MIT Introduction to Statistics 18.05 Reading 4 - Questions

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Contents

1 References and License

We are answering questions in the material from MIT OpenCourseWare course 18.05, Introduction to Probability and Statistics.
Please see the references section for detailed citation information.

We are answering the questions that Orloff and Bloom ask in [5].
We use documentation in [6], [2], and [1] to write LaTeXsource code of this document.

2 Problem 1

2.1 Expected Value of a Bernoulli Random Variable

In [4] Orloff and Bloom show that the expected value of a Bernoulli random variable X where $X \sim \text{Bernoulli}(p)$ is p.
So, if $X \sim \text{Bernoulli}\left(\frac{1}{3}\right)$, then the expected value of X is $\frac{1}{3}$.

2.2 Expected Value of the square of a Bernoulli Random Variable

We use the same technique Orloff and Bloom use in their proof for the expected value of a Bernoulli random variable in [4].

Proof. In [4] Orloff and Bloom give the formula for the expected value of a random variable. It is:

$$E\left(X\right) = \sum_{j=1}^{n} p\left(x_j\right) x_j \tag{1}$$

$X \sim$ Bernoulli $\left(\frac{1}{3}\right)$, so $p\left(1\right) = \frac{1}{3}$, and $p\left(0\right) = \frac{2}{3}$
Therefore,

$$E\left(X^2\right) = \frac{1}{3} 1^2 + \frac{2}{3} 0^2 = \frac{1}{3} \tag{2}$$

□

3 Expected Value of a Binomial Random Variable

In this section we will calculate the expected value of a binomial random variable $E\left(Y\right)$ where $Y \sim \textit{binomial}\left(12, \frac{1}{3}\right)$.
In [3] Orloff and bloom give the general probability mass functions for k successes out of n trials.
For reference, we reproduce these formulas here:

values a	0	1	2	...	k	...	n
pmf	$(1-p)^n$	$\binom{n}{1}p^1\left(1-p\right)^{n-1}$	$\binom{n}{2}p^2\left(1-p\right)^{n-2}$	...	$\binom{n}{k}p^k\left(1-p\right)^{n-k}$	...	p^n

We first substitute the value 12 for n in the table above, and then use refexpectedValFormula to write a formula for $E\left(Y\right)$:

$$
\begin{aligned}
E\left(Y\right) = \left(\frac{2}{3}\right)^{12} + \binom{12}{1}\left(\frac{1}{3}\right)^{1}\left(\frac{2}{3}\right)^{11} + \binom{12}{2}\left(\frac{1}{3}\right)^{2}\left(\frac{2}{3}\right)^{10} + \binom{12}{3}\left(\frac{1}{3}\right)^{3}\left(\frac{2}{3}\right)^{9} \\
+\binom{12}{4}\left(\frac{1}{3}\right)^{4}\left(\frac{2}{3}\right)^{8} + \binom{12}{5}\left(\frac{1}{3}\right)^{5}\left(\frac{2}{3}\right)^{7} + \binom{12}{6}\left(\frac{1}{3}\right)^{6}\left(\frac{2}{3}\right)^{6} \\
+\binom{12}{7}\left(\frac{1}{3}\right)^{7}\left(\frac{2}{3}\right)^{5} + \binom{12}{8}\left(\frac{1}{3}\right)^{8}\left(\frac{2}{3}\right)^{4} + \binom{12}{9}\left(\frac{1}{3}\right)^{9}\left(\frac{2}{3}\right)^{3} \\
+\binom{12}{10}\left(\frac{1}{3}\right)^{10}\left(\frac{2}{3}\right)^{2} + \binom{12}{11}\left(\frac{1}{3}\right)^{11}\left(\frac{2}{3}\right)^{1} + \left(\frac{1}{3}\right)^{12}
\end{aligned} \tag{3}
$$

We can use R to do the arithmetic on the right hand side of equation 3 in a sussinct manner:

```
> sum(choose(12,0:12) * (1/3)^(0:12) * (2/3)^(12:0) * c(0:12))
[1] 4
```

However, we can make an informal argument appealing to our intuition. Since $Y \sim \text{binom}\left(12, \frac{1}{3}\right)$, we expect about 4 out of 12 successes in each trial. We define the following sample space:

$$
\Omega = \{(x_1, x_2, \ldots, x_{12}) \mid x_i \in \{0, 1\}, i \in \{i, 2, \ldots, 12\}\} \tag{4}
$$

And let Y take the value of the sum of the number of ones in $\omega \in \Omega$. Beause we expect $\frac{1}{3}$ of the 12 elements in ω to have the value one, the expected value is 4.

