

MIT Introduction to Statistics 18.05 Reading 6A

Think Questions

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1 References and License

We are answering questions in the material from MIT OpenCourseWare course 18.05, Introduction to Probability and Statistics.

In this document we are answering questions Orloff and Bloom ask in [2].

Please see the references section for detailed citation information.

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2 Questions about X

In this section we answer questions Orloff and Bloom ask in [3] regarding a random variable X .

Orloff and Bloom specify that X is defined on $[0, 1]$, and the pdf of X is cx^2 .

2.1 Value of c

Orloff and Bloom ask us to calculate the value of c . We will use rules and properties for integration from [1] in order to calculate the value for c .

We know

$$\int_0^1 cx^2 dx = 1. \quad (1)$$

Therefore

$$c \int_0^1 x^2 dx = 1. \quad (2)$$

The anti-derivative of x^2 is $\frac{x^3}{3} + C$, so we can replace the integral in the equation above with:

$$c \left(\frac{x^3}{3} \Big|_0^1 \right) = 1. \quad (3)$$

We then evaluate the anti-derivative over the interval $[0, 1]$ to obtain:

$$c \left(\frac{1^3}{3} \right) = 1. \quad (4)$$

This implies $c = 3$.

2.2 Mean, variance, and standard deviation of X

2.2.1 Mean

We use the definition of mean value that Orloff and Bloom give in [3]. The mean value of X is

$$\mu = \int_0^1 x (x^2) dx \quad (5)$$

References

- [1] Michael Dougherty. *Chapter 6 Basic Integration*. Available at <http://faculty.swosu.edu/michael.dougherty/book/chapter06.pdf> (2012/11/20).
- [2] Jeremy Orloff and Jonathan Bloom. *Continuous Expectation and Variance, the Law of Large Numbers, and the Central Limit Theorem 18.05 Spring 2014* Jeremy Orloff and Jonathan Bloom. Available at https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/class-slides/MIT18_05S14_class6slides.pdf (Spring 2014).
- [3] Jeremy Orloff and Jonathan Bloom. *Expectation, Variance and Standard Deviation for Continuous Random Variables Class 6, 18.05, Spring 2014* Jeremy Orloff and Jonathan Bloom. Available at https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading6a.pdf (Spring 2014).