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Modeling and Forecasting Crude Oil Price Volatility: Evidence from Historical and Recent Data*

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Abstract: This paper uses the Markov-switching multifractal (MSM) model and generalized autoregressive conditional heteroscedasticity (GARCH)-type models to forecast oil price volatility over the time periods from January 02, 1875 to December 31, 1895 and from January 03, 1977 to March 24, 2014. Based on six different loss functions and by means of the superior predictive ability (SPA) test, we evaluate and compare their forecasting performance at short and long horizons. The empirical results indicate that none of our volatility models can uniformly outperform other models across all six different loss functions. However, the new MSM model comes out as the model that most often across forecasting horizons and subsamples cannot be outperformed by other models, with long memory GARCH-type models coming out second best.

Keywords *Crude oil prices, GARCH, Multifractal processes, SPA test*

JEL classification C52, C53, C22

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1 Introduction

The recent literature shows a growing interest in modeling and forecasting oil price volatility due to its impact on the global and regional economies (cf. Wang et al., 2012; Rahman and Serletis, 2012). How oil price shocks may affect economic growth is well-documented in a large body of research. Different transmission mechanisms were developed in the literature. Examples include Rotemberg and Woodford (1996) and Finn (2000), among others. Papers by Hamilton (1983) Davis and Haltiwanger (2001), and Lee and Ni (2002) clearly demonstrated that positive oil price shocks induce a slow-down in aggregate measures of growth or employment and that negative oil price shocks lead to an increase in aggregate measures of growth or employment. Recently, Elder and Serletis (2010) found that increased uncertainty about oil price changes causes a significant drop in real output and heavily affects measures of durable consumption and fixed investment in the United States. Their finding is also confirmed by Rahman and Serletis (2012) for the Canadian economy. In his seminal paper, Hamilton (2003) confirmed the existence of a strong relationship between oil price changes and GDP growth and showed that this relationship is of a nonlinear nature. Jones and Kaul (1996) and Sadorsky (1999) showed that oil price shocks have direct or indirect influence on financial markets. According to Backus and Crucini (2000) they may be responsible for fluctuations in the international terms of trade. Oil price volatility also represents an important input for macro-econometric models (cf. Ferderer, 1996), pricing of derivatives (cf. Wang et al., 2008) and portfolio selection models (cf. Geman and Kharoubi, 2008). So, it is of primary importance for firms, financial market participants and policy makers to have models available that can properly reproduce the *stylized facts* of oil price volatility and provide accurate forecasts.

The widespread tool used in the literature to analyze oil price volatility consists in GARCH-type models (cf. Kang et al., 2009; Cheong, 2009; Mohammadi and Su, 2010; Wei et al., 2010). All these papers have attempted to find the most appropriate GARCH-type models, linear or nonlinear, that can properly reproduce the stylized facts of oil price volatility, and thus, produce accurate forecasts. While some results speak in favor of fractionally integrated GARCH (FIGARCH) models (cf. Kang et al., 2009), others provide evidence that the standard GARCH and FIAPARCH (cf. Cheong, 2009), and the APARCH models (cf. Mohammadi and Su, 2010) could be more appropriate. In contrast to the previous papers, Wei et al. (2010) consider nine GARCH-type models and compare their forecasting performance based on six different loss functions. They found that none of these models can consistently outperform each other, despite the fact that the nonlinear models can properly capture long memory volatility and/or the asymmetric leverage effect in volatility.

This paper extends the work of Wei et al. (2010) in two important respects: (i) we add to the set of GARCH models used in Wei et al. (2010) a new type of volatility model, namely the Markov switching multifractal (MSM) model, (ii) we consider a large data set that contains oil price observations of the pre- and post-1900 eras. Our objective is to compare the forecasting performance of the MSM model with that of GARCH models. Availability of daily data for a twenty-year period within the 19th century provides the valuable opportunity to compare the statistical features of the modern oil market with those of a much earlier phase of the same market. The multifractal[5] model provides a completely new approach to the modeling of financial volatility which it conceives as a multiplicative, hierarchically structured process. Via its particular principles of construction, it allows to estimate a Markov-switching model with a high number of states without falling victim to the curse of dimensionality. This structure gives it an intermediate nature between "true" long-memory processes and simple regime-switching processes allowing to modulate the temporal dependency via its parameters and the number of hierarchical components. The flexible regime-switching nature makes it attractive for time series that show pronounced differences between highly volatile and more tranquil periods (as oil prices do). Research on stock and foreign exchange markets has documented superior forecasting capabilities of MSM against traditional GARCH models (Calvet and Fisher, 2004; Lux and Kaizoji, 2007; Lux et al., 2014). It seems interesting to explore in how far these findings can be confirmed with important commodities such as oil. As in Wei et al. (2010), we also use six different loss functions as criteria for comparison, and then apply the predictive ability test of Hansen (2005) in order to infer whether one particular model is outperformed by others or not. Here we prefer the predictive ability test of Hansen (2005) to other powerful evaluation techniques existing in the literature (cf. Diebold and Mariano, 1995; West, 1996; White, 1996) due to its robustness, and the fact that it allows to compare a benchmark (possibly nested) model for a whole set of competitors.

The remainder of the paper is organized as follows. Section 2 presents the descriptive statistics of our data sets. Section 3 introduces the different volatility models. The forecasting evaluation methodologies are presented in Section 4 and results are provided in Section 5. Finally, Section 6 concludes.

2 Data

We use daily closing oil prices (in US dollars per barrel) of West Texas Intermediate (WTI) over two different sample periods. The first one covers the period from January 02, 1875 to December 31, 1895 and the second one runs from January 03, 1977 to March 24, 2014. For the more recent era, we also split the sample into two different parts. This will help us to better observe the time evolution of oil prices. The samples are driven purely by availability of daily data at the time of writing this paper, with the data being sourced from the Global Financial Database, https://www.globalfinancialdata.com. We

[5]The term multifractal refers to the fractal structure of the resulting volatility process. The MSM has actually been adapted from very similar models that have first been developed for turbulent flows (cf. Mandelbrot, 1974). Fractality is also a concept that plays an important role in geophysical research and petroleum geology (cf. Barton and La Pointe, 1995), but it seems unlikely that the two aspects - fractality of oil fields and fractality of oil price volatility - are materially related to each other.

compute the percent continuously compounded returns r_t as

$$r_t = 100 * [\ln(p_t) - \ln(p_{t-1})], \tag{1}$$

where p_t denotes the oil price at the end of period t and p_{t-1} is the oil price on the previous day.

To get some first impression of our data sets we first plot the oil prices, their log-returns and squared log-returns (cf. *Figs.* 1 through 8). Their descriptive statistics are reported in Tables 1, 2, 3 and 4. The data sets exhibit high variability, in other words the standard deviations are very high compared to the sample means. We observe positive skewness for the data set of pre-1900 and a negative one for the data set of post-1900. Both data sets exhibit excess kurtosis. These results show that the computed log-returns do not follow a Normal distribution. This observation is confirmed by the Jarque-Bera test, which rejects the null hypothesis of Normally distributed log-returns at any level of significance. We also apply the augmented Dickey-Fuller (ADF) unit-root test of Dickey and Fuller (1979) to oil returns and the results clearly speak for the stationarity of both data sets. The Hurst indices reported in Tables 1, 2, 3 and 4 are computed via Detrended Fluctuation Analysis (DFA) (cf. Weron, 2002). The Hurst index values for log-returns are close to 0.5 and not significantly different from this value at the 95% confidence level, implying absence of long memory features in oil price returns. For absolute and squared returns the Hurst index values are significantly above 0.5, indicating the presence of long memory in oil price volatility. Finally, in order to show the decay of the unconditional distribution of oil price returns in its extremal region, we compute the so-called Hill estimator for the tail index (cf. Hill, 1975). We find that the estimates for the tail indices are in the vicinity of 3 and these results are in harmony with typical findings for other commodities and financial assets, cf. Tables 1, 2, 3 and 4.

Figs. 2, 4, 6 and 8 depict the autocorrelation functions of log-returns, absolute and squared log-returns. We observe that the absolute and squared log-returns are highly correlated and this observation is in conformity with the Ljung-Box statistics, $Q(10)$ and $Q(20)$. The Ljung-Box tests also reject the null hypothesis of no serial correlation for raw log-returns at the 5% significance level. This indicates the presence of some serial dependence in the oil price log-returns. The higher statistics of the Ljung-Box statistics for the raw returns in the 19th century might indicate a lower degree of "financialisation" of this commodity at earlier times.

3 Model Framework

In this section we briefly present the volatility models used for our forecasting exercises. In general, financial returns in these models are formalized as

$$r_t = \mu_t + \sigma_t e_t, \tag{2}$$

where $r_t = 100 * [\ln(P_t) - \ln(P_{t-1})]$, $\ln(P_t)$ is the log asset price, $\mu_t = \mathbb{E}_{t-1}[r_t]$ is the conditional mean of the return series, σ_t is the volatility process and e_t is standard Normally distributed. Defining $x_t = r_t - \mu_t$, the *centered* returns are given by

$$x_t = \sigma_t e_t. \tag{3}$$

In this paper we assume that μ_t follows an AR(1) process and consider two different types of volatility models for describing σ_t, namely the linear and nonlinear GARCH-type models and the Markov switching multifractal (MSM) model.

3.1 GARCH-type Models

The underlying idea of the autoregressive conditional heteroskedasticity (ARCH) model was developed by Engle (1982) in his seminal paper. The ARCH model and its subsequent generalized versions are well known in the literature for their ability to capture the most important *stylized facts* (e.g. clustering effects, long-memory and short-memory effects, asymmetric leverage effects) observed in all measures of volatility (e.g. absolute log-returns, squared log-returns, etc...). In the following we list the eight different GARCH models used in this study.

3.1.1 The GARCH and IGARCH Models

Introduced by Bollerslev (1986) the linear GARCH model is the most popular volatility model in the literature. In the simple, but effective GARCH(1,1) (cf. Bollerslev et al., 1994) the conditional variance is modeled as

$$\sigma_t^2 = \omega + \alpha x_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{4}$$

where $\omega > 0$, $\alpha > 0$, $\beta > 0$ and $\alpha + \beta < 1$. The nonnegativity constraints on ω, α and β guarantee the positivity of σ_t^2.

h-step ahead forecasts from GARCH(1,1) are obtained recursively as

$$\begin{aligned}
\hat{\sigma}_{t+h}^2 &= \omega + (\alpha + \beta)\,\hat{\sigma}_{t+h-1}^2 \\
&= \bar{\sigma}^2 + (\alpha + \beta)\left(\hat{\sigma}_{t+h-1}^2 - \bar{\sigma}^2\right) \\
&= \bar{\sigma}^2 + (\alpha + \beta)^{h-1}\left(\hat{\sigma}_{t+1}^2 - \bar{\sigma}^2\right).
\end{aligned} \tag{5}$$

where $\bar{\sigma}^2 = \omega\,(1 - \alpha - \beta)^{-1}$ is the unconditional variance. As $h \to \infty$, it is clear that the volatility forecast in *eq.* (5) approaches the unconditional variance $\bar{\sigma}^2$ and $(\alpha + \beta)$ dictates the speed of the mean reversion.

If $\alpha + \beta = 1$, the GARCH(1,1) reduces to the IGARCH(1,1) model proposed by Engle and Bollerslev (1986) in order to account for infinite persistence in the conditional variance. The h-step ahead forecast representation becomes

$$\hat{\sigma}^2_{t+h} = \hat{\omega} + \hat{\sigma}^2_{t+h-1}$$
$$\hat{\sigma}^2_{t+h} = \hat{\omega}h + \hat{\sigma}^2_t. \tag{6}$$

3.1.2 The Exponential GARCH Model

The exponential GARCH (EGARCH) model was proposed by Nelson (1991) with the aim to capture the asymmetric relation between stock returns and volatility changes noted by Black (1976). The conditional variance in the EGARCH (1,1) model is given by

$$\ln\left(\sigma^2_t\right) = \omega + \alpha e_{t-1} + \gamma\left(|e_{t-1}| - \mathbb{E}\left[|e_{t-1}|\right]\right) + \beta \ln\left(\sigma^2_{t-1}\right), \tag{7}$$

where γ represents the asymmetric leverage parameter that quantifies the degree of the volatility leverage effect in the model and α the magnitude. As in *eq.* (2), $e_t \sim N(0,1)$ with $\mathbb{E}\left[|e_{t-1}|\right] = \sqrt{2/\pi}$. The model parameters are free from nonnegativity constraints.

Following the same procedures as with GARCH(1,1), the h-step ahead forecast formula of the EGARCH(1,1) can be expressed as

$$\ln \hat{\sigma}^2_{t+h} = \bar{\sigma}^2 + \beta^{h-1}\left(\ln \hat{\sigma}^2_{t+1} - \bar{\sigma}^2\right), \tag{8}$$

where $\bar{\sigma}^2 = (\omega - \gamma/\sqrt{2/\pi})/(1-\beta)$.

3.1.3 The Glosten/Jagannathan/Runkle GARCH Model

The GJR-GARCH model developed by Glosten et al. (1993) is designed in a way that allows the model to account for the potential larger impact of negative shocks on return volatility. The conditional variance in the GJR-GARCH(1,1) can be formalized as

$$\sigma^2_t = \omega + \left[\alpha + \gamma D(x_{t-1} < 0)\right] x^2_{t-1} + \beta\sigma^2_{t-1}, \tag{9}$$

where $D(.)$ is an indicator function that takes the value 1 if $x_{t-1} < 0$ (bad news), and 0 (good news) otherwise. The parameter γ quantifies the magnitude of the asymmetric leverage effect. The h-step ahead forecast representation of the GJR-GARCH(1,1) can be formalized as

$$\hat{\sigma}^2_{t+h} = \bar{\sigma}^2 + \left(\alpha + \beta + \frac{\gamma}{2}\right)^{h-1}\left(\hat{\sigma}^2_{t+1} - \bar{\sigma}^2\right), \tag{10}$$

where $\bar{\sigma}^2 = \omega/(1-\alpha-\beta-\gamma/2)$ is the unconditional or long run variance.

