Fórmulas Matemáticas E

Geometria

Círculo de raio r: circunferência $= 2\pi r$; área $= \pi r^2$.

Esfera de raio r: área $= 4\pi r^2$; volume $= \frac{4}{3}\pi r^3$.

Cilindro circular reto de raio r e altura h:
área $= 2\pi r^2 + 2\pi rh$; volume $= \pi r^2 h$.

Triângulo de base a e altura h: área $= \frac{1}{2}ah$.

Fórmula de Báskara

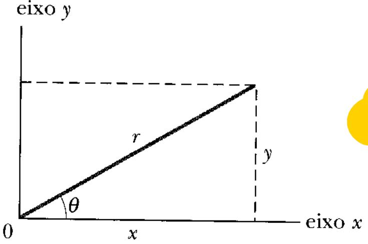
Se $ax^2 + bx + c = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Funções Trigonométricas do Ângulo θ

$\operatorname{sen}\theta = \dfrac{y}{r}$ $\quad \cos\theta = \dfrac{x}{r}$

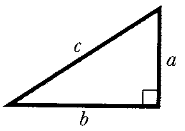
$\tan\theta = \dfrac{y}{x}$ $\quad \cot\theta = \dfrac{x}{y}$

$\sec\theta = \dfrac{r}{x}$ $\quad \csc\theta = \dfrac{r}{y}$



Teorema de Pitágoras

Neste triângulo retângulo,
$a^2 + b^2 = c^2$



Triângulos

Ângulos: A, B, C

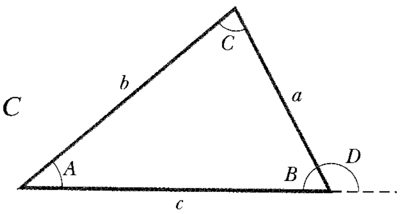
Lados opostos: a, b, c

$A + B + C = 180°$

$\dfrac{\operatorname{sen} A}{a} = \dfrac{\operatorname{sen} B}{b} = \dfrac{\operatorname{sen} C}{c}$

$c^2 = a^2 + b^2 - 2ab\cos C$

Ângulo externo $D = A + C$



Sinais e Símbolos Matemáticos

$=$ igual a

$\approx$ aproximadamente igual a

$\sim$ da ordem de grandeza de

$\neq$ diferente de

$\equiv$ idêntico a, definido como

$>$ maior que ($\gg$ muito maior que)

$<$ menor que ($\ll$ muito menor que)

$\geq$ maior ou igual a (não menor que)

$\leq$ menor ou igual a (não maior que)

$\pm$ mais ou menos

$\propto$ proporcional a

Σ somatório de

$x_{\text{méd}}$ valor médio de x

Identidades Trigonométricas

$\operatorname{sen}(90° - \theta) = \cos\theta$

$\cos(90° - \theta) = \operatorname{sen}\theta$

$\operatorname{sen}\theta / \cos\theta = \tan\theta$

$\operatorname{sen}^2\theta + \cos^2\theta = 1$

$\sec^2\theta - \tan^2\theta = 1$

$\csc^2\theta - \cot^2\theta = 1$

$\operatorname{sen} 2\theta = 2\operatorname{sen}\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \operatorname{sen}^2\theta = 2\cos^2\theta - 1 = 1 - 2\operatorname{sen}^2\theta$

$\operatorname{sen}(\alpha \pm \beta) = \operatorname{sen}\alpha\cos\beta \pm \cos\alpha\operatorname{sen}\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \operatorname{sen}\alpha\operatorname{sen}\beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\operatorname{sen}\alpha \pm \operatorname{sen}\beta = 2\operatorname{sen}\frac{1}{2}(\alpha \pm \beta)\cos\frac{1}{2}(\alpha \mp \beta)$

$\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$

$\cos\alpha - \cos\beta = -2\operatorname{sen}\frac{1}{2}(\alpha + \beta)\operatorname{sen}\frac{1}{2}(\alpha - \beta)$

