MIT Introduction to Statistics 18.05 Reading 7B Questions

John Hancock

April 14, 2017

Contents

1 References and License

We are answering questions in the material from MIT OpenCourseWare course 18.05, Introduction to Probability and Statistics.
In this document we are answering questions Orloff and Bloom ask in [3].
Please see the references section for detailed citation information.
The material for the course is licensed under the terms at `http://ocw.mit.edu/terms`.
We use documentation in [1] to write LaTeXsource code for this document.

2 Non-zero covariance

The first question Orloff and Bloom ask is: if $\mathrm{Cov}(X, Y) \neq 0$, are X, and Y independent?
In [2] Orloff and Bloom state that, if X, and Y are independent, then $\mathrm{Cov}(X, Y) = 0$.
We use symbolic logic to answer this question. The validity of our answer rests on Peter Williams' work in [5]
Let p be the statement, "X, and Y are independent", and q be the statement, "covariance is 0."

Then, for this question, we know that p implies q.
We use a result from [4] that tells us that p implies q, if and only if $\neg q$ implies $\neg p$.
In this case $\neg q$ is the statement, "covariance $\neq 0$," and $\neg p$ is the statement, "X, and Y are not independent."
Therefore we know that if $\text{Cov}(X, Y) \neq 0$, then X, and Y are not independent.

3 Zero covariance

For this question, Orloff and Bloom ask us a variation on the first, "Suppose $\text{Cov}(X, Y) = 0$ are X and Y independent?"
In [2] Orloff and Bloom state that if X and Y are independent, then their covariance is 0. They state this as a property of covariance.
However, in [2] Orloff and Bloom give an example where two random variables are not independent, and have a covariance of 0.
Therefore cases exist where two independent variables have a covariance of 0, and cases exist where two variables that are not independent have a covariance of 0.
Therefore the answer to this question is maybe.

4 Joint-distributed discrete random variables

For these problems, Orloff and Bloom give us the following joint distribution of discrete random variables, X, and Y:

$\frac{X}{Y}$	0	2	
0	0.25	0.3	0.55
2	0.25	0.2	0.45
	0.5	0.5	

4.1 Independence

The first question Orloff and Bloom have for us on the joint distriubtion is to ask us whether or not X, and Y are independent.
In [2] Orloff and Bloom state that variables X and Y of a joint distribution are independent if each probability $p(x_i, y_j)$ in their joint distribution table is the product of its respective marginal probabilities $p(x_i)$ and $p(y_j)$.
Therefore, if we find one probability in the table above that is not equal to the product of its respective marginal probabilities, then X and Y are not independent.
The joint probability for $X = 0$, and $Y = 0$ is 0.25. The respective marginal probabilities for this joint probability are $p(x_1) = 0.55$ and p $(y_1) = 0.5$. $0.5 \times 0.55 = 0.275$. $0.275 \neq 0.25$, so X annd Y are not independent.

4.2 Covariance

We use the formula for covariance of discrete random variables from [2] to compute the covariance of X and Y:

$$\mathrm{Cov}\left(X, Y\right) = \left(\sum_{i=1}^{n}\sum_{j=1}^{m} p\left(x_i, y_j\right) x_i, y_j\right) - \mu_X \mu_Y. \tag{1}$$

In order to use equation 1 we must compute μ_X, and μ_Y.
μ_X is the expected value of X, which is the sum of the products of the values of X and their respective probabilities.
X takes values 0, and 2, so only the product of 2 and the probability of $X = 2$ will contribute to the expected value of X. There is a probability of 0.3 that X will have the value 2 when $Y = 0$, or a probability of 0.2 that X will will have the value 2 when $Y = 2$. We apply the law of total probability to compuute the probability of $X = 2$. The probability of $X = 2$ is therefore $0.2 + 0.3 = 0.5$. Therefore the expected value of X, $\mu_X = 0.5 \times 2 = 1$.
We apply reasoning similar to what we do above to compute the expected value μ_Y of Y. $\mu_Y = 0.9$.
The next step in our calculation of Cov(X, Y), is to compute the value of the double sum

$$\left(\sum_{i=1}^{n}\sum_{j=1}^{m} p\left(x_i, y_j\right) x_i, y_j\right) = (0.25 \times 0 \times 0)+(0.3 \times 2 \times 0)+(0.25 \times 0 \times 0)+(0.2 \times 2 \times 2) \tag{2}$$

The terms on the right hand side of the equation above sum to 0.8.
Now we know the values of the double sum in equation 1 and μ_X, and μ_Y, so we can calculate Cov$(X, Y) = -0.1$.

References

[1] Latex.org user alainremillard. *Logical Not Symbol.* Available at `http://www.math.csusb.edu/notes/logic/lognot/node2.html` (2012/3/16).

[2] Jeremy Orloff and Jonathan Bloom. *Covariance and Correlation Class 7, 18.05, Spring 2014 Jeremy Orloff and Jonathan Bloom.* Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading7b.pdf` (Spring 2014).

[3] Jeremy Orloff and Jonathan Bloom. *Reading Questions 7b.* Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/reading-questions-7b/` (Spring 2014).

[4] Peter Williams. *Logical equivalence and implication*. Available at http://www.math.csusb.edu/notes/logic/lognot/node2.html (1996/9/2).

[5] Peter Williams. *Statements, truth values and truth tables*. Available at http://www.math.csusb.edu/notes/logic/lognot/node1.html\#SECTION00010000000000000000 (1996/9/2).