3.1.4 The Asymmetric Power ARCH Model

The asymmetric power ARCH (APARCH) model introduced by Ding et al. (1993) aims to reproduce both leverage and the *Taylor* effect, named after Taylor (1986) who first documented the fact that the sample autocorrelation of absolute returns was usually larger than that of squared returns. The conditional variance in the APARCH(1,1) model is given by

$$\sigma_t^\delta = \omega + \alpha\left(|x_{t-1}| - \gamma x_{t-1}\right)^\delta + \beta\sigma_{t-1}^\delta, \tag{11}$$

where $\delta > 0$ and γ is the leverage coefficient. The APARCH(1,1) model reduces to GARCH(1,1) when $\delta = 2$ and $\gamma = 0$.

The h-step ahead forecast formula of the APARCH(1,1) is given by

$$\begin{aligned}\hat{\sigma}_{t+h}^\delta &= \omega + \left(\alpha\mathbb{E}_t\left[(|e_{t+h-1}| - \gamma e_{t+h-1})^\delta\right] + \beta\right)\hat{\sigma}_{t+h-1}^\delta \\ &= \kappa + (\alpha c + \beta)^{h-1}\left(\hat{\sigma}_{t+1}^\delta - \kappa\right),\end{aligned} \tag{12}$$

where $\kappa = \omega(1 - \alpha c - \beta)^{-1}$ is the long run variance to the power δ and $c = \mathbb{E}_t\left[(|e_{t+h-1}| - \gamma e_{t+h-1})^\delta\right]$ is given by

$$c = \frac{1}{\sqrt{2\pi}}\left[(1+\gamma)^\delta + (1-\gamma)^\delta\right]2^{\frac{\delta-1}{2}}\Gamma\left(\frac{\delta+1}{2}\right).$$

3.1.5 The Fractionally Integrated GARCH Model

By introducing fractional differences in the GARCH process Baillie et al. (1996) obtained the FIGARCH model that can reproduce the long memory property of financial returns volatility. The FIGARCH(1,d,1) model volatility can be expressed as

$$\sigma_t^2 = \omega + \left[1 - \beta(L) - \phi(L)(1-L)^d\right]x_t^2 + \beta\sigma_{t-1}^2, \tag{13}$$

where $\omega > 0$, $\phi < 1$, $\beta < 1$, $0 \le d \le 1$. L denotes the lag operator and d is the parameter of fractional differentiation. The parameters have to fulfill the following conditions:

$$\beta - d \le \phi \le \frac{(2-d)}{3} \tag{14}$$

and

$$d\left[\phi - \frac{(1-d)}{2}\right] \le \beta(d - \beta + \phi). \tag{15}$$

We can rewrite *eq.* (13) as follows

$$\begin{aligned}\sigma_t^2 &= \omega(1-\beta)^{-1} + \left[1 - (1-\beta)\phi(L)(1-L)^d\right]x_t^2 \\ &= \omega(1-\beta)^{-1} + \eta(L)x_t^2,\end{aligned} \tag{16}$$

where $\eta(L) = \eta_1 L + \eta_2 L^2 + \ldots, \eta_j \ge 0$ for $j = 1, 2, \ldots$.

$\eta(L)$ can be computed from the recursions:

$$\begin{cases} \eta_1 = & \hat{\phi} - \hat{\beta} + \hat{d}, \\ \vdots & \vdots \\ \eta_j = & \hat{\beta}\eta_{j-1} + \left[\left(j - 1 - \hat{d}\right) j^{-1} - \hat{\phi}\right] \pi_{j-1} \end{cases} \tag{17}$$

where $\pi_j \equiv \pi_{j-1}\left(j - 1 - \hat{d}\right) j^{-1}$ are the coefficients in the MacLaurin series expansion of the fractional differencing operator $(1 - L)^d$. As in previous research, we set the truncation order of the infinite series $(1 - L)^d$ to 1000 lags.

The FIGARCH model reduces to the GARCH model when $d = 0$ and the IGARCH model when $d = 1$.

From *eq.* (16) one can easily derive the one-step ahead forecast of σ_t^2

$$\hat{\sigma}_{t+1}^2 = \omega(1 - \beta)^{-1} + \eta_1 x_t^2 + \eta_2 x_{t-1}^2 + \dots \tag{18}$$

Using recursive substitution described above the h-step ahead forecasts of the FIGARCH(1,d,1) are obtained as

$$\hat{\sigma}_{t+h}^2 = \omega\,(1 - \beta)^{-1} + \sum_{i=1}^{h-1} \eta_i \hat{\sigma}_{t+h-i}^2 + \sum_{j=0}^{\infty} \eta_{h+j} x_{t-j}^2. \tag{19}$$

3.1.6 The Hyperbolic GARCH Model

Recently developed by Davidson (2004), the hyperbolic GARCH (HYGARCH) model is constructed in a way that allows the model not only to reproduce long memory features in volatility of many financial time series, but also (unlike FIGARCH) to be covariance stationary. The HYGARCH(1,d,1) process models the conditional variance as

$$\begin{aligned} \sigma_t^2 &= \omega + \left\{1 - \beta(L) - \phi(L)\left[(1 - \tau) + \tau(1 - L)^d\right]\right\} x_t^2 + \beta\sigma_{t-1}^2 \\ &= \omega(1 - \beta)^{-1} + \lambda(L) x_t^2 \end{aligned} \tag{20}$$

where $\lambda(L) = \left\{1 - (1 - \beta(L))\phi(L)\left[(1 - \tau) + \tau(1 - L)^d\right]\right\}$, $\omega > 0$, $\phi < 1$, $\beta < 1$, $0 \le d \le 1$ and $\tau \ge 0$. $\lambda(L) = \lambda_1 L + \lambda_2 L^2 + \dots$, $\lambda_j \ge 0$ for $j = 1, 2, \dots$. L is the lag operator and the HYGARCH model reduces to FIGARCH and IGARCH when $\tau = 1$ and $\tau = 0$, respectively. *Eqs.* (14) and (15) become

$$\beta - \tau d \le \phi \le \frac{(2 - d)}{3} \tag{21}$$

and

$$\tau d\left[\phi - \frac{(1 - d)}{2}\right] \le \beta(\tau d - \beta + \phi). \tag{22}$$

We refer the reader to Conrad (2010) for more details on the non-negativity conditions for the HYGARCH model and for the proof for the covariance stationarity of the process. The h-step ahead forecasts of the HYGARCH(1,d,1) are easily obtained by following the same procedures used for FIGARCH(1,d,1).

3.1.7 The Fractionally Integrated APARCH Model

Inspired by the FIGARCH model Tse (1998) incorporates fractional differences into the asymmetric power ARCH model of Ding et al. (1993) to obtain the fractionally intergrated APARCH model. The FIAPARCH(1,d,1) model is defined as

$$\sigma_t^{\delta} = \omega + \left[1 - \beta(L) - \phi(L)(1-L)^d\right](|x_{t-1}| - \gamma x_{t-1})^{\delta} + \beta\sigma_{t-1}^{\delta}, \tag{23}$$

where $\omega > 0$, $\phi < 1$, $\beta < 1$, $0 \leq d \leq 1$ and $-1 < \gamma < 1$.

The FIAPARCH process seems to be a promising model due to the fact that it is able to simultaneously capture long memory and asymmetric leverage effects in the data. The FIAPARCH model encompasses the FIGARCH model for $\gamma = 0$ and $\delta = 2$. Following the same procedures described above the forecasts for future variance can be easily obtained.

Note that the parameters in all formulas for forecasting future volatility have to be replaced by their corresponding estimates. All GARCH-type models are estimated via (quasi-) maximum likelihood as it is customary in the literature.

3.2 The Markov-Switching Multifractal Model

The recently introduced Markov-switching multifractal models are characterized by a multiplicative rather than additive structure of the volatility process. In the MSM framework instantaneous volatility is modeled as a product of k volatility components or multipliers $M_t^1, M_t^2, \ldots, M_t^k$ and a positive scale factor σ^2 (cf. Calvet and Fisher, 2001, 2004; Lux, 2008). Formally, we have

$$\sigma_t^2 = \sigma^2 \prod_{i=1}^{k} M_t^{(i)}. \tag{24}$$

The multipliers or volatility components are assumed to be independent of each other at any time and satisfy $\mathbb{E}\left[M_t^i\right] = 1$. Each multiplier M_t^i is renewed at time t with probability γ_i depending on its rank within the hierarchy of multipliers and remains unchanged with probability $1-\gamma_i$. In their seminal paper Calvet and Fisher (2001) derived a formalization for the transition probabilities, γ_i, that guarantee the convergence of the discrete-time MSM to a Poisson multifractal process in the continuous-time limit. Here we are not interested in the continuous-time process, and therefore, we prefer to use the pre-specified transition probabilities proposed by Lux (2008) that are given by

$$\gamma_i = 2^{i-k}. \tag{25}$$

To fully specify the MSM model we assume that the random multipliers follow a Lognormal[6] distribution with parameters λ and ν, i.e.,

$$M_t^i \sim LN(-\lambda, \nu). \tag{26}$$

We normalize the distribution of the multipliers to guarantee $\mathbb{E}\left[M_t^i\right] = 1$ which leads to

$$\exp\left(-\lambda + \frac{1}{2}\nu^2\right) = 1. \tag{27}$$

From *eq.* (27) it is obvious that the shape parameter ν can be expressed as: $\nu = \sqrt{2\lambda}$. With this restriction the Lognormal distribution of multipliers is fully defined by the scale parameter λ. So, the parameters to be estimated in the Lognormal MSM (LMSM) are only λ and σ. We carry out their estimation for all specifications $k = 2, \ldots, 20$ using the GMM approach proposed by Lux (2008). We then choose the specification with the lowest GMM criterion as our preferred model for the subsequent forecasting exercise. Note that higher k increases the number of regimes (which is 2^k), and generates proximity to long memory over a larger number of lags, but comes at no additional computational cost in our approach. The pertinent moments used for the estimation can be found in Lux (2008). Note that maximum likelihood would be possible only for MSM models with a finite, discrete support of the multipliers, and computationally feasible only for a limited number of hierarchical components up to about 8.

We perform the out-of-sample forecasting on the base of the LMSM model using the standard approach for best linear forecasts outlined in Brockwell and Davis (1991) together with the generalized Levinson-Durbin algorithm proposed by Brockwell and Dahlhaus (2004). The forecasting procedure is performed in two steps.

1. In the first step: We compute the following zero-mean time series

$$Z_t = x_t^2 - \mathbb{E}\left[x_t^2\right] = x_t^2 - \sigma^2, \tag{28}$$

   where $\hat{\sigma}$ is the estimate of the scale factor σ.

2. In the second step: Assuming that the oil price volatility data follow the stationary process $\{Z_t\}$ defined in the first step, h-step best linear forecasts are given by

$$\hat{Z}_{n+h} = \sum_{i=1}^{n} \psi_{ni}^{(h)} Z_{n+1-i} = \Psi_n^{(h)} \mathbf{Z}_n, \tag{29}$$

   where the vectors of weights $\Psi_n^{(h)} = \left(\psi_{n1}^{(h)}, \psi_{n2}^{(h)}, \ldots, \psi_{nn}^{(h)}\right)'$ are solutions of

[6] Other distributional assumptions such as Binomial, Gamma can be used as well, but have been found to make little difference in previous literature, cf. Liu et al. (2007), Lux (2008).

$$\Gamma\Psi_n^{(h)} = \gamma_n^h, \tag{30}$$

with $\gamma_n^h = (\gamma(h), \gamma(h+1), \ldots, \gamma(n+h-1))'$ being the auto-covariances for the data generating process of Z_t at lags h and beyond, and $\Gamma_n = [\gamma(i-j)]_{i,j=1,\ldots,n}$ the pertinent variance-covariance matrix. The pertinent auto-covariances for the multifractal model can be found in Lux (2008).

In sum, our portfolio of volatility models includes two linear GARCH models (GARCH, IGARCH), six nonlinear GARCH models (EGARCH, GJR-GARCH, APARCH, FIGARCH, HYGARCH, FIAPARCH) and one multifractal model (LMSM).

4 Forecast Evaluation Methodologies

To obtain our forecasts we proceed as follows: We first split the pre-1900 data set containing oil price observations from January 3, 1875 to December 31, 1895 into two subgroups. The first one covers the period from January 3, 1875 to December 31, 1892 and is used as in-sample data for model estimation. The second one contains oil prices of the last three years, i.e., from January 3, 1893 to December 31, 1895 and serves as out-of-sample data that we use for evaluation purposes. The estimation period is rolled forward by adding one observation and removing one day by day, so that the size of the data set used for the estimation remains fixed over the out-of-sample period. Forecasts are computed for horizons of various lengths: 1, 5, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100 days.

Second, we take a portion of the post-1900 data that contains oil price observations from January 6, 1992 to December 31, 2009 and split the oil price observations into in-sample data for volatility estimation covering the period from January 6, 1992 to December 29, 2006 and out-of-sample data stretching over the period from January 2, 2007 to December 31, 2009, which is in line with Wei et al. (2010). The great recession of 2008-2009 after the global financial crisis of 2007-2008 caused a demand contraction of oil and oil prices fluctuated from USD 145.31 (July 03, 2008) to USD 30.28 per barrel (December 23, 2008). Therefore, we find that this period should be interesting for testing the performance of our volatility models.

Third, we consider the extended data set covering the period from January 06, 1992 to March 24, 2014. This period of time does not cover only the great recession of 2008-2009, but also the subsequent recovery of the world economy. During this period the oil price stabilized at about USD 100 per barrel. We use oil price observations from January 6, 1992 until December 31, 2009 as in-sample data and the remaining observations, i.e., oil prices from January 4, 2010 to March 24, 2014 as of-out-sample data.

Finally, we also take the whole post-1900 data, i.e., from January 3, 1977 to March 24, 2014 to evaluate the contribution of a longer in-sample set. We use oil price observations from January 3, 1977 until December 31, 2009 as in-sample data and the remaining observations, i.e., oil prices from January 4, 2010 to March 24, 2014 as of-out-sample data. Note that forecasts in the second, third and fourth forecasting experiments are computed as previously done in the first one.

4.1 Forecasting Evaluation Criteria

We evaluate the forecasting ability of our volatility models in all four forecasting experiments by means of the following six different loss functions:

$$\mathrm{MSE} = T^{-1}\sum_{i=1}^{T}\left(\sigma_{f,t}^{2} - \sigma_{a,t}^{2}\right)^{2}, \tag{31}$$

$$\mathrm{MAE} = T^{-1}\sum_{i=1}^{T}\left|\sigma_{f,t}^{2} - \sigma_{a,t}^{2}\right|, \tag{32}$$

$$\mathrm{HMSE} = T^{-1}\sum_{i=1}^{T}\left(1 - \frac{\sigma_{a,t}^{2}}{\sigma_{f,t}^{2}}\right)^{2}, \tag{33}$$

$$\mathrm{HMAE} = T^{-1}\sum_{i=1}^{T}\left|1 - \frac{\sigma_{a,t}^{2}}{\sigma_{f,t}^{2}}\right|, \tag{34}$$

$$\mathrm{QLIKE} = T^{-1}\sum_{i=1}^{T}\left[\ln\left(\sigma_{f,t}^{2}\right) + \frac{\sigma_{a,t}^{2}}{\sigma_{f,t}^{2}}\right], \tag{35}$$

$$\mathrm{RLOG} = T^{-1}\sum_{i=1}^{T}\left[\ln\left(\frac{\sigma_{a,t}^{2}}{\sigma_{f,t}^{2}}\right)\right]^{2}, \tag{36}$$

where $\sigma_{f,t}^{2}$ denotes the volatility forecast obtained using a GARCH-type model or MSM model, $\sigma_{a,t}^{2}$ is the daily actual volatility that is computed using the daily squared returns, and T denotes the number of out-of-sample observations. MSE and MAE are the mean square error and mean absolute error, respectively, and HMSE and HMAE are their corresponding heteroscedasticity adjusted statistics. QLIKE quantifies the loss implied by a Gaussian likelihood and RLOG puts more weight on small observations (cf. Bollerslev et al., 1994).

All the above-mentioned loss functions are well known in the literature and each of them can be used depending on the contexts and the objective of the users. However, based only on these loss function criteria, it is difficult to conclude that the forecasting performance of one model dominates that of the other one. To draw such conclusions, we need statistical tests that can provide more reliable information. In the next section, we briefly describe the superior predictive ability (SPA) test of Hansen (2005).

4.2 Superior Predictive Ability Test

The superior predictive ability (SPA) test of Hansen (2005) sheds light on the relative performance of a particular model in comparison with its competitors. In other words, it answers the question whether any of the alternative models are better than the particular benchmark model in terms of expected loss. The null hypothesis that the benchmark model is not dominated by any of the other competitive models is postulated as follows

$$H_0 : \max_{i=1,\dots,K} \mathbb{E}\left[d_t\right] \leq 0, \tag{37}$$

where $d_t = (d_{i,t}, \dots, d_{K,t})'$ is a vector of relative performances, $d_{i,t}$, that are computed as $d_{i,t} = L^{(0)}_{t,h} - L^{(i)}_{t,h}$. K is the number of the competitive models, h denotes the forecasting horizon and $L^{(0)}_{t,h}$ and $L^{(i)}_{t,h}$ are the loss functions at time t for a benchmark model M_0 and for its competitor models, $M_{i_{(i=1,\dots,K)}}$, respectively.

The associated test statistic is given by

$$\text{SPA} = \max_{i=1,\dots,K} \frac{\sqrt{T}\bar{d}_i}{\sqrt{\lim_{T\to\infty} \mathbb{V}ar(\sqrt{T}\bar{d}_i)}}, \tag{38}$$

where $\bar{d} = T^{-1}\sum d_t$. We use a stationary bootstrap procedure to obtain the p-values of the SPA. A high p-value indicates non-rejection of the null hypothesis that a particular model is not outperformed by its competitors. We refer the reader to Hansen (2005) for more details on technical issues.

5 Empirical Results

5.1 Estimation Results

We estimate the GARCH models via the ML approach and the results are reported in Tables 5, 6, 7 and 8. Overall the estimates of β in GARCH, IGARCH, EGARCH, GJR-GARCH and APARCH models are close to 1 and significant at the 1% level. While the asymmetric leverage parameters are significant at the 1% level in the EGARCH model in Tables 5, 6 and 7 and not significant at any level in Table 8, they are insignificant at any level in the GJR-GARCH and APARCH models.

With the pre-1900 oil price data, the estimate of τ in the HYGARCH model is quite close to 1 and significant at the 1% level. The estimates of δ are 1.748 in the APARCH model and 1.315 in the FIAPARCH model. In contrast to the APARCH model the asymmetric leverage parameter in the FIAPARCH model is significant at the 1% level. The estimates of d in FIGARCH, HYGARCH and FIAPARCH models are significant at the 1% level and give evidence of the presence of long memory effects in oil price volatility.

With the post-1900 oil price data, we first estimate the GARCH models using oil price observations from January 6, 1992 to December 31, 2009. Here the estimate of τ in the HYGARCH model is significant at the 1% level and different from 1. By expanding the estimation sample, i.e. from January 6, 1992 to March 24, 2014, we do not observe a dramatic change in the estimation results. The estimates of d are significant at the 1% level in all long memory GARCH models.

Finally, we estimate the whole post-1900 oil price data. The estimates of d in FI-GARCH, HYGARCH and FIAPARCH models are now equal to 1 and significant at the 1% confidence level. These results indicate the presence of infinite persistence in the oil price data post-1900.

When we look at the estimation diagnostics, it seems that the three long memory GARCH models perform better in terms of fitting oil price observations over all different periods of time. In sum, the Log(L), AIC and BIC for the long memory models are smaller than those of short memory models. Furthermore, the Ljung-Box tests on the squared residuals and the ARCH tests also speak in favor of the long memory models. For all three long memory models the Ljung-box tests mostly cannot reject the null hypothesis of no serial correlation in the squared standardized residuals at the 5% level and the ARCH tests mostly accept the null hypothesis that the standardized residuals consist of independent identically distributed (*i.i.d*) Gaussian disturbances.

We now turn to the estimation of the Lognormal MSM. The best GMM objective function implies a high number of hierarchical levels, $k = 20$. The estimates of the Lognormal parameter, $\hat{\lambda}$, and the scale factor parameter, $\hat{\sigma}$, are reported in Table 9. Higher $\hat{\lambda}$ in the pre-1900 era indicates a higher degree of fractality of the series in the 19th than the 20th and 21st centuries, i.e. more pronounced changes between tranquil and turbulent phases which is in harmony with the visual impression of more "spikyness" in the years 1875-1895.

5.2 Forecasting Results

The results of the SPA test for our three forecasting exercises for all our volatility models are reported in Tables 10, 11, 12, 13, 14, 15, 16, and 17. The first column in each table contains the benchmark models and each model is tested against the remaining eight models. It also contains individual model combination forecasts that are tested against all nine single models. The p-values of the SPA test are computed based on 5000 bootstrap samples in the empirical test under any pre-specified loss function. First, we observe that in each case of the four forecasting exercises none of our volatility models can outperform all other models at short and long horizons across all six different loss functions. The forecasting performance of our volatility models also differs from one sample period to another. Often, a volatility model that provides relatively accurate forecasts for a period of time might perform poorly in terms of forecasting performance when expanding or reducing the sample size. However, all in all it seems that the long memory volatility models are more appropriate to forecast oil price volatility. We also observe that for the more standard loss functions such as MSE or MAE, the MSM model mostly cannot be outperformed. Based on the SPA results, we count for each of our volatility models the cases where it cannot be outperformed by others across all time spans and criteria. The results indicate that in 99 cases the LMSM cannot be outperformed by its competitor models at the 10% confidence level, followed by HYGARCH (94 cases), FIAPARCH (89 cases), GARCH (74 cases), EGARCH (70 cases), IGARCH (49 cases), FIGARCH (45 cases), GJR-GARCH (42 cases), and APARCH (28 cases). Overall, the new multifractal model, therefore, appears to perform better on average than any particular model from the GARCH family. This is particularly remarkable as (i) it has fewer parameters than all GARCH-type models (i.e., only two while the second best, the HYGARCH model, comes with five parameters that have to be estimated), (ii) our estimation and forecasting methods used for the multifractal model are not the most efficient ones, while we have used the most efficient ML estimates and conditional expectations based upon those to compute forecasts for the GARCH family. Across time periods and criteria we find the following tendencies: First, the MSM and FIAPARCH do well and cannot be rejected as non-dominated models for the 19th century data and for the 2010-2014 out-of-sample period. Both do not perform well for the 2007-2009 out-of-sample period. The HYGARCH model gains its prominent rank particularly from its better performance in this period, but also other short-memory GARCH-type models do better in this period than in the others. Presumably, the higher volatility in the crisis period rewards a concentration on the short-run dynamics rather than long trends in volatility. Across criteria, the RLOG statistic is typically an outlier in its patterns of SPA results which is not surprising given the higher weight it attributes to small rather than large events.

The difficulty to discover a uniformly best model across all six different loss functions at short and long horizons motivates us to also try simple average forecast combinations. Granger and Teräsvirta (1999) and Aiolfi and Timmermann (2006) pointed out that it is often preferable to combine forecasts from competitive models in a linear way and thereby generate hopefully superior predictions. Following this idea, we adopt two different combination strategies. The first combination strategy is given by equally weighted linear combinations of short memory GARCH-type models (GARCH, IGARCH, GJR-GARCH, EGARCH and APARCH) and long memory GARCH-type and MSM models (FIGARCH, HYGARCH, FIAPARCH and LMSM). The second one is also obtained by equally weighted linear combinations of long memory GARCH-type models and the LMSM. Both combination strategies shed light on the complementarities of the short- and long-memory GARCH models on the one side and the complementarities of two classes of volatility models, GARCH-type and MSM, on the other side. In fact, both strategies lead to a high number of forecast combinations. To reduce the number of forecast combinations we only considered GARCH-type models that have the highest p-values according to the SPA test results for our single volatility models. Note that this selection criterion does not hold for the LMSM, so that we always combined the best GARCH-type models in terms of their p-values with the LMSM in order to explore their complementarities. This selection criterion for GARCH-type models led to different forecast combinations for different loss functions. The new predictor is tested against the single models and the test results are reported in Tables 10, 11, 12, 13, 14, 15, 16, and 17. The results are diverse: First, one often observes that forecast combinations of two relatively successful models do not necessarily improve performance against single models. This holds particularly for combinations of short-memory GARCH specifications. Combinations of long-memory GARCH models with the MSM model are more often successful, but we nevertheless find cases where the combination of well performing single models can be outperformed by the forecasts from one or more of those single models. This exercise underlines that forecast combination is a delicate operation: There is apparently no guarantee that two good models are complementary in their virtues, they could also lead to an overall deterioration when applied in combination. This underscores the necessity of finding more elaborate rules for combinations that are data-driven and react on the single models' advantages and disadvantages.

6 Conclusion

This paper has analyzed the forecasting performance of two classes of volatility models, namely the GARCH-type models and the MSM model via six different loss functions and the superior predictive ability test. The analysis is performed by using a large sample of oil prices of the pre- and post-1900 period. Results were largely uniform for the data of the 19th century and the later record of the 20th/21st centuries with the crisis period 2007 - 2009 showing somewhat unusual behavior. Empirical results of the SPA test indicate that none of the volatility models including the MSM model can outperform their competitor models under all loss criteria. As it turned out, however, the new MSM model most often cannot be outperformed when standard loss functions are used. Across all forecasting horizons and subsamples used, it is the model that in the highest number of cases cannot be outperformed by any other models, and, in this respect, it beats all simple models from our broad selection of GARCH-type processes. Forecast combination exercises point to more robustness of combinations of long-memory GARCH and MSM models rather than short-memory GARCH models. However, superior forecast performance of combined models against their single components is in no way guaranteed.

All in all, the MSM model appears a valuable addition to the toolbox of volatility models not only for financial assets, but also for commodities like oil. Given its highest number of non-rejections by the SPA test, it comes out as the more robust model compared to any GARCH specification, and it also is the most parsimonious one among all candidates considered.

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Table 1: Descriptive statistics of the data pre-1900

	Log-returns	Absolute returns	Squared returns
6376 observations (from January 02,1875 to Decem 31, 1895)			
Minimum	-16.186	0	0
Maximum	33.647	33.647	1.132E+3
Mean	-0.007	1.439	5.129
Standard deviation	2.265	1.749	21.291
Skewness	0.752	3.715	29.944
Kurtosis	18.240	34.497	1.460E+3
Hurst index	0.540	0.842^{***}	0.868^{***}
Hill tail index at 5% tail	2.547 **[2.485 2.610]**		
Q(10)	79.177	2.467E+3	659.798
Q(20)	100.685	3.169E+3	712.731
JB	5.343E+4		
ADF	- 73.875		

Note: *** indicates 1% significance of Hurst coefficients based on the simulated boundary values of Weron (2002) for Wiener Brownian motion. For the tail index estimates, the brackets contain the 95% percent confidence intervals of the point estimate based upon the limiting distribution of the estimator.

Table 2: Descriptive statistics of the data post-1900 containing oil prices from Jan 06,1992 to December 31, 2009

	Log-returns	Absolute returns	Squared returns
4521 observations (from January 06,1992 to December 31, 2009)			
Minimum	-17.092	0	0
Maximum	16.414	17.092	292.129
Mean	0.031	1.748	6.078
Standard deviation	2.466	1.739	16.326
Skewness	-0.154	2.700	8.361
Kurtosis	8.222	15.256	99.210
Hurst index	0.490	0.856^{***}	0.905^{***}
Hill tail index at 5% tail	2.797 **[2.716 2.879]**		
Q(10)	31.015	1099.7	875.701
Q(20)	42.686	2051.1	1556.1
JB	5155.4		
ADF	- 68.396		

Note: *** indicates 1% significance of Hurst coefficients based on the simulated boundary values of Weron (2002) for Wiener Brownian motion. For the tail index estimates, the brackets contain the 95% percent confidence intervals of the point estimate based upon the limiting distribution of the estimator.

