MIT Introduction to Statistics 18.05 Problem Set 1

John Hancock

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An element of $\omega \in \Omega$ is a sequence of birthdays.
We gave a formal definition of Ω in
A birthday is an integer from 1 through 365. All birthdays are equally likely, and two or more people can be born on the same day, so any ω of length n is a sample with replacement from the set $\{1, 2, 3, ..., 365\}$. We will use 365^n for the number of $\omega \in \Omega$ for a given n is.
Using 365^n for the total number of $\omega \in \Omega$ means that the order of birthdays in a given ω is important. To see this, consider that for $n = 2$ two elements of Ω are $(5, 6)$ and $(6, 5)$.
Therefore 365^n is the denomenator we use when we are dividing our countof events by the total number of events to calculate a probability.
It will be easier to calculate the probability $\bar{p}$ that for an ω containing n birthdays, none of the birthdays are the same. Then the probability p that some two birthdays in ω are the same will be $1 - \bar{p}$.
It is easier to calculate $1 - \bar{p}$ because in order to calculate p directly, we have to take into account that there are *binomn*2 ways to select two birthdays in ω to be the same, or *binomn*3 ways to selct three birthdays in ω to be the same, or so on.
To calculate $1 - \bar{p}$ we must count the number of ways to select n birthdays from a set of 365 birthdays where no two birthdays are the same.
Recall that we are using 365^n as the total number of $\omega \in \Omega$, and the order of birthdays in a given ω is important to us.
The number of samples of size n of 365 elements, where the order is important is ${}_{365}P_n$. These samples of size n meet the definition of elements of Ω.
Therefore the probability *hatp* that some $\omega \in \Omega$ has n distinct birthdays is:

$$\bar{p} = \frac{{}_{365}P_n}{365^n} \tag{1}$$