Studio 5

John Hancock

July 8, 2017

Contents

1 References and License

We are answering questions in the material from MIT OpenCourseWare course 18.05, Introduction to Probabil-

ity and Statistics.
In this document we are answering questions Orloff and Bloom ask in [1].
Please see the references section for detailed citation information.
The material for the course is licensed under the terms at `http://ocw.mit.edu/terms`.
We use documentation in to write the LaTeXsource code for this document.

2 Compare posterior distributions

Orloff and Bloom ask us to change the prior probability distribution from a uniformly distributed probability of 0.2 for each type of dice to a probability of 0.05 for each type of dice, except the 20-sided die. We assign a 0.8 probability for selecting the 20-sided die.
This table shows the posterior distribution with uniformly distributed probability of selecting any kind of die:

```
[1] "Bayes table after one roll 1 : roll = 3"
  dice prior likelihood posterior.prenormalize poster
1    4   0.2 0.25000000                0.05000000
0.37037037
2    6   0.2 0.16666667                0.03333333
0.24691358
```

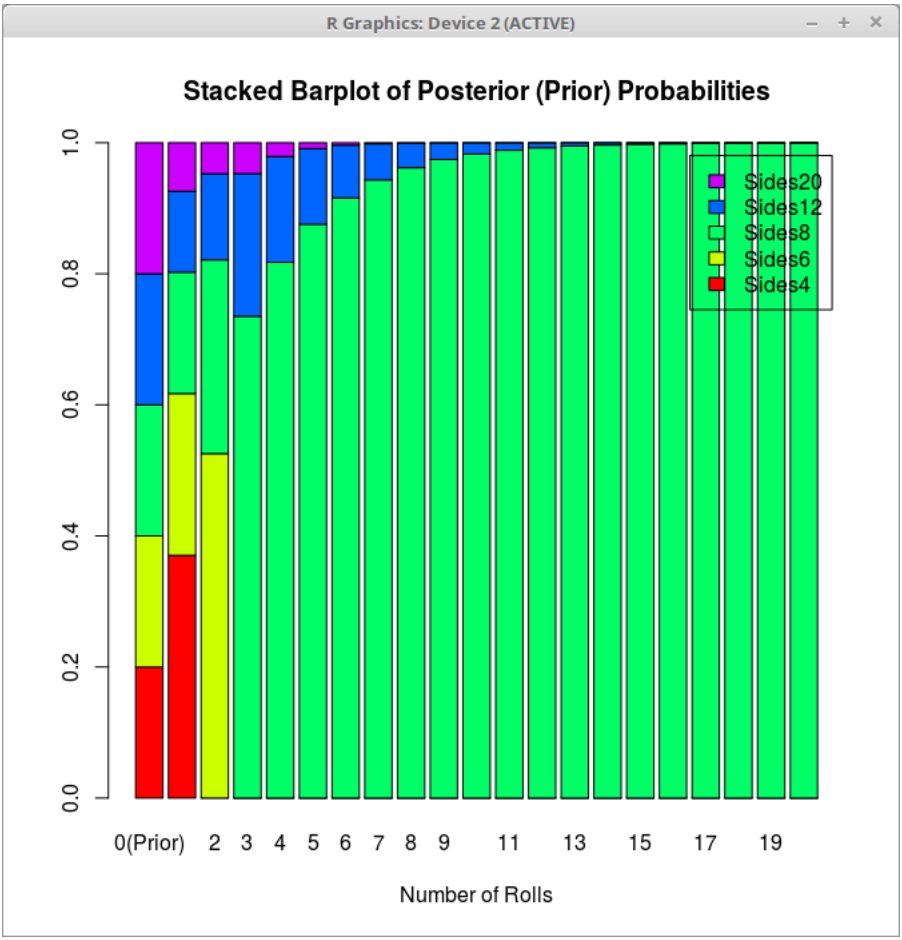
```
3     8    0.2 0.12500000                0.02500000
0.18518519
4    12    0.2 0.08333333                0.01666667
0.12345679
5    20    0.2 0.05000000                0.01000000
0.07407407
```