Teorema Binomial

$$(1+x)^n = 1+\frac{nx}{1!}+\frac{n(n-1)x^2}{2!}+\cdots \qquad (x^2<1)$$

Expansão Exponencial

$$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$$

Expansão Logarítmica

$$\ln(1+x) = x-\tfrac{1}{2}x^2+\tfrac{1}{3}x^3-\cdots \qquad (|x|<1)$$

Expansões Trigonométricas (θ em radianos)

$$\operatorname{sen}\theta = \theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}-\cdots$$

$$\cos\theta = 1-\frac{\theta^2}{2!}+\frac{\theta^4}{4!}-\cdots$$

$$\tan\theta = \theta+\frac{\theta^3}{3}+\frac{2\theta^5}{15}+\cdots$$

Regra de Cramer

Duas equações lineares simultâneas com incógnitas x e y,

$$a_1x+b_1y=c_1 \quad \text{e} \quad a_2x+b_2y=c_2,$$

têm como soluções

$$x=\frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}=\frac{c_1b_2-c_2b_1}{a_1b_2-a_2b_1}$$

e

$$y=\frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}=\frac{a_1c_2-a_2c_1}{a_1b_2-a_2b_1}.$$

Produtos de Vetores

Sejam $\hat{\mathrm{i}}$, $\hat{\mathrm{j}}$ e $\hat{\mathrm{k}}$ vetores unitários nas direções x, y e z, respectivamente. Nesse caso,

$$\hat{\mathrm{i}}\cdot\hat{\mathrm{i}}=\hat{\mathrm{j}}\cdot\hat{\mathrm{j}}=\hat{\mathrm{k}}\cdot\hat{\mathrm{k}}=1, \quad \hat{\mathrm{i}}\cdot\hat{\mathrm{j}}=\hat{\mathrm{j}}\cdot\hat{\mathrm{k}}=\hat{\mathrm{k}}\cdot\hat{\mathrm{i}}=0,$$

$$\hat{\mathrm{i}}\times\hat{\mathrm{i}}=\hat{\mathrm{j}}\times\hat{\mathrm{j}}=\hat{\mathrm{k}}\times\hat{\mathrm{k}}=0,$$

$$\hat{\mathrm{i}}\times\hat{\mathrm{j}}=\hat{\mathrm{k}}, \qquad \hat{\mathrm{j}}\times\hat{\mathrm{k}}=\hat{\mathrm{i}}, \qquad \hat{\mathrm{k}}\times\hat{\mathrm{i}}=\hat{\mathrm{j}}$$

Qualquer vetor $\vec{a}$ de componentes a_x, a_y e a_z ao longo dos eixos x, y e z pode ser escrito na forma

$$\vec{a}=a_x\hat{\mathrm{i}}+a_y\hat{\mathrm{j}}+a_z\hat{\mathrm{k}}.$$

Sejam $\vec{a}$, $\vec{b}$ e $\vec{c}$ vetores arbitrários de módulos a, b e c. Nesse caso,

$$\vec{a}\times(\vec{b}+\vec{c})=(\vec{a}\times\vec{b})+(\vec{a}\times\vec{c})$$

$$(s\vec{a})\times\vec{b}=\vec{a}\times(s\vec{b})=s(\vec{a}\times\vec{b}) \qquad (s = \text{escalar}).$$

Seja θ o menor dos dois ângulos entre $\vec{a}$ e $\vec{b}$. Nesse caso,

$$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}=a_xb_x+a_yb_y+a_zb_z=ab\cos\theta$$

$$\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}=\begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$=\hat{\mathrm{i}}\begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix}-\hat{\mathrm{j}}\begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix}+\hat{\mathrm{k}}\begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$$

$$=(a_yb_z-b_ya_z)\hat{\mathrm{i}}+(a_zb_x-b_za_x)\hat{\mathrm{j}}$$
$$+(a_xb_y-b_xa_y)\hat{\mathrm{k}}$$

$$|\vec{a}\times\vec{b}|=ab\operatorname{sen}\theta$$

$$\vec{a}\cdot(\vec{b}\times\vec{c})=\vec{b}\cdot(\vec{c}\times\vec{a})=\vec{c}\cdot(\vec{a}\times\vec{b})$$

$$\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$$

Derivadas e Integrais

Nas fórmulas a seguir as letras u e v representam duas funções de x, e a e m são constantes. A cada integral indefinida deve-se somar uma constante de integração arbitrária. O *Handbook of Chemistry and Physics* (CRC Press Inc.) contém uma tabela mais completa.

