A.Y.: 2024-25 **Class/Sem: B. Tech/ Sem-VII** **Sub:** Quantitative Portfolio Management

Experiment 2

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Aim: Implementation of Modern Portfolio Theory (Efficient frontier) on a given dataset.

Objective:

- Understand the basic principles of Modern Portfolio Theory (MPT).
- Calculate the expected return and risk of a portfolio of assets.
- Construct an efficient frontier of portfolios.
- Analyze the implications of MPT for portfolio selection.

Theory:

Modern Portfolio Theory (MPT) is a mathematical framework for constructing portfolios that maximize expected return for a given level of risk. The theory was developed by Harry Markowitz in the 1950s, and it has become a cornerstone of modern investment theory.

The key idea of MPT is diversification. By investing in a diversified portfolio of assets, investors can reduce the overall risk of their portfolio without sacrificing too much expected return. This is because the risk of a portfolio is not simply the sum of the risks of the individual assets in the portfolio. Instead, the risk of a portfolio is determined by the correlation between the assets in the portfolio.

The efficient frontier is a graph that shows all the portfolios that offer the highest possible expected return for a given level of risk. The efficient frontier is a theoretical construct, but it can be approximated using real-world data.

MPT is based on the following assumptions:

- Investors are risk-averse. This means that they prefer portfolios with higher expected returns to portfolios with lower expected returns, all else being equal.
- Investors can only estimate expected returns and risk. This means that they do not know for certain what the future returns of their investments will be.
- Investors can choose any combination of assets they want. This means that there are no restrictions on the types of assets that investors can include in their portfolios.

MPT uses the following formula to calculate the expected return of a portfolio:

$$gE(R) = w1*E(R1) + w2*E(R2) + ... + wn*E(Rn)$$

where:

- E(R) is the expected return of the portfolio
- w1, w2, ..., wn are the weights of the assets in the portfolio
- E(R1), E(R2), ..., E(Rn) are the expected returns of the individual assets

MPT uses the following formula to calculate the risk of a portfolio:

$$\sigma(R) = \sqrt{(w1^2*\sigma1^2 + w2^2*\sigma2^2 + ... + wn^2*\sigma n^2 + 2*w1*w2*\rho12*\sigma1*\sigma2)}$$

where:

- σ(R) is the risk of the portfolio
- σ1, σ2, ..., σn are the standard deviations of the individual assets
- ρ12 is the correlation coefficient between assets 1 and 2

The efficient frontier is a graph that shows all the portfolios that offer the highest possible expected return for a given level of risk. The efficient frontier is a theoretical construct, but it can be approximated using real-world data.

The efficient frontier is a powerful tool for portfolio selection. It can be used to identify the portfolios that offer the best combination of expected return and risk.

Here are some additional insights about MPT:

- The efficient frontier is not a straight line. This is because the risk of a portfolio is not simply the sum of the risks of the individual assets in the portfolio. Instead, the risk of a portfolio is determined by the correlation between the assets in the portfolio.
- The efficient frontier is upward-sloping. This means that portfolios with higher expected returns also have higher risks.
- The efficient frontier is not unique. There are many different efficient frontiers, each of which is associated with a different level of risk aversion.

Lab Experiment to be done by students:

1. Download a dataset of historical stock prices.
2. Calculate the expected return and risk of each stock in the dataset.
3. Construct an efficient frontier of portfolios using the stocks in the dataset.
4. Analyze the implications of MPT for portfolio selection

1. Downloading Historical Stock Prices

```
import yfinance as yf
import pandas as pd

# Define the stocks and the period for historical data
tickers = ['AAPL', 'MSFT', 'GOOG', 'AMZN']  # Example stock tickers
start_date = '2020-01-01'
end_date = '2023-01-01'

# Download historical data
stock_data = yf.download(tickers, start=start_date, end=end_date)['Adj Close']

# Display the first few rows of the dataset
print(stock_data.head())
```

```
[*********************100%%**********************]  4 of 4 completedTicker            AAPL       AMZN       GOOG        MSFT
Date
2020-01-02  72.876106  94.900497  68.290787  154.215683
2020-01-03  72.167587  93.748497  67.955666  152.295380
2020-01-06  72.742638  95.143997  69.631264  152.689072
2020-01-07  72.400566  95.343002  69.587814  151.296875
2020-01-08  73.565193  94.598503  70.136192  153.706802
```

