# MIT Introduction to Statistics 18.05 Problem Set 2

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## Contents



## 1 References and License

We are answering questions in the material from MIT OpenCourseWare course 18.05, Introduction to Probability and Statistics.
Please see the references section for detailed citation information.
The material for the course is licensed under the terms at `http://ocw.mit.edu/terms`.
We are answering the questions that Orloff and Bloom ask in [2].
We use documentation in to write LaTeXsource code for this document.

## 2 Variance of a random variable

In this section we answer the question that Orloff and Bloom ask regarding the variance of a discrete random variable.

The following table describes a discrete random variable $X$:

| $X$ | 1 | 2 | 4 |
|---|---|---|---|
| PMF $p(x)$ | 0.2 | 0.3 | 0.5 |

We will use the formula from [3] for variance:

$$Var(X) = E\left(X^2\right) - E(X)^2 \tag{1}$$

First we compute $E(X)$:

$$E(X) = 1 \times 0.2 + 2 \times 0.3 + 4 \times 0.5 = 0.2 + 0.6 + 2 = 2.8 \tag{2}$$

Now we compute $E\left(X^2\right)$:

$$E\left(X^2\right) = 1 \times 0.2 + 4 \times 0.3 + 16 \times 0.5 = 0.2 + 1.2 + 8 = 9.4 \tag{3}$$

Now we apply the formula for variance in equation 1:

$$Var(X) = 9.4 - (2.8)^2 = 9.4 - 7.84 = 1.56 \tag{4}$$

# 3 Apply properties of variance

The second question in [2] is in several parts; Orloff and Bloom ask us several questions where we apply the properties of the variance of discrete random variables to answer.
Orloff and Bloom write that $X$ is a random variable with mean 2 and variance 3.

## 3.1 Variance of constant multiple

Orloff and Bloom ask is for us to compute $Var(3X)$
In [3] Orloff and Bloom show for constants $a$, and $b$

$$Var(aX + b) = a^2 Var(X) \tag{5}$$

Therefore $Var(3X) = 9 \times 3 = 27$

## 3.2 Variance of multiple plus constant

Orloff and Bloom pose a second question that is a variation on the previous question. They ask us to compute $Var(3X + 8)$
However, by 5 the answer to this question is no different than the answer to the previous question. Therefore, $Var(3X + 8) = 27$

### 3.3 Expected value of square

Orloff and Bloom ask us to compute $E\left(X^2\right)$.
We apply equation 1 to compute the answer to this question.

$$Var\left(X\right) = E\left(X^2\right) - E\left(X\right)^2 \tag{6}$$

This implies

$$3 = E\left(X^2\right) - 4 \tag{7}$$

We rewrite the equation above placing unknown terms on the left hand side, and known terms on the right hand side of the equation below:

$$E\left(X^2\right) = 4 + 3 = 7 \tag{8}$$

## 4 Further Questions on Same Random Variable

For this problem, Orloff and Bloom continue to ask us questions on the same random variable they define in the previous question.

### 4.1 Variance of square

In this section, Orloff and Bloom ask us if we can compute a value for $Var\left(X^2\right)$.
We cannot compute this value because the only information we have about $X$ is the value of its mean and variance. Terms in the sum to compute $Var\left(X\right)$, $\left(x_i - \mu\right)^2$ might be very different from terms in the sum to compute $Var\left(X^2\right)$, $\left(x_i^2 - \mu\right)^2$.

### 4.2 Negative Values

Orloff and Bloom ask if $X$ can take negative values.
The short answer is yes. However we can construct an example with the aid of R.

```
> x = c(-2, -1, 0, 1, 2)
> mean(x)
[1] 0
> var(x)
[1] 2.5
> var(sqrt(3/2.5) * x)
[1] 3
> var(sqrt(3/2.5) * x + 2)
[1] 3
> mean(sqrt(3/2.5) * x + 2)
```

```
[1] 2
> sqrt(3/2.5) * x + 2
[1] -0.1908902  0.9045549  2.0000000  3.0954451  4.1908902
>
```

To get started, we create a list, and assume the contents of the list are the product $p(x_i)\,x_i$, of a probability mass function and the value of a random variable. Values of a probability mass function are always positive, so the negative values in the list must be negative values of the random variable.
We use the properties of variance that Orloff and Bloom show in [3] to transform the list into a list that has a variance of 3 and a mean of 2.
Hence; the list satisfies the parameters of $X$ given in problem 2 of [2]; it has a mean of 2 and a variance of 3.
On the last line of the listing we print out the values of the transformed list. One sees that the first value in the list is negative. Since we assumed that the elements of the list are products of a random variable's value and its probability mass function's value for that value, the random variable's value must be negative. Therefore, in the long way of answering this question, $X$ can have negative values.

# 5 Sums of variances and means

In this section we explore sums of means textemvs. sums of variances.

## 5.1 Sums of means

Orloff and Bloom ask whether or not, for any random variables $X$, and $Y$, $E(X+Y) = E(X) + E(Y)$.
The answer to this question is yes. In fact, Orloff and Bloom prove this in [1].

## 5.2 Sums of variances

As a follow up question, Orloff and Bloom ask if $Var(X+Y) = Var(X) + Var(Y)$ for any random variables $X$ and $Y$. In [3] Orloff and Bloom state that this is true when $X$, and $Y$ are independent. Therefore $Var(X+Y) = Var(X) + Var(Y)$ is not true for any random variables $X$, and $Y$.

## 5.3 Standard deviation of Bernoulli random variable

Orloff and Bloom ask that if $X \sim \text{Bernoulli}(0.8)$, what is the standard deviation of $X$.
In [3] Orloff and Bloom show that, if $X \sim \text{Bernoulli}(p)$, then $Var(X) = (1-p)\,p$.

Also in [3], Orloff and Bloom state that the definition of the standard deviation of a random variable is the square root of the variance.
Therefore the standard deviation of $X$ is $\sqrt{(1-0.8)\,0.8}$, which is the square root of 0.16, which is 0.4.

## References


[1] Jeremy Orloff and Jonathan Bloom. *Discrete Random Variables: Expected Value Class 4, 18.05, Spring 2014 Jeremy Orloff and Jonathan Bloom*. Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading4b.pdf` (Spring 2014).

[2] Jeremy Orloff and Jonathan Bloom. *Reading Questions 5a*. Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/reading-questions-5a/` (Spring 2014).

[3] Jeremy Orloff and Jonathan Bloom. *Variance of Discrete Random Variables Class 5, 18.05, Spring 2014 Jeremy Orloff and Jonathan Bloom*. Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading5a.pdf` (Spring 2014).