The University of Texas at Austin
Mechanical Engineering Department
**MODELING OF PHYSICAL SYSTEMS**

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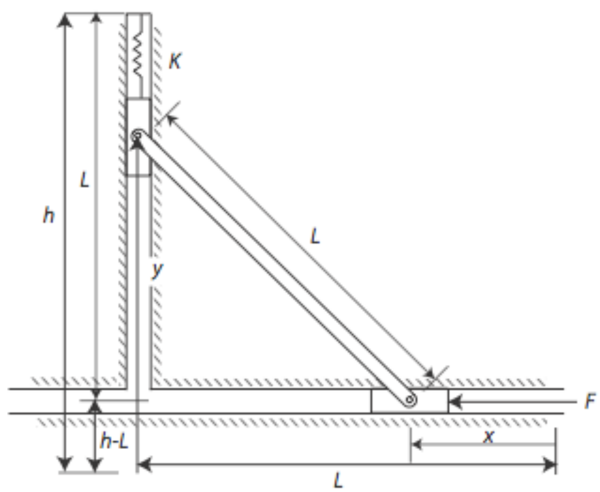
| J.J. Beaman | ME 383Q.4 | Spring 2022 |
| Assigned 01/26/2022 | **Assignment 1** | Due 2/3/2022 |

Read Chapters 1 and 2 in notes

1. In steady state a good weather vane points into the wind, but when the wind shifts, the vane cannot always be trusted to be pointing into the wind. Identify inputs, outputs, and system parameters which affect its response to the wind.

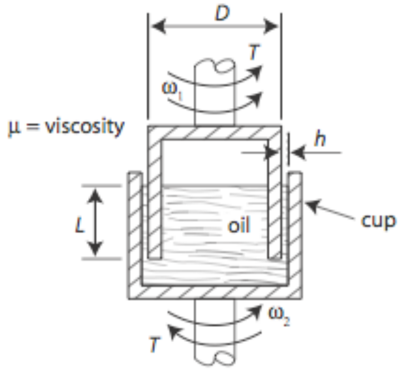
2. A nonlinear spring is constructed as shown below from a massless linkage and a linear spring.




The unstressed length of the linear spring is $h$ and the spring constant is $K$. The linkage is just slightly up at the left end when the force $F$ is zero.

a. Determine the force $F$ as a function of $x$ where $x$ is measured from the unstressed position of the system. Sketch the function on a graph. What is the region of validity?
b. Derive an expression for the energy stored in the system as a function of $x$. Show graphically the area represented by the energy.

3. A drag cup is built by immersing a thin cylinder in a cup of oil of viscosity $\mu$.




a. Derive an expression for the function relating the relative angular velocity, $\Delta\omega = \omega_1 - \omega_2$, to the torque $T$ in terms of the geometry of the cup and viscosity.
b. What ideal element is best represented by this system?
c. Derive an expression for the power dissipated in the system as a function of $\Delta\omega$.
d. If the speeds $\omega_1$ and $\omega_2$ are not constant but can change quickly, what other elements might be required to represent this device?

4. An iron-core inductor is built by winding 500 turns of copper wire around a steel torus with a mean diameter $D$ of 3 in. and a cross-sectional area of 0.2 in$^2$ (Figure 2-61). If we assume that the coil resistance is negligible and that flux is uniform throughout the iron core, we see that Faraday's law gives

$$v = N\frac{d\phi}{dt} = \frac{d\lambda}{dt} \qquad \text{volts.}$$

The flux density $B$ in the iron is defined as $\phi/A$, