# Data Retrieval With Elementary Data Structures

Recapping Insertion/Deletion/Search Algorithms With Arrays And Lists

## Elementary Data Retrieval

- Sequence of operations of mixed types
  - Insertion/deletion/search of items
- Collection of items: Accessed by an attribute (key)
  - Managed as arrays, linked lists (should be familiar to all already)
  - Binary search trees for better performance
- Time complexities of those operations on different data structures

## Elementary Data Structures

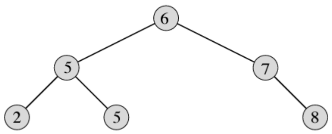
- Stacks, queues, linked lists: Undergrad prerequisites
  - Study CLRS Ch. 10 to recap or equivalent book
  - Focus on linked lists for general data retrieval operations (insert/delete/search)
  - Everyone should be able to write code for insert/delete/search on singly/doubly linked lists with *pointers*
- Binary tree representation using *pointers* (CLRS 10.4)
- Time complexities of insert/delete/search algorithms on sorted/unsorted arrays/linked lists
  - Everyone should be able to derive all these

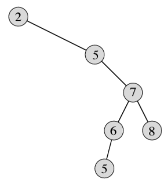
# Binary Search Trees

Average-Case Logarithmic Insert/Delete/Search/Minimum/Maximum Operations

# What is a Binary Search Tree (BST)?

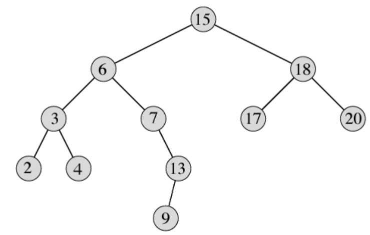
- Recursive definition
  - An empty tree is a BST.
  - A binary tree with root node $r$ is a BST if and only if:
    - $r$'s left/right subtree is a BST.
    - All values in $r$'s left subtree are less than or equal to $r$.
    - All values in $r$'s right subtree are greater than ("or equal to" included in CLRS) $r$.




# Querying Binary Search Tree

- Searching for a key
  - Very similar to binary search of a sorted array
    - The mid entry is just replaced with the tree node.
    - left = mid + 1: Traversing to the right subtree
    - right = mid – 1: Traversing to the left subtree
- Experiment BST searches at http://visualgo.net/bst
- All are $O(h)$, where $h$ is the tree height.



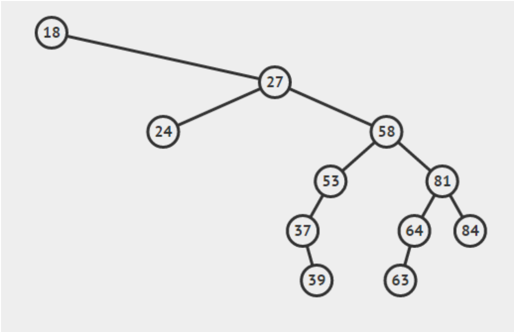

# Inserting in a Binary Search Tree

- Add a new leaf that continues to meet the BST property
- Start like search, but don't stop at a match
  - Continue until hitting a nil node
  - Add a new leaf there with the inserted value.
- http://visualgo.net/bst
- Still $O(h)$


# Deleting From Binary Search Tree

- Of course search first. Return if not found.
- If the found node (call it $z$) is a leaf, trivial.
- If $z$ has only one child, almost trivial.
- If $z$ has both children,
  - Find $z$'s right subtree's minimum ($z$'s successor). Call it $y$.
  - $y$ should be moved to $z$'s position.
  - Filling in $y$'s vacancy is almost trivial, as $y$ must have no left child.
- Experiment at http://visualgo.net/bst
- Actual code (even pseudocode) can be tricky. Study CLRS 12.3 code.

## BST Deletion Examples




## Time Complexities of BST Operations

- All are $O(h)$
  - $h = n - 1$ in the worst case.
    - Totally skewed to one side, or zig-zag
    - Therefore, worst case BST operations are all $\Theta(n)$.
- Average case tree height
  - Expected height of a randomly built BST
  - Another probability and random variable analysis
    - See Proof of Theorem 12.4 in CLRS pp. 300-303
- Theorem 12.4: Expected height of a randomly built BST on $n$ distinct keys is $O(\lg n)$.