Table 3: Descriptive statistics of the data post-1900 containing oil prices from Jan 06,1992 to March 24, 2014

	Log-returns	Absolute returns	Squared returns
5590 observations (from January 06,1992 to March 24, 2014)			
Minimum	-17.092	0	0
Maximum	16.414	17.092	292.129
Mean	0.030	1.654	5.482
Standard deviation	2.341	1.657	15.030
Skewness	-0.145	2.753	8.889
Kurtosis	8.525	15.993	113.634
Hurst index	0.471	0.868***	0.910***
Hill tail index at 5% tail	2.899 **[2.823 3.975]**		
Q(10)	31.447	1.419E+3	1.127E+3
Q(20)	41.038	2.647E+3	1.999E+3
JB	7.129E+3		
ADF	-75.996		

Note: *** indicates 1% significance of Hurst coefficients based on the simulated boundary values of Weron (2002) for Wiener Brownian motion. For the tail index estimates, the brackets contain the 95% percent confidence intervals of the point estimate based upon the limiting distribution of the estimator.

Table 4: Descriptive statistics of the complete data post-1900

	Log-returns	Absolute returns	Squared returns
9417 observations (from January 03,1977 to March 24, 2014)			
Minimum	-40.204	0	0
Maximum	19.861	40.204	1614.4
Mean	0.021	1.363	4.922
Standard deviation	2.219	1.751	22.929
Skewness	-0.832	3.962	40.176
Kurtosis	22.738	42.191	2624.4
Hurst index	0.517	0.938***	0.944***
Hill tail index at 5% tail	2.668 **[2.614 2.722]**		
Q(10)	52.030	7.804E+3	1.458E+3
Q(20)	75.992	1.458E+4	1.681E+3
JB	1.540E+5		
ADF	-98.326		

Note: *** indicates 1% significance of Hurst coefficients based on the simulated boundary values of Weron (2002) for Wiener Brownian motion. For the tail index estimates, the brackets contain the 95% percent confidence intervals of the point estimate based upon the limiting distribution of the estimator.

Table 5: Estimation results using oil prices from January 2, 1875 to December 31, 1895

	GARCH	IGARCH	EGARCH	GJR-GARCH	APARCH	FIGARCH	HYGARCH	FIAPARCH
ω	0.066	0.065	0.083	0.066	0.074	0.301	0.277	0.224
	(0.033)	**(0.029)**	**(0.016)**	**(0.035)**	**(0.052)**	**(0.040)**	**(0.054)**	**(0.051)**
α	0.127	0.128	0.007	0.127	0.131			
	(0.027)	**(0.033)**	**(0.012)**	**(0.028)**	**(0.029)**			
β	0.871	0.872	0.961	0.872	0.869	0.550	0.550	0.524
	(0.030)	**(0.033)**	**(0.011)**	**(0.031)**	**(0.038)**	**(0.080)**	**(0.078)**	**(0.097)**
γ			0.302	0.001	0.002			-0.239
			(0.037)	**(0.003)**	**(0.001)**			**(0.062)**
δ					1.748			1.315
					(0.776)			**(0.114)**
ϕ						0.254	0.255	0.302
						(0.052)	**(0.051)**	**(0.056)**
d						0.493	0.489	0.396
						(0.049)	**(0.047)**	**(0.062)**
τ							1.012	
							(0.019)	
				Diagnostic				
Log(L)	-12806	-12806	-12757	-12806	-12802	-12749	-12749	-12729
AIC	25618	25618	25522	25620	25614	25506	25508	25470
BIC	25638	2538	25550	25647	25648	25533	25541	25511
$Q(20)$	37.248	37.240	34.517	37.263	37.349	32.905	32.782	32.309
	[0.011]	**[0.011]**	**[0.023]**	**[0.011]**	**[0.011]**	**[0.035]**	**[0.036]**	**[0.040]**
$Q^2(20)$	28.591	28.561	30.376	28.585	31.990	16.956	16.862	22.671
	[0.090]	**[0.097]**	**[0.064]**	**[0.096]**	**[0.043]**	**[0.656]**	**[0.662]**	**[0.305]**
Arch(20)	27.544	27.525	29.137	27.536	30.653	16.490	16.395	22.105
	[0.121]	**[0.121]**	**[0.085]**	**[0.121]**	**[0.060]**	**[0.686]**	**[0.692]**	**[0.335]**

Note: The numbers in parentheses are standard errors of the estimations. Log(L) is the logarithm maximum likelihood function. AIC and BIC are the Akaike and Bayesian information criterion respectively. $Q(20)$ and $Q^2(20)$ are the Ljung-Box Q-statistics of order 20 obtained from the standardized residuals and squared standardized residuals respectively. ARCH(20) denotes the no conditional heteroscedasticity statistic of order 20. The values reported in square brackets are the p-values of the statistics.

Table 6: Estimation results using oil prices from January 06,1992 to December 31, 2009

	GARCH	IGARCH	EGARCH	GJR-GARCH	APARCH	FIGARCH	HYGARCH	FIAPARCH
ω	0.052	0.033	0.027	0.051	0.046	0.569	0.255	0.197
	(0.022)	**(0.017)**	**(0.016)**	**(0.022)**	**(0.026)**	**(0.132)**	**(0.106)**	**(0.186)**
α	0.064	0.068	-0.005	0.068	0.072			
	(0.017)	**(0.019)**	**(0.008)**	**(0.029)**	**(0.019)**			
β	0.929	0.932	0.988	0.930	0.928	0.469	0.414	0.382
	(0.017)	**(0.019)**	**(0.028)**	**(0.019)**	**(0.002)**	**(0.083)**	**(0.026)**	**(0.090)**
γ			0.156	-0.008	0.003			-0.132
			(0.019)	**(0.028)**	**(0.021)**			**(0.068)**
δ					1.63			1.889
					(0.241)			**(0.175)**
ϕ						0.214	0.204	0.211
						(0.092)	**(0.056)**	**(0.072)**
d						0.364	0.290	0.261
						(0.046)	**(0.059)**	**(0.052)**
τ							1.112	
							(0.281)	
				Diagnostic				
Log(L)	-10024	-10026	-10022	-10023	-10020	-10014	-10013	-10008
AIC	20054	20056	20052	20055	20054	20037	20035	20029
BIC	20073	20069	20078	20081	20082	20062	20068	20067
$Q(20)$	19.655	19.418	21.498	19.502	20.389	22.225	22.506	22.991
	[0.480]	**[0.495]**	**[0.368]**	**[0.489]**	**[0.434]**	**[0.328]**	**[0.314]**	**[0.289]**
$Q^2(20)$	43.085	42.259	49.062	43.338	45.733	30.682	29.907	30.403
	[0.002]	**[0.003]**	**[<0.001]**	**[0.002]**	**[<0.001]**	**[0.060]**	**[0.071]**	**[0.064]**
Arch(20)	37.525	36.912	41.822	37.727	39.417	28.478	27.943	28.193
	[0.010]	**[0.012]**	**[0.003]**	**[0.010]**	**[0.006]**	**[0.099]**	**[0.111]**	**[0.105]**

Note: The numbers in parentheses are standard errors of the estimations. Log(L) is the logarithm maximum likelihood function. AIC and BIC are the Akaike and Bayesian information criterion respectively. $Q(20)$ and $Q^2(20)$ are the Ljung-Box Q-statistics of order 20 obtained from the standardized residuals and squared standardized residuals respectively. ARCH(20) denotes the no conditional heteroscedasticity statistic of order 20. The values reported in square brackets are the p-values of the statistics.

Table 7: Estimation results oil prices from from January 06,1992 to March 24, 2014

	GARCH	IGARCH	EGARCH	GJR-GARCH	APARCH	FIGARCH	HYGARCH	FIAPARCH
ω	0.041	0.028	0.022	0.041	0.039	0.431	0.149	4.982E-5
	(0.020)	**(0.013)**	**(0.010)**	**(0.019)**	**(0.071)**	**(0.105)**	**(0.153)**	**(3.558E−5)**
α	0.063	0.067	0.144	0.057	0.069			
	(0.017)	**(0.017)**	**(0.038)**	**(0.021)**	**(0.034)**			
β	0.931	0.933	0.989	0.932	931	0.491	0.431	0.135
	(0.018)	**(0.017)**	**(0.005)**	**(0.017)**	**(0.053)**	**(0.138)**	**(0.124)**	**(0.049)**
γ			0.221	0.011	-0.017			-0.999
			(0.026)	**(0.020)**	**(0.009)**			**(0.312)**
δ					1.610			1.574
					(0.817)			**(0.195)**
ϕ						0.251	0.238	0.111
						(0.132)	**(0.127)**	**(0.041)**
d						0.360	0.285	0.070
						(0.044)	**(0.054)**	**(0.020)**
τ							1.113	
							(0.069)	
				Diagnostic				
Log(L)	-12074	-12076	-12069	-12073	-12068	-12061	-12059	-12069
AIC	24154	24156	24147	24154	24146	24130	24128	24149
BIC	24174	24170	24173	24181	24179	24157	24161	24189
$Q(20)$	17.345	17.195	19.311	17.682	18.507	20.055	20.352	22.334
	[0.631]	**[0.640]**	**[0.502]**	**[0.608]**	**[0.554]**	**[0.455]**	**[0.436]**	**[0.323]**
$Q^2(20)$	47.267	46.878	55.311	47.102	50.154	29.610	28.514	36.200
	[<0.001]	**[<0.001]**	**[<0.001]**	**[<0.001]**	**[<0.001]**	**[0.076]**	**[0.098]**	**[0.015]**
Arch(20)	41.457	41.263	47.483	41.304	43.473	27.511	26.755	33.973
	[0.003]	**[0.003]**	**[<0.001]**	**[0.003]**	**[0.001]**	**[0.122]**	**[0.142]**	**[0.026]**

Note: The numbers in parentheses are standard errors of the estimations. Log(L) is the logarithm maximum likelihood function. AIC and BIC are the Akaike and Bayesian information criterion respectively. $Q(20)$ and $Q^2(20)$ are the Ljung-Box Q-statistics of order 20 obtained from the standardized residuals and squared standardized residuals respectively. ARCH(20) denotes the no conditional heteroscedasticity statistic of order 20. The values reported in square brackets are the p-values of the statistics.

Table 8: Estimation results oil prices from from January 03,1977 to March 24, 2014

	GARCH	IGARCH	EGARCH	GJR-GARCH	APARCH	FIGARCH	HYGARCH	FIAPARCH
ω	3.852E-4	3.846E-4	0.031	3.791E-4	2.098E-4	0.004	0.004	0.011
	(1.986E−4)	**(1.946E−4)**	**(0.006)**	**(1.949E−4)**	**(4.187E−4)**	**(0.002)**	**(0.002)**	**(0.013)**
α	0.079	0.079	0.221	0.074	0.088			
	(0.008)	**(0.009)**	**(0.027)**	**(0.015)**	**(0.017)**			
β	0.921	0.921	0.987	0.921	912	0.924	0.914	0.921
	(0.009)	**(0.009)**	**(0.004)**	**(0.009)**	**(0.033)**	**(0.014)**	**(0.018)**	**(0.016)**
γ			-0.016	0.010	-0.029			-0.203
			(0.015)	**(0.019)**	**(0.268)**			**(0.016)**
δ					2.250			1.591
					(0.893)			**(0.097)**
ϕ						4.632E-8	3.123E-8	1.898E-7
						(1.405E−8)	**(1.173E−8)**	**(9.175E−7)**
d						1.000	1.000	1.00
						(0.023)	**(0.038)**	**(0.031)**
τ							1.017	
							(0.006)	
				Diagnostic				
Log(L)	-16182	-16180	-15963	-16180	-16131	-16114	-16047	-16030
AIC	32370	32365	31933	32368	32273	32236	32103	32072
BIC	32392	32379	31962	32397	32308	32264	32139	32115
$Q(20)$	177.286	177.450	132.202	179.483	177.855	187.193	193.544	204.216
	[0.000]	**[0.000]**	**[0.000]**	**[0.000]**	**[0.000]**	**[0.000]**	**[0.000]**	**[0.000]**
$Q^2(20)$	12.904	12.866	7.523	13.313	12.836	11.969	9.567	8.820
	[0.882]	**[0.883]**	**[0.995]**	**[0.864]**	**[0.884]**	**[0.917]**	**[0.974]**	**[0.984]**
Arch(20)	12.708	12.672	7.530	13.110	13.073	11.764	9.807	8.824
	[0.890]	**[0.891]**	**[<0.0.994]**	**[0.873]**	**[0.874]**	**[0.924]**	**[0.972]**	**[0.985]**

Note: The numbers in parentheses are standard errors of the estimations. Log(L) is the logarithm maximum likelihood function. AIC and BIC are the Akaike and Bayesian information criterion respectively. $Q(20)$ and $Q^2(20)$ are the Ljung-Box Q-statistics of order 20 obtained from the standardized residuals and squared standardized residuals respectively. ARCH(20) denotes the no conditional heteroscedasticity statistic of order 20. The values reported in square brackets are the p-values of the statistics.

Table 9: Estimation results of LMSM model

Parameters	Jan 2, 1875 to Dec 31, 1895	Jan 6, 1992 to Dec 31, 2009	Jan 6, 1992 to March 24, 2014	Jan 3, 1977 to March 24, 2014
λ	1.320	1.016	1.034	1.011
σ	2.252	2.465	2.341	2.218

Note that the optimal objective function of the GMM estimation is obtained for $k = 20$.