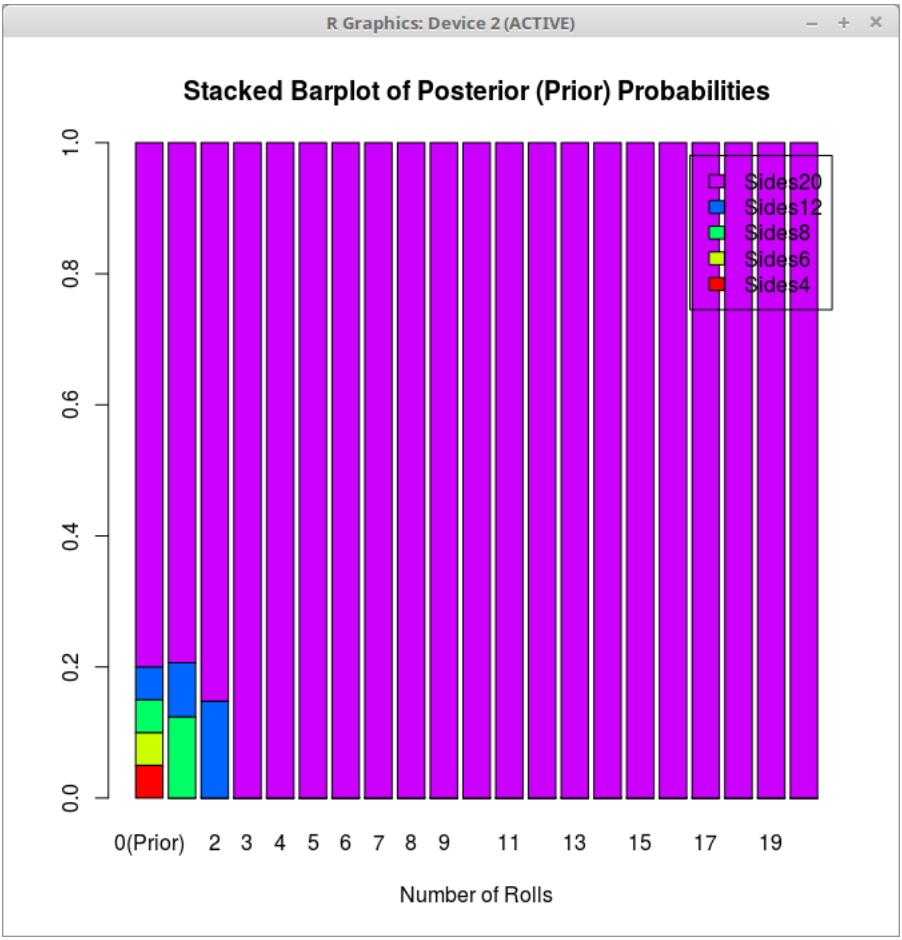
This table show the posterior distribution where we alter the prior distrubtion so that it is far more probable that we select the 20-sided die:

```
[1] "Bayes table after one roll 1 : roll = 8"
  dice prior likelihood posterior.prenormalize poster
1    4  0.05 0.00000000                0.000000000
0.00000000
2    6  0.05 0.00000000                0.000000000
0.00000000
3    8  0.05 0.12500000                0.006250000
0.12396694
4   12  0.05 0.08333333                0.004166667
0.08264463
5   20  0.80 0.05000000                0.040000000
0.79338843
```

We see that the posterior distribution changes to reflect the increased probabiliy of selecting the 20 sided die.
The stacked bar charts below show how the posteriror distributions change as we update as we obtain more data