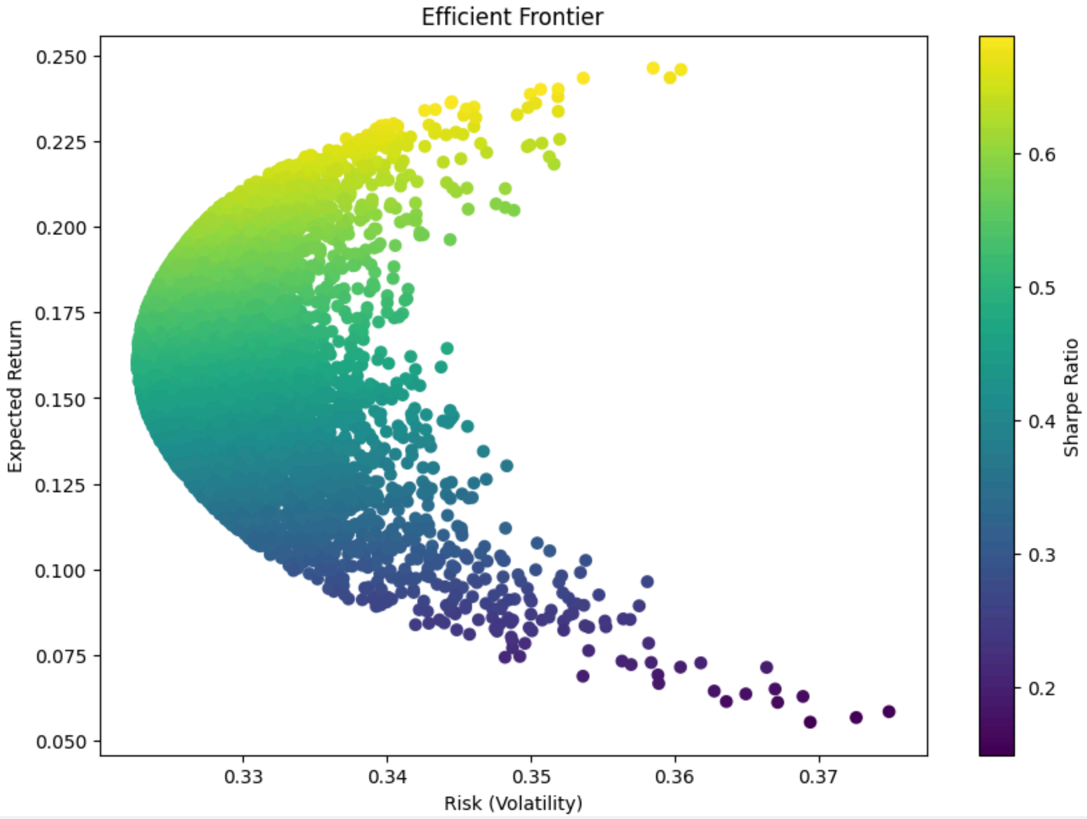
2. Calculating Expected Return and Risk

```
# Calculate daily returns
returns = stock_data.pct_change().dropna()

# Calculate expected return (mean) and risk (standard deviation)
expected_returns = returns.mean() * 252  # Annualize the returns
risk = returns.std() * (252 ** 0.5)      # Annualize the risk (volatility)

# Display the expected return and risk
print("Expected Returns:\n", expected_returns)
print("\nRisk (Volatility):\n", risk)
```

```
Expected Returns:
 Ticker
AAPL    0.258011
AMZN    0.035559
GOOG    0.146087
MSFT    0.203655
dtype: float64

Risk (Volatility):
 Ticker
AAPL    0.369334
AMZN    0.390844
GOOG    0.343706
MSFT    0.347572
dtype: float64
```

