MIT Introduction to Statistics 18.05 Problem Set 2

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Contents

1 References and License

We are answering questions in the material from MIT OpenCourseWare course 18.05, Introduction to Probability and Statistics.
In this document we are answering questions Orloff and Bloom ask in [1] after writing the word, "Think," in bold face font.
Please see the references section for detailed citation information.
The material for the course is licensed under the terms at `http://ocw.mit.edu/terms`.
We use documentation in [2] to write LaTeXsource code for this document.

2 Total area under pdf

The first question Orloff and Bloom ask in [1] is, "What is the total area under the pdf $f(x)$?"
In [1], Orloff and Bloom state that it is a property of probability density functions f that

$$\int_{-infty}^{infty} f(x)dx = 1 \tag{1}$$

Therefore, the total area under the pdf $f(x)$ is 1.

3 Pdf's textemvs probabilities

In [1], Orloff and bloom ask, ": In the previous example f(x) takes values greater than 1. Why does this not violate the rule that probabilities are always between 0 and 1?"
This does not violate the rule that probabilities are always between 0 and 1 because the value of a pdf is not a probability. The value of the integral of a pdf over an interval is a probability. Orloff and Bloom state this above where they ask this question in [1].

4 Probabilities of points

In [1] Orloff and Bloom ask three related questions about CDF's for the probabilities of intervals of length 0 for continuous random variables.
The value of any integral of a continuous function over a region is the value of the antiderivative evaluated at the end of the region, minus the value of the antiderivative at the beginning of the region. Therefore, if the beginning of the region is the same as the end of the region, the value of the integral will be 0. Since the probablity is defined to be the value of the integral of the pdf, the probability of a pdf having the value of a specific point is always 0.
However, this does not mean that the pdf never attains the value that it is defined on for a point. The value of a function at a point, and the value of the integral of a function over the interval that begins and ends at the same point are totally different things.

References

[1] Jeremy Orloff and Jonathan Bloom. *Continuous Random Variables Class 5, 18.05, Spring 2014 Jeremy Orloff and Jonathan Bloom.* Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading5b.pdf` (Spring 2014).

[2] Sharelatex. *Integrals, sums and limits.* Available at `https://www.sharelatex.com/learn/Integrals,_sums_and_limits` (2017).