

Answers To Questions in Conditional Probability, Independence, Bayes Theorem 18.05

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1 References and License

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In this document, we are answering the questions in [1].

We use documentation in [3] for properly writing the \LaTeX source code for this document.

2 Probability At Least 3 Heads, Given First Toss Tails

We are tossing a coin four times. Therefore we define the sample space

$$\Omega = \{(x_1, x_2, x_3, x_4), x_1, x_2, x_3, x_4 \in \{H, T\}\} \quad (1)$$

$$|\Omega| = 16$$

We assume all outcomes are equally likely.

A is the event that at we toss heads at least 3 times.

B is the event that we toss tails the first time.

We use the definition of conditional probability to caculate $P(A | B)$.

$$P(A | B) = \frac{P(A \cap B)}{P(B)}, \text{ Provided } P(B) \neq 0 \quad (2)$$

$P(A) = \frac{5}{16}$ since there are 5 elements of Ω that represent the event that we toss heads at least three times, and we assume all outcomes are equally likely.

These are: $(T, H, H, H), (H, T, H, H), (H, H, T, H), (H, H, H, T), (H, H, H, H)$.

The elements of B are $(T, T, T, T), (T, T, T, H), (T, T, H, T), (T, T, H, H), (T, H, T, T), (T, H, T, H), (T, H, H, T), (T, H, H, H)$.

By inspection $A \cap B$ is the element (T, H, H, H) .

We substitute values into 2 to get

$$P(A | B) = \frac{\frac{1}{16}}{\frac{8}{16}} = \frac{1}{8} \quad (3)$$

3 Probability First Toss Tails, Given At Least 3 Heads

We use 2 and definitions of the sets Ω , A , $A \cap B$, and B that we define in section 1. In addition we assume all outcomes are equally likely.

We use 2 to get

$$P(B | A) = \frac{P(B \cap A)}{P(A)}, \text{ Provided } P(A) \neq 0 \quad (4)$$

The \cap operator is commutative, so $P(A \cap B) = P(B \cap A)$, and we discover $P(A \cap B)$ in section 1.

Therefore,

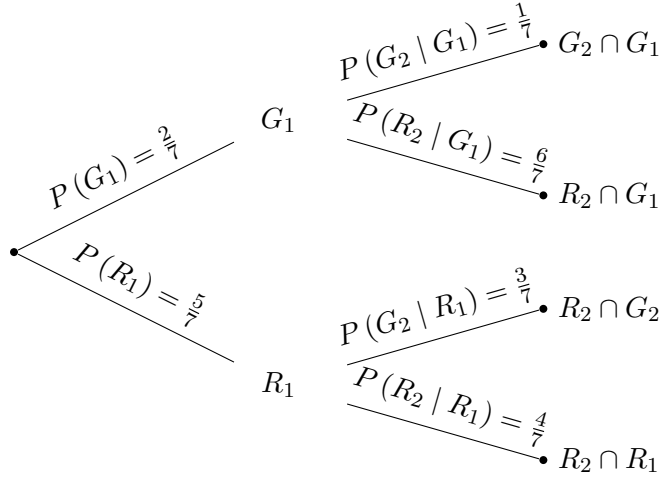
$$P(B | A) = \frac{\frac{1}{16}}{\frac{5}{16}} = \frac{1}{5} \quad (5)$$

4 Probability Urn

4.1 Probability Second Ball Red

Orloff and Bloom ask the following question in [1], "What is the probability the second ball is red?"

We are given the probability tree below:



We use the law of total probability to write an equation for $P(R_2)$.

$$P(R_2) = P(R_1 \cap R_2) + P(G_1 \cap R_2) \quad (6)$$

Now we can use the definition of conditional probability to rewrite 6:

$$P(R_2) = P(R_2 | R_1) P(R_1) + P(R_2 | G_1) P(G_1) \quad (7)$$

Orloff and Bloom give us the probabilities in 7 in the probability tree above, so we can use them to compute $P(R_2)$.

$$P(R_2) = \left(\frac{4}{7}\right) \left(\frac{5}{7}\right) + \left(\frac{6}{7}\right) \left(\frac{2}{7}\right) = \frac{20}{49} + \frac{12}{49} = \frac{32}{49} \approx 0.653 \quad (8)$$

4.2 Probability First Ball Red, Given second Ball Red

To answer the question, "What is the probability the first ball was red given the second ball was red?" [1]

This question is asking for $P(R_1 | R_2)$.

