The University of Texas at Austin

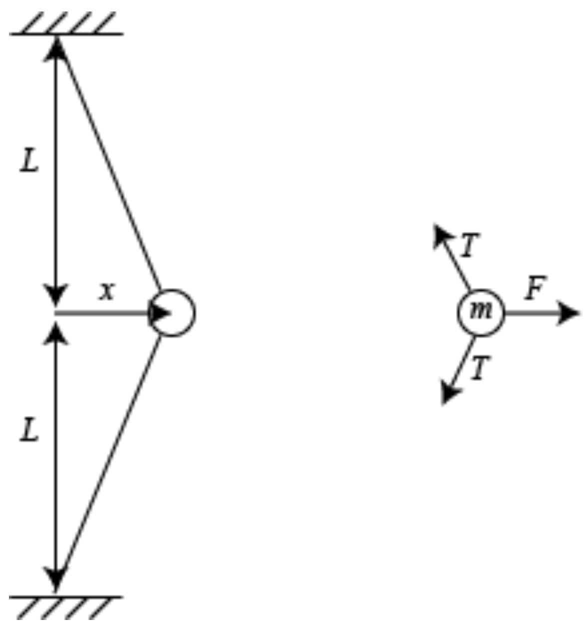
Mechanical Engineering Department

MODELING OF PHYSICAL SYSTEMS

J.J. Beaman	ME 383Q.4	Spring 2022
Assigned 3/12/2022	**Assignment 4**	Due 3/24/2022

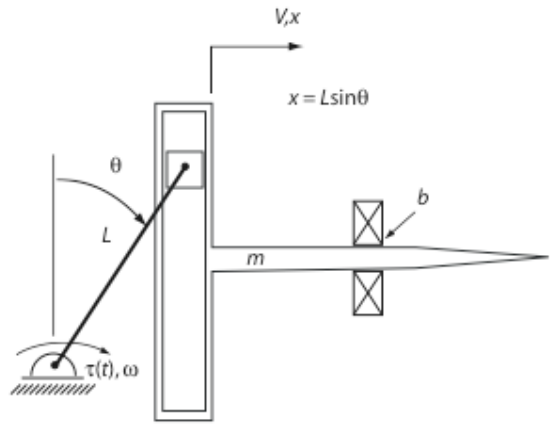
Read Chapter 4.6, 4.7, 4.8, 6

1. Shown below is a mass m constrained by two equal extensible strings. The strings are put into a relatively constant tension by a vertical force T. There is a mass m between the two strings which can be pulled a distance x by a horizontal force F. The tension force T remains constant during this pull. The mass is pulled out a distance x_0 and released from rest.



   (a) Derive nonlinear state equations for this mass string system in terms of distance x and mass momentum p.
   (b) Linearize the nonlinear state equations for a small value of $x = \delta x$.
   (c) For $L = 1$ m, $m = 0.1$ kg, and $T = 100$ N, simulate your nonlinear and linear state equations with $x_0 = .05$, .1, .5, and $x_0 = 1$ meters for time span of 1 sec. Comment on the accuracy of the linearized model with respect to initial distance pulled x_0.

2. In a certain type of sewing machine, a scotch yoke mechanism is used to drive a reciprocating needle.
   (a) Develop a bond graph model for this system (Use just the elements indicated by the parameters and the relations in the figure with torque input).
   (b) Obtain state equations for your system.



3. Read the material on the energy-saving hydraulic power supply in Doeblin.pdf.

(a) Develop a bond graph model of this system using the article's description.
Use the following variables in your model.

F_K = spring force A = piston area and transformer modulus
x = spring deflection Q_{piston} = piston volume flow τ_{Be} = motor friction torque
P = supply pressure Q_p = pump volume flow $T_l(t)$ = motor load torque
F_L = lever force Q_{pl} = pump leakage L = lever length
V_L = lever tip velocity $\omega_p(t)$ = pump angular velocity
F_p = piston force $d_p\phi = Q_p/\omega_p$ pump modulated transformer modulus
τ_L = lever torque τ_m = motor torque
h_ϕ = lever angular momentum Q_{ml} = motor leakage
ϕ = lever angle $d_m = \tau_m/P = Q_m/\omega_m$ motor transformer modulus
τ_B = lever friction torque ω_m = motor angular velocity
ω_L = lever angular velocity h_m = motor angular momentum

(b) Simulate your model with the following parameters
J_i = .1 in-lb-sec^2 pump angular inertia
B_i = in-lb-sec   pump angular damping
K = 100 lb/in   spring constant
L = 1 in   lever length
$K_{si} = KL^2$
J_m = .043 in-lb-sec^2 motor angular inertia
B_m = 20 in-lb-sec   motor angular damping
C_f = .0001 lb/in^5 or .001 lb/in^5 pipe fluid compliance
d_m = .486 in^3 transformer modulus
d_p = 1.0 in^3/radian   modulated transformer modulus
A = .5 in^2 piston area and transformer modulus
K_{pl} = .0006 in^5/sec   pump leakage loss coefficient
$\omega_p(t)$ = 180 radian/sec pump angular velocity step input
$T_l(t)$ = 0 in-lb motor load torque
K_{ml} = .0004 in^5/sec   motor leakage loss coefficient

(c) Develop a set of state equations from your bond graph.

(d) What is wrong with the state equations as given in this article?

(e) With J_i = .1, L = 1, K = 100., J_m = .043, B_m = 20, d_m =.486, d_p = 1 A = .5, K_{pl} = .0006, ω_p = 180, T_l = 0, and K_{ml} = .0004, perform a digital simulation with B_i = 50 and C_f = .0001.

Compare with the results from the article. Repeat for $B_i = 20$ and $C_f = .001$. Compare with the results from the article.