very far from the mean.



2.3 Autocorrelations

Autocorrelations

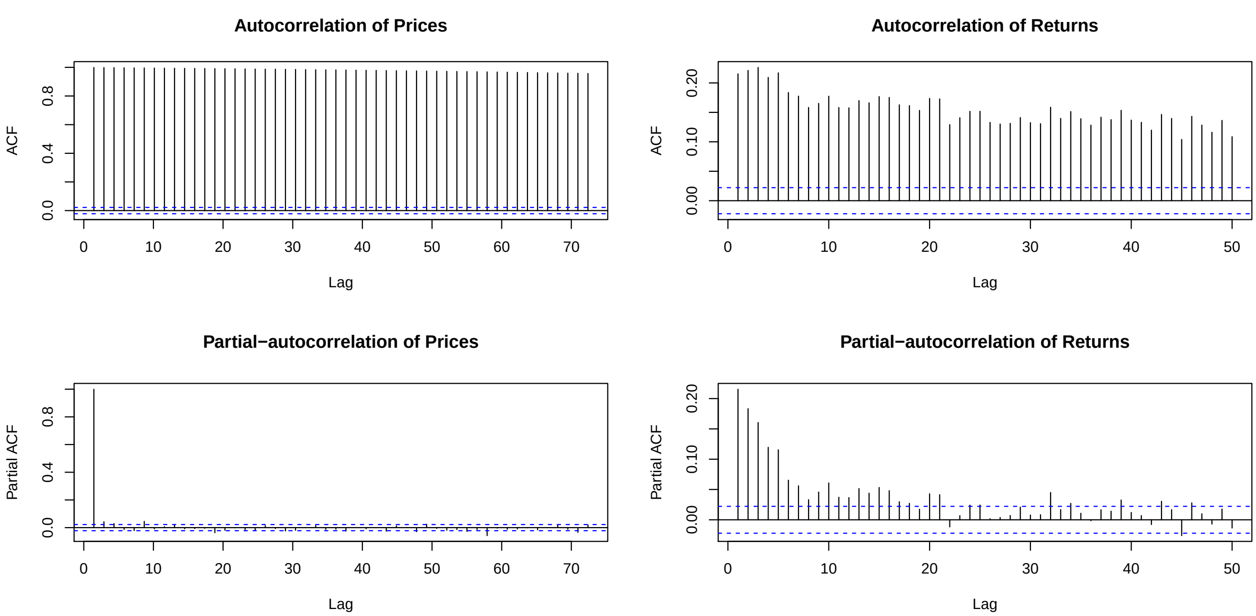


Figure 2: Autocorrelation plots for prices and returns.

3 Price-Volatility Regression Analysis

Relating price level and the measures of volatility at each time in the series is a simple exploration of the research problem. The covariance and correlation measures of vectors representing price versus returns, 30-day, and 5-day historic volatility indicate a weak, negative relationship (Table 2). However, the unscaled return series indicates a stronger positive covariance and correlation with the price series.

Table 2: Covariance and correlation of price level and returns, 30-day, and 5-day historic volatility.

	Covariance	Correlation
Return Series	0.0102677	0.0134110
Unscaled Return Series	12.0871446	0.4393373
30-day Historic Volatility	-1.0525476	-0.1860878
5-day Historic Volatility	-0.8388984	-0.1123822

In quantitative finance, a process where volatility scales with price is a lognormal process. When volatility is independent of price, the process is normal (Ho and Lee, 2003). Regression of the price level and squared returns is identified as the method of distinguishing lognormal from normal processes in the literature. Figure 4 shows scatter plots of the two return series and two historic volatility measures fitted with a linear regression model.

