MIT Introduction to Statistics 18.05 Problem Set 3

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Contents

1 References and License

We are answering questions in the material from MIT OpenCourseWare course 18.05, Introduction to Probability and Statistics.
In this document we are answering questions Orloff and Bloom ask in [1].
Please see the references section for detailed citation information.

We use documentation in [6], [5] to write LaTeXsource code for this document.

2 Independence

In this section we answer a problem in [1] that involves rolling two six sided dice.

2.1 Pairwise and mutual independence

We define two events, A, and B to be pairwise independent if $P(A \cap B) = P(A)P(B)$.

For this problem Orloff and Bloom give us the definition of mutual independence for three events, A, B, and C. A, B, and C are mutally independent if

$$P(A \cap B \cap C) = P(A)\, P(B)\, P(C) \tag{1}$$

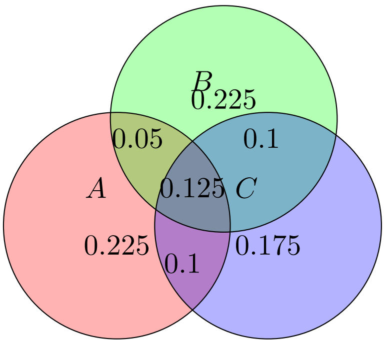
In this section, Orloff and Bloom give the following definitions for events A, B, and C:

- A is the event that we roll an odd number with the first die.
- B is the event that we roll an odd number with the second die.
- C is the event that the sum of the numbers we roll is odd.

A, B, and C are not mutually independent. Whatever the A, B, and C probabilities of A, B, and C are individually, the probability of $P(A \cap B \cap C)$ is 0 since the sum of two odd numbers is always an even number.

2.2 Venn diagram

Orloff and Bloom give the following Venn diagram:



And ask us whether or not the events in the Venn diagram above are mutually independent.
These events are not mutually independent because

$$P(A)\, P(B)\, P(C) = 0.225 \times 0.225 \times 0.175 = 0.008859375. \tag{2}$$

However, in the Venn diagram above, Orloff and Bloom give us that $P(A \cap B \cap C) = 0.125$
Therefore the events are not mutually independent.

2.3 How many kids

For this question we use the same assumptions about the probability of the gender that a child is born with that Orloff and Bloom use in example 9 of [3].
We define the following events:

- A is the event that the children in a family are both boys and girls.
- B is the event that at most one of the children is a girl.
- $C_{i,b}$ is the event that child number i is a boy.
- $C_{i,g}$ is the event that child number i is a girl.

Our goal is to construct a sample space such that A and B are independent. The definition of independent events is in [2].
We rely on the same assumption that Orloff and Bloom make in [3] regarding the probability of the genders of sequences of children.
Therefore we assume $P(C_{i,b}) = 0.5$, and $P(C_{i,g}) = 0.5$, independent of the event that any other child is a boy or a girl.
We write the following table to discover the number of children where A, and B will meet the definition of independent events.
We fill in one cell in the table below for each possible sequnce of three children in the family being boys or girls.

$C_{1,b}C_{2,b}C_{3,b}$	$C_{1,b}C_{2,b}C_{3,g}$	$C_{1,b}C_{2,g}C_{3,b}$	$C_{1,b}C_{2,g}C_{3,g}$
$C_{1,g}C_{2,b}C_{3,b}$	$C_{1,g}C_{2,b}C_{3,g}$	$C_{1,g}C_{2,g}C_{3,b}$	$C_{1,g}C_{2,g}C_{3,g}$

In the table above there are 6 sequences that are in A, so $P(A) = \frac{6}{8}$.
Also, there are 4 sequences in B, so $P(B) = \frac{4}{8}$.
Moreover, there are 3 sequences where there is at most one girl, and the children are both boys and girls. Therefore $P(A \cap B) = \frac{3}{8}$.
A and B are indpendent since

$$P(A)P(B) = \left(\frac{6}{8}\right)\left(\frac{4}{8}\right) = \frac{24}{64} = \frac{3}{8}. \tag{3}$$

Therefore $P(A)P(B) = P(A \cap B)$, so A and B must be independent events.
We made these calculations assuming that there are 3 children, therefore the number of children we require in order for A, and B to be independent events is 3.

3 Dice

In this section we will deal with problems that Orloff and Bloom ask about the random variable X, that is equal to the value we roll with a fair 4-sided die, the random variable Y, that is equal to the value we roll with a fair 6 sided die, and the random variable Z, that is equal to the average of X and Y.

3.1 Standard deviation of X, Y, and Z

We use the definition of variance and standard deviation in [4] to calculate the standard deviations $\sigma(X)$, $\sigma(Y)$.

$$\sigma(X) = 1.708. \tag{4}$$

$$sigma(Y) = \tag{5}$$

References

[1] Jeremy Orloff and Jonathan Bloom. *18.05 Problem Set 3, Spring 2014*. Available at https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/assignments/MIT18_05S14_ps3.pdf (Spring 2014).

[2] Jeremy Orloff and Jonathan Bloom. *Conditional Probability, Independence and Bayes Theorem Class 3, 18.05, Spring 2014*. Available at https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading3.pdf (Spring 2014).

[3] Jeremy Orloff and Jonathan Bloom. *Discrete Random Variables Class 4, 18.05, Spring 2014 Jeremy Orloff and Jonathan Bloom*. Available at https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading4a.pdf (Spring 2014).

[4] Jeremy Orloff and Jonathan Bloom. *Variance of Discrete Random Variables Class 5, 18.05, Spring 2014 Jeremy Orloff and Jonathan Bloom*. Available at https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading5a.pdf (Spring 2014).

[5] StackExchange.com user Paul Gessler. *TikZ: How to set a node on an exact position on a line?* Available at http://tex.stackexchange.com/questions/147052/tikz-how-to-set-a-node-on-an-exact-position-on-a-line (2015/2/08).

[6] Texample.net user Till Tantau. *Example: Venn diagram*. Available at http://www.texample.net/tikz/examples/venn-diagram/ (2006/11/08).