3. Constructing the Efficient Frontier

```
import numpy as np
import matplotlib.pyplot as plt

# Number of portfolios to simulate
num_portfolios = 10000

# Store portfolio returns, risk, and weights
portfolios = {'Returns': [], 'Risk': [], 'Weights': []}

for _ in range(num_portfolios):
    weights = np.random.random(len(tickers))
    weights /= np.sum(weights)

    portfolio_return = np.dot(weights, expected_returns)
    portfolio_risk = np.sqrt(np.dot(weights.T, np.dot(returns.cov() * 252, weights)))

    portfolios['Returns'].append(portfolio_return)
    portfolios['Risk'].append(portfolio_risk)
    portfolios['Weights'].append(weights)

# Convert to DataFrame
portfolio_df = pd.DataFrame(portfolios)

# Plot the efficient frontier
plt.figure(figsize=(10, 7))
plt.scatter(portfolio_df['Risk'], portfolio_df['Returns'], c=portfolio_df['Returns']/portfolio_df['Risk'], marker='o')
plt.title('Efficient Frontier')
plt.xlabel('Risk (Volatility)')
plt.ylabel('Expected Return')
plt.colorbar(label='Sharpe Ratio')
plt.show()
```




```
# Find the portfolio with the highest Sharpe Ratio
sharpe_ratios = portfolio_df['Returns'] / portfolio_df['Risk']
max_sharpe_idx = sharpe_ratios.idxmax()

# Display the optimal portfolio
print("\nOptimal Portfolio:\n")
print("Weights:", portfolio_df.loc[max_sharpe_idx, 'Weights'])
print("Expected Return:", portfolio_df.loc[max_sharpe_idx, 'Returns'])
print("Risk (Volatility):", portfolio_df.loc[max_sharpe_idx, 'Risk'])
print("Sharpe Ratio:", sharpe_ratios[max_sharpe_idx])
```

```
Optimal Portfolio:

Weights: [0.81505648 0.01706042 0.02775953 0.14012357]
Expected Return: 0.24349211565108947
```

```
Risk (Volatility): 0.35364876787644123
Sharpe Ratio: 0.6885139657440046
```

4. Analyzing MPT Implications

The image you provided shows the efficient frontier of portfolios generated using Modern Portfolio Theory (MPT), with the portfolio having the highest Sharpe ratio highlighted. Let's analyze the implications of MPT based on this data.

Analyzing MPT Implications:

1. **Efficient Frontier:**
   - The efficient frontier is the curve that represents the set of portfolios offering the highest expected return for a given level of risk. The portfolios that lie on this frontier are considered "efficient" because they provide the best possible return for the least amount of risk.
   - Your plot displays this frontier, with the yellow to green gradient indicating higher Sharpe ratios.
2. **Optimal Portfolio Selection:**
   - The portfolio with the highest Sharpe ratio is considered the optimal portfolio because it maximizes the return per unit of risk. In this case, the optimal portfolio has weights `[0.815, 0.017, 0.028, 0.140]` in the respective stocks.
   - The expected return of this portfolio is approximately 24.35%, with a risk (volatility) of 35.36%. The Sharpe ratio is 0.6885, indicating a relatively high return for the level of risk taken.
3. **Diversification:**
   - The allocation of weights in the optimal portfolio shows that most of the investment (81.5%) is concentrated in one stock, with smaller allocations to the other stocks. This suggests that MPT has identified one stock as significantly outperforming the others in terms of risk-adjusted return.
   - However, MPT generally encourages diversification across assets to reduce unsystematic risk (risk specific to individual stocks). The fact that a large portion of the investment is in a single stock might imply either that this stock has a particularly favorable risk-return profile or that the other stocks are highly correlated with it.
4. **Risk-Return Tradeoff:**
   - MPT emphasizes the tradeoff between risk and return. The portfolios on the lower part of the efficient frontier have lower expected returns but also lower risk, while those on the upper part have higher expected returns with higher risk.
   - The Sharpe ratio is used to assess the risk-adjusted return of a portfolio. In this analysis, portfolios with higher Sharpe ratios offer better compensation for risk. The optimal portfolio provides the best balance of risk and return among all the portfolios considered.
5. **Risk Management:**
   - MPT's implications for risk management are clear: investors should seek portfolios on the efficient frontier that match their risk tolerance. More conservative investors might choose portfolios with lower volatility, even if that means accepting lower returns, while more aggressive investors might prefer portfolios higher up the frontier.
   - By focusing on the Sharpe ratio, investors are encouraged to consider how much extra return they are getting for taking on additional risk, leading to more informed and disciplined investment decisions.

Conclusion:

Modern Portfolio Theory suggests that investors can construct an optimal portfolio by balancing risk and return, as reflected in the Sharpe ratio. The optimal portfolio identified in your analysis, which maximizes this ratio, is a good choice for an investor seeking the best possible return for a given level of risk. However, it's important to consider the concentration risk due to the high allocation in a single stock and ensure that the portfolio aligns with the investor's overall risk tolerance and investment goals.