4 Expected Value of Functions of Random Variables

In this section, we use the previous sections' definition of X and Y.

4.1 $E\left(4X + 7\right)$

in [4] Orloff and Bloom prove that

$$
E\left(aX + b\right) = aE\left(X\right) + b \tag{5}
$$

Therefore $E\left(4X + 7\right) = 4\frac{1}{3} + 7 \approx 8.333$.

4.2 $E\left(X + Y\right)$

As Orloff and Bloom direct in [5], we assume X and Y are random variables on the same sample space.

In [4] Orloff and Bloom prove that

$$E\left(X+Y\right)=E\left(X\right)+E\left(Y\right) \tag{6}$$

We use the values of X and Y that we calculated above to get $E\left(X+Y\right)=\frac{1}{3}+4\approx 4.333$

5 Random Variable Defined *via* Table

In [5] Orloff and Bloom give the following table for the random variable T:

values a	-4	-2	0	2	4
pmf $p\left(a\right)$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{3}{10}$

5.1 $E\left(T\right)$

We use equation 1 to compute $E\left(T\right)$:

$$E\left(T\right)=-4\frac{3}{10}+-2\frac{2}{10}+0\frac{1}{10}+2\frac{1}{10}+4\frac{3}{10}=-\frac{2}{10}=-0.2 \tag{7}$$

5.2 Value of T as a Payoff Function

In [5] Orloff and Bloom ask us to consider T as a payoff function for some game. If T returns a positive value, the house pays the player T's value in dollars. If T returns a negative value, the player pays the house T's value in dollars.

Orloff and Bloom ask if we would rather be the player or the house.

We would rather be the house because the expected value of T is a negative number.

6 Expected Value of a Product

In [5] Orloff and Bloom give $E\left(W\right)=-1$, and ask for the value of $E\left(W^2\right)$.

There is not enough information to give an example.

We reference the formula for the expected value of a random variable in equation 1.

We do not know how we multiply the values x_j of W with their respective probability mass functions $p\left(x_j\right)$ so that their sum is -1.

Therefore we cannot say how substituting the squares of the various values of W into equation 1 will change the value of that sum.

Furthermore, in [5], Orloff and Bloom make a note that in general, $E\left(h\left(X\right)\right)\neq h\left(E\left(X\right)\right)$

References

[1] Wikibooks.org users 109.192.80.194 et al. *LaTeX/Theorems*. Available at `https://en.wikibooks.org/w/index.php?title=LaTeX/Theorems&oldid=3039766` (2012/10/3).

[2] tex.stackexchange.com member Leo Liu. *How to break a long equation? [duplicate], Leo Liu Answer*. Available at `http://tex.stackexchange.com/questions/8936/how-to-break-a-long-equation` (2011/1/14).

[3] Jeremy Orloff and Jonathan Bloom. *Discrete Random Variables Class 4, 18.05, Spring 2014 Jeremy Orloff and Jonathan Bloom*. Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading4a.pdf` (Spring 2014).

[4] Jeremy Orloff and Jonathan Bloom. *Discrete Random Variables: Expected Value Class 4, 18.05, Spring 2014 Jeremy Orloff and Jonathan Bloom*. Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading4b.pdf` (Spring 2014).

[5] Jeremy Orloff and Jonathan Bloom. *Reading Questions 4b*. Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/reading-questions-4b/` (Spring 2014).

[6] tex.stackexchange.com member user11232. *How do I draw a tilde in math mode? [duplicate], user11232 Answer*. Available at `http://tex.stackexchange.com/questions/75237/how-do-i-draw-a-tilde-in-math-mode` (2012/10/3).