MIT Introduction to Statistics 18.05 Slides 4 - Questions

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Contents

1 References and License

We are answering questions in the material from MIT OpenCourseWare course 18.05, Introduction to Probability and Statistics.
Please see the references section for detailed citation information.

We are answering the questions that Orloff and Bloom ask in [2].
We use documentation in [4] to write LaTeXsource code for this document.

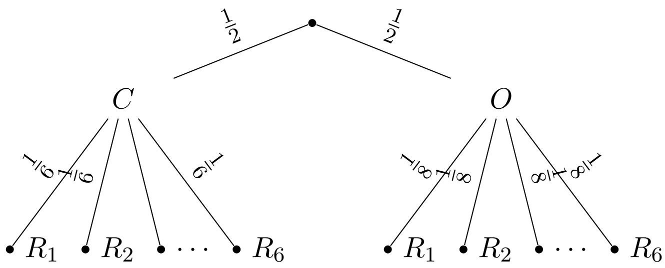
2 Conditional Probability of Unknown Die

The first question Orloff and Bloom give in [2] is:

> 1. The Randomizer holds the 6-sided die in one fist and the 8-sided die in the other.
> 2. The Roller selects one of the Randomizers fists and covertly takes the die.
> 3. The Roller rolls the die in secret and reports the result to the table.
>
> Given the reported number, what is the probability that the 6-sided die was chosen?

Note: we needed to see the solution in [3] in order to write the answer to this question.

We have two cases.
The first case is the Roller reports a 7 or an 8. Then the probability that the 6-sided die was chosen is 0.
The second case is the Roller reports a number with a value from 1 to six.
We draw a probability tree to get started on a solutionn. We refer to the section titled, "Shorthand vs. precise trees," in [1] for guidance on drawing the tree below.
C is the event that the Roller selects the cube shaped 6-sided die.
O is the event that the Roller selects the octohedron shaped 8-sided die.
$R1$, $R2$, ..., $R6$ are the events that the Roller reports one, two, or so on to six.



Since the probabilities on all edges in the tree connected to C are $\frac{1}{6}$, and the probabilities on all edges in the tree connected to O are $\frac{1}{8}$, we can calculate $P(C \mid R_1)$, and the result will be the same for any of the other leaf nodes in the tree above. This is because the calculation will involve the same numbers $\frac{1}{2}$, $\frac{1}{6}$, and $\frac{1}{8}$, and the same operations on these numbers.
Using the tree above, we can calculate $P(R_1 \mid C)$.
Now, we use Bayes' theorem in [1] to calculate $P(C \mid R_1)$

$$P(C \mid R_1) = \frac{P(R_1 \mid C)\, P(C)}{P(R_1)} \tag{1}$$

Now, we apply definitions for values on various parts of probability trees using the section titled "Shorthand vs. precise trees," in [1] to obtain values for the numerator and denominator on the righthand side of 1.
From the probability tree,

$$P(R_1 \mid C) = \frac{1}{6} \tag{2}$$

$P(C) = \frac{1}{2}$. Note: we are assuming the Roller uses either die with equal probability.
We apply Bayes rule [1] and the Law of Total Probability [1] to compute $P(R_1)$.

$$P\left(R_1\right) = P\left(R_1 \cap C\right) + P\left(R_1 \cap O\right) = P\left(R_1 \mid C\right) P\left(C\right) + P\left(R_1 \mid O\right) P\left(O\right) = \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{8}\right)\left(\frac{1}{2}\right) \tag{3}$$

Now we have values for the numerator and denominator of the right hand side of 1.

$$P\left(C \mid R_1\right) = \frac{\left(\frac{1}{6}\right)\left(\frac{1}{2}\right)}{\left(\frac{1}{6}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{8}\right)\left(\frac{1}{2}\right)} \tag{4}$$

References

[1] Jeremy Orloff and Jonathan Bloom. *Conditional Probability, Independence and Bayes Theorem Class 3, 18.05, Spring 2014*. Available at https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading3.pdf (Spring 2014).

[2] Jeremy Orloff and Jonathan Bloom. *Discrete Random Variables; Expectation 18.05 Spring 2014 Jeremy Orloff and Jonathan Bloom*. Available at https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/class-slides/MIT18_05S14_class4slides.pdf (Spring 2014).

[3] Jeremy Orloff and Jonathan Bloom. *Discrete Random Variables; Expectation 18.05 Spring 2014 Jeremy Orloff and Jonathan Bloom*. Available at https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/class-slides/MIT18_05S14_class4_slides.pdf (Spring 2014).

[4] ShareLaTex. *Typesetting quotations*. Available at https://www.sharelatex.com/learn/Typesetting_quotations (Spring 2014).