$$\frac{\mathcal{E}_{i,L}^{k+1}-\mathcal{E}_{i,L}^{n}}{\Delta t^n} = -\frac{1}{2}\frac{2}{h_i}\left(\mathcal{F}_i^n - \mu^-\Psi_{i-1/2}^{-,n} - \mu^+\Psi_{i-1/2}^{+,n}\right) - \frac{1}{2}\frac{2}{h_i}\left(\mathcal{F}_i^{k+1} - \mu^-\Psi_{i-1/2}^{-,k+1} - \mu^+\Psi_{i-1/2}^{+,k+1}\right)$$

$$- \frac{1}{2}\sigma_{t,i,L}^n c\,\mathcal{E}_{i,L}^n - \frac{1}{2}\sigma_{t,i,L}^k c\,\mathcal{E}_{i,L}^{k+1}$$

$$+ \frac{1}{2}\sigma_{s,i,L}^n c\,\mathcal{E}_{i,L}^n + \frac{1}{2}\sigma_{s,i,L}^k c\,\mathcal{E}_{i,L}^{k+1}$$

$$+ \frac{1}{2}Q_{0,i,L}^n + \frac{1}{2}Q_{0,i,L}^{k+1}$$

$$Q_{0,i,L}^{k+1} = \sigma_{a,i,L}^k a c (T_{i,L}^{k+1})^4 - \sigma_{t,i,L}^k \frac{u_{i,L}^k}{c}\left(\mathcal{F}_{i,L}^k - \frac{4}{3}\mathcal{E}_{i,L}^k u_{i,L}^k\right)$$

$$= -\frac{1}{2}\frac{2}{h_i}\left(\mathcal{F}_i^n - \mu^-\Psi_{i-1/2}^{-,n} - \mu^+\Psi_{i-1/2}^{+,n}\right) - \frac{1}{2}\frac{2}{h_i}\left(\mathcal{F}_i^{k+1} - \mu^-\Psi_{i-1/2}^{-,k+1} - \mu^+\Psi_{i-1/2}^{+,k+1}\right)$$

$$+ \frac{1}{2}\left[\sigma_a^k c\left(aT^4\right)^4 - \mathcal{E}\right]_{i,L}^{k+1} + \frac{1}{2}\left[\sigma_a^n c\left(aT^4 - \mathcal{E}\right)\right]_{i,L}^n$$

$$\oplus \frac{1}{2}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_{i,L}^n \oplus \frac{1}{2}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_{i,L}^k$$

Mult by $\frac{h_i}{2}$ and add L and R:

$$h_i\frac{\left(\mathcal{E}_i^{k+1} - \mathcal{E}_i^n\right)}{\Delta t^n} = -\frac{1}{2}\left(\mu^-\Psi_{i-1/2}^{-,n} - \mu^+\Psi_{i-1/2}^{+,n} + \mu^-\Psi_{i+1/2}^{-,n} + \mu^+\Psi_{i+1/2}^{+,n}\right)$$

$$- \frac{1}{2}\left(\mu^-\Psi_{i-1/2}^{-,k+1} - \mu^+\Psi_{i-1/2}^{+,k+1} + \mu^-\Psi_{i+1/2}^{-,k+1} + \mu^+\Psi_{i+1/2}^{+,k+1}\right)$$

$$+ \frac{1}{2}h_i\left[\sigma_a^n c\left(aT^4 - \mathcal{E}\right)\right]_i^n + \frac{1}{2}h_i\left[\sigma_a^k c\left(aT^4 - \mathcal{E}\right)^{k+1}\right]_i$$

$$\oplus \frac{1}{2}h_i\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_i^n \oplus \frac{1}{2}h_i\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_i^k$$

Adding all elements:

$$\sum_i h_i \mathcal{E}_i^{k+1} = \sum_i h_i \mathcal{E}_i^n + \Delta t^n\left(\frac{1}{2}\left(\mu^+\Psi_{1/2}^{+,n} + \mu^-\Psi_{1/2}^{-,n} - \mu^+\Psi_{N+1/2}^{+,n} - \mu^-\Psi_{N+1/2}^{-,n}\right)\right.$$

$$+ \frac{1}{2}\left(\quad^{k+1}\quad\right)$$

$$\left. + \text{blah}\right)$$

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$$\sum_i h_i\left(E_i^{k+1} + \mathcal{E}_i^{k+1}\right) = \sum_i h_i\left(E_i^n + \mathcal{E}_i^n\right) + \Delta t^n\left(F_{1/2}^{E,n+1/2} - F_{N+1/2}^{E,n+1/2}\right.$$

$$+ \frac{1}{2}\left(\mu^+\Psi_{1/2}^{+,n} + \mu^-\Psi_{1/2}^{-,n} - \mu^+\Psi_{N+1/2}^{+,n} - \mu^-\Psi_{N+1/2}^{-,n}\right)$$

$$\left. + \frac{1}{2}\left(\mu^+\Psi_{1/2}^{+,k+1} + \mu^-\Psi_{1/2}^{-,k+1} - \mu^+\Psi_{N+1/2}^{+,k+1} - \mu^-\Psi_{N+1/2}^{-,k+1}\right)\right.$$

