MIT Introduction to Statistics 18.05 Reading 6A Think Questions

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Contents

1 References and License

We are answering questions in the material from MIT OpenCourseWare course 18.05, Introduction to Probability and Statistics.
In this document we are answering questions Orloff and Bloom ask in [4].
We use documentation in [2] in order to write the LaTeXcode for this document.
Please see the references section for detailed citation information.

2 The mean does not divde probability mass in half.

The mean value of a continuous random variable X does not necessarily divide the area under the curve of its probability mass function because we may have the case where more values X occur where X is greater (or less than) the median value of X.

3 Questions on $X \sim \mathbf{U}(0,4)$

3.1 Mean and variance of $\mathbf{U}(0,4)$

We use the definition of the mean value of a continuous random variable from [3].
The mean, μ, of $\mathrm{U}(0,4)$ is

$$\mu = \int_0^4 \frac{x}{4}\, dx. \tag{1}$$

We use the power rule for integrals [1] to replace the right hand side of 1 with its anti-derivative:

$$\mu = \left.\frac{x^2}{8}\right|_0^4. \tag{2}$$

Now we can evaluate the antiderivative over the interval $[0,4]$:

$$\left.\frac{x^2}{8}\right|_0^4 = \frac{16}{8} - \frac{0}{8} = 2. \tag{3}$$

Therefore the mean value of $\mathrm{U}(0,4)$ is 2.
Now we turn to computing the variance of $\mathrm{U}(0,4)$.
In [3] Orloff and Bloom define the variance of a continuous random variable X with mean μ as $E\left((X-\mu)\right)$, where E is the function for computing the expected value of X.
Therefore

$$\mathrm{Var}\left(X\right) = \int_0^4 \frac{(x-\mu)^2}{4}\, dx \tag{4}$$

Note:

$$(x-\mu)^2 = x^2 - 2x\mu + \mu^2. \tag{5}$$

Therefore:

$$\int_0^4 \frac{(x-\mu)^2}{4}\, dx = \int_0^4 \frac{x^2 - 2x\mu + \mu^2}{4}\, dx \tag{6}$$

Now we can use theorem 6.1.1 from [1] to obtain:

$$\int_0^4 \frac{x^2 - 2x\mu + \mu^2}{4}\, dx = \frac{1}{4}\left(\int_0^4 x^2\, dx - 2\int_0^4 \mu x\, dx + \int_0^4 \mu^2\, dx\right). \tag{7}$$

Next we apply the power rule for integrals [1] to the equation above to get:

$$\frac{1}{4}\left(\int_0^4 x^2\, dx - 2\int_0^4 \mu x\, dx + \int_0^4 \mu^2\, dx\right) = \frac{1}{4}\left(\left.\frac{x^2}{2}\right|_0^4 - \left.\mu x^2\right|_0^4 + \left.\mu^2 x\right|_0^4.\right) \tag{8}$$

References

[1] Michael Dougherty. *Chapter 6 Basic Integration*. Available at http://faculty.swosu.edu/michael.dougherty/book/chapter06.pdf (2012/11/20).

[2] StackExchange.com user Hendrik Vogt. *What's the proper way to typeset a differential operator?* Available at http://tex.stackexchange.com/questions/14821/whats-the-proper-way-to-typeset-a-differential-operator (2011/4/3).

[3] Jeremy Orloff and Jonathan Bloom. *Expectation, Variance and Standard Deviation for Continuous Random Variables Class 6, 18.05, Spring 2014 Jeremy Orloff and Jonathan Bloom*. Available at https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading6a.pdf (Spring 2014).

[4] Jeremy Orloff and Jonathan Bloom. *Reading Questions 6a*. Available at https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/reading-questions-6a/ (Spring 2014).