MIT Introduction to Statistics 18.05 Reading 6A Think Questions

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1 References and License

We are answering questions in the material from MIT OpenCourseWare course 18.05, Introduction to Probability and Statistics.
In this document we are answering questions Orloff and Bloom ask in [3].
We use documentation in in order to write the LaTeXcode for this document.
Please see the references section for detailed citation information.

2 Histogram with a lot of values

In the first question on reading 6b, Orloff and Bloom ask us if a histogram for a random variable X that follows some unknown distribution with $100,000$ values should be close to a graph of the probability density function of whatever distribution X follows.
We do not know anything about the distribution that X follows, so it might take a lot more than $100,000$ values for the histogram to converge on the probability distribution function for X.

3 Standardization

For the second question, Orloff and Bloom give us a the mean and standard deviation of a random variable X. The mean μ of X is 2 and the standard deviation of X is 2.
By definition [1], the standardization of X is

$$Z = \frac{X - 2}{2} \tag{1}$$

4 Mean

For the third problem, Orloff and Bloom give us that Y is a standard normal random variable, and that $W = 3Y + 4$. They then ask us for the mean of W.
In [2] Orloff and Bloom show that if X, and Y are continuous random variables defined on a sample space Ω, and a, and b are constants, then:

$$E\left(aX + b\right) = aE\left(X\right) + b \tag{2}$$

Y is a standard normal random variable, so Y has a mean of 0 [2].
Now we can substitute the information Orloff and Bloom give us for this problem into equation 2 to conclude that the mean of W is:

$$E\left(3Y + 4\right) = 3E\left(Y\right) + 4 = 3 \times 0 + 4 = 4. \tag{3}$$

5 Use Central Limit Theorem

For this last problem, Orloff and Bloom ask us to apply the Central Limit Theorem to estimate a probability.
Orloff and Bloom ask us to estimate the probability that we toss heads more than 40 times in 64 coin tosses.
We use the same technique Orloff and Bloom use in [1] to estimate the probability. Specifically we follow the technique Orloff and Bloom use in Example 2 and Example 3 of [1], and replace the numbers they use with 40 and 64 to get started.
For this problem, $\mu = 0.5 \times 64 = 32$, and $\sigma = \sqrt{\mathrm{Var}\left(S\right)} = \sqrt{\frac{64}{4}} = 4$.
Let S be the sum of the number of times we toss heads.
Then we wish to estimate $P\left(S > 40\right)$.
If we apply the Central Limit Theorem like Orloff and Bloom do in [1], then we can write

$$P\left(S > 40\right) = P\left(\frac{S - 32}{4} > \frac{40 - 32}{4}\right) \approx P\left(Z > 2\right). \tag{4}$$

In [3], Orloff and Bloom encourage us to use the rules of thumb for standard normal probabilities to approximate $P(Z > 2)$.
We know from [1], Example 3, $P(Z > 2) \approx 0.025$.
Therefore our estimate of the probability of tossing heads 40 times in 64 coin tosses is 0.025.

References

[1] Jeremy Orloff and Jonathan Bloom. *Expectation, Variance and Standard Deviation for Continuous Random Variables Class 6, 18.05, Spring 2014 Jeremy Orloff and Jonathan Bloom*. Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading6b.pdf` (Spring 2014).

[2] Jeremy Orloff and Jonathan Bloom. *Expectation, Variance and Standard Deviation for Continuous Random Variables Class 6, 18.05, Spring 2014 Jeremy Orloff and Jonathan Bloom*. Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading6a.pdf` (Spring 2014).

[3] Jeremy Orloff and Jonathan Bloom. *Reading Questions 6b*. Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/reading-questions-6b/` (Spring 2014).