60009210033

CSE(Data Science)

Experiment 6

(Dynamic Programming)

Aim: Implementation of coin change problem using dynamic programming.

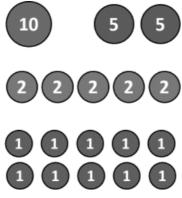
Theory:

Making Change problem is to find change for a given amount using a minimum number of coins from a set of denominations.

Explanation: If we are given a set of denominations $D = \{d0, d1, d2, ..., dn\}$ and if we want to change for some amount N, many combinations are possible. Suppose $\{d1, d2, d5, d8\}$, $\{d0, d2, d4\}$, $\{d0, d5, d7\}$ all feasible solutions.

The aim of making a change is to find a solution with a minimum number of coins / denominations. Clearly, this is an optimization problem.

This problem can also be solved by using a greedy algorithm. However, greedy does not ensure the minimum number of denominations.



Various denominations for amount 10

General assumption is that infinite coins are available for each denomination. We can select any denomination any number of times.

Mathematical Formulation:

$$C[i,j] = \begin{cases} 1 + C \ [1,j-d_1], & \text{, if } i = j \\ C[i-1,j], & \text{if } j < d_i \\ min \ (C \ [i-1,j], \ 1 + C \ [i,j-d_i]), & \text{otherwise} \end{cases}$$

Pseudocode:

```
Algorithm MAKE_A_CHANGE(d,N)
// d[1...n] = [d1,d2,...,dn] is array of n denominations
// C[1...n, 0...N] is n x N array to hold the solution of sub problems
// N is the problem size, i.e. amount for which change is required
for i \leftarrow 1 to n do
 C[i, 0] \leftarrow 0
end
for i \leftarrow 1 to n do
 for j \leftarrow 1 to N do
  if i = 1 \& \& j < d[i] then
        C[i, j] \leftarrow \infty
  else if i == 1 then
        C[i, j] \leftarrow 1 + C[1, j - d[1])
  else if j < d [i] then
        C[i, j] \leftarrow C[I-1, j]
  else
        C[i, j] \leftarrow \min (C[i-1, j], 1 + C[i, j-d[i])
  end
end
end
return C[n, N]
Algorithm TRACE_MAKE_A_CHANGE(C)
// When table C is filled up, i = n and j = N
Solution = { }
while (j > 0) do
 if (C[i, j] == C[i - 1, j] then
  i \leftarrow i - 1
 else
  j \leftarrow j - di
  Solution = Solution U {d[i] }
 end
```

Complexity:

end

Best Case Time Complexity: O(n)

Lab Assignment:

Write a C Program and consider the set of denominations, D=1,4,6. Achieve the sum of 8 and calculate the number of coins required and the actual denominations needed using dynamic programming.

CODE: #include <stdio.h>

```
#include mits.h>
#define N 3
#define SUM 8
int min(int a, int b) {
return a < b ? a : b;
void findCoins(int coins[], int dp[][SUM+1]) {
int i = N, j = SUM, count = 0;
while (j > 0) {
if (dp[i][j] == dp[i-1][j]) {
i--;
} else {
coins[count++] = i;
i = i;
int main() {
int denominations[N] = \{1, 4, 6\};
int dp[N+1][SUM+1], coins[SUM], count = 0;
for (int i = 0; i \le N; i++) {
for (int j = 0; j \le SUM; j++) {
if (j == 0) {
dp[i][j] = 0;
} else {
dp[i][j] = INT\_MAX;
for (int i = 1; i \le N; i++) {
for (int i = 1; i \le SUM; i++) {
if (j < denominations[i-1]) {
dp[i][j] = dp[i-1][j];
} else {
dp[i][j] = min(dp[i-1][j], 1 + dp[i][j-denominations[i-1]]);
}
printf("60009210033 Jhanvi Parekh\n");
printf("Number of coins required: %d\n", dp[N][SUM]);
findCoins(coins, dp);
printf("Actual denominations needed: ");
for (int i = 0; i < dp[N][SUM]; i++) {
printf("%d ", denominations[coins[i]-1]);
return 0;
```



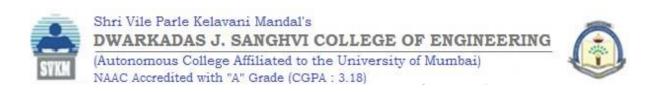
```
main.c
1 #include <stdio.h>
 2 #include <limits.h>
 3 #define N 3
4 #define SUM 8
 5 - int min(int a, int b) {
6 return a < b ? a : b;
7 }
8 - void findCoins(int coins[], int dp[][SUM+1]) {
9 int i = N, j = SUM, count = 0;
10 - \text{ while } (j > 0)  {
11 - if (dp[i][j] == dp[i-1][j]) {
12 i--;
13 - } else {
14 coins[count++] = i;
15 j -= i;
16 }
17 }
18 }
19 - int main() {
20 int denominations[N] = {1, 4, 6};
21 int dp[N+1][SUM+1], coins[SUM], count = 0;
22 - \text{for (int } i = 0; i \le N; i++)  {
23 - \text{for (int } j = 0; j \le \text{SUM; } j++)  {
24 - if (j == 0) {
25 dp[i][j] = 0;
26 - } else {
27 dp[i][j] = INT_MAX;
28 }
29 }
30 }
31 - for (int i = 1; i \le N; i++) {
32 - \text{for (int } j = 1; j \le \text{SUM; } j++)  {
33 - if (j < denominations[i-1]) {
34 dp[i][j] = dp[i-1][j];
```

