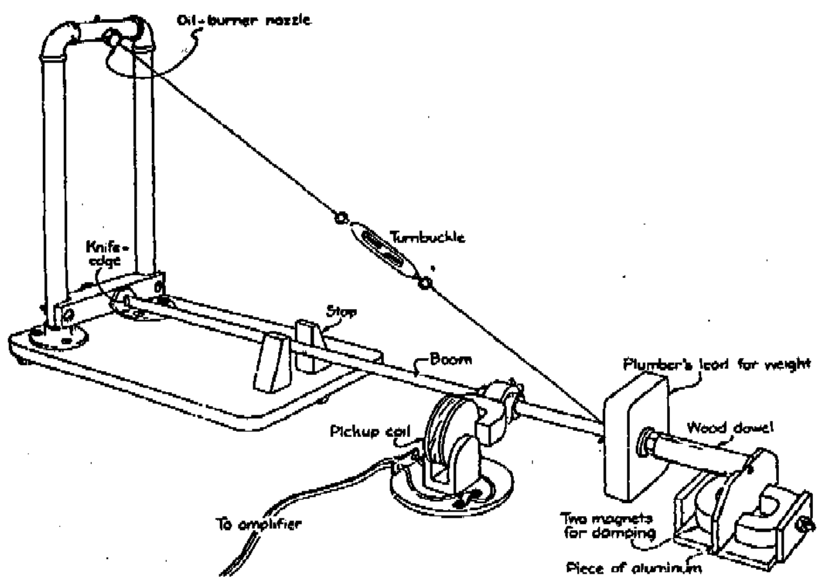
The University of Texas at Austin
Mechanical Engineering Department
MODELING OF PHYSICAL SYSTEMS

J.J. Beaman	ME 383Q.4	Spring 2022
Assigned 4/1/2022	**Assignment 5**	Due 4/12/2022

Read Chapter's 6, 8, and 9

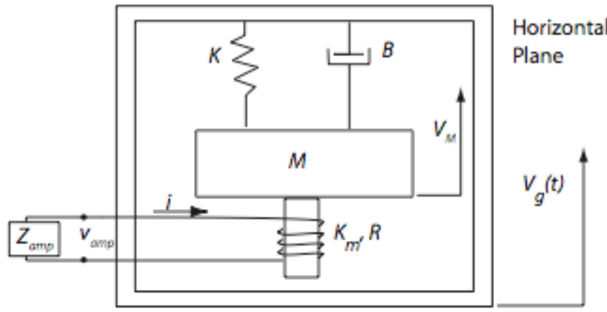
1. Modeling of a seismometer
   In 1979 Lehman[1] described how to build a simple seismometer that could be built at home. A figure from his Scientific American article is shown below.



Picture from 1979 Scientific American article

A simplified model for this system is shown schematically in following figure.

[1] Lehman, *How to build a simple seismograph to record earthquake waves at home,* Scientific American Magazine July, 1979.



In this figure, the electric coil is attached to the frame and a cylindrical permanent magnet moves inside of this coil. The constant K_m relates the voltage v_{amp} to the relative velocity of the magnet to the coil, $V_r = V_g - V_m$, and $v_{amp} = K_m V_r$. The symbol Z_{amp} refers to the impedance of the electronic amplifier.

(a) Develop a bond graph model for this system.

(b) Voltage amplifiers are designed so that the input current $i \approx 0$ and the amplifier impedance $Z_{amp} \to \infty$. Derive a set of set of state and output equations in matrix form that relates the input ground velocity V_g to the output amplifier voltage v_{amp}.

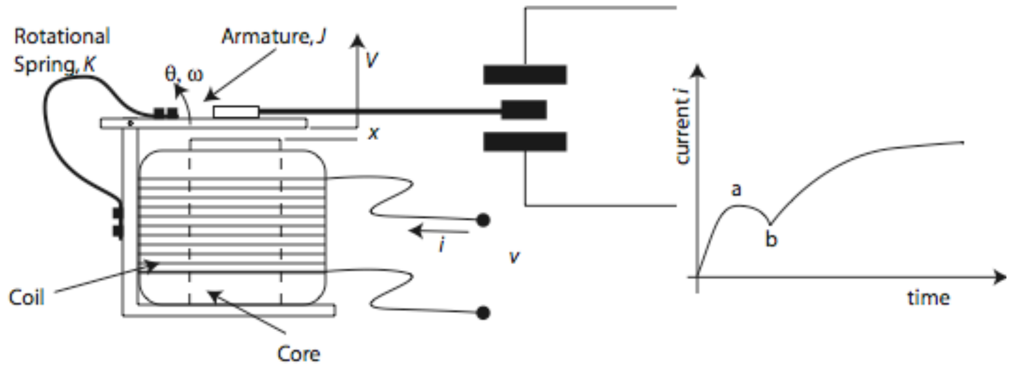
(c) Obtain the transfer function $\frac{v_{amp}}{V_g}$ from the state equations (b) above.

(d) Obtain an impedance bond graph which includes amplifier impedance Z_{amp}.

(e) Use impedance methods and your impedance bond graph to obtain the transfer functions $\frac{F_g}{V_g}, \frac{F_c}{F_g}$, and $\frac{i}{F_c}$, where F_g is total force to the ground and F_c is electronic coil force. Use these to obtain the transfer function $\frac{v_{amp}}{V_g}$.

(f) If K = .005 N/cm, M = 2 kg, K_m = 50 mV-sec/cm, b = 20 mN-sec/cm, and $Z_{amp} \to \infty$ to obtain the frequency response of the seismometer. What is the longest time period of a seismic wave this device will work for?

2. The dc relay shown below is energized with a battery of voltage v.
   a. Assuming that the permeability of the core and armature is so high that all the magnetic energy of the core and armature is stored in the air gap so that the magnetic energy can be expressed as $T_{\lambda x} = \frac{\Phi^2 x}{2\mu_0 A}$, where Φ is magnetic flux, A is the horizontal area of the air gap, x is the vertical separation of the core and the armature at the air gap, and μ_0 is the permeability of free space. In addition, assume for small angle that $V = L\omega$ where L is the distance from the pivot to the middle of the core. The number of turns of the coil is N and has resistance R.
   b. Develop a bond graph model for this system.
   c. With state variables of magnetic flux Φ, armature angular momentum h, angle θ, and gap x, obtain a set of state equations for this system. The spring has stiffness K and the armature has moment of inertia J about the pin joint.
   d. A typical current-time trace is shown below. Account for the dip in the current from a to b. It is desirable for control purposes to now when the armature strikes the core. At what time does this occur?



3. Shown below is a simple pump driven by a piston and a crankshaft. The poppet valves of the pump have constitutive relation shown in fig. b below.
    a. Formulate a detailed bond graph model of this system if the inertia of the crank links can be neglected and the mass of the piston is m_p.
    b. Repeat part (a) assuming that the drive link has moment of inertia J_d about its fixed pivot and the floating link has moment of inertia J_f about its center of gravity and mass m_f.
    c. With $\tau(t)$ given, obtain state equations for (a) and (b) above using direct methods.
    d. Repeat (b) using a Lagrange bond graph. Obtain state equations using the Lagrange bond graph.