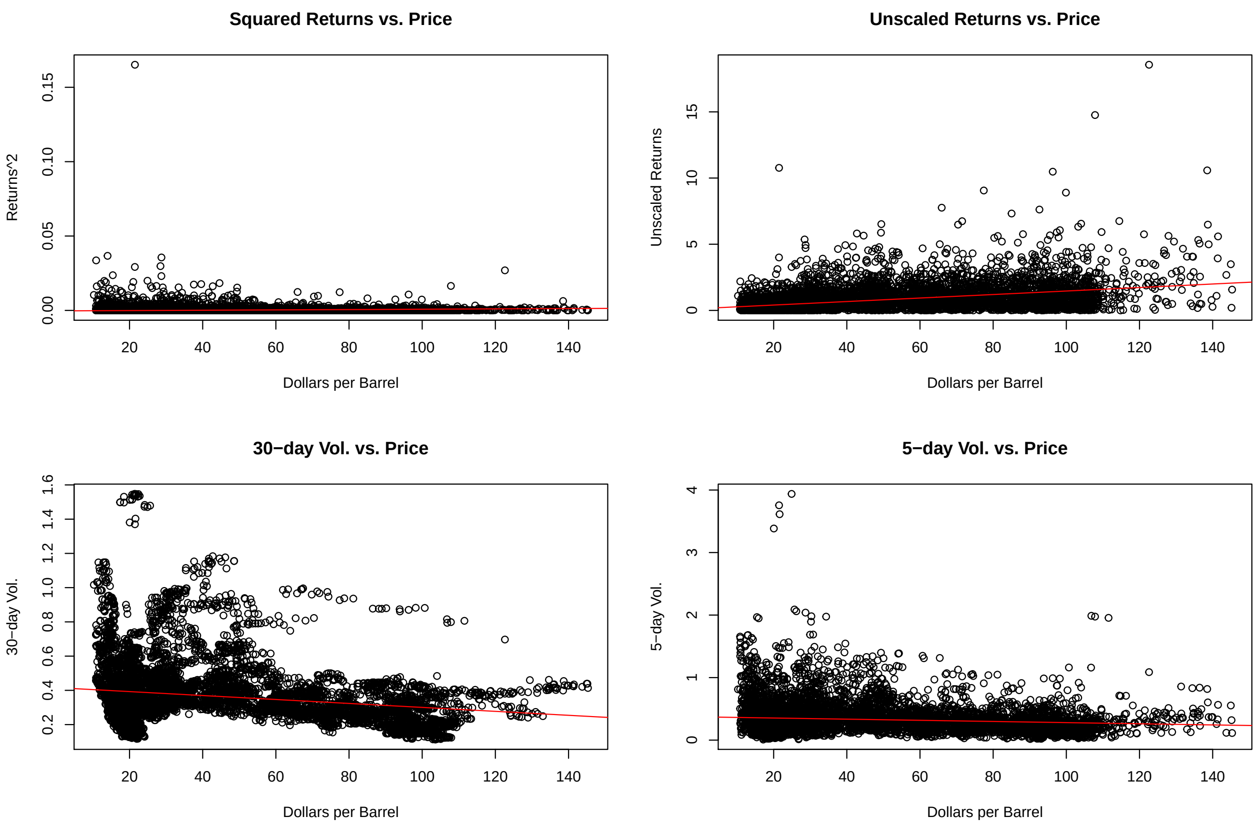
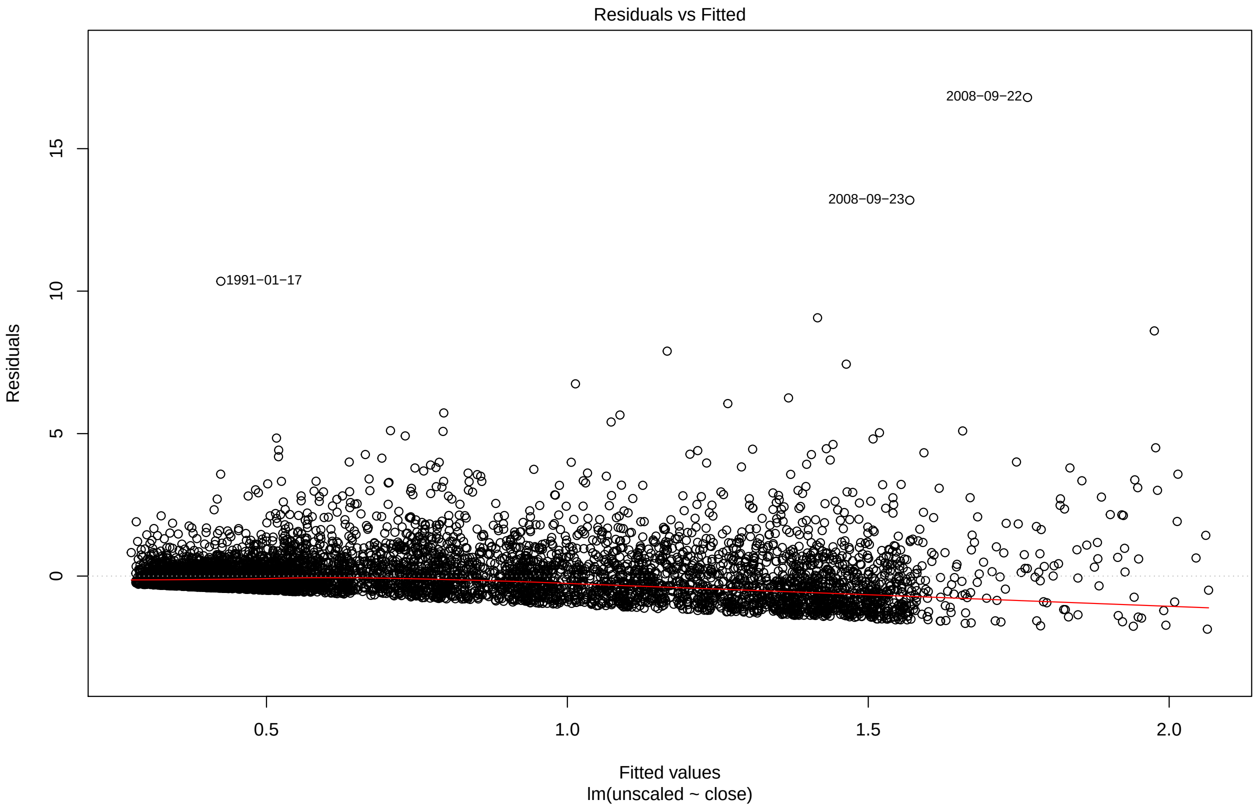
