MIT Introduction to Statistics 18.05 - answers to questions in second class 5

John Hancock

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Contents

1 References and License

We are answering questions in the material from MIT OpenCourseWare course 18.05, Introduction to Probability and Statistics.
In this document we are answering questions Orloff and Bloom ask in [3].
We use documentation in [4] to write the LaTeXsource code for this document.
Please see the references section for detailed citation information.

We use documentation in to write LaTeXsource code for this document.

2 Probability within distances from the mean

2.1 $\mu = 0$, $\sigma = 1$

In [2] Orloff and Bloom state that for the continuous random variable $Z \sim N(0, 1)$:

- $P(-1 \leq Z \leq 1) \approx 0.68$
- $P(-2 \leq Z \leq 2) \approx 0.95$
- $P(-3 \leq Z \leq 3) \approx 0.99$

2.2 $\mu = 0$, $\sigma = 3$

Orloff and Bloom ask a question similar to the previous, but change the standard deviation of the distribution to 3.
We use R to compute how much probability is within σ of μ, 2σ of μ, and 3σ of μ

```
> pnorm(3, 0, 3) - pnorm(-3, 0, 3)
[1] 0.6826895
> pnorm(6, 0, 3) - pnorm(-6, 0, 3)
[1] 0.9544997
> pnorm(9, 0, 3) - pnorm(-9, 0, 3)
[1] 0.9973002
```

Interestingly enough, these are the same results as for the normal distribution with mean 0 and standard deviation 1.
Next, Orloff and Bloom ask us if changing μ changes our answer to problem 2. Indeed it does. We use the pnorm function in R to see how the answer changes when we set $\mu = 10$:

```
> pnorm(3, 10, 3) - pnorm(-3, 10, 3)
[1] 0.009807985
> pnorm(6, 10, 3) - pnorm(-6, 10, 3)
[1] 0.09121117
> pnorm(9, 10, 3) - pnorm(-9, 10, 3)
[1] 0.3694413
```

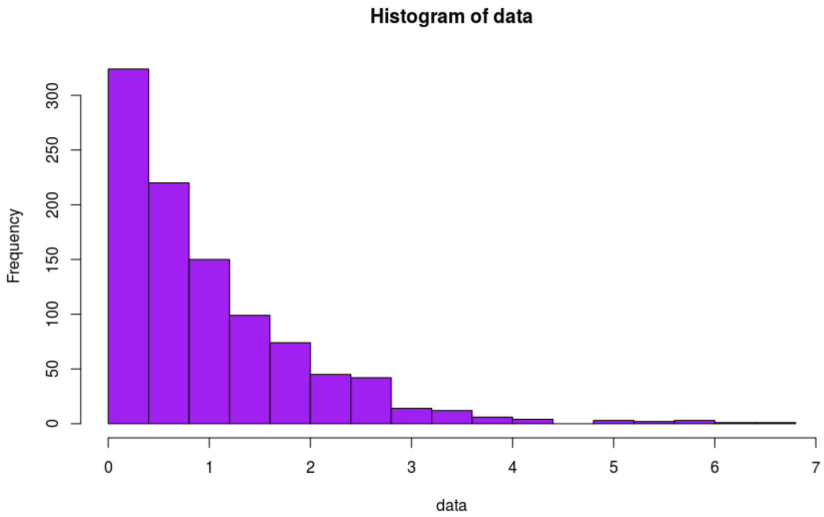
3 Histogram of exp(1) random variable

We make a small modification to the code in [1] to plot a histogram of $1,000$ sambles of a random variable that follows an exp(1) distribution:

```
data = rexp(1000,1)
binwidth = .4
```

```
bins = seq(min(data), max(data)+binwidth, binwidth)
hist(data, breaks=bins, col='purple')
```

We run this code in RStudio to produce the following graphic:

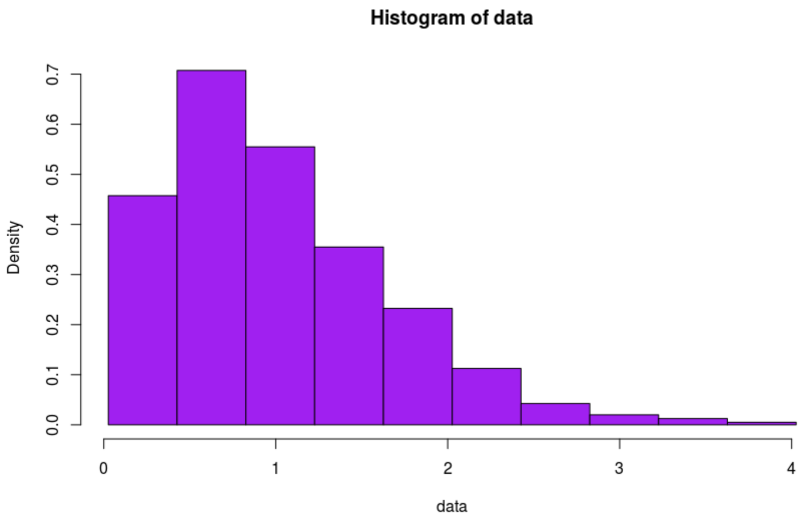


4 Histogram of average of 2 exp(1) random variables

Orloff and Bloom ask us to draw a histogram of the average of two independent variables that follow the exp(1) distribution.
We make a small modification to the code in [1] to draw the histogram. We run the following code in RStudio:

```
data = (rexp(1000,1) + rexp(1000,1))/2
binwidth = .4
bins = seq(min(data), max(data)+binwidth, binwidth)
hist(data, breaks=bins, col='purple', freq=FALSE)
```