## Average Case Insert/Delete/Search

| Operations | Unsorted arrays | Sorted arrays | Unsorted singly linked lists | Sorted singly linked lists | Unsorted doubly linked lists | Sorted doubly linked lists | BST |
|---|---|---|---|---|---|---|---|
| INSERT(A/L, i/n) | $O(n)$ | | $O(n)$ | | $O(1)$ | | |
| INSERT(A/L, k) | $O(n)$ | $O(n)$ | $O(1)$ | $O(n)$ | $O(1)$ | $O(n)$ | $O(\lg n)$ |
| DELETE(A/L, i/n) | $O(n)$ | | $O(n)$ | | $O(1)$ | | |
| DELETE(A/L, k) | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(\lg n)$ |
| SEARCH(A/L, k) | $O(n)$ | $O(\lg n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(\lg n)$ |
| MINIMUM(A/L) | $O(n)$ | $O(1)$ | $O(n)$ | $O(1)$ | $O(n)$ | $O(1)$ | $O(\lg n)$ |
| MAXIMUM(A/L) | $O(n)$ | $O(1)$ | $O(n)$ | $O(1)$ | $O(n)$ | $O(1)$ | $O(\lg n)$ |

## Worst Case Insert/Delete/Search

| Operations | Unsorted arrays | Sorted arrays | Unsorted singly linked lists | Sorted singly linked lists | Unsorted doubly linked lists | Sorted doubly linked lists |
|---|---|---|---|---|---|---|
| INSERT(A/L, i/n) | | | | | | |
| INSERT(A/L, k) | | | | | | |
| DELETE(A/L, i/n) | | | | | | |
| DELETE(A/L, k) | | | | | | |
| SEARCH(A/L, k) | | | | | | |
| MINIMUM(A/L) | | | | | | |
| MAXIMUM(A/L) | | | | | | |

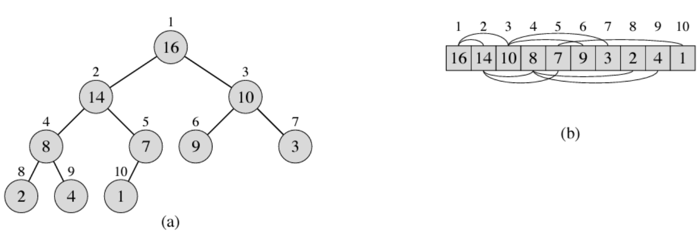
# Heaps And Heapsort

When We Want O(1) MAXIMUM() (Or MINIMUM()) All The Time

## What Is a Heap?

- A data structure that's specialized for retrieving minimum (or maximum) in $O(1)$ time. This is called Priority Queue.
  - Many applications for "priority queues" in many other algorithms
  - BST can only give us $O(\lg n)$ (Even balanced BST for worst case)
- Utilize binary tree, but make sure it's as balanced as possible
  - *Complete* binary tree
    - As balanced as possible, all leaves packed to the left
  - With heap property
    - For each node, its value is less than (for min-heap) or great than (for max-heap) both of its children
  - Implemented using an array
    - No need for pointer operations/traversals

## Max Heap Example (CLRS Fig. 6.1)




$A[\text{PARENT}(i)] \geq A[i]$

PARENT($i$)
1 **return** $\lfloor i/2 \rfloor$

LEFT($i$)
1 **return** $2i$

RIGHT($i$)
1 **return** $2i + 1$

## Building A Max-Heap

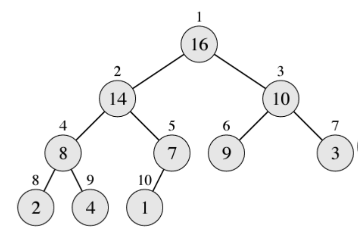
- Given an array of arbitrary values, build a max-heap.
- Two approaches:
  - Insert item by item starting from an empty heap
    - After each insertion, the resulting array must form a max-heap.
    - So fix up each inserted (appended) item by "trickling-up".
    - $n$ insertions, each insertion possibly taking $O(h)$, resulting in $O(n \lg n)$
  - Consider the original array as a heap
    - Of course it's not really a heap, so fix one-by-one, from bottom up, but we do "trickling-down" here.
    - Each fix-up could possibly take $O(h)$, and there are $n$ fix-ups possible, so this looks like another $O(n \lg n)$
    - Turns out that this is not a tight bound. It's actually $O(n)$.
      - Analysis in CLRS 6.3