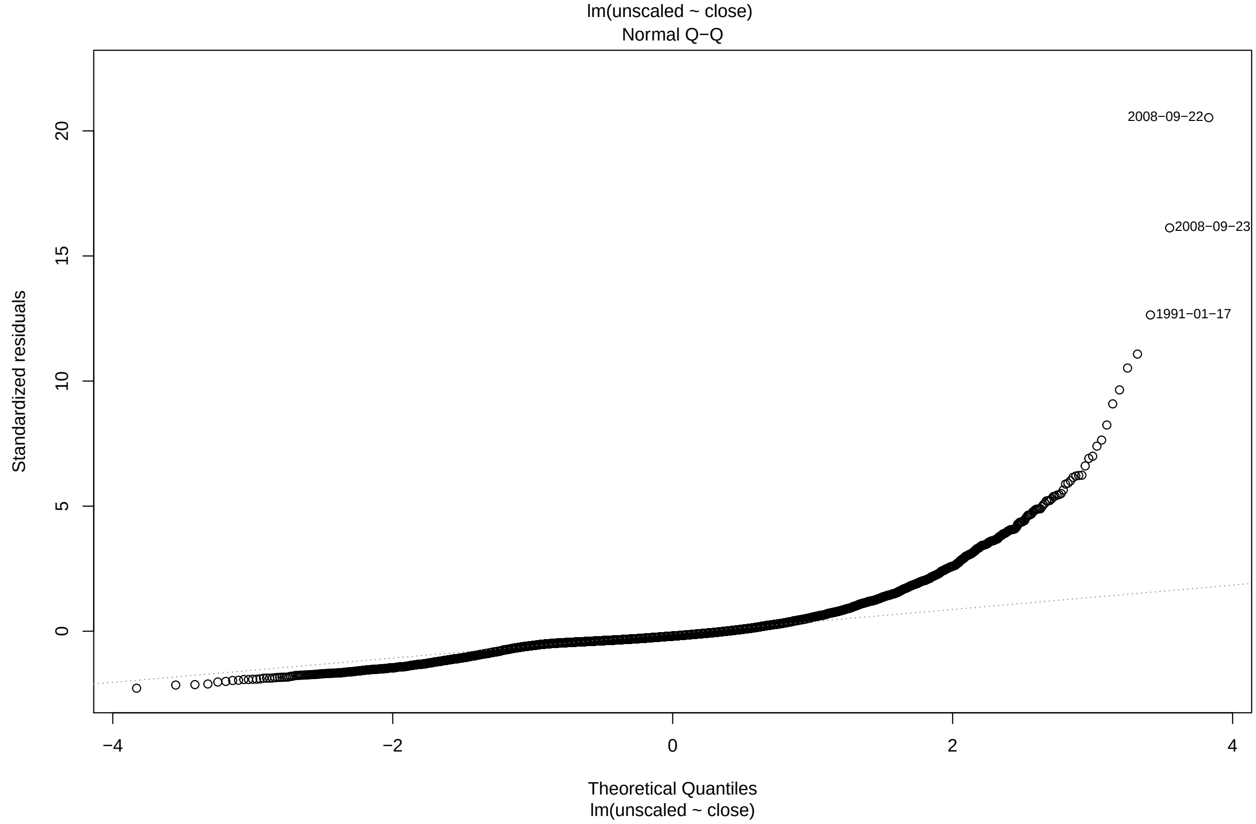