$$F = \mu^+ \psi^+ + \mu^- \psi^-$$

$$\frac{1}{c}\frac{F_{i,L}^{k+1} - F_{i,L}^{n}}{\Delta t} = -\frac{1}{2}\frac{2}{3}\frac{1}{h_i}\left(\phi_i^n - \psi_{i-1,R}^{+,n} - \psi_{i,L}^{-,n}\right) - \frac{1}{2}\frac{2}{3}\frac{1}{h_i}\left(\phi_i^{k+1} - \psi_{i-1,R}^{+,k+1} - \psi_{i,L}^{-,k+1}\right)$$

$$-\frac{1}{2}\sigma_{t,i,L}^{n} F_{i,L}^{n} - \frac{1}{2}\sigma_{t,i,L}^{k} F_{i,L}^{k+1} + \frac{1}{2}\left(\mu^+ Q_{i,L}^{+,n} + \mu^- Q_{i,L}^{-,n}\right) + \frac{1}{2}\left(\mu^+ Q_{i,L}^{+,k} + \mu^- Q_{i,L}^{-,k}\right)$$

$$Q^{\pm} = \frac{Q^{\pm}}{2} = \frac{Q_0}{2} + \frac{3}{2}\mu^{\pm} Q_1 \qquad Q_{1,i,L}^{n} \qquad Q_{1,i,L}^{k}$$

Multiply by $\frac{h_i}{2}$ and add L and R:

$$\frac{h_i}{c}\frac{F_i^{k+1} - F_i^{n}}{\Delta t} = -\frac{1}{2}\frac{1}{3}\left(\phi_i^n - \psi_{i-1,R}^{+,n} - \psi_{i,L}^{-,n} + \psi_{i,R}^{+,n} + \psi_{i+1,L}^{-,n}\right)$$

$$-\frac{1}{2}\frac{1}{3}\left(-\psi_{i-1,R}^{+,k+1} - \psi_{i,L}^{-,k+1} + \psi_{i,R}^{+,k+1} + \psi_{i+1,L}^{-,k+1}\right)$$

$$-\frac{1}{2}h_i\left(\frac{1}{2}\sigma_{t,i,L}^{n} F_{i,L}^{n} + \frac{1}{2}\sigma_{t,i,R}^{n} F_{i,R}^{n}\right)$$

$$-\frac{1}{2}h_i\left(\frac{1}{2}\sigma_{t,i,L}^{k} F_{i,L}^{k+1} + \frac{1}{2}\sigma_{t,i,R}^{k} F_{i,R}^{k+1}\right)$$

$$+\frac{1}{2}h_i\left(\frac{Q_{1,i,L}^{n}}{2} + \frac{Q_{1,i,R}^{n}}{2}\right)$$

$$+\frac{1}{2}h_i\left(\frac{Q_{1,i,L}^{k}}{2} + \frac{Q_{1,i,R}^{k}}{2}\right)$$

$$Q_1 = \frac{4}{3}\sigma_t \mathcal{E} u$$

$$Q_{1,i,L}^{k} = \frac{4}{3}\sigma_{t,i,L}^{k}\mathcal{E}_{i,L}^{k} u_{i,L}^{k}$$

Multiply by $\frac{1}{c}$ and sum over i:

$$\frac{1}{c^2}\sum_i h_i F_i^{k+1} = \frac{1}{c^2}\sum_i h_i F_i^{n} + \Delta t^n\left(\frac{1}{2}\left(\frac{\psi_{inc}^{+,n}}{3c} - \frac{\psi_{inc}^{-,n}}{3c}\right) + \frac{1}{2}\left(\frac{\psi_{inc}^{+,k+1}}{3c} - \frac{\psi_{inc}^{-,n+1}}{3c}\right)\right.$$

$$\left. + \psi_{1,L}^{-,n} - \psi_{N,R}^{+,n} + \psi_{1,L}^{-,k+1} - \psi_{N,R}^{+,k+1}\right)$$

$$-\frac{1}{2}\sum_i h_i\left(\frac{1}{2}\frac{\sigma_{t,i,L}^{n}}{c} F_{i,L}^{n} + \frac{1}{2}\frac{\sigma_{t,i,R}^{n}}{c} F_{i,R}^{n}\right) - \frac{1}{2}\sum_i h_i\left(\frac{1}{2}\frac{\sigma_{t,i,L}^{k}}{c} F_{i,L}^{k+1} + \frac{1}{2}\frac{\sigma_{t,i,R}^{k}}{c} F_{i,R}^{k+1}\right)$$

$$+\frac{1}{2}\sum_i \frac{h_i}{c}\left(\frac{Q_{1,i,L}^{n}}{2} + \frac{Q_{1,i,R}^{n}}{2}\right) + \frac{1}{2}\sum_i \frac{h_i}{c}\left(\frac{Q_{1,i,L}^{k}}{2} + \frac{Q_{1,i,R}^{k}}{2}\right)$$

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$$-\frac{1}{2}\sum_i h_i\left(\frac{1}{2}\frac{\sigma_{t,i,L}^{n}}{c} F_{0,i,L}^{n} + \frac{1}{2}\frac{\sigma_{t,i,R}^{n}}{c} F_{0,i,R}^{n}\right) - \frac{1}{2}\sum_i h_i\left(\frac{1}{2}\frac{\sigma_{t,i,L}^{k}}{c} F_{0,i,L}^{k+1} + \frac{1}{2}\frac{\sigma_{t,i,R}^{k}}{c} F_{0,i,R}^{k+1}\right)$$