## Building Max-Heap By Item-By-Item Insertions

- Given array $A = [4, 1, 3, 2, 16, 9, 10, 14, 8, 7]$,

## Building Max-Heap By Node-By-Node Fix-ups

- Given array $A = [4, 1, 3, 2, 16, 9, 10, 14, 8, 7]$,




```
BUILD-MAX-HEAP(A)
A.heap-size = A.length
for i = ⌊A.length/2⌋ downto 1
    MAX-HEAPIFY(A, i)
```

```
MAX-HEAPIFY(A, i)
l = LEFT(i)
r = RIGHT(i)
if l ≤ A.heap-size and A[l] > A[i]
    largest = l
else largest = i
if r ≤ A.heap-size and A[r] > A[largest]
    largest = r
if largest ≠ i
    exchange A[i] with A[largest]
    MAX-HEAPIFY(A, largest)
```

## Time Complexity Of BUILD-MAX-HEAP($A$)

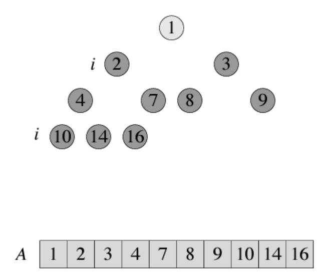
- Naïve/loose analysis: $O(\lg n)$ for each MAX-HEAPIFY($A, i$), $n/2$ times, so easily $O(n \lg n)$, but this is not tight as shown below:
- Note that MAX-HEAPIFY($A, i$) is not on the root (at height $h = \lfloor \lg n \rfloor$) all the time, but mostly on nodes at lower heights!
  - Up to $n/2$ nodes at height 0 (leaf), $n/4$ nodes at height 1, $n/8$ nodes at height 2, … ➔ Up to $\lceil n/2^{n+1} \rceil$ nodes at height $h$, where $0 \le h \le \lfloor \lg n \rfloor$
- Therefore, actual # operations is:

$$\sum_{h=0}^{\lfloor \lg n \rfloor} \left\lceil \frac{n}{2^{h+1}} \right\rceil O(h) = O\left(n \sum_{h=0}^{\lfloor \lg n \rfloor} \frac{h}{2^h}\right) = O\left(n \sum_{h=0}^{\infty} \frac{h}{2^h}\right) = O(n) .$$

$$\sum_{h=0}^{\infty} \frac{h}{2^h} = \frac{1/2}{(1-1/2)^2} = 2 .$$

## Heapsort By Repeatedly Deleting (Extracting) Max (CLRS Fig. 6.4)

- The root of a max-heap is always the maximum of all values!
  - Remove root. Its sorted position is that of the last node of the heap.
  - Move last node in heap to root, fix-up the heap (trickle-down)
  - Then repeat this whole process until there's no node left in the heap.
- Complexity: $O(n \lg n)$ obviously.
- Experiment all heap operations at http://visualgo.net/heap



# Heap as Priority Queue

- INSERT($S, x$)
  - Insert $x$ into queue so that GET-MAX() and EXTRACT-MAX() is efficient.
  - Place $x$ at the end of array (last node in the heap), trickle it up. This is $O(\log n)$.
- GET-MAX($S$): Always root. $O(1)$.
- EXTRACT-MAX($S$): Removes & returns max of all values in queue
  - Remove root, move last heap node to root, trickle it down. This is $O(\log n)$.
- Many applications in various computer science specialty areas
  - Especially in scheduling & simulation: All about temporal priorities.
  - Also used frequently in many graph algorithms (e.g., shortest paths)