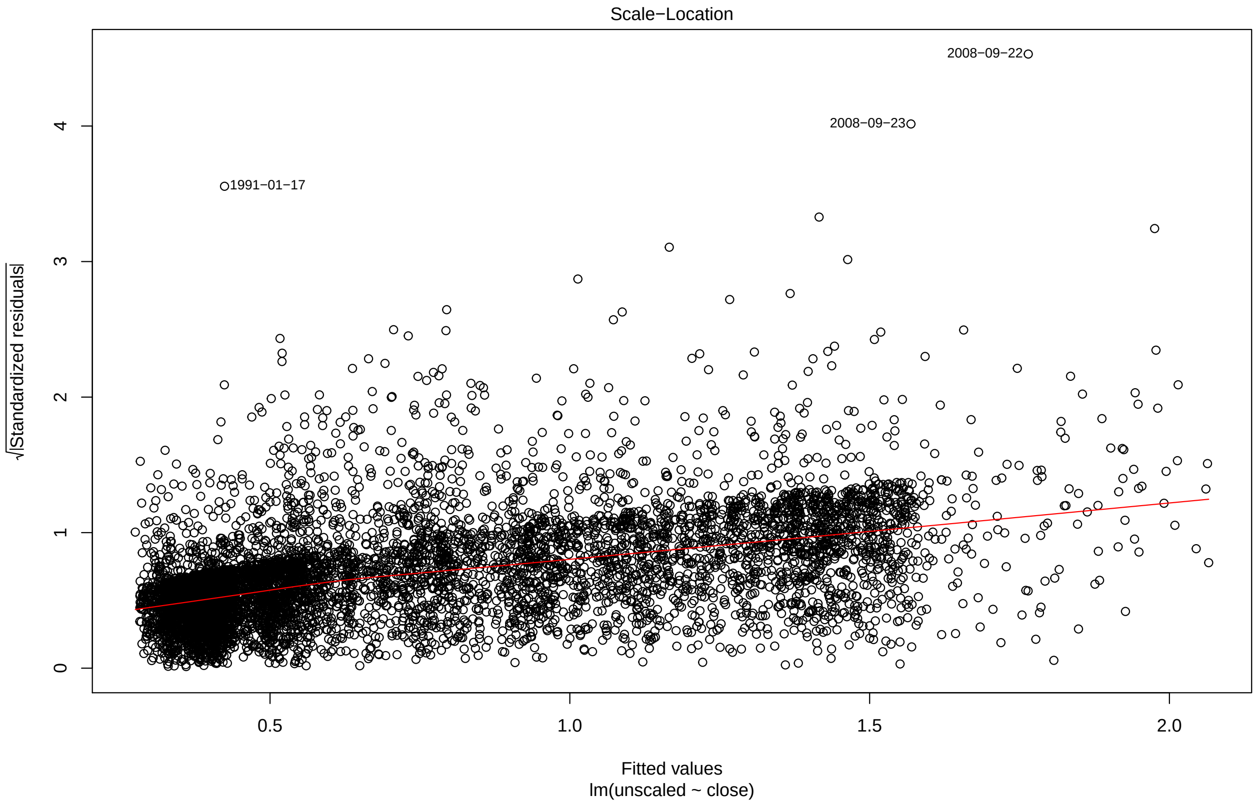
Figure 3: Scatterplots with a linear model relating spot oil price with returns, 30-day, and 5-day historic volatility.