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```
main.c
15 j -= i;
16 }
17 }
18 }
19 - int main() {
20 int denominations[N] = {1, 4, 6};
21 int dp[N+1][SUM+1], coins[SUM], count = 0;
22 - \text{for (int } i = 0; i \le N; i++)  {
23 - \text{for (int } j = 0; j \le SUM; j++)  {
24 - if (j == 0) {
25 dp[i][j] = 0;
26 - } else {
27 dp[i][j] = INT_MAX;
28 }
29 }
30 }
31 - for (int i = 1; i \le N; i++) {
32 - \text{for (int } j = 1; j \le \text{SUM; } j++)  {
33 - if (j < denominations[i-1]) {
34 dp[i][j] = dp[i-1][j];
35 - } else {
36 dp[i][j] = min(dp[i-1][j], 1 + dp[i][j-denominations[i-1]]);
37 }
38 }
39 }
40 printf("60009210033 Jhanvi Parekh\n");
41 printf("Number of coins required: %d\n", dp[N][SUM]);
42 findCoins(coins, dp);
43 printf("Actual denominations needed: ");
44 - \text{for (int } i = 0; i < dp[N][SUM]; i++) {
45 printf("%d ", denominations[coins[i]-1]);
46 }
47 return 0;
48 }
```



Output

/tmp/gJE9jFWs5r.o

60009210033 Jhanvi Parekh Number of coins required: 2

Actual denominations needed: 4 4



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	required of idilj i=0 i=1, di=1	0 0	u acti	2 0 2 2	usui	ations 4	ber of using	6 0 6	7 0 7	
	regulated of i = 0 $i = 0$ $i = 1$, $di = 1$ $i = 2$, $di = 4$ $i = 3$, $di = 6$	0 0 0	le acti	2 0 2 2 2	3 0 3 3	ations 4	ber of using	6 0 6	7 0 7 4	
	required of idilj i=0 i=1, di=1 i=2, di=4 i=3, di=6 c[111] =	0 0 0 0 0	le acti	2 2 2 2	3 0 3 3	ations 4	ber of using	6 0 6	7 0 7 4	
	required of i dilj i = 0 i = 1, di = 1 i = 2, di = 4 i = 3, di = 6 c [1] 1] = case 1 acc	0 0 0 0 0 i=1,	le acti	2 2 2 2 2	3 3 3 3	ations 4	ber of using	6 0 6	7 0 7 4	
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	required of idilj i=0 i=1, di=1 i=2, di=4 i=3, di=6 c[1,1] = case 1 acc c[1,1] = 1+ = 1+	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	di = 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	2 2 2 2 1 1 j = 1	the the enomination of 3 3 3 $i = = 1$	ations 4	ber of using	6 0 6	7 0 7 4	
	required of idilj i=0 i=1, di=1 i=2, di=4 i=3, di=6 C[1,1] = case 1 acc C[1,1] = 1 + = 1 +	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	di = 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	2 2 2 2 1 1 j = 1 j	3 3 3 3 i==)	ations 4	ber of using	6 0 6	7 0 7 4	
	required of i = 0 i = 1, di = 1 i = 2, di = 4 i = 3, di = 6 $C[1,1] = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$	0 0 0 0 0 0 i=1, epted c[1, -[1,0]	di = 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	2 2 2 2 1 1 j = 1	3 0 3 3 3 i==)	ations 4	ber of using	6 0 6	7 0 7 4	



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$c[211] = i = 2$, $d_2 = 4$, $d_3 = 1$ $case 2$ accepted $c[211] = c[2-i]$ $= c[111] = 1$ $c[211] = i = 3$, $j = 1$, $d_1 = 6$ $case 2$; $c[i-1]$	3 to 1
case 2 accepted $c[2,1] = c[2-i,1]$ $= c[1,1] = 1$ $c[3,1] = i = 3, j = 1, di = 6$ $case 2 : c[i-1,j]$	3 /- 1
c[2,1] = c[2-c,1] $= c[1,1] = 1$ $c[3,1] = c = 3, j = 1, d = 6$ $case$	3 /- 1