Since we know $P(R_2 | R_1)$, we apply Bayes Theorem to compute $P(R_1 | R_2)$.

$$P(R_1 | R_2) = \frac{P(R_2 | R_1) P(R_1)}{P(R_2)} \quad (9)$$

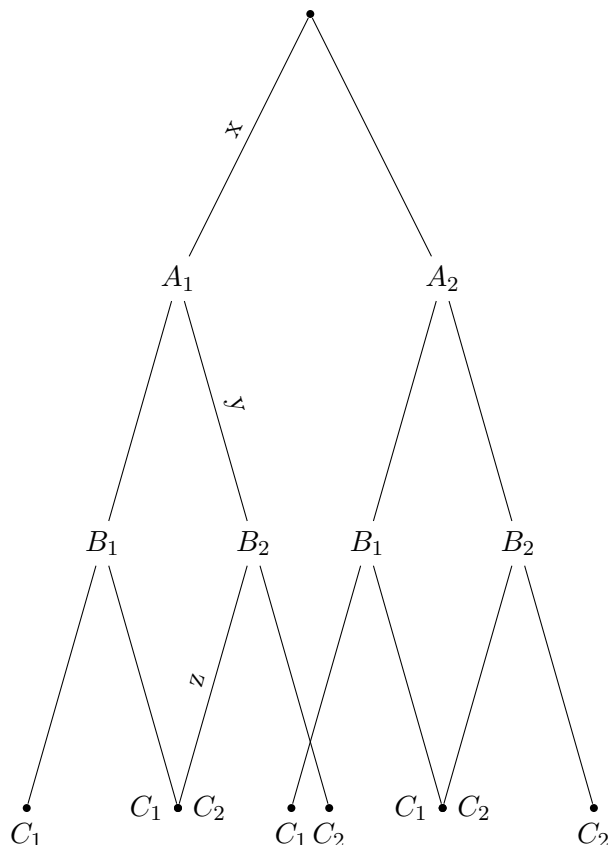
Bloom and Orloff give us the values for all of the probabilities in the right hand side of 10 in the probability tree above.

Therefore

$$P(R_1 | R_2) = \frac{\left(\frac{4}{7}\right) \left(\frac{5}{7}\right)}{\frac{32}{49}} = \left(\frac{20}{49}\right) \left(\frac{49}{32}\right) = \frac{20}{32} \approx 0.625 \quad (10)$$

5 Concept Questions on Probability Trees

In this section, we answer questions about the probability tree that Orloff and Bloom give in [1].



The edge labeled, "x," in the figure above represents $P(A_1)$.

The edge labeled, "y," in the figure above represents $P(B_2 | A_1)$.

The edge labeled, "z," in the figure above represents $P(C_1 | A_1 \cap B_2)$. We read section 5.1, titled, "Shorthand vs. precise trees," in [1] in order to understand what the edge labeled, "z," represents. This section explains that nodes having a distance of two or more edges to the root of the tree represent the probabilities of intersections of sets of outcomes.

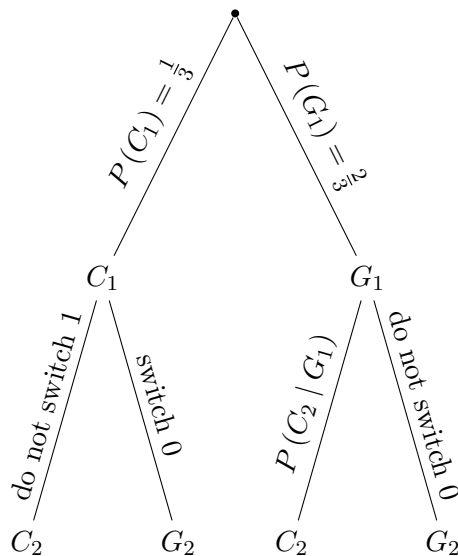
The node labeled, "C₁," at the end of the path, A_1, B_2, C_2 , represents the event, $A_1 \cap B_2 \cap C_1$. This is also clear when we read section 5.1, titled, "Shorthand vs. precise trees," in [1].

6 Monty Hall Problem

In this section we draw a probability tree to answer the question of which action we should take at the last step of the Monty Hall problem.

Note: we had to look at [2] to write the answer to this question.
 We give the following definitions of events:

- C is the event we choose the door with the car behind it.
- G is the event we choose the door with a goat behind it.



We want to know

References

- [1] Jeremy Orloff and Jonathan Bloom. *Conditional Probability, Independence, Bayes Theorem 18.05*. Available at https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/class-slides/MIT18_05S14_class3slides.pdf (Spring 2014).
- [2] Jeremy Orloff and Jonathan Bloom. *Conditional Probability, Independence, Bayes Theorem 18.05 S*. Available at https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/class-slides/MIT18_05S14_class3_slides.pdf (Spring 2014).
- [3] Stack Exchange User Michael Underwood. *What is the correct way of embedding text into math mode? (Micheal Underwood's answer)*. Available at <http://tex.stackexchange.com/questions/3415/what-is-the-correct-way-of-embedding-text-into-math-mode> (2010/9/23).