Problem 1. For the following unity feedback system with,

$$G(s) = \frac{10K}{s(s+1)(s+5)}$$

used frequency domain methods to design a lead compensator that will yield a phase margin of 60 degrees with a static velocity error constant, $K_v = 5$.

Solution:

Step 1: Find the gain K to give the desired static velocity error constant. Recall, $K_v = \lim_{s\to 0} sG(s)$ (Ogata, p. 277), so,

$$K_v = \lim_{s\to 0} sGH = \lim_{s\to 0} s\frac{10K}{s(s+1)(s+5)} = \frac{10K}{5}$$

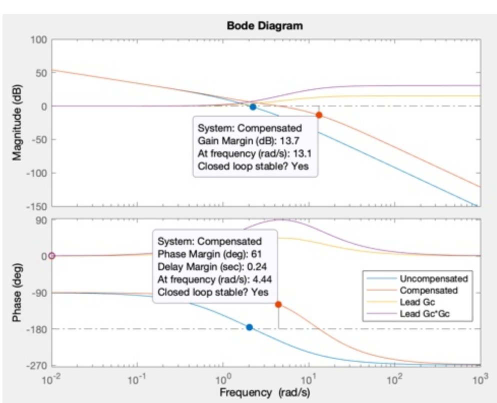
So, for $K_v = 5$, require $K = K_v/2 = 2.5$.

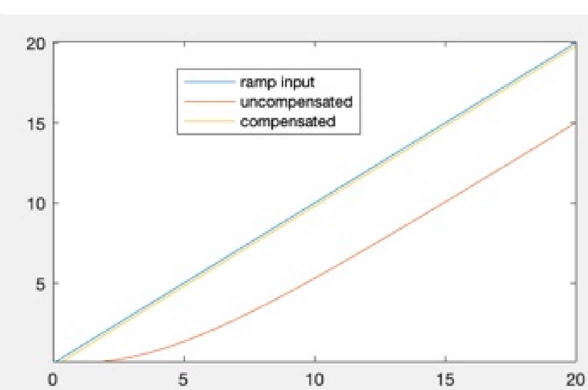
Step 2: From the bode plot or by directly solving from the gain-adjusted ucompensated system, the system is found to be stable but with a gain margin $\gamma = 3.94$ degrees (less than the 60 degrees required).

Step 3: A lead compensator requires a ϕ_m of at least 56 degrees, but it ends up not being sufficient and soon you find you need a very small $\alpha < 0.07$, which is not recommended, and it still not sufficient to give you the right phase margin. Instead, try by putting two lead compensators in series with a ϕ_m that leads to a reasonable $\alpha > 0.07$. Iterating, found that two lead compensators with $\phi_m = 45$ degrees provides the results as indicated in next steps.

Step 4: For $\phi_m = 45$ degrees, $\alpha = 0.172$. Turns out we need to iterate a little on the new crossover frequency, ω_m. Using the approach proposed by Ogata give $\omega_m = 3.11$ rad/sec, but this does not give good results. If we slide over the ω_m to higher values, we find that a value about 50% higher or $\omega_m = 4.68$ rad/sec gives much better phase and gain margin of 61 degrees and 14 dB, respectively.

Step 5: Each compensator is $G_c(s) = (Ts+1)/(\alpha Ts+1)$, with $\alpha = 0.172$ and $T = 0.5154$ seconds. The final compensator is formed by the series set, $G_c(s) \cdot G_c(s)$.





Problem 2. A unity feedback system with,

$$G(s) = \frac{K}{s(s+5)(s+8)}$$

is operating with a 20% overshoot given a step input. Using frequency domain methods, design a lag compensator to yield a fivefold improvement in steady state error with a ramp input without appreciably changing its transient response.

Solution:

Step 1: This problem requires you to find the original gain, K, which was not specified and which leads to about 20% overshoot. You can find this several ways, and a value from 105 to 110 is good. I used 110, found by iterating on the step response until the peak response was 20% (as measured on the graph).

Step 2: Now, to "...yield a fivefold improvement in steady state error with a ramp input without appreciably changing its transient response" means you want to adjust the K so that the steady-state error to a ramp input improves five times. To do this, see below:

$$e_{ss} = \lim_{s \to 0} s \cdot \underbrace{\frac{e}{r}}_{\text{error ref.}} \cdot \underbrace{\frac{1}{s^2}}_{\text{ramp input}}$$

$$\frac{e}{r} = \frac{1}{1 + G \cdot H} = \frac{1}{1 + \frac{K}{s(s+5)(s+8)} \cdot 1}$$

$$\frac{e}{r} = \frac{s(s+5)(s+8)}{s(s+5)(s+8) + K}$$

$$e_{ss_0} = \lim_{s \to 0} \frac{(s+5)(s+8)}{s(s+5)(s+8) + K} = \frac{40}{K} = \frac{40}{110} = \underline{0.364}.$$

Error due to a ramp input.

Desired error

$$\text{Want} \quad e_{ss_1} = \frac{e_{ss_0}}{5} = \underline{0.0727}.$$

five fold improvement

$$\therefore \text{ This requires } \quad K = \frac{40}{0.0727} = \underline{550}.$$

You find that this K gives a steady state error of 3.6%. The lag compensator must reduce this error about five times. The phase margin is however is -1.55 degrees and the gain crossover frequency is 6.5 rad/sec.

Step 3: We want to retain the original phase margin so that the transient response is not appreciably changed, and this value was 48 degrees. This means, we want to find the phase equal to -180 + 48 + about 5 to 12. Say, this is -127 and find the frequency where phase takes this value, set it to new gain crossover. We find this is $\omega_{g,new} = 2.07$ rad/sec.

Step 4: Now set the value of $\omega_1 = 0.1\omega_{g,new}$, and $T = 1/\omega_1$. This helps us find the value of β (attenuation factor) as follows: a) find β that will bring the magnitude at $\omega_{g,new}$ to 0 dB by first finding the magnitude at this frequency, $|G(\omega_{g,new})|$, b) then from $20\log_{10}(1/\beta) = -20\log_{10}(|G(\omega_{g,new})|)$, solve for β. This now gives you $\omega_2 = 1/(\beta T)$.

Step 5: The compensator takes the form: $G_c(s) = (Ts+1)/(\beta Ts+1)$, with $T = 4.827$ and $\beta = 5.935$.