c[3 1] = i = 3, j = 1, di = 6 $case$	
c[311] = i = 3, j = 1, d1 = 6 $case 2 : c[i-1,j]$	e + f
case 2 : c[i-1,j]	8 (1)
c [3-1,1]	
e [211]	
(E,1-0	
(8)	(3.5
c[1,2] = i = 1, di = 1, j = 2	
Case 1: 1+C[1,2-1]	
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2 (8,1-8	
(818)	
c[2,2]: i=2,j=2, di=4	
case 2 : c [i-11j]	- C
c[2-1/2]	
C[1,2]	
2	
Carl Ja	
c[3,2]: i=3, j=2, d3=6	
couse 2: c[i-1,j]	
c[2-1,2] 1 = ch 1 = 1 = 2	
(F) b - C[112] - 1 ([1-1])	
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Departm	nent of Computer Science and Engineering (Data Science)
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	o [1,3]: i=1, j=3, di=1
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	1 + c[12]
	1+2
	3
	100,1-3150
	c[2:3]: i=2, j=3, d2=4
	case a: c[i-1,j]
	c[2-1,3]
	c [113]
	3 0-1-10-10-1-3-00-10-1
	. [1-0,1] 3 +1 (1)
	$c[3,3] = c=3, j=3, d_3 = 6/11/3 = 1$
	case 2: c[i-1,j]
	c[3-1,3]
	C[2 3]
	3 1 - 20 2 - 212 - 2 2 5 2 1 2 1 2 1 2 2 2 2 2 2 2 2 2 2 2
	e[1,4]: i=1; j=4, d,=1 [201-2]
	case 1: 1+ c[1,j-di]
	1+0[1,4-1]
	1+ c[1,3]
	4
	C[2,4]: $i=2,j=4,d2=4$
	case 3: min (c[i-1,j], 1+c[i,j-di])
	12 + 1 1 1 1 1 1 1 1 1 1
	(4,1+0)



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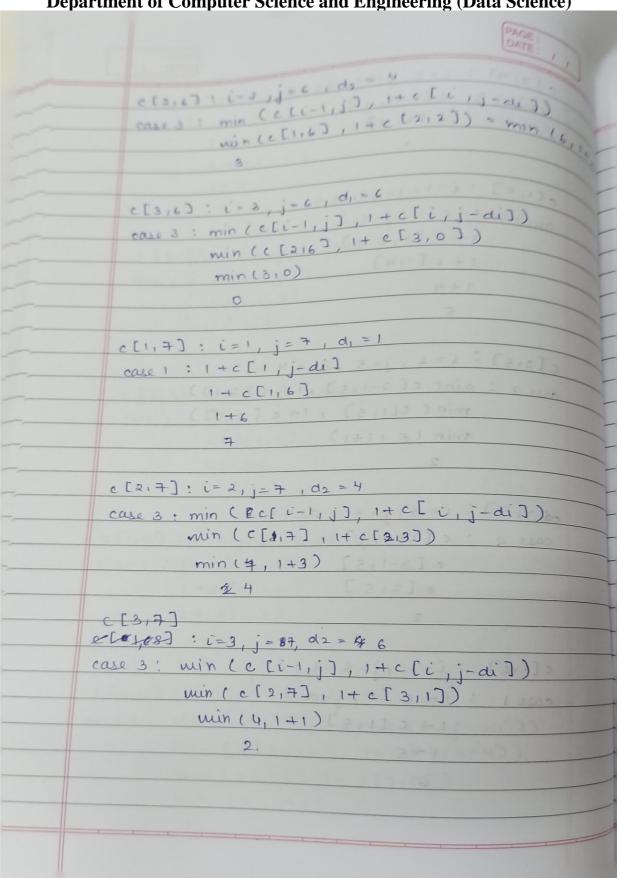
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1 1 1	1100 F (C C 21 4) 1 , C 311 1 3 7 mins
	1
	1,5]: i=1, j=5, d1=1
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	1+4
	5
	1 · 10 1 F = 1 1 = 3 : (F,17)
c E	215] : i=2, j=5, d2=41] >+1
case	3 = min(c[2-1,5],[1+,c[211])
	min (c[1,5], 1+c[211])
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cat 3	15]: i=8, j=5[, d3=6]] om
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e[118]: i=1 1j=8	, d, =1
case 1: 1+c [1,j-	ai 7
1+ c[1,8-	
17 0 [1] 7	
1+7	
8	4 - 1 h - 3 custs
c[218]: [-2, j=8]	, d2 = 4
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2	at a street of the
	ch - j - t - j - 1
c[3,8]: i=3, j=8,	013=6
case 2: c[i-1,j]	
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2	fr. 43 wa assista
from the last cell of to	ble
i=3, j=8	
check c[i,j] = c[i-	-1,33
as nothing is added in .	the previous solution go to the
previous step by reducin	g the value of i by 1
i = i-1	•
= 3-1	
i = 2	
Now , i= 2 , j=8	
check cli, j] = cli-	1,;]
as the condition is balle	
Denomination di = d2 = .	4 was added in the provious
solution	
Add di en solution set	4 suduce the problem size
di L (j-di)	



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	4
ace set - 8	3 1 3 4 1
j=8, d2=4	3-4-4
j - j-d2	2+1
- ij=4	8
i=2, i=	= c[i-1,j] as the condition is to de = 4 was added in the previous
Now clinia	de = 4 was added in the previous de = 4 was added in the previous est 4 wednes the problem size j-di
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1 -12 in CON	n . See
sol. set = f	4,43
j= 4, d2 > 4	
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At j=0 the a	Igorithm will stop and final denomina
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