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CEN 6405
Homework #1
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12.2

Exercise 12.2 gives us the following pmf for the distribution of traffic arriving at a network gateway:

$$f(x) = (1-p)^{x-1} p,\ x = 1, 2, 3, \ldots, \infty$$

First, we are required to compute the mean, variance, standard deviation, and coefficient of variation of the burst size:

Mean:

The general formula for the mean value is

$$\mu = E(x) = \sum_{i=i}^{n} p_i x_i \tag{1}$$

In this case, the p_i are $(1-p)^{x-1} p$ for a given value of x in $\{1, 2, 3, ..., \infty\}$, and the x_i are the values $\{1, 2, 3, \ldots, \infty\}$. Therefore the mean value is:

$$\mu = \sum_{i=1}^{\infty} i (1-p)^{i-1} p$$

Variance:

The general formula for variance is:

$$Var(x) = E\left[(x-\mu)^2\right] = \sum_{i=1}^{n} p_i (x_i - \mu)^2 \tag{2}$$

Again, p_i is $(1-p)^{x-1} p$ for a given value of x in $\{1, 2, 3, ..., \infty\}$. We know the value of μ from 1. We can therefore rewrite 2 as :

$$Var(x) = \sum_{i=1}^{\infty} \left[(1-p)^{i-1} p\right] \left[i - \left(i (1-p)^{i-1} p\right)\right]^2 \tag{3}$$

Standard Deviation:

The square root of the variance is the standard deviation. Therefore the standard deviation is the square root of 3 :

$$\sigma = \sqrt{\sum_{i=1}^{\infty} \left[(1-p)^{i-1} p\right] \left[i - \left(i (1-p)^{i-1} p\right)\right]^2} \tag{4}$$

Coefficient of variation:

The coefficient of variation is the standard deviation divided by the mean. In this case it will be μ from equation 1 divided by σ from 4:

$$\frac{\sigma}{\mu} = \frac{\sqrt{\sum_{i=1}^{\infty} \left[(1-p)^{i-1} p\right] \left[i - \left(i (1-p)^{i-1} p\right)\right]^2}}{\sum_{i=1}^{\infty} i (1-p)^{i-1} p}$$

To complete the exercise we must also plot the probability mass function (pmf) and cumulative density function (CDF) for $p = 0.2$

pmf plot

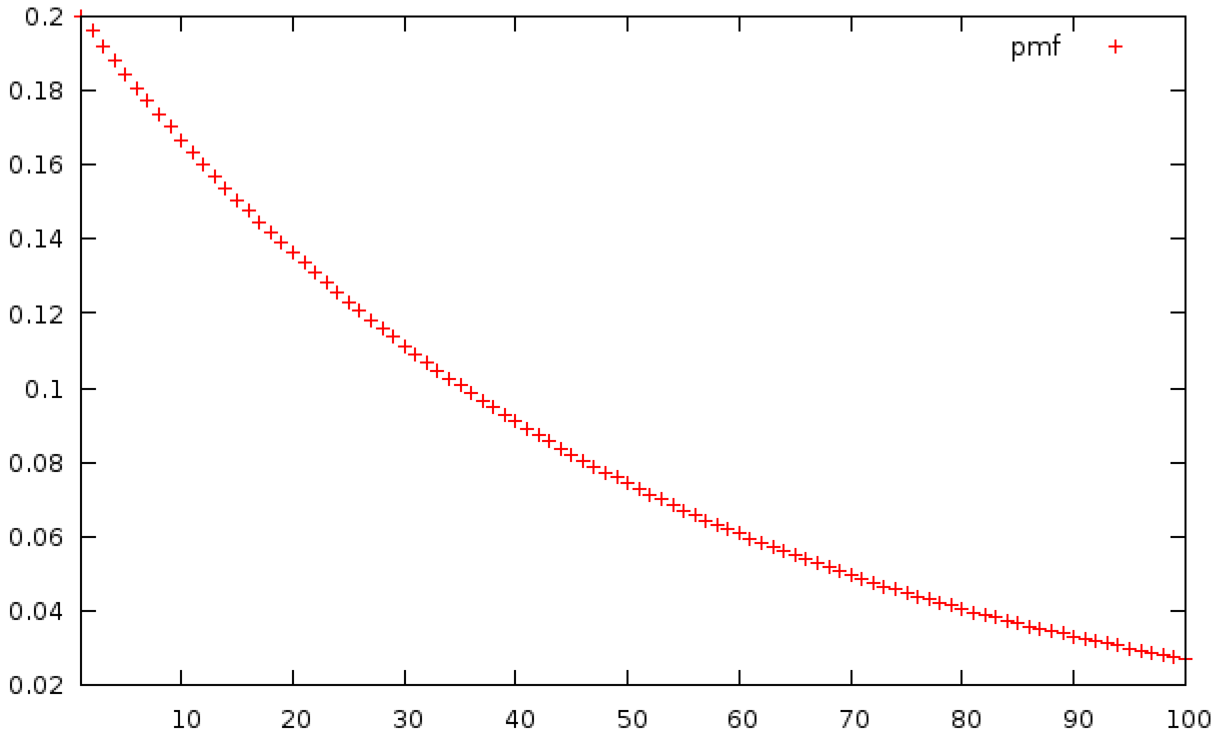
To plot the pmf we plug 0.2 into the pmf and draw a graph of it as a function of x. Plugging 0.2 into

