Problem Set 4

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1 References and License

We are answering questions in the material from MIT OpenCourseWare course 18.05, Introduction to Probability and Statistics.
In this document we are answering questions Orloff and Bloom ask in [3].
Please see the references section for detailed citation information.

We use documentation in [**logicNot**], [**proofs**], [**bars**], [**packageClash**], [**curlyBrace**], [**cases**] to write the LaTeXsource code for this document.

2 Time to failure

The first group of problems Orloff and Bloom have for us involve some random variables that follow an exponential distribution.
The exponential distribution they give us to work with has probability density function (pdf):

$$f(x) = \lambda e^{-\lambda x}, x \geq 0. \tag{1}$$

2.1 $P(X \geq x)$

We know how to calculate $P(X < x)$ as a definite integral [1], therefore we will find $P(X < x)$, and our final result will be to find $P(X \geq x) = 1 - P(X < x)$.

In order to calculate this probability, we will do a change of variable similar to the technique Orloff and Bloom show in section 3.4 of [2].
We change the variable in the pdf $f(x)$ to u; therefore we rewrite the pdf as $f(u)$:

$$f(u) = \lambda e^{-\lambda u}. \tag{2}$$

We use this definition, the fact that f is defined for $x \geq 0$, and the definition of probability of continuous random variables [**readingx5b**] to write this equation:

$$P(X < x) = \int_0^x \lambda e^{-\lambda u}\, du. \tag{3}$$

We substitute the integral on the right hand side of the previous equation with its antiderivative to get:

$$P(X < x) = -e^{-\lambda u}\Big|_{u=0}^{x}. \tag{4}$$

We evaluate the the antiderivative at the limits of integration:

$$P(X < x) = -e^{-\lambda x} - -e^{-\lambda 0}. \tag{5}$$

Now we simplify the previous equation:

$$P(X < x) = -e^{-\lambda x} + 1. \tag{6}$$

Now, we apply the identity:

$$P(X \geq x) = 1 - P(X <). \tag{7}$$

Therefore

$$P(X > x) = 1 - \left(-e^{-\lambda x} + 1\right). \tag{8}$$

The previous equation simplifies to:

$$P(X > x) = e^{-\lambda x}. \tag{9}$$

References

[1] Jeremy Orloff and Jonathan Bloom. *Continuous Random Variables Class 5, 18.05, Spring 2014 Jeremy Orloff and Jonathan Bloom*. Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading5b.pdf` (Spring 2014).

[2] Jeremy Orloff and Jonathan Bloom. *Continuous Random Variables Class 5, 18.05, Spring 2014 Jeremy Orloff and Jonathan Bloom*. Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading5b.pdf` (Spring 2014).

[3] Jeremy Orloff and Jonathan Bloom. *Joint Distributions, Independence Covariance and Correlation 18.05 Spring 2014 Jeremy Orloff and Jonathan Bloom*. Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/class-slides/MIT18_05S14_class7slides.pdf` (Spring 2014).