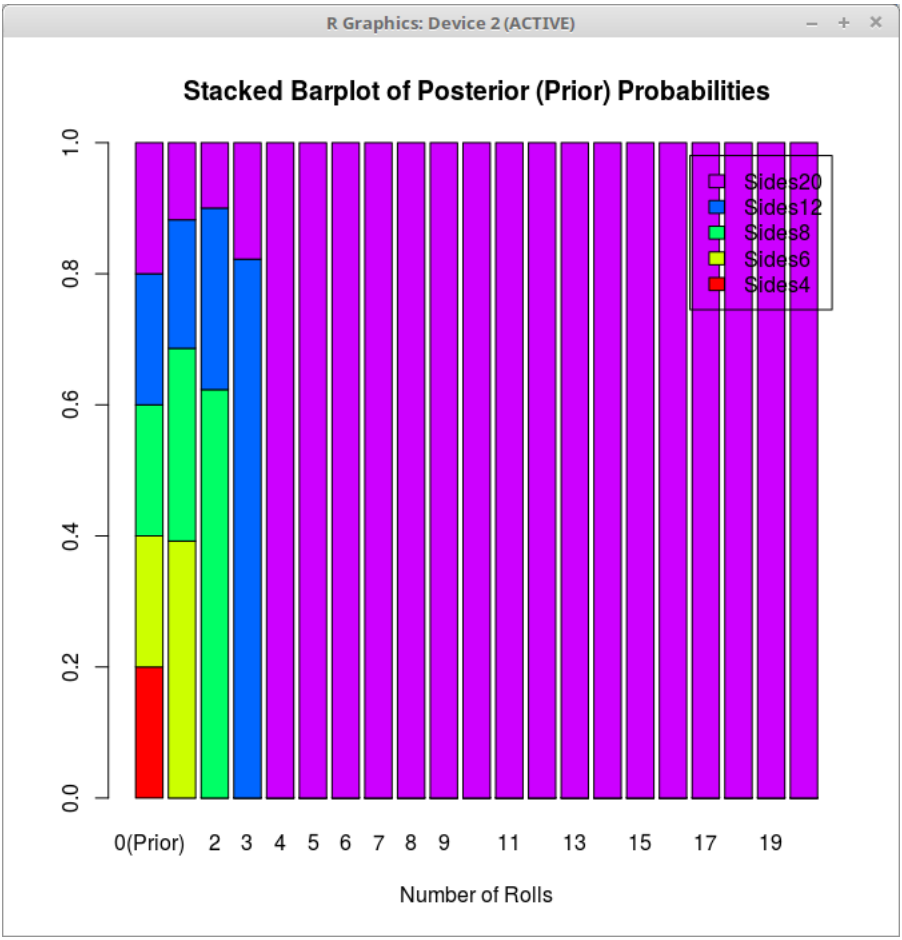
from rolling the die.


R Graphics: Device 2 (ACTIVE)
Stacked Barplot of Posterior (Prior) Probabilities
Sides20
Sides12
Sides8
Sides6
Sides4
1.0
0.8
0.6
0.4
0.2
0.0
0(Prior)
2
3
4
5
6
7
8
9
11
13
15
17
19
Number of Rolls


R Graphics: Device 2 (ACTIVE)
Stacked Barplot of Posterior (Prior) Probabilities
Sides20
Sides12
Sides8
Sides6
Sides4
1.0
0.8
0.6
0.4
0.2
0.0
0(Prior)
2
3
4
5
6
7
8
9
11
13
15
17
19
Number of Rolls

3 Uniform distribution, force 20-sided die

Orloff and Bloom ask us to run the simulation where we set the die that the software selects to be the 20-sided die. The image below shows how the updating changes when we assume there is a 20% chance of selecting any die, and update probabilities as we obtain more data.


R Graphics: Device 2 (ACTIVE)
Stacked Barplot of Posterior (Prior) Probabilities
Sides20
Sides12
Sides8
Sides6
Sides4
1.0
0.8
0.6
0.4
0.2
0.0
0(Prior)
2
3
4
5
6
7
8
9
11
13
15
17
19
Number of Rolls

We see that the updating takes a bit longer to converge on the high probability that we are seeing data from rolling a 20-sided die.

4 Censored data

In this section, Orloff and Bloom ask us to consider censoring the data such that we record a zero if we roll anything other than a one. If we roll a one with any of the dice, we record a one.

4.1 All possible hypotheses

The possible hypothesis do not change; therefore the hypotheses are:

- $\mathcal{H}_4$ the die has four sides
- $\mathcal{H}_6$ the die has six sides
- $\mathcal{H}_8$ the die has eight sides
- $\mathcal{H}_{12}$ the die has twelve sides
- $\mathcal{H}_{20}$ the die has twenty sides

4.2 Likelihood table

Here is the likelihood table for one roll, y_j:

Hypothesis	$y_j = 0$	$y_j = 1$
$\mathcal{H}_4$	$\frac{3}{4}$	$\frac{1}{4}$
$\mathcal{H}_6$	$\frac{5}{6}$	$\frac{1}{6}$
$\mathcal{H}_8$	$\frac{7}{8}$	$\frac{1}{8}$
$\mathcal{H}_{12}$	$\frac{11}{12}$	$\frac{1}{12}$
$\mathcal{H}_{20}$	$\frac{19}{20}$	$\frac{1}{20}$

References

[1] Jeremy Orloff and Jonathan Bloom. *Studio 5: Bayesian Updating 18.05 Spring 2014*. Available at `https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/studio-resources/MIT18_05S14_studio5slides.pdf` (Spring 2014).