MIT Introduction to Statistics 18.05 Reading 3 - Questions

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Contents

1 References and License

We are answering questions in the material from MIT OpenCourseWare course 18.05, Introduction to Probability and Statistics.
Please see the references section for detailed citation information.
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We are answering the questions in [1].
We use documentation in [2], [3], [4], and [5] for properly writing the LaTeXsource code for this document.

2 Problem 1

You roll two dice. Consider the following events.
A = 'first die is 3'
B = 'sum is 7'
C = 'sum is greater than or equal to 7'

2.1 Compute $P(B)$; Dice Sum to 7

We are rolling two dice so the sample space, Ω, is $\{(x, y) \mid x, y \in \{1, 2, 3, 4, 5, 6\}\}$
Then B is $\{(x, y) \in \Omega \mid x + y = 7\}$.

Therefore by inspection $B = \{(1,6),(6,1),(5,2),(2,5),(3,4),(4,3)\}$
There are 36 sequences of integers (x,y) for $(x,y) \in \{1,2,3,4,5,6\}$. There are 6 elements in B, so $P(B) = \frac{6}{36} \approx 0.1667$.

2.2 Compute $P(B \mid A)$

$$P(B \mid A) = \frac{P(B \cap A)}{P(A)} \tag{1}$$

We defined elements of B in the previous section, and listed them out.
We define A:

$$A = \{(x,y) \mid x = 3, y \in \{1,2,3,4,5,6\}\} \tag{2}$$

A has six elements.
B has one element where the first element of the sequence is 3, $(3,4)$.
Therefore $(A \cap B) = (3,4)$.
Now, we have all the information we need to calculate $P(B \mid A)$.
We Coninue from 2:

$$\frac{P(B \cap A)}{P(A)} = \frac{|(B \cap A)|}{|B|} = \frac{1}{6} \approx 0.167 \tag{3}$$

2.3 Compute $P(B \mid C)$

We define B in the previous section. C is:

$$C = \{(x,y) \mid x + y \geq 7, x, y \in \{1,2,3,4,5,6\}\} \tag{4}$$

We count the number of elements of C to find a vaule for $|C|$.

$$\begin{aligned} C = &\{(1,6),(2,5),(2,6),(3,4),(3,5),(3,6),(4,3),(4,4),(4,5),(4,6)\} \\ \cup &\{(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\} \end{aligned} \tag{5}$$

We count elements of C listed above to find that $|C| = 21$. However, it behooves us to note that

$$|C| = \sum_{i=1}^{6} i = \frac{6 \times 7}{2} = \frac{42}{2} = 21. \tag{6}$$

Therefore:

$$P(B \mid C) = \frac{|(B \cap C)|}{|C|} = \frac{6}{21} \approx 0.286 \tag{7}$$

References

[1] Jeremy Orloff and Jonathan Bloom. *Reading Questions 3*. Available at https://ocw.mit.edu/courses/mathematics/ 18-05-introduction-to-probability-and-statistics-spring-2014/readings/reading-questions-3/(Spring 2014).

[2] Scott Pakin. *The Comprehensive Latex Symbol List*. Available at `https://math.uoregon.edu/wp-content/uploads/2014/12/compsymb-1qyb3zd.pdf`(2002/10/8).

[3] Stack Exchange Users Will Robertson and Steven. *How can I split an equation over two lines (Answer)*. Available at http://tex.stackexchange.com/questions/3782/how can-i-split-an-equation-over-two-lines(Spring 2014).

[4] ShareLaTex. *Bold, Italics and Underlining*. Available at `https://www.sharelatex.com/learn/Bold,_italics_and_underlining`(2017).

[5] Stack Exchange User zhouyiyu. *How to get LaTeX symbol in document (Answer)*. Available at http://tex.stackexchange.com/questions/7546/how-to-get-latex-symbol-in-document(Spring 2014).