MIT Introduction to Statistics 18.05 Reading 7B Questions

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1 References and License

We are answering questions in the material from MIT OpenCourseWare course 18.05, Introduction to Probability and Statistics.
In this document we are answering questions Orloff and Bloom ask in [3].
Please see the references section for detailed citation information.
The material for the course is licensed under the terms at `http://ocw.mit.edu/terms`.
We use documentation in [1] to write LaTeXsource code for this document.

2 Non-zero covariance

The first question Orloff and Bloom ask is: if $\mathrm{Cov}(X, Y) \neq 0$, are X, and Y independent?
In [2] Orloff and Bloom state that, if X, and Y are independent, then $\mathrm{Cov}(X, Y) = 0$.
We use symbolic logic to answer this question. The validity of our answer rests on Peter Williams' work in [5]
Let p be the statement, "X, and Y are independent", and q be the statement, "covariance is 0."
Then, for this question, we know that p implies q.
We use a result from [4] that tells us that p implies q, if and only if $\neg q$ implies $\neg p$.

In this case $\neg q$ is the statement, "covariance $\neq 0$," and $\neg p$ is the statement, "X, and Y are not independent."
Therefore we know that if $\text{Cov}(X,Y) \neq 0$, then X, and Y are not independent.

3 Zero covariance

For this question, Orloff and Bloom ask us a variation on the first, "Suppose $\text{Cov}(X,Y) = 0$ are X and Y independent?"
In [2] Orloff and Bloom state that if X and Y are independent, then their covariance is 0. They state this as a property of covariance.
However, in [2] Orloff and Bloom give an example where two random variables are not independent, and have a covariance of 0.
Therefore cases exist where two independent variables have a covariance of 0, and cases exist where two variables that are not independent have a covariance of 0.
Therefore the answer to this question is maybe.

References

[1] Latex.org user alainremillard. *Logical Not Symbol.* Available at `http://www.math.csusb.edu/notes/logic/lognot/node2.html` (2012/3/16).

[2] Jeremy Orloff and Jonathan Bloom. *Covariance and Correlation Class 7, 18.05, Spring 2014 Jeremy Orloff and Jonathan Bloom.* Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading7b.pdf` (Spring 2014).

[3] Jeremy Orloff and Jonathan Bloom. *Reading Questions 7b.* Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/reading-questions-7b/` (Spring 2014).

[4] Peter Williams. *Logical equivalence and implication.* Available at `http://www.math.csusb.edu/notes/logic/lognot/node2.html` (1996/9/2).

[5] Peter Williams. *Statements, truth values and truth tables.* Available at `http://www.math.csusb.edu/notes/logic/lognot/node1.html\#SECTION00010000000000000000` (1996/9/2).