Table 10: Superior predictive ability (SPA) test results using oil price observations from January 3, 1875 to December 31, 1892 as in-sample and from January 3, 1893 to December 31, 1895 as out-of-sample.

Base model	Forecast horizons											
	1	5	10	20	30	40	50	60	70	80	90	100
						MSE						
GARCH	0.020	0.062	**0.156**	0.090	0.070	0.040	0.074	0.046	**0.134**	**0.432**	0.012	0.004
IGARCH	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GJR-GARCH	0.008	0.032	0.016	0.022	0.084	0.058	**0.338**	0.080	**0.226**	**0.226**	**0.604**	**0.632**
EGARCH	0.020	**0.118**	**0.740**	**0.644**	**0.656**	**0.688**	**1.000**	**0.766**	**0.980**	**0.752**	**0.578**	**0.170**
APARCH	0.004	0.014	0.002	0.004	0.018	0.014	**0.542**	0.032	**0.234**	**0.400**	**0.928**	**0.836**
FIGARCH	0.012	0.092	**0.700**	**0.554**	0.040	0.016	0.076	0.066	0.088	0.018	0.004	0.000
HYGARCH	**1.000**	**0.380**	**0.148**	0.032	0.024	0.038	0.038	0.034	0.038	0.026	0.020	0.014
FIAPARCH	0.044	**0.732**	**0.602**	**0.450**	**0.176**	0.052	0.078	0.050	0.050	0.028	0.022	0.018
LMSM	0.042	**0.170**	**0.880**	**0.924**	**0.404**	**0.312**	**0.316**	**0.270**	**0.240**	**0.204**	**0.130**	**0.126**
FCOM1	0.004	0.016	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FCOM11	0.026	0.202	**1.000**	**0.931**	**0.992**	**1.000**	**0.846**	**0.305**	**0.348**	**0.305**	**0.185**	**0.139**
FCOM111	0.049	**0.216**	**0.999**	**0.864**	**0.468**	0.086	**0.209**	**0.127**	**0.153**	**0.231**	0.047	0.012
						MAE						
GARCH	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
IGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GJR-GARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
EGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
APARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FIGARCH	0.000	0.002	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
HYGARCH	**1.000**	**0.512**	**0.360**	0.098	0.060	0.066	0.058	0.046	0.052	0.080	0.062	0.080
FIAPARCH	0.020	**0.488**	**0.984**	**1.000**	**0.574**	**0.292**	**0.344**	**0.276**	**0.192**	**0.170**	**0.142**	**0.144**
LMSM	0.000	0.018	**0.250**	0.056	**0.486**	**0.708**	**0.656**	**0.724**	**0.808**	**1.000**	**1.000**	**1.000**
FCOM2	0.000	0.000	0.005	0.000	0.007	0.048	0.015	0.062	**0.131**	**0.102**	0.073	0.076
FCOM21	0.000	0.045	**0.460**	**0.207**	**0.725**	**0.485**	**0.988**	**0.505**	**0.355**	**0.222**	**0.198**	**0.193**
						HMSE						
GARCH	**0.102**	**0.118**	0.070	0.058	0.042	0.062	0.034	0.054	0.014	0.022	0.038	0.032
IGARCH	0.024	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
GJR-GARCH	0.020	0.032	0.066	0.046	0.008	0.022	0.010	0.036	0.022	0.016	0.032	0.018
EGARCH	0.008	0.056	0.042	0.038	0.018	0.032	0.024	0.028	0.020	0.016	0.028	0.024
APARCH	0.008	0.036	0.076	0.050	0.014	0.030	0.018	0.040	0.008	0.020	0.034	0.030
FIGARCH	0.084	**0.130**	0.072	0.058	0.032	0.062	0.032	0.054	0.020	0.022	0.042	0.030
HYGARCH	**1.000**	0.048	0.082	0.064	0.022	0.088	0.010	0.050	0.006	0.034	0.058	0.030
FIAPARCH	0.040	**0.112**	0.072	0.052	0.014	0.052	0.008	0.026	0.020	0.030	0.056	0.014
LMSM	0.068	0.040	0.032	0.036	0.026	0.050	0.036	0.046	0.022	0.016	0.034	0.030
FCOM3	0.007	0.232	0.079	0.061	0.033	0.053	0.036	0.047	0.032	0.035	0.041	0.021

Note: The table entries are the p-values of the SPA test of Hansen (2005). The null hypothesis is that a base model cannot be outperformed by other competitor models. The values in bold face represent the p-values that are greater than or equal to the 10% confidence level under a pre-specified loss function. We combine: FCOM1=EGARCH+APARCH, FCOM11=EGARCH+LMSM, FCOM111=FIAPARCH+LMSM, FCOM2=HYGARCH+FIAPARCH+LMSM, FCOM21=FIAPARCH+LMSM, and FCOM3=IGARCH+HYGARCH+LMSM.

Table 11: Superior predictive ability (SPA) test results using oil price observations from January 3, 1875 to December 31, 1892 as in-sample and from January 3, 1893 to December 31, 1895 as out-of-sample.

Base model	Forecast horizons											
	1	5	10	20	30	40	50	60	70	80	90	100
						HMAE						
GARCH	0.012	0.060	0.046	0.046	0.018	0.044	0.012	0.020	0.004	0.012	0.006	0.006
IGARCH	0.006	**1.000**	**1.000**	**1.000**	**1.000**	0.016	0.008	0.012	0.000	0.002	0.012	0.002
GJR-GARCH	0.010	0.020	0.040	0.034	0.012	0.014	0.010	0.020	0.010	0.014	0.022	0.016
EGARCH	0.006	0.006	0.006	0.006	0.005	0.030	0.012	0.006	0.004	0.004	0.006	0.002
APARCH	0.006	0.008	0.020	0.016	0.006	0.026	0.008	0.012	0.004	0.004	0.022	0.008
FIGARCH	0.002	0.064	0.042	0.038	0.006	0.028	0.008	0.014	0.004	0.006	0.014	0.002
HYGARCH	**1.000**	0.010	0.018	0.010	0.000	0.026	0.000	0.004	0.000	0.002	0.004	0.000
FIAPARCH	0.050	0.094	0.042	0.020	0.004	0.018	0.002	0.002	0.000	0.002	0.004	0.000
LMSM	0.002	0.008	0.006	0.006	0.004	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
FCOM4	0.000	**0.695**	0.089	0.058	0.017	0.025	0.011	0.031	0.013	0.018	0.019	0.000
FCOM41	0.000	**0.165**	0.056	0.044	0.015	0.024	0.007	0.017	0.004	0.007	0.012	0.000
						QLIKE						
GARCH	0.000	**0.566**	**0.960**	**0.846**	**0.160**	0.032	**0.260**	**0.154**	**0.362**	**0.500**	0.066	0.024
IGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GJR-GARCH	0.000	0.000	0.000	0.006	**0.540**	**0.606**	**0.778**	**0.664**	**0.508**	**0.546**	**0.650**	**0.870**
EGARCH	0.000	0.060	**0.520**	**0.250**	**0.992**	**0.962**	**0.570**	**0.846**	**0.714**	**0.878**	**0.432**	**0.162**
APARCH	0.000	0.000	0.000	0.004	**0.600**	**0.524**	**0.402**	**0.308**	**0.632**	**0.604**	**0.524**	**0.302**
FIGARCH	0.000	**0.518**	**0.220**	**0.172**	0.072	0.052	0.014	0.022	0.006	0.014	0.022	0.000
HYGARCH	**1.000**	**0.786**	**0.224**	**0.128**	0.062	0.062	0.022	0.030	0.010	0.014	0.024	0.004
FIAPARCH	0.000	**0.156**	**0.146**	0.082	0.042	0.042	0.018	0.024	0.012	0.014	0.014	0.004
LMSM	0.000	**0.164**	**0.930**	**0.616**	**0.396**	**0.366**	**0.344**	**0.238**	**0.192**	**0.128**	0.082	0.076
FCOM5	0.000	0.002	**0.893**	**0.322**	**0.997**	**0.676**	**0.742**	**0.666**	**0.758**	**0.600**	**0.324**	0.075
FCOM51	0.000	0.000	**0.939**	**0.275**	**0.775**	**0.997**	**0.671**	**0.698**	**0.554**	**0.462**	**0.347**	**0.239**
FCOM511	0.000	0.001	**0.793**	**0.382**	**1.000**	**1.000**	**1.000**	**0.540**	**0.619**	**0.593**	**0.560**	**0.330**
FCOM5111	0.000	**0.494**	**0.870**	**0.676**	**0.455**	**0.143**	**0.247**	**0.100**	**0.155**	0.088	0.074	0.038
						RLOG						
GARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
IGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GJR-GARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
EGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
APARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FIGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
HYGARCH	0.000	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
FIAPARCH	**0.834**	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LMSM	0.023	**0.902**	**0.453**	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FCOM6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FCOM61	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FCOM611	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: The table entries are the p-values of the SPA test of Hansen (2005). The null hypothesis is that a base model cannot be outperformed by other competitor models. The values in bold face represent the p-values that are greater than or equal to the 10% confidence level under a pre-specified loss function. We combine: FCOM4=IGARCH+HYGARCH+LMSM, FCOM41=HYGARCH+LMSM, FCOM5=GARCH+GJR-GARCH+EGARCH, FCOM51=GJR-GARCH+EGARCH+HYGARCH, FCOM511=EGARCH+GJR-GARCH+LMSM, FCOM5111=HYGARCH+LMSM, FCOM6=GARCH+HYGARCH, FCOM61=HYGARCH+FIAPARCH, and FCOM611=HYGARCH+LMSM.

Table 12: Superior predictive ability (SPA) test results using oil price observations from January 6, 1992 to December 29, 2006 as in-sample and from January 2, 2007 to December 31, 2009 as out-of-sample.

Base model	Forecast horizons											
	1	5	10	20	30	40	50	60	70	80	90	100
							MSE					
GARCH	0.002	**0.282**	**0.780**	**0.820**	**0.294**	**0.532**	**0.410**	**0.894**	**0.302**	**0.314**	**0.100**	**0.100**
IGARCH	0.002	0.004	0.002	0.004	0.000	0.004	0.016	0.088	**0.960**	**0.938**	**0.550**	**0.828**
GJR-GARCH	0.004	0.008	0.004	0.002	0.000	0.004	0.000	0.000	0.000	0.000	0.000	0.000
EGARCH	0.002	0.000	0.004	0.006	0.000	0.006	0.010	0.036	**0.128**	**0.260**	**0.626**	**0.172**
APARCH	0.000	0.002	0.002	0.004	0.000	0.002	0.000	0.000	0.008	0.002	0.004	0.040
FIGARCH	0.000	**0.122**	0.020	0.016	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
HYGARCH	**1.000**	**0.838**	**0.332**	**0.222**	**0.706**	**0.536**	**0.654**	**0.170**	**0.294**	0.010	0.000	0.000
FIAPARCH	0.000	0.006	0.004	0.000	0.000	0.000	0.004	0.000	0.000	0.000	0.000	0.000
LMSM	0.002	0.080	0.018	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FCOM1	0.003	0.068	0.027	0.042	0.012	0.014	0.089	0.006	0.000	0.000	0.001	0.002
FCOM11	0.002	**0.235**	**0.111**	**0.124**	**0.383**	**0.767**	**0.573**	**0.286**	0.004	0.000	0.000	0.000
FCOM111	0.003	**0.310**	**0.500**	**0.670**	**0.636**	**0.993**	**0.994**	**0.302**	0.012	0.000	0.000	0.000
							MAE					
GARCH	0.000	0.016	0.016	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
IGARCH	0.000	**0.826**	**0.680**	**0.268**	**0.810**	**0.238**	**0.180**	0.002	0.002	0.000	0.000	0.000
GJR-GARCH	0.000	**0.596**	**0.420**	**0.114**	**0.930**	**0.970**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
EGARCH	0.000	0.074	**0.254**	**0.798**	**0.786**	**0.444**	0.064	0.012	0.000	0.000	0.000	0.000
APARCH	0.000	**0.202**	**0.760**	**0.184**	**0.310**	0.058	0.000	0.000	0.000	0.000	0.000	0.000
FIGARCH	0.000	0.006	0.002	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
HYGARCH	**0.124**	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FIAPARCH	**0.902**	0.084	0.054	0.028	**0.518**	**0.144**	0.078	0.056	0.038	**0.126**	0.036	**0.116**
LMSM	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FCOM2	0.002	**0.342**	0.002	**0.194**	**0.858**	**0.706**	0.086	0.015	0.000	0.000	0.000	0.000
FCOM21	0.002	**0.588**	**0.166**	0.017	**0.909**	**0.394**	**0.255**	0.016	0.009	0.000	0.000	0.000
FCOM211	0.025	**0.456**	**0.297**	0.043	**0.868**	**0.640**	**0.426**	**0.333**	**0.252**	**0.414**	**0.179**	**0.382**
							HMSE					
GARCH	0.038	**0.152**	**0.796**	**0.104**	**1.000**	**0.854**	**0.148**	**0.806**	**0.390**	**0.282**	0.086	0.000
IGARCH	0.000	0.002	0.002	0.000	0.000	0.022	0.018	**0.104**	**0.308**	**0.874**	**0.408**	**0.734**
GJR-GARCH	0.000	0.002	0.002	0.000	0.000	0.004	0.000	0.008	0.006	0.000	0.002	0.002
EGARCH	0.000	0.002	0.002	0.000	0.000	0.000	0.010	0.030	0.036	**0.512**	**0.592**	**0.266**
APARCH	0.000	0.000	0.006	0.000	0.000	0.002	0.006	0.012	0.008	0.010	0.008	0.004
FIGARCH	0.008	**0.128**	0.096	0.010	0.054	0.028	0.020	0.024	0.026	0.008	0.006	0.000
HYGARCH	**1.000**	**1.000**	**0.204**	**1.000**	**0.108**	**0.146**	**0.852**	**0.240**	**0.874**	**0.144**	0.056	0.020
FIAPARCH	0.000	0.086	0.070	0.018	0.040	0.028	0.002	0.000	0.002	0.010	0.000	0.010
LMSM	0.078	**0.122**	0.020	0.026	0.050	0.034	0.022	0.016	0.034	0.018	0.012	0.006
FCOM3	0.000	0.001	0.015	0.002	0.011	0.033	**0.175**	0.032	0.007	0.011	0.016	0.007
FCOM31	0.000	0.027	0.057	0.031	0.022	0.064	**0.831**	**0.158**	0.013	0.020	0.019	0.012

Note: The table entries are the p-values of the SPA test of Hansen (2005). The null hypothesis is that a base model cannot be outperformed by other competitor models. The values in bold face represent the p-values that are greater than or equal to the 10% confidence level under a pre-specified loss function. We combine: FCOM1=GARCH+IGARCH, FCOM11=GARCH+IGARCH+HYGARCH, FCOM111=HYGARCH+LMSM, FCOM2=GJR-GARCH+EGARCH, FCOM21=GJR-GARCH+IGARCH, FCOM211=GJR-GARCH+FIAPARCH, FCOM3=GARCH+IGARCH and FCOM31=GARCH+IGARCH+HYGARCH.