$$f(x) = (1-p)^{x-1}\, p,\; x = 1, 2, 3, \ldots, \infty$$

gives us

$$f(x) = (1-0.2)^{x-1}\, 0.2,\; x = 1, 2, 3, \ldots, \infty$$

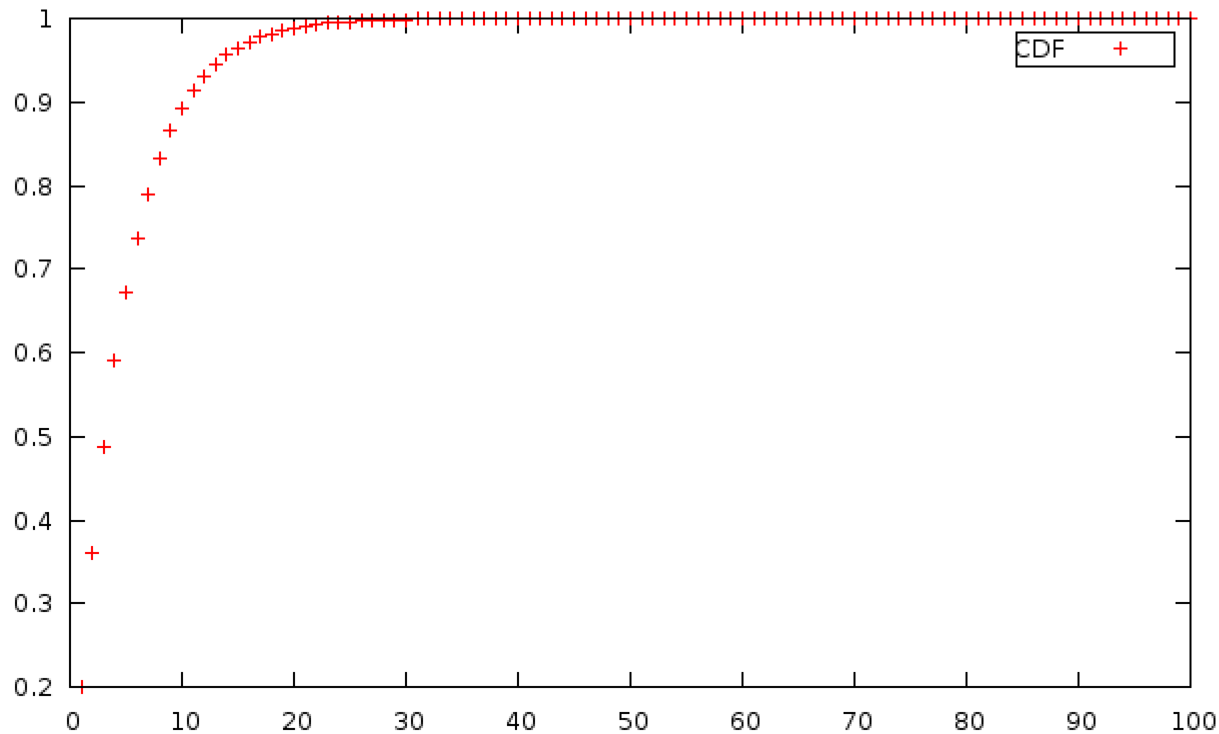
This function is pictured below:



The CDF of $f(x)$ is the function:

$$F(x) = \sum_{i=1}^{x} \left[(1-0.2)^{i-1}\, 0.2 \right],\; x = 1, 2, 3, \ldots, \infty$$

This is a plot of the CDF:



12.11

The index of central tendency we would use for the list of disk I/O's given in exercise 12.11 is the mean. We chose the mean because not the total number of disk I/O's is not categorical, and the total number of disk I/O's is a meaningful number.