MIT Introduction to Statistics 18.05 Problem Set 1

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2 Problem 1: Poker Hands

2.1 Two-Pair

We calculate the probability of the poker dealer dealing us a hand that is a two-pair hand. First we count the number of two-pair hands, then we divide the number of two-pair hands by the total number of hands to calculate the probability of the dealer dealing us a two pair hand.
The definition of a two-pair hand is, "Two cards have one rank, two cards have another rank, and the remaining card has a third rank. e.g.$\{2\heartsuit, 2\spadesuit, 2\clubsuit, 5\clubsuit, K\diamondsuit\}$." [1]
We take a combinations approach similar to the approach Orloff and Bloom take to calculate the probability of a one-pair hand in [2].
First we choose the ranks of the pairs. There are 13 ranks, so there are $\binom{13}{2}$ ways to choose the ranks of the pairs.

Next we choose the suits for the cards in the pairs. There are $\binom{4}{2}$ ways to select the suits for the cards in the first pair, and $\binom{4}{2}$ ways to select the suits for the cards in the second pair.
To complete the hand we select one card. We have 11 ranks to choose from for the fifth card, and $\binom{4}{1}$ ways to select its suit.
We apply the rule of product to count the number of two-pair hands:

$$\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1} = 78 \times 6 \times 6 \times 11 \times 4 = 123552 \tag{1}$$

The number of all poker hands is the number of ways to select 5 items from a set of 52 items. Therefore the number of all poker hands is $\binom{52}{5} = 2598960$. Therefore the probability of a two-pair hand is $\frac{123552}{2598960} \approx 0.048$.

2.2 Three-of-a-Kind

Orloff and Bloom give the definition of a three-of-a-kind hand as, "Three cards have one rank and the remaining two cards have two other ranks. e.g. $\{2\heartsuit, 2\spadesuit, 2\clubsuit, 5\clubsuit, K\diamondsuit\}$." [1]
We use the same approach as above.
First we select the rank for the three cards that have the same rank.
There are 13 ranks, so there are $\binom{13}{1}$ ways to select this rank.
Next we select the suits for the three cards that have the same rank. There are 4 suits, and we choose one for each card, so there are $\binom{4}{3}$ ways to select the suits for the 3 cards.
We have $\binom{12}{2}$ ways to select the ranks for the fourth and fifth cards, and $\binom{4}{1}^2$ ways to select their suits.
Now we apply the rule of product to count the number of three-of-a-kind hands:

$$\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}^2 = 13 \times 4 \times 66 \times 16 = 54912 \tag{2}$$

Therefore the probability of a three-of-a-kind hand is $\frac{54912}{2598960} \approx 0.021$.

3 Problem 2: Non-transitive Dice

3.1 Probability White Beats Green

We follow the method Orloff and Bloom use to calculate the probability that red beats white[2].
We write probability tables for white dice and green dice:

Green Die		
outcomes	1	4
probability	$\frac{1}{6}$	$\frac{5}{6}$

White Die		
outcomes	2	5
probability	$\frac{1}{2}$	$\frac{1}{2}$

Next we write the probability table for the product sample space of white and green dice:

		Green Die	
		1	4
White Die	2	$\frac{1}{12}$	$\frac{5}{12}$
	5	$\frac{1}{12}$	$\frac{5}{12}$

The pairs in the table above where the outcome for white is greater than the number for green correspond to outcomes in the product sample space where white wins. These are: $\{white = 2, green = 1\}$, $\{white = 5, green = 1\}$, and $\{white = 5, green = 4\}$.
We then add the corresponding probabilities for these outcomes where white wins to to calcuate the probability that white wins:

$$\frac{1}{12} + \frac{1}{12} + \frac{5}{12} = \frac{7}{12} \approx 0.583 \tag{3}$$

References

[1] Jeremy Orloff and Jonathan Bloom. *18.05 Problem Set 1, Spring 2014*. Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/assignments/MIT18_05S14_ps1.pdf` (Spring 2014).

[2] Jeremy Orloff and Jonathan Bloom. *Probability: Terminology and Examples 18.05 Spring 2014*. Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/class-slides/MIT18_05S14_class2slides.pdf`(Spring 2014).