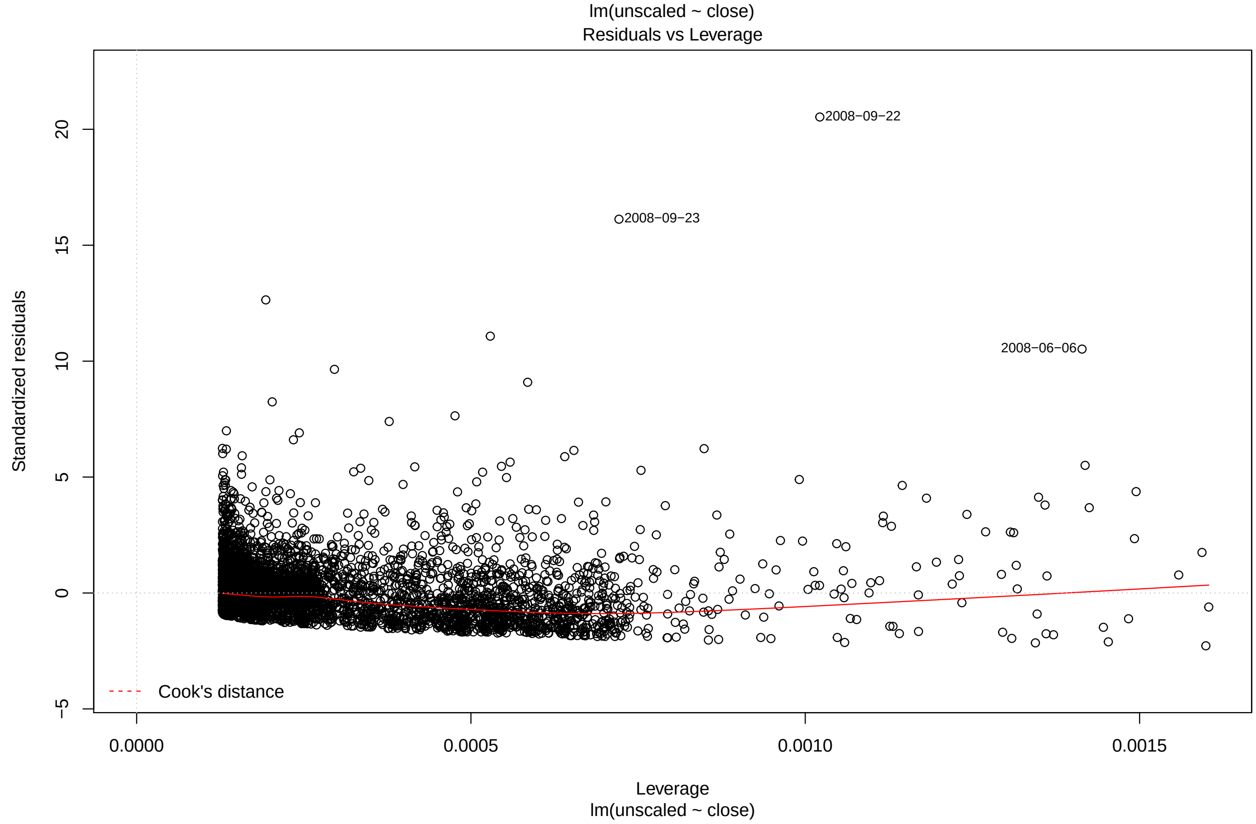
The case of 30-day historic volatility indicates a negative relationship between price level and volatility.

However, this result appears to be due to a cluster high volatility around $20 per barrel, creating a leverage point. Residual analysis indicates that this is not a good relationship to model with linear regression.









As seen in Figure 1, nominal oil prices have spent time as high as $145 per barrel. However, the majority of the time series is far lower, with a median price of $28 per barrel. This means that the dataset is unbalanced and higher price levels represent a smaller portion of the dataset. In addition, oil price (and financial time series in general) exhibits volatility clustering. Therefore, it is anticipated that this simple regression model based on price and volatility is not the best possible solution to the question of characterizing the dependency of volatility on price. A second time series was created by limiting WTI prices to the January 3, 1998 through December 31, 2016. This results in a distribution that, while significantly different than the normal distribution, is more evenly distributed across the price range from $10.82 per barrel to $145.30 per barrel (Figure 5).

Given the ever evolving economic, political, and technological landscape affect commodity prices, there is some tradeoff between looking at a large historic price record, which covers a larger data set of possible system states (and higher statistical power), and limiting to a more recent price record, which more fully represents the system in its current state. However, the linear models relating price level to returns and volatility resulting from the recent time series does not differ much from using the full data set. In general there is a weak, negative correlation between volatility and price, and a weak positive correlation between returns and price.

This simple method of exploring the relationship between price level and volatility does not provide a satisfactory explanation of whether we can expect more volatility, indicating more commodity system uncertaintly, at high price levels. Volatility in financial time series tend to cluster. Typically, some event (called a "shock") occurs which results in an extremely high price movement. These are the large noticeable movements in the return series (Figure 1). Subsequent returns are also of higher magnitude than the typical return size, but taper off over time, eventually returning to a value close to the average return for the price series. This is referred to as volatility clustering, and indicates heterosketasticity in the variance component of a time series. This violates the assumption in the most frequently used time series model, the autoregressive integrated moving average (ARIMA) model.

These results using simple linear regression models to study how daily returns and historic volatility relate to price level may not adequately deal with this time-varying variance structure. If, for a given price level, we have a price shock, we expect the volatility to return back to its normal level. So at that price level we may have relatively few measurements indicating high volatility, resulting in regression models heavily weighted towards the baseline level of volatlity. This "one to one" view of the price level - volatility relationship leaves the question open as to whether we can anticipate more volatility or larger price shocks at higher price levels.

4 Comparing Volatility across Price Regimes

In order to capture the volatility dynamics of the WTI price series, including volatility clustering and changes in base price, change point detection was used to break the series into price regimes. Change point detection aims to detect the point or points where the statistical properties of a sequence of observations change (Killick and Eckley, 2014). Change point detection was used to estimate changes in the oil price mean throughout the period of record using the Pruned Exact Linear Time (PELT) algorithm (Killick et al. 2012). The time series between these changepoints represent "price regimes" (i.e. time series between changepoints) which have generally similar mean oil price compared to the entire record.

Changepoint detection proceeds by minimizing a cost function over possible locations and number of changepoints. The cost function:

```
##
##  Fligner-Killeen test of homogeneity of variances
##
## data:  wti.regimes.ret$returns and wti.regimes.ret$regime
## Fligner-Killeen:med chi-squared = 587.38, df = 14, p-value <
## 2.2e-16
```

```
## [1] "statistic" "parameter" "p.value"   "method"    "data.name"
```

The Fligner-Killeen test of homogeneity of variances determines if it is appropriate to reject the null hypothesis that the variances between each regime are the same. This test was chosen due to it's suitability for non-normal data. The chi-squared value of the test was 587.38 and the p-value was 0, indicating it is appropriate to conclude the variances across the regimes are different.

Then, the standard deviation of the prices within a regime was compared to the median price of the regime. The resulting dataset describing the regimes were then modeled using linear regression. The resulting model indicates a slope of 0.08, meaning that for every dollar increase in the median price of a price regime, we expect an increase in the standard deviation of that regime of 0.08. This model has an adjusted r-squared value of 0.45.

Table 3: Linear regression resulting from price as the predictor variable and regime variance as the response variable.

term	estimate	std.error	statistic	p.value
(Intercept)	1.9666240	1.5125362	1.300216	0.2161068
med	0.0828545	0.0252947	3.275565	0.0060244

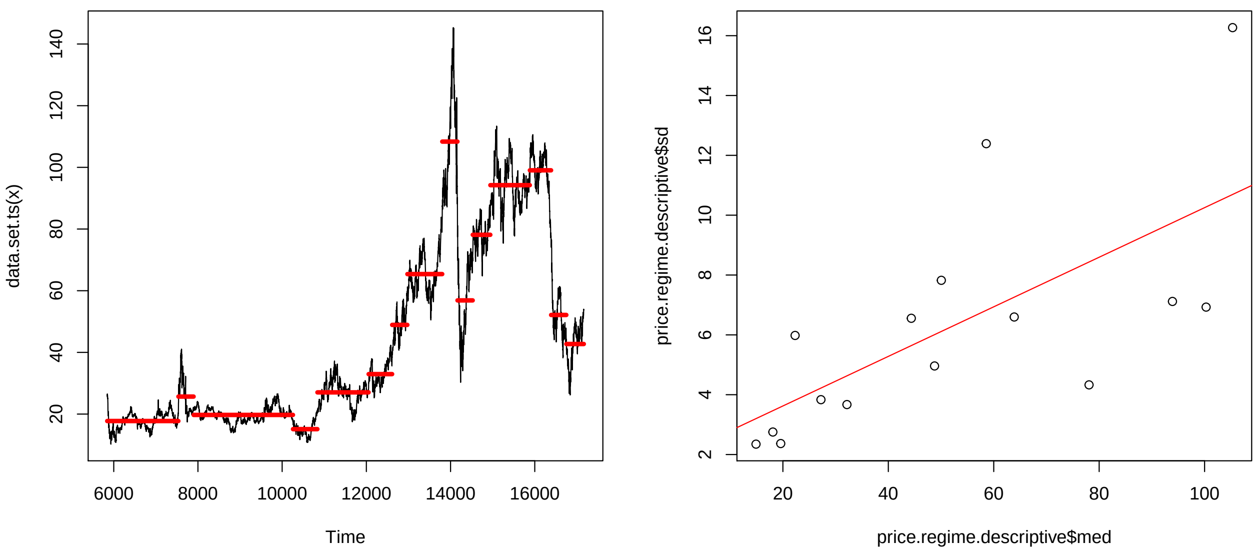
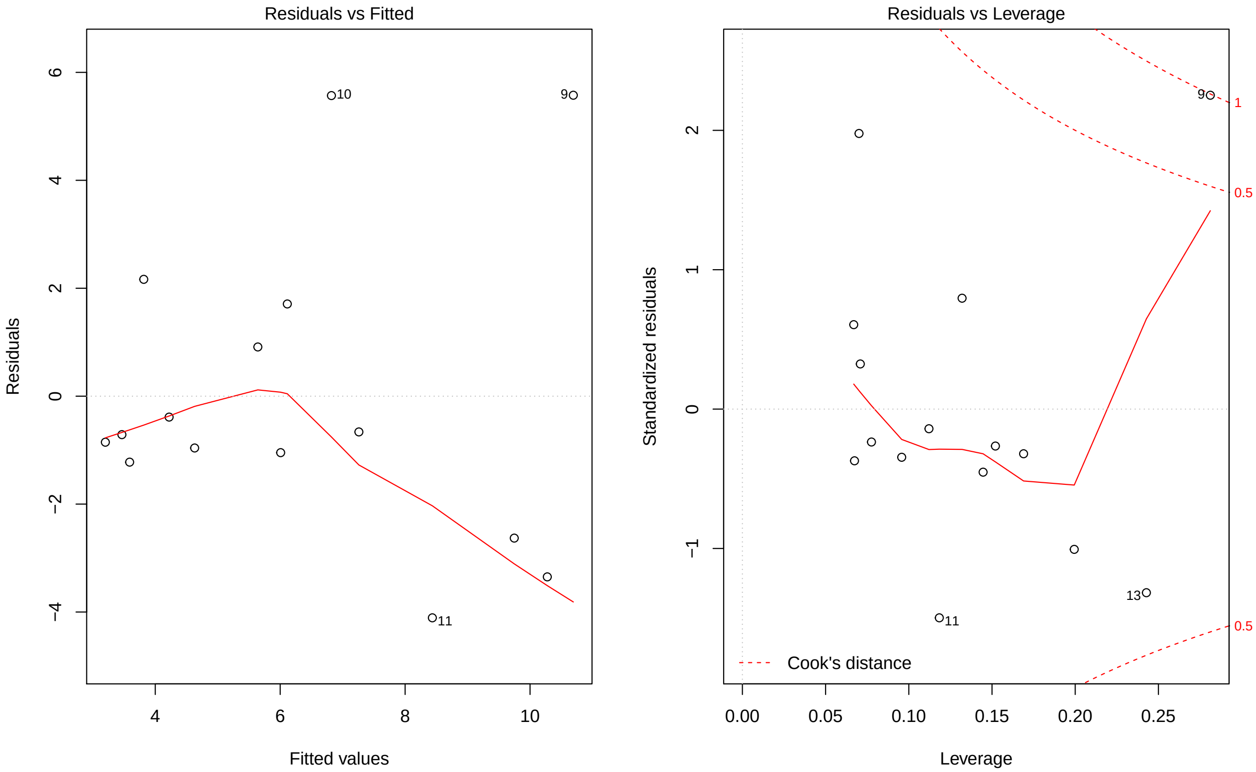