Table 13: Superior predictive ability (SPA) test results using oil price observations from January 6, 1992 to December 29, 2006 as in-sample and from January 2, 2007 to December 31, 2009 as out-of-sample.

Base model	Forecast horizons											
	1	5	10	20	30	40	50	60	70	80	90	100
							HMAE					
GARCH	0.000	**0.136**	**0.292**	0.062	**0.594**	**0.490**	0.060	**0.518**	**0.216**	**0.758**	0.068	0.000
IGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.038	**0.666**	**0.798**	**1.000**
GJR-GARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
EGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.004	**0.144**	**0.312**	0.026
APARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FIGARCH	0.000	0.030	0.002	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
HYGARCH	**1.000**	**1.000**	**0.708**	**1.000**	**0.406**	**0.510**	**1.000**	**0.482**	**0.784**	**0.412**	0.010	0.000
FIAPARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LMSM	0.000	0.016	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FCOM4	0.000	**0.302**	**0.721**	**0.216**	**0.837**	**1.000**	**0.517**	**0.167**	0.000	0.000	0.000	0.000
FCOM41	0.000	0.018	0.002	0.003	0.005	0.003	0.001	0.012	0.001	0.000	0.000	0.000
							QLIKE					
GARCH	0.000	**0.286**	**0.662**	**0.158**	**0.808**	**0.764**	**0.116**	**0.772**	**0.546**	0.068	0.010	0.002
IGARCH	0.000	0.000	0.000	0.000	0.000	0.002	0.014	0.098	**0.752**	**1.000**	**0.560**	**1.000**
GJR-GARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
EGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.034	**0.138**	**0.238**	**0.440**	0.084
APARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FIGARCH	0.000	**0.144**	0.022	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
HYGARCH	**1.000**	**0.790**	**0.338**	**0.842**	**0.192**	**0.236**	**0.884**	**0.316**	**0.564**	0.076	0.000	0.000
FIAPARCH	0.000	0.004	0.002	0.002	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LMSM	0.000	0.022	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FCOM5	0.000	0.034	0.011	0.026	0.013	0.015	0.090	0.005	0.000	0.000	0.001	0.001
FCOM51	0.000	**0.379**	**0.995**	**0.360**	**0.532**	**0.641**	**0.925**	0.054	0.000	0.001	0.002	0.000
FCOM511	0.000	**0.170**	**0.129**	**0.131**	**0.167**	**0.141**	0.025	0.043	0.001	0.001	0.002	0.000
							RLOG					
GARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.002
IGARCH	0.002	0.000	0.000	0.000	0.009	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GJR-GARCH	0.000	0.000	0.000	0.002	0.014	0.006	0.002	0.004	0.016	0.012	0.012	0.008
EGARCH	**0.742**	**0.882**	**1.000**	**1.000**	0.082	0.004	0.000	0.000	0.000	0.000	0.000	0.000
APARCH	**0.458**	**0.118**	0.008	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FIGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
HYGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FIAPARCH	**0.444**	0.000	0.000	0.080	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
LMSM	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FCOM6	**1.000**	0.006	0.031	**0.200**	**0.180**	0.016	0.002	0.002	0.006	0.006	0.002	0.000

Note: The table entries are the p-values of the SPA test of Hansen (2005). The null hypothesis is that a base model cannot be outperformed by other competitor models. The values in bold face represent the p-values that are greater than or equal to the 10% confidence level under a pre-specified loss function. We combine: FCOM4=GARCH+HYGARCH, FCOM41=HYGARCH+LMSM, FCOM5=GARCH+IGARCH, FCOM51=GARCH+HYGARCH, FCOM511=HYGARCH+LMSM and FCOM6=EGARCH+FIAPARCH.

Table 14: Superior predictive ability (SPA) test results using oil price observations from January 6, 1992 to December 31, 2009 as in-sample and from January 4, 2010 to March 24, 2014 as out-of-sample.

Base model	Forecast horizons											
	1	5	10	20	30	40	50	60	70	80	90	100
						MSE						
GARCH	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
IGARCH	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GJR-GARCH	0.004	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
EGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
APARCH	0.000	0.020	0.040	0.016	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FIGARCH	0.002	0.024	0.002	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000
HYGARCH	**1.000**	0.032	0.000	0.012	0.008	**0.100**	0.094	0.076	0.086	0.002	0.042	0.076
FIAPARCH	0.018	**0.106**	**0.160**	**0.186**	**0.196**	**0.370**	**0.694**	**0.526**	**0.728**	**0.754**	**0.864**	**1.000**
LMSM	0.004	**1.000**	**1.000**	**0.866**	**0.872**	**0.704**	**0.306**	**0.558**	**0.334**	**0.246**	**0.136**	0.082
FCOM1	**0.104**	**0.495**	**0.400**	**0.387**	**0.484**	**0.824**	**0.852**	**0.850**	**0.997**	**0.967**	**0.678**	**0.716**
FCOM11	0.018	**0.652**	**0.606**	**0.695**	**0.808**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
						MAE						
GARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
IGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GJR-GARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
EGARCH	0.000	0.000	0.000	0.000	0.000	0.006	0.000	0.000	0.000	0.000	0.000	0.000
APARCH	0.000	0.000	0.000	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FIGARCH	0.000	0.007	0.000	0.000	0.000	0.004	0.000	0.000	0.000	0.000	0.008	0.006
HYGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.011
FIAPARCH	**1.000**	0.000	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.000	0.000	0.000
LMSM	0.000	**1.000**	0.000	0.000	0.000	0.000	0.000	0.000	0.000	**1.000**	**0.876**	**0.901**
FCOM2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FCOM21	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FCOM211	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
						HMSE						
GARCH	0.054	0.068	0.008	**0.178**	**0.194**	**0.560**	**0.450**	**0.314**	**0.104**	0.098	0.072	**0.110**
IGARCH	0.004	0.002	0.004	0.008	0.008	0.004	0.010	0.018	0.022	0.012	0.044	0.020
GJR-GARCH	0.004	0.002	0.004	0.006	0.006	0.008	0.008	0.014	0.004	0.008	0.018	0.008
EGARCH	0.006	0.002	0.006	0.012	0.004	0.028	0.032	**0.720**	**1.000**	**1.000**	**1.000**	**1.000**
APARCH	0.006	0.004	0.004	0.008	0.004	0.014	0.006	0.010	0.000	0.026	0.054	0.066
FIGARCH	0.052	**0.194**	**1.000**	**0.908**	**0.826**	**0.440**	**0.762**	**0.156**	0.036	0.052	0.086	0.060
HYGARCH	**1.000**	**0.888**	0.040	**0.124**	0.076	0.064	0.090	0.032	0.026	0.024	0.046	0.058
FIAPARCH	0.004	0.032	0.022	0.014	0.008	0.010	0.000	0.004	0.000	0.000	0.000	0.002
LMSM	0.012	**0.212**	0.088	0.018	0.044	0.084	0.076	0.054	0.056	0.036	0.054	0.038
FCOM3	0.035	0.005	0.014	0.013	0.005	0.097	**0.134**	**0.995**	0.067	0.066	0.086	0.084
FCOM31	0.039	0.006	0.016	0.019	0.012	**0.130**	0.092	**0.957**	0.051	0.061	0.078	0.083
FCOM311	0.002	**0.482**	0.068	**0.134**	0.076	**0.270**	**0.111**	0.040	0.083	0.065	0.072	0.085

Note: The table entries are the p-values of the SPA test of Hansen (2005). The null hypothesis is that a base model cannot be outperformed by other competitor models. The values in bold face represent the p-values that are greater than or equal to the 10% confidence level under a pre-specified loss function. We combine: FCOM1=HYGARCH+FIAPARCH, FCOM11=FIAPARCH+LMSM, FCOM2=GARCH+FIAPARCH, FCOM21=GARCH+LMSM, FCOM211=FIAPARCH+LMSM, FCOM3=GARCH+EGARCH, FCOM31=EGARCH +FIGARCH and FCOM311=FIGARCH+HYGARCH.

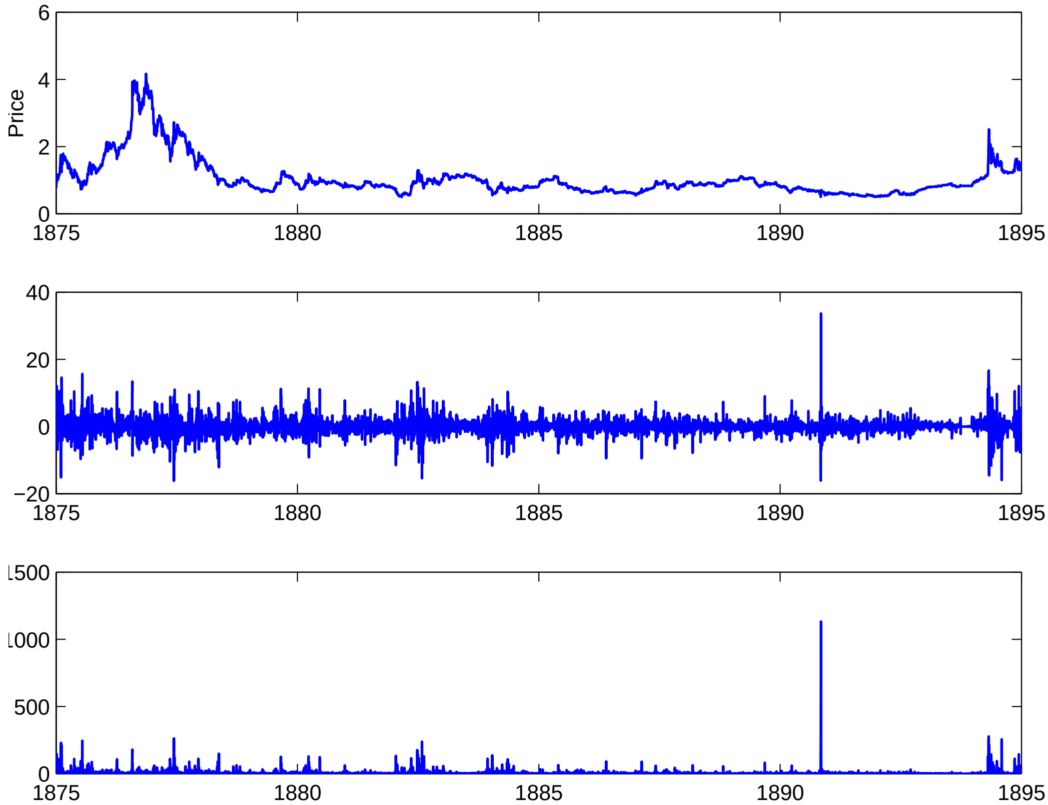

Figure 1: Plot of oil prices, log-returns and squared returns (from January 2, 1875 to December 31, 1895)

Table 15: Superior predictive ability (SPA) test results using oil price observations from January 6, 1992 to December 31, 2009 as in-sample and from January 4, 2010 to March 24, 2014 as out-of-sample.

Base model	Forecast horizons											
	1	5	10	20	30	40	50	60	70	80	90	100
							HMAE					
GARCH	0.000	0.026	0.010	**0.124**	**0.116**	0.074	**0.200**	**0.488**	**0.170**	**0.200**	**0.314**	**0.242**
IGARCH	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.004
GJR-GARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
EGARCH	0.000	0.000	0.000	0.002	0.000	0.002	0.000	0.008	**0.186**	**0.330**	**0.280**	**0.308**
APARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.004
FIGARCH	0.000	0.034	**1.000**	**1.000**	**0.884**	**1.000**	**0.800**	**0.512**	**0.854**	**0.752**	**0.896**	**0.890**
HYGARCH	**1.000**	**1.000**	**0.132**	0.038	0.008	0.006	0.004	0.000	0.002	0.000	0.002	0.014
FIAPARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LMSM	0.000	0.038	0.000	0.000	0.002	0.004	0.004	0.002	0.000	0.004	0.002	0.008
FCOM4	0.000	0.058	**0.324**	**0.121**	0.018	0.048	0.010	0.027	0.044	0.003	0.029	0.088
FCOM41	0.000	**0.469**	**0.526**	**0.496**	0.052	0.022	0.005	0.048	0.046	0.009	0.066	0.017
							QLIKE					
GARCH	0.012	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
IGARCH	0.000	0.002	0.002	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GJR-GARCH	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
EGARCH	0.002	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
APARCH	0.000	0.000	0.004	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FIGARCH	0.022	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
HYGARCH	**1.000**	0.000	0.000	**0.126**	0.076	**0.686**	**0.628**	**0.764**	**0.796**	**0.624**	**1.000**	**1.000**
FIAPARCH	0.034	0.006	0.002	0.012	0.008	0.024	0.050	0.014	0.026	0.036	0.052	0.070
LMSM	0.004	**1.000**	**1.000**	**0.874**	**1.000**	**0.314**	**0.372**	**0.236**	**0.204**	**0.376**	0.080	0.018
FCOM5	0.000	0.058	0.020	**0.180**	**0.408**	**0.916**	**0.587**	**0.966**	**0.986**	**0.971**	**0.137**	0.050
FCOM51	**0.202**	**0.602**	**0.628**	**0.904**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
							RLOG					
GARCH	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
IGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GJR-GARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
EGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
APARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FIGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
HYGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FIAPARCH	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
LMSM	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FCOM6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FCOM61	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: The table entries are the p-values of the SPA test of Hansen (2005). The null hypothesis is that a base model cannot be outperformed by other competitor models. The values in bold face represent the p-values that are greater than or equal to the 10% confidence level under a pre-specified loss function. We combine: FCOM4=FIGARCH+HYGARCH, FCOM41=FIGARCH+LMSM, FCOM5=HYGARCH+LMSM, FCOM51=FIAPARCH+LMSM, FCOM6=GARCH+FIAPARCH and FCOM61=FIAPARCH+LMSM.

