Udacity Intro To Statistics Problem Set 6

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Contents

1 Ratio of Correlation to Regression

Given:

$$b = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \tag{1}$$

and:

$$r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}} \tag{2}$$

find a simplified form for $\frac{b}{r}$
let $f(x, y) =$

$$\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \tag{3}$$

then $\frac{b}{r} =$

$$\frac{\frac{f(x,y)}{\sum_{i=1}^{n} (x_i - \bar{x})^2}}{\frac{f(x,y)}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}}} \tag{4}$$

$$\implies \frac{b}{r} = \frac{f(x, y)}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \frac{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}}{f(x, y)} \tag{5}$$

$$\implies \frac{b}{r} = \frac{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \tag{6}$$

$$\implies \frac{b}{r} = \frac{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2}}{\sum_{i=1}^{n}(x_i-\bar{x})^2} \tag{7}$$

$$\implies \frac{b}{r} = \frac{(\sum_{i=1}^{n}(x_i-\bar{x})^2)^{\frac{1}{2}}\sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2}}{\sum_{i=1}^{n}(x_i-\bar{x})^2} \tag{8}$$

$$\implies \frac{b}{r} = \left(\sum_{i=1}^{n}(x_i-\bar{x})^2\right)^{\frac{1}{2}} \left(\sum_{i=1}^{n}(x_i-\bar{x})^2\right)^{-1} \sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2} \tag{9}$$

$$\implies \frac{b}{r} = \left(\sum_{i=1}^{n}(x_i-\bar{x})^2\right)^{-\frac{1}{2}} \sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2} \tag{10}$$

$$\implies \frac{b}{r} = \frac{\sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2}}{\sqrt{\left(\sum_{i=1}^{n}(x_i-\bar{x})^2\right)}} \tag{11}$$

$$\implies \frac{b}{r} = \sqrt{\frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}} \tag{12}$$