# MIT Introduction to Statistics 18.05 Problem Set 2

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## Contents



## 1 References and License

We are answering questions in the material from MIT OpenCourseWare course 18.05, Introduction to Probability and Statistics.
Please see the references section for detailed citation information.
The material for the course is licensed under the terms at `http://ocw.mit.edu/terms`.
We are answering the questions that Orloff and Bloom ask in [1].
We use documentation in to write LaTeXsource code for this document.

## 2 Variance of a random variable

In this section we answer the question that Orloff and Bloom ask regarding the variance of a discrete random variable.
The following table describes a discrete random variable $X$:

| $X$ | 1 | 2 | 4 |
|---|---|---|---|
| PMF $p(x)$ | 0.2 | 0.3 | 0.5 |

We will use the formula from [2] for variance:

$$Var\left(X\right) = E\left(X^2\right) - E\left(X\right)^2 \tag{1}$$

First we compute $E\left(X\right)$:

$$E\left(X\right) = 1 \times 0.2 + 2 \times 0.3 + 4 \times 0.5 = 0.2 + 0.6 + 2 = 2.8 \tag{2}$$

Now we compute $E\left(X^2\right)$:

$$E\left(X^2\right) = 1 \times 0.2 + 4 \times 0.3 + 16 \times 0.5 = 0.2 + 1.2 + 8 = 9.4 \tag{3}$$

Now we apply the formula for variance in equation 1:

$$Var\left(X\right) = 9.4 - \left(2.8\right)^2 = 9.4 - 7.84 = 1.56 \tag{4}$$

# 3 Apply properties of variance

The second question in [1] is in several parts; Orloff and Bloom ask us several questions where we apply the properties of the variance of discrete random variables to answer.
Orloff and Bloom write that $X$ is a random variable with mean 2 and variance 3.

## 3.1 Variance of constant multiple

Orloff and Bloom ask is for us to compute $Var\left(3X\right)$
In [2] Orloff and Bloom show for constants $a$, and $b$

$$Var\left(aX + b\right) = a^2 Var(X) \tag{5}$$

Therefore $Var\left(3X\right) = 9 \times 3 = 27$

## 3.2 Variance of multiple plus constant

Orloff and Bloom pose a second question that is a variation on the previous question. They ask us to compute $Var\left(3X + 8\right)$
However, by 5 the answer to this question is no different than the answer to the previous question. Therefore, $Var\left(3X + 8\right) = 27$

## 3.3 Expected value of square

Orloff and Bloom ask us to compute $E\left(X^2\right)$.
We apply equation 1 to compute the answer to this question.

$$Var\left(X\right) = E\left(X^2\right) - E\left(X\right)^2 \tag{6}$$

This implies

$$3 = E\left(X^2\right) - 4 \tag{7}$$

We rewrite the equation above placing unknown terms on the left hand side, and known terms on the right hand side of the equation below:

$$E\left(X^2\right) = 4 + 3 = 1 \tag{8}$$

# 4 Further Questions on Same Random Variable

For this problem, Orloff and Bloom continue to ask us questions on the same random variable they define in the previous question.

## 4.1 Variance of square

In this section, Orloff and Bloom ask us if we can compute a value for $Var\left(X^2\right)$.
We cannot compute this value because the only information we have about $X$ is the value of its mean and variance. Terms in the sum to compute $Var\left(X\right)$, $\left(x_i - \mu\right)^2$ might be very different from terms in the sum to compute $Var\left(X^2\right)$, $\left(x_i^2 - \mu\right)^2$.

# References


[1] Jeremy Orloff and Jonathan Bloom. *Reading Questions 5a*. Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/reading-questions-5a/` (Spring 2014).

[2] Jeremy Orloff and Jonathan Bloom. *Variance of Discrete Random Variables Class 5, 18.05, Spring 2014 Jeremy Orloff and Jonathan Bloom.* Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading5a.pdf` (Spring 2014).