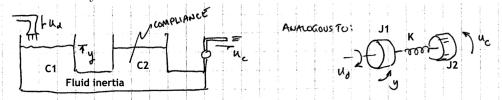
**Problem 1.** The two systems shown below were discussed in lecture.



When modeled, these lead to similar state equations. Choose one of these and practice deriving the state equations as shown in lecture. Note that the controlled input is applied at the 2nd tank (or inertia), and there is a disturbance on the first tank (or inertia). Ignore any losses that could exist in such systems and make all parameter values unity to simplify the form. The equations were presented in lecture on 11/30/21. What are the A, B, C, and D matrices?

**Problem 2.** For the system in Problem 1, show that the TF relating state  $x_1$  to the control input  $u_c(t)$  is,

$$\frac{x_1}{u_d} = \frac{-1}{s(s^2 + 2)}$$

**Problem 3.** Use a root locus to show that the system cannot be stabilized for any gain, K. Use Matlab to confirm your work.

**Problem 4.** Find the controllability matrix  $P_c$  for the system in Problem 1 and show that the system is completely controllable (i.e., determinant of  $P_c \neq 0$ ).

**Problem 5.** Find the observability matrix for the system in Problem 1 and show that the system is completely observable.

**Problem 6.** Work through Example 11.2 in Dorf and Bishop.

**Problem 7.** Work through Example 11.5 in Dorf and Bishop.

**Problem 8.** Show how the state equations for the 'simple' inverted pendulum presented in Dorf and Bishop's Example 11.6 are derived. Let g = l = 1, and show that the system is completely controllable.

**Problem 9.** Work Dorf and Bishop's Example 11.6 design for stabilizing the simple inverted pendulum. Show

**Problem 10.** If Problem 1 shows the system is controllable, apply Ackermann's formula for determining the feedback gains for the simple inverted pendulum of Problems 8 and 9.