Figure 4: Price regimes and linear model relating median oil price for the regime to standard deviation within that price regime.

Analyzing the residuals of this model indicate some problems with the fit. Namely, residuals are not evenly distributed and the model is subject to suspected leverage point observations. This is typical in linear models with small sample sizes.



In addition, this analysis was performed for the time series limited to 1998 - 2016, but this had little affect on the relationship between price regime volatlitity.

This section's analysis using changepoint analysis to break the oil price series into price regimes as defined by changes in the series mean is highly dependent upon model parameters. The PELT change point detection method identifies different regimes depending on the minimum segment length and penalty parameters, and based on the section of the time series used. An interactive Shiny App was created and may be access in order to try different parameterizations of the change point model and view the effects on the relationship between medial regime price and standard deviation within that regime.

5 Multivariate GARCH Model

Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) models attempt to model the variance component of a time series. It accounts for asymetric, clustered variance

6 Conclusion

This analysis suggests that the question "does oil price volatility scale with price?" requires further clarification to address.

Expected Returns at a Given Price Level It is apparent from the regression of unscaled returns to price level that we may expect returns to increase linearly with increasing price. The linear model indicates that for every one dollar increase in price, we may anticipate a 1.3 cent increase in expected returns. However, at

$R_t = 0.013P_t + 0.14$

```
##
## Call:
```

```
## lm(formula = unscaled ~ close, data = wti.combined)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.8635 -0.3511 -0.1594  0.1853 16.7954
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.1394237  0.0161217   8.648   <2e-16 ***
## close       0.0132547  0.0003071  43.163   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8185 on 7789 degrees of freedom
## Multiple R-squared:  0.193,  Adjusted R-squared:  0.1929
## F-statistic:  1863 on 1 and 7789 DF,  p-value: < 2.2e-16
```

Scaled Volatiily

7 Acknowledgements

I would like to thank John Kemp, Thompson Reuters energy journalist, for posing the question investigated in this research paper. In addition, I thank Tancred Lidderdale and Mason Hamilton from the U.S. Energy Information Administration for additional information pertaining to the question.

8 Bibliography

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