MIT Introduction to Statistics 18.05 Reading 3 - *Think* Questions

John Hancock

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1 References and License

2 *Think* What is $P\left(S_2 \mid S_1^{\complement}\right)$

S_1 = 'first card is a spade' S_2 = 'second card is a spade'
Therefore $S_2^{\complement}$ = 'second card is not a spade'
We will calculate

$$P\left(S_2 \mid S_1^{\complement}\right) \tag{1}$$

That is, we will calculate the probability that the first card is a spade given that the second card is not a spade.
We will apply the same method Orloff and Bloom use in [2], section 3, "Multiplication Rule."
If the first card is a spade then of the 51 cards remaining, $3 \times 13 = 39$ cards are not spades.

Therefore

$$P\left(S_2 \mid S_2^{\complement}\right) = \frac{39}{51} \tag{2}$$

We use the multiplication rule from [2] to compute the same probability.
We apply the multiplcation rule letting $A = S_1$ and $B = S_2^{\complement}$

$$P\left(S_1 \cap S_2^{\complement}\right) = P\left(S_1 \mid S_2^{\complement}\right) \cdot P\left(S_2^{\complement}\right) \tag{3}$$

$P(S_1) = \frac{13}{52}$ This is because 13 of the 52 cards are spades.
$P\left(S_2^{\complement}\right) = \frac{39}{52}$ because 39 cards are not spades, and we are dividing by 52 because any of the 39 out of 52 cards are equallly likely in all possible sequences of two cards.
$P\left(S_1 \cap S_2^{\complement}\right) = \frac{13}{52}\frac{39}{51}$ Because there are 13 out of 52 spades to draw for the first card, and 39 out of 51 cards that are not spades to draw for the second card.
We computed $P\left(S_1 \mid S_2^{\complement}\right)$ in equation 2. Now we will solve equation 4 for $P\left(S_1 \mid S_2^{\complement}\right)$:

$$P\left(S_1 \mid S_2^{\complement}\right) = \frac{P\left(S_1 \cap S_2^{\complement}\right)}{P\left(S_2^{\complement}\right)} = \frac{\left(\frac{13}{52}\frac{39}{51}\right)}{\frac{39}{52}} = \frac{13}{52}\frac{52}{39}\frac{39}{51} = \frac{13}{51} \tag{4}$$

This is the same result we found in equation 2.

3 *Think:* For what other value(s) of P(A) is A independent of itself?

Any value of $P(A)$ where A will be independent of itself will satisfy the equation

$$P(A) = P(A)^2 \tag{5}$$

We find this equation in [1].
Therefore members of the set $\left\{x \mid x = x^2, x \in \Re\right\}$ are the values of P we seek.
We solve for x:

$$x = x^2 \tag{6}$$

$$\iff -x^2 + x = 0 \tag{7}$$

$$\iff -x(x-1) = 0 \tag{8}$$

$$\iff x(x-1) = 0 \tag{9}$$

$$\iff x \in \{0, 1\} \tag{10}$$

Therefore another value for $P(A)$ that is independent of itself is $P(A) = 1$. Furthermore, because we solved 6 for $\{x \in \Re\}$, these are the only values for $P(A)$ for which A is independent of itself. $\blacksquare$

References

[1] Luther Martin. *Events that are independent of Themselves.* Available at https://www.voltage.com/math-2/events-that-are-independent-of-themselves/(2011/6/26).

[2] Jeremy Orloff and Jonathan Bloom. *Conditional Probability, Independence and Bayes Theorem Class 3, 18.05, Spring 2014.* Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading3.pdf` (Spring 2014).