1. $\dfrac{dx}{dx} = 1$
2. $\dfrac{d}{dx}(au) = a\dfrac{du}{dx}$
3. $\dfrac{d}{dx}(u+v) = \dfrac{du}{dx} + \dfrac{dv}{dx}$
4. $\dfrac{d}{dx}x^m = mx^{m-1}$
5. $\dfrac{d}{dx}\ln x = \dfrac{1}{x}$
6. $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
7. $\dfrac{d}{dx}e^x = e^x$
8. $\dfrac{d}{dx}\operatorname{sen} x = \cos x$
9. $\dfrac{d}{dx}\cos x = -\operatorname{sen} x$
10. $\dfrac{d}{dx}\tan x = \sec^2 x$
11. $\dfrac{d}{dx}\cot x = -\csc^2 x$
12. $\dfrac{d}{dx}\sec x = \tan x \sec x$
13. $\dfrac{d}{dx}\csc x = -\cot x \csc x$
14. $\dfrac{d}{dx}e^u = e^u\dfrac{du}{dx}$
15. $\dfrac{d}{dx}\operatorname{sen} u = \cos u\dfrac{du}{dx}$
16. $\dfrac{d}{dx}\cos u = -\operatorname{sen} u\dfrac{du}{dx}$

1. $\int dx = x$
2. $\int au\, dx = a\int u\, dx$
3. $\int (u+v)\, dx = \int u\, dx + \int v\, dx$
4. $\int x^m\, dx = \dfrac{x^{m+1}}{m+1}(m \neq -1)$
5. $\int \dfrac{dx}{x} = \ln|x|$
6. $\int u\dfrac{dv}{dx}dx = uv - \int v\dfrac{du}{dx}dx$
7. $\int e^x\, dx = e^x$
8. $\int \operatorname{sen} x\, dx = -\cos x$
9. $\int \cos x\, dx = \operatorname{sen} x$
10. $\int \tan x\, dx = \ln|\sec x|$
11. $\int \operatorname{sen}^2 x\, dx = \frac{1}{2}x - \frac{1}{4}\operatorname{sen} 2x$
12. $\int e^{-ax}\, dx = -\dfrac{1}{a}e^{-ax}$
13. $\int xe^{-ax}\, dx = -\dfrac{1}{a^2}(ax+1)e^{-ax}$
14. $\int x^2 e^{-ax}\, dx = -\dfrac{1}{a^3}(a^2x^2 + 2ax + 2)e^{-ax}$
15. $\int_0^\infty x^n e^{-ax}\, dx = \dfrac{n!}{a^{n+1}}$
16. $\int_0^\infty x^{2n} e^{-ax^2}\, dx = \dfrac{1\cdot 3\cdot 5\cdots(2n-1)}{2^{n+1}a^n}\sqrt{\dfrac{\pi}{a}}$
17. $\int \dfrac{dx}{\sqrt{x^2+a^2}} = \ln(x + \sqrt{x^2+a^2})$
18. $\int \dfrac{x\, dx}{(x^2+a^2)^{3/2}} = -\dfrac{1}{(x^2+a^2)^{1/2}}$
19. $\int \dfrac{dx}{(x^2+a^2)^{3/2}} = \dfrac{x}{a^2(x^2+a^2)^{1/2}}$
20. $\int_0^\infty x^{2n+1} e^{-ax^2}\, dx = \dfrac{n!}{2a^{n+1}}(a>0)$
21. $\int \dfrac{x\, dx}{x+d} = x - d\ln(x+d)$