Answers To Questions in Conditional Probability, Independence, Bayes Theorem 18.05

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Contents

1 References and License

In this document, we are answering the questions in [1].
We use documentation in for properly writing the LaTeXsource code for this document.

2 Probability At Least 3 Heads, Given First Toss Tails

We are tossing a coin four times. Therefore we define the sample space

$$\Omega = \{(x_1, x_2, x_3, x_4), x_1, x_2, x_3, x_4 \in \{H, T\}\} \tag{1}$$

$|\Omega| = 16$
We assume all outcomes are equally likely.
A is the event that at we toss heads at least 3 times.
B is the event that we toss tails the first time.
We use the definition of conditional probability to cacluate $P(A \mid B)$.

$$P(A \mid B) = fracP(AcapB)P(B), ProvidedP(B) \neq 0 \tag{2}$$

$P(A) = frac516$ since there are 5 elements of Ω that represent the event that we toss heads at least three times, and we assume all outcomes are equally likely.
These are:(T, H, H, H), (H, T, H, H), (H, H, T, H), (H, H, H, T), (H, H, H, H).
The elements of B are (T, T, T, T), (T, T, T, H), (T, T, H, T), (T, T, H, H), (T, H, T, T), (T, H, T, H), (T, H, H, T), (T, H, H, H).
By inspection $A \cap B$ is the element (T, H, H, H).
We substitute values into 2 to get

$$P(A \mid B) = \frac{\frac{1}{16}}{\frac{8}{16}} = frac18 \tag{3}$$

3 Probability First Toss Tails, Given At Least 3 Heads

We use 2 and definitions of the sets Ω, A, $A \cap B$, and B that we define in section 1. In addition we assume all outcomes are equally likely.
We use **??** to get

$$P(B \mid A) = fracP(B \cap A)P(A), ProvidedP(A) \neq 0 \tag{4}$$

The $\cap$ operator is commutative, so $P(A \cap B) = P(B \cap A)$, and we discover $P(A \cap B)$ in the previous section 1.
Therefore,

$$P(B \mid A) = frac\frac{1}{16}\frac{5}{16} = \frac{1}{5} \tag{5}$$

4 Probability Second Ball Red

References

[1] Jeremy Orloff and Jonathan Bloom. *Conditional Probability, Independence, Bayes Theorem 18.05*. Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/class-slides/MIT18_05S14_class3slides.pdf` (Spring 2014).