Problem 18.

Proof. Note that

$$\mathbb{P}(x = k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

We seek the value of k for which this is maximized. Consider

$$\frac{\mathbb{P}(x = k+1)}{\mathbb{P}(x = k)} = \frac{\frac{e^{-\lambda}\lambda^{k+1}}{k+1!}}{\frac{e^{-\lambda}\lambda^k}{k!}} = \frac{\lambda}{k+1}$$

This is equal to 1 at $\lambda = k + 1$, giving us the extrema. □

Problem 19.

Proof. Note that

$$\mathbb{P}(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

and that

$$E(X^n) = \sum_{k=0}^{\infty} k^n \frac{e^{-\lambda}\lambda^k}{k!}$$

But this is simply

$$E(X^n) = \lambda \sum_{k=0}^{\infty} (k-1)^n \frac{e^{-\lambda}\lambda^{k-1}}{(k-1)!} = \lambda E((X-1)^{n-1})$$

□