MIT Introduction to Statistics 18.05 Reading 4B - *Think* Questions

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Contents

1 References and License

We are answering questions in the material from MIT OpenCourseWare course 18.05, Introduction to Probability and Statistics.
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We are answering the questions that Orloff and Bloom ask after the word,"think," in [3].
We use documentation in [1], and [2] to write LaTeXsource code of this document.

2 Would We Be Willing to Play?

Would we be willing to play a game of chance where the average expected loss is \$ 69.44?
No.
Over a large number of trials, we would expect to win about once every 36 times. Hence loosing about $\$3,500$ before winning the $\$1,000$; we would eventually run out of money.
If we were to play once, we would have a $\frac{1}{36}$ chance of winning, which we do not consider to be good odds.

3 Expected Value of the Sum of Two Dice

Orloff and Bloom state that the expected value of rolling one die is 3.5.
In this think question, Orloff and Bloom ask us what is the expected value for rolling two dice.
It is $3.5 + 3.5 = 7$.
We can use the formula for expected value that Orloff and Bloom give in [3] to verify.
We define X to be the random variable whose value is the sum of the values we roll with two dice.
We give the probabilities for each of the sums we could roll:

Value a	2	3	4	5	6	7	8	9	10	11	12
pmf $p(a)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Then, the expected value, $E(X)$ is:

$$E(X) = 2\cdot\frac{1}{36}+3\cdot\frac{2}{36}+4\cdot\frac{3}{36}+5\cdot\frac{4}{36}+6\cdot\frac{5}{36}+7\cdot\frac{6}{36}+8\cdot\frac{5}{36}+9\cdot\frac{4}{36}+10\cdot\frac{3}{36}+11\cdot\frac{2}{36}+12\cdot\frac{1}{36} \tag{1}$$

We use R to do the arithmetic operations on the numbers in the right hand side of 1 to get the value 7:

```
> 2*1/36 + 3*2/36 + 4*3/36 + 5*4/36 + 6*5/36 + 7*6/36 + 8*5/36 + 9*4/36
+ 10*3/36 + 11*2/36 +12*1/36
[1] 7
```

4 If $Y = h(X)$ does $E(Y) = h(E(X))$?

In [3] Orloff and Bloom state that this is not true in general.
They also ask us if it is true for example 13 in [3].
We see in 3 that the expected value $E(X)$ for the sum of rolls of two dice is 7.
In example 13 $h(X) = X^2 - 6X + 1$, so $h(E(X)) = 49 - 42 + 1 = 8$
However, in example 13, Orloff and Bloom show that $E(h(x)) \approx 13.833$
Therefore it is not true in example 13 that $E(h(x)) = h(E(X))$
Therefore it is not true that $E(Y) = h(E(X))$ if $Y = h(X)$

References

[1] Stack Exchange User ArTourter. *Quote marks are backwards (using texmaker/PDFLatex) [duplicate] ArTouter's Answer*. Available at `http://tex.stackexchange.com/questions/52351/quote-marks-are-backwards-using-texmaker-pdflatex` (2012/4/18).

[2] Latex community.org user localghost. *Getting the Dollar Sign in the Output*. Available at http://latex-community.org/forum/viewtopic.php?t=5833 (2007/2/2).

[3] Jeremy Orloff and Jonathan Bloom. *Discrete Random Variables: Expected Value Class 4, 18.05, Spring 2014 Jeremy Orloff and Jonathan Bloom*. Available at https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading4b.pdf (Spring 2014).