Table 16: Superior predictive ability (SPA) test results using oil price observations from January 3, 1977 to December 31, 2009 as in-sample and from January 4, 2010 to March 24, 2014 as out-of-sample.

Base model	Forecast horizons											
	1	5	10	20	30	40	50	60	70	80	90	100
MSE												
GARCH	0.015	0.007	0.053	**0.125**	**0.392**	**0.232**	**0.749**	**0.772**	**0.360**	**0.303**	**0.538**	**0.571**
IGARCH	0.029	0.094	0.046	0.053	0.037	**0.158**	**0.261**	0.010	0.010	0.026	0.005	0.003
GJR-GARCH	0.021	**0.106**	0.028	**0.127**	**0.773**	**0.218**	**0.517**	**0.364**	**0.178**	0.058	0.014	0.007
EGARCH	0.005	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
APARCH	0.012	0.044	0.008	0.082	**0.368**	0.016	**0.144**	0.084	0.004	0.002	0.000	0.000
FIGARCH	0.014	0.084	0.031	**0.156**	**0.581**	**0.160**	**0.817**	**0.192**	**0.272**	0.026	**0.219**	**0.259**
HYGARCH	**1.000**	0.006	0.001	0.005	0.002	0.009	0.000	0.001	0.002	0.000	0.000	0.000
FIAPARCH	0.021	0.013	0.015	0.040	0.071	0.096	**0.323**	**0.131**	**0.298**	**0.397**	**0.379**	**0.524**
LMSM	0.009	**1.000**	**1.000**	**1.000**	**0.814**	**1.000**	**0.730**	**0.732**	**0.884**	**0.812**	**0.872**	**0.817**
FCOM1	0.030	**0.242**	**0.201**	**0.322**	**0.695**	**0.272**	**0.803**	**0.883**	**0.415**	**0.488**	**0.895**	**0.884**
FCOM11	0.017	**0.117**	**0.163**	**0.253**	**0.645**	**0.350**	**0.846**	**1.000**	**0.731**	**0.775**	**0.850**	**0.771**
MAE												
GARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
IGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GJR-GARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
EGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000
APARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FIGARCH	0.000	0.000	0.000	0.000	0.000	0.005	0.000	0.000	0.000	0.000	0.000	0.000
HYGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FIAPARCH	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.021	0.005	0.004	0.006	0.000
LMSM	0.000	0.000	0.000	0.000	0.000	0.000	0.000	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
FCOM2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FCOM21	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
HMSE												
GARCH	0.019	0.003	0.028	0.004	0.067	0.030	0.034	0.000	0.016	0.021	0.074	0.026
IGARCH	0.015	0.013	0.025	0.018	0.087	0.070	0.049	0.018	0.059	0.054	0.074	0.009
GJR-GARCH	0.016	0.013	0.012	0.005	0.086	0.068	0.047	0.020	0.067	0.053	0.074	0.008
EGARCH	0.032	0.023	**0.847**	**0.863**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
APARCH	0.091	0.031	0.003	0.034	0.086	0.011	0.023	0.023	0.043	0.035	0.030	0.120
FIGARCH	0.035	0.001	0.019	0.026	0.052	0.056	0.053	0.017	0.028	0.064	0.069	0.022
HYGARCH	**1.000**	0.036	0.025	0.010	0.038	0.001	0.056	0.009	0.030	0.055	0.073	0.119
FIAPARCH	0.011	0.024	0.018	0.041	0.073	0.015	0.000	0.014	0.003	0.000	0.000	0.004
LMSM	0.023	**1.000**	**0.153**	**0.137**	0.046	0.018	0.019	0.026	0.019	0.030	0.036	0.050
FCOM3	0.006	0.042	0.046	0.015	0.059	0.026	0.069	0.040	0.047	0.069	**0.119**	**0.140**

Note: The table entries are the p-values of the SPA test of Hansen (2005). The null hypothesis is that a base model cannot be outperformed by other competitor models. The values in bold face represent the p-values that are greater than or equal to the 10% confidence level under a pre-specified loss function. We combine: FCOM1=GARCH+GJR-GARCH+FIGARCH, FCOM11=GARCH+GJR-GARCH+LMSM, FCOM2=GJR-GARCH+FIAPARCH, FCOM21=FIAPARCH+LMSM and FCOM3=EGARCH+HYGARCH+LMSM.

Table 17: Superior predictive ability (SPA) test results using oil prices observation from January 3, 1977 to December 31, 2009 as in-sample and from January 4, 2010 to March 24, 2014 as out-of-sample.

Base model	Forecast horizons											
	1	5	10	20	30	40	50	60	70	80	90	100
							HMAE					
GARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
IGARCH	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.004	0.001
GJR-GARCH	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.006	0.001
EGARCH	0.000	0.002	**0.426**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**0.868**	**0.789**	**0.546**	0.384
APARCH	0.000	0.000	0.000	0.000	0.004	0.001	0.001	0.012	0.011	0.010	0.012	0.012
FIGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
HYGARCH	**1.000**	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.003	0.001
FIAPARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LMSM	0.000	**1.000**	**0.574**	0.045	0.006	0.020	0.029	0.083	**0.132**	**0.211**	**0.454**	**0.616**
FCOM4	0.000	0.045	**0.877**	**0.159**	0.043	**0.158**	**0.312**	**0.646**	**0.825**	**0.901**	**0.773**	**0.604**
							QLIKE					
GARCH	0.002	0.003	0.042	0.022	0.052	0.032	0.067	**0.135**	0.045	0.079	**0.118**	0.076
IGARCH	0.002	0.082	**0.208**	**0.343**	**0.140**	**0.419**	**0.538**	**0.980**	**0.315**	**0.835**	**0.256**	**0.817**
GJR-GARCH	0.003	**0.309**	**0.485**	**0.630**	**0.542**	**0.569**	**0.305**	**0.466**	**0.172**	**0.372**	**0.175**	**0.415**
EGARCH	0.000	**0.104**	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
APARCH	0.003	**0.131**	**0.348**	**0.315**	**0.489**	**0.188**	**0.416**	**0.393**	0.068	0.036	0.038	0.001
FIGARCH	0.004	**0.351**	**0.135**	**0.462**	**0.639**	**0.465**	**0.780**	**0.871**	**0.697**	**0.692**	**0.986**	**0.869**
HYGARCH	**1.000**	**0.373**	**0.470**	**0.529**	**0.735**	**0.777**	**0.766**	**0.396**	**0.664**	**0.647**	**0.779**	**0.345**
FIAPARCH	0.000	0.001	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LMSM	0.000	**0.916**	**0.854**	**0.866**	**0.833**	**0.714**	**0.793**	**0.534**	**0.783**	**0.710**	**0.497**	**0.524**
FCOM5	0.003	0.097	**0.186**	**0.388**	**0.166**	**0.618**	**0.457**	**0.722**	**0.193**	**0.607**	**0.275**	**0.659**
FCOM51	0.000	**0.584**	**0.652**	**0.775**	**0.844**	**1.000**	**1.000**	**0.880**	**0.902**	**0.873**	**0.868**	**0.890**
							RLOG					
GARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
IGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GJR-GARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
EGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
APARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FIGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
HYGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FIAPARCH	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
LMSM	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FCOM6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FCOM61	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: The table entries are the p-values of the SPA test of Hansen (2005). The null hypothesis is that a base model cannot be outperformed by other competitor models. The values in bold face represent the p-values that are greater than or equal to the 10% confidence level under a pre-specified loss function. We combine: FCOM4=EGARCH+LMSM, FCOM5=IGARCH+GJR-GARCH, FCOM51=FIGARCH+HYGARCH+LMSM, FCOM6=HYGARCH+FIAPARCH and FCOM61=FIAPARCH+LMSM.

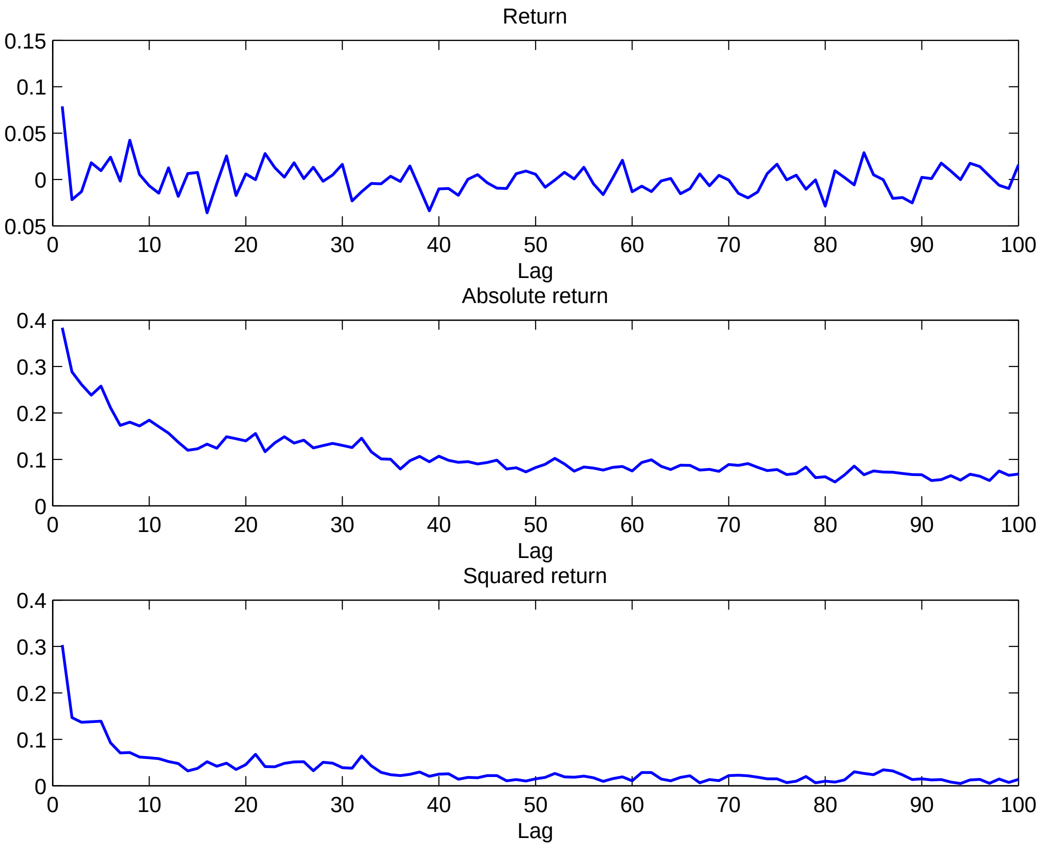

Figure 2: Plot of autocorrelation functions of log-returns, absolute and squared log-returns (from January 2, 1875 to December 31, 1895)

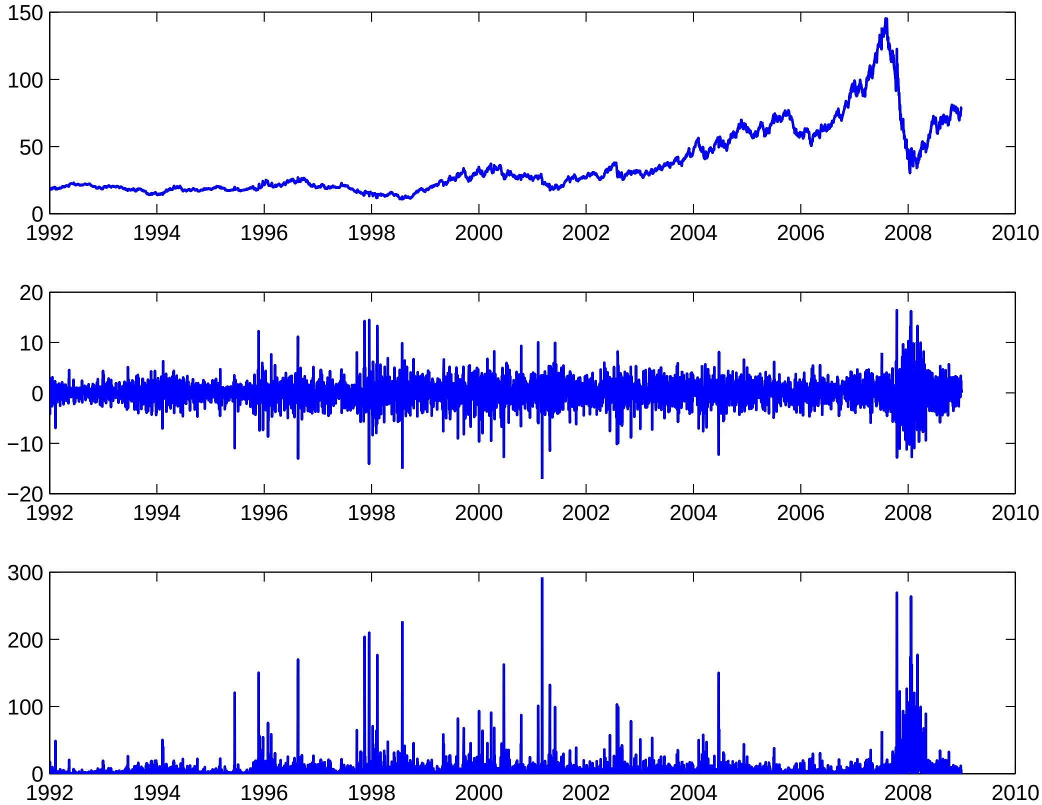

Figure 3: Plot of oil prices, log-returns and squared returns (from January 6, 1992 to December 31, 2009)