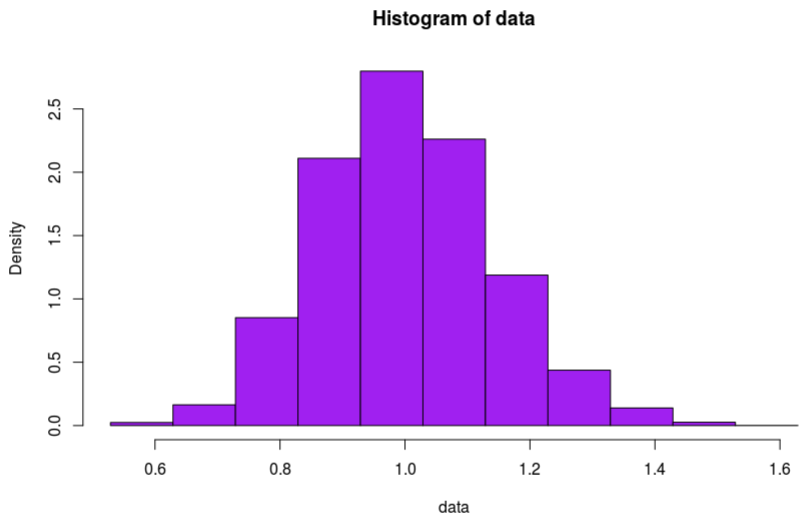
To produce this histogram:



5 Histogram of 50 exp(1) random variables

We make another modificaiton to the R code that Orloff and Bloom give us in [1] to plot the histogram. We run the code here:

```
data = colMeans(matrix(rexp(50*10000), 50, 10000))
binwidth = .1
bins = seq(min(data), max(data)+binwidth, binwidth)
hist(data, breaks=bins, col='purple', freq=FALSE)
```



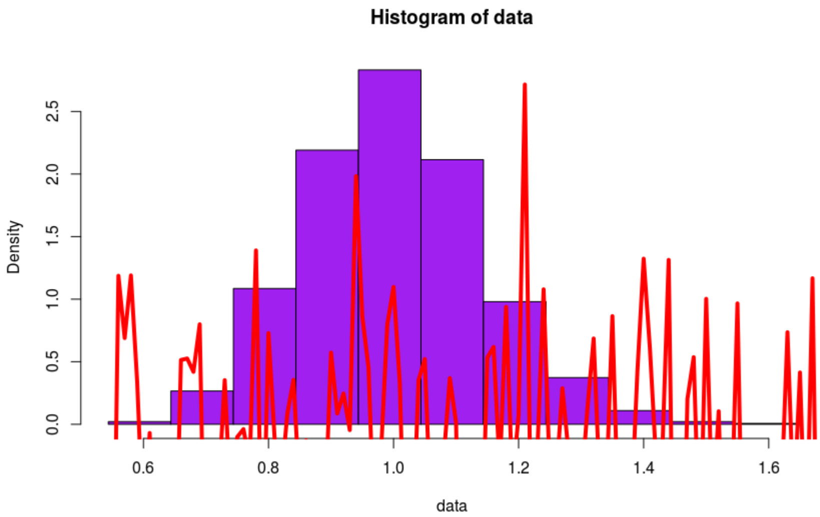
To produce this histogram

6 Superimpose plot of normal distribution

This is our attempt to superimpose a plot of the normal distribution $N\left(1, \frac{1}{50}\right)$. We modify code in [1]:

```
        data = colMeans(matrix(rexp(50*10000), 50, 10000))
binwidth = .1
bins = seq(min(data), max(data)+binwidth, binwidth)
hist(data, breaks=bins, col='purple', freq=FALSE)
x = seq(0,max(data)+1,.01)
lines(x,rnorm(x,1/50), col='red', lwd=4)
```

We run this code in RStudio to produce the following graphic:



References

[1] Jeremy Orloff and Jonathan Bloom. *Code for Studio 3*. Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/studio-resources/studio3.zip` (Spring 2014).

[2] Jeremy Orloff and Jonathan Bloom. *Gallery of Continuous Random Variables Class 5, 18.05, Spring 2014 Jeremy Orloff and Jonathan Bloom*. Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading5c.pdf` (Spring 2014).

[3] Jeremy Orloff and Jonathan Bloom. *Studio 3 18.05 Spring 2014 Jeremy Orloff and Jonathan Bloom*. Available at https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/class-slides/MIT18_05S14_cl5contslides.pdf (Spring 2014).

[4] ShareLatex.com. *Inserting Images*. Available at https://www.sharelatex.com/learn/Inserting_Images (2017).