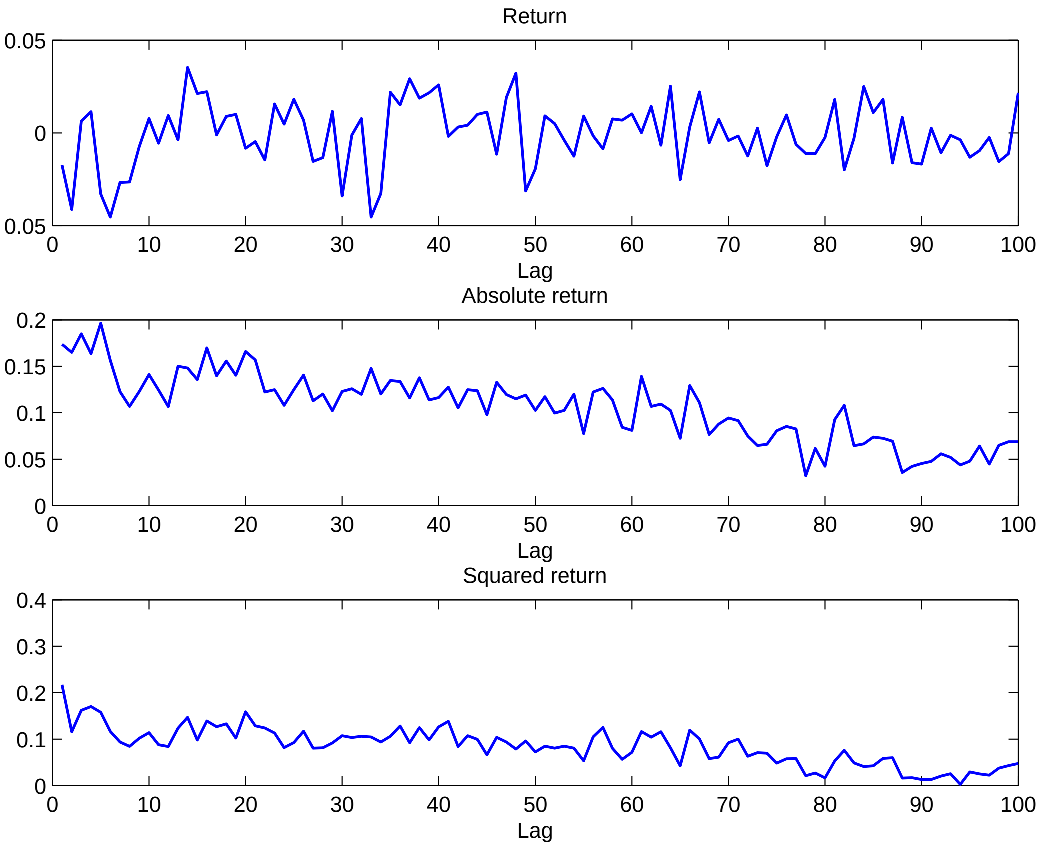

Figure 4: Plot of autocorrelation functions of log-returns, absolute and squared log-returns (from January 6, 1992 to December 31, 2009)

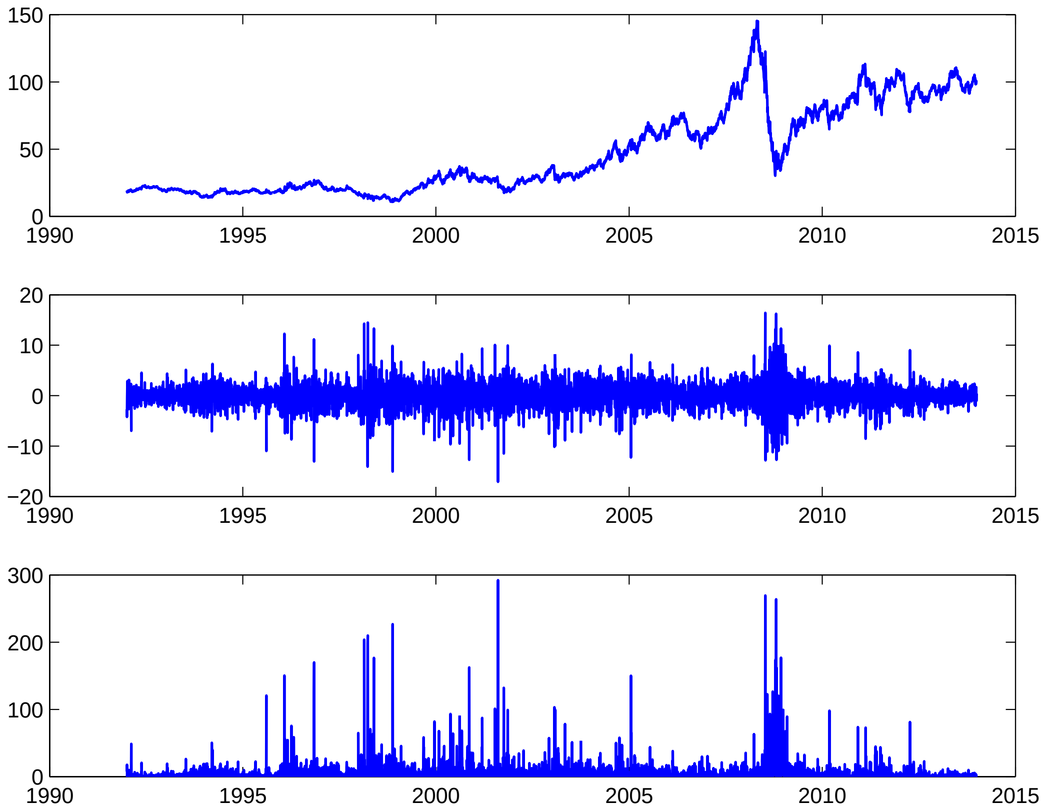

Figure 5: Plot of oil prices, log-returns and squared returns (from January 6, 1992 to March 24, 2014)

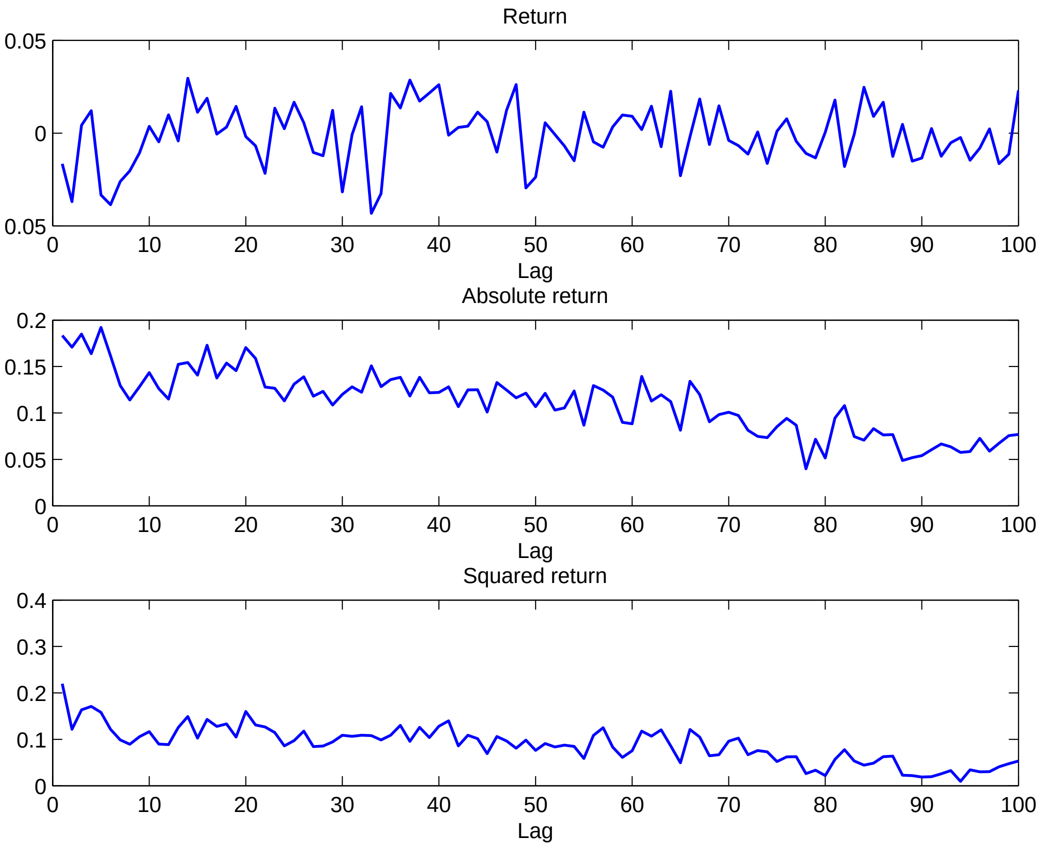

Figure 6: Plot of autocorrelation functions of log-returns, absolute and squared log-returns (from January 6, 1992 to March 24, 2014)

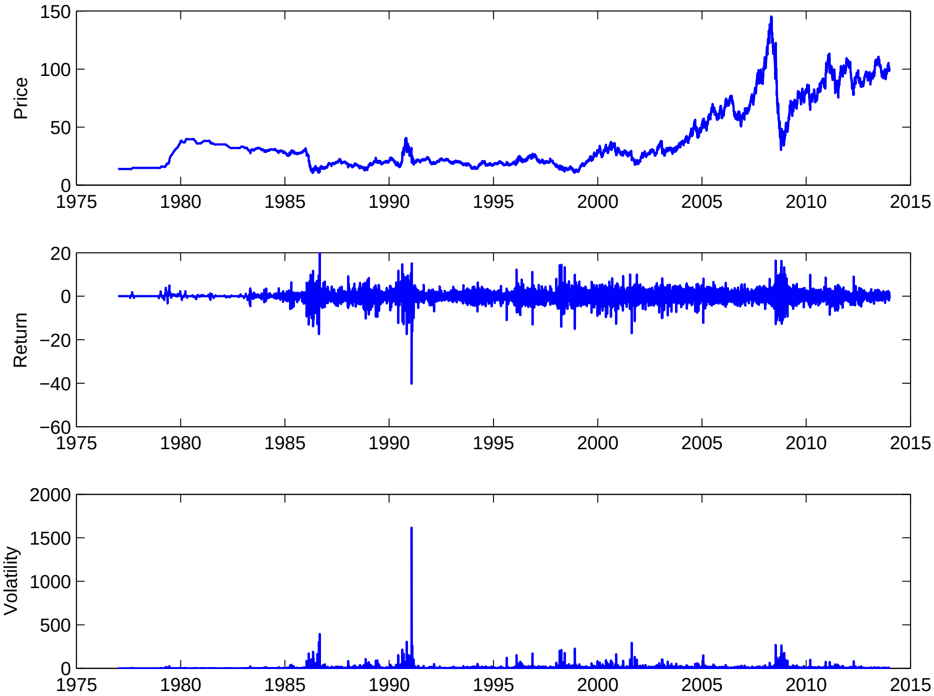

Figure 7: Plot of oil prices, log-returns and squared returns (from January 6, 1977 to March 24, 2014)

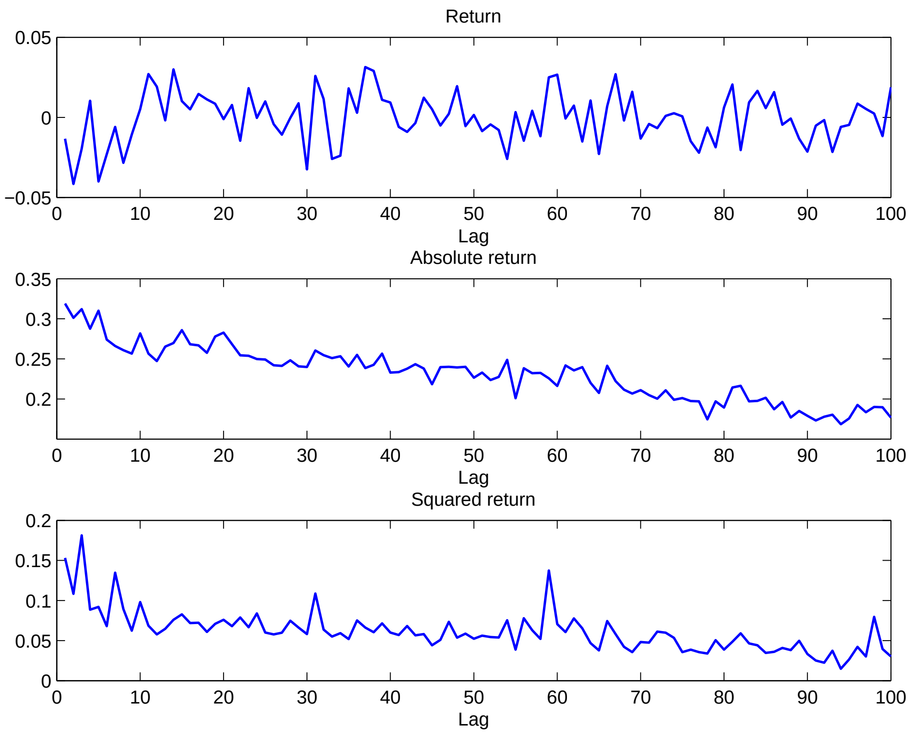

Figure 8: Plot of autocorrelation functions of log-returns, absolute and squared log-returns (from January 6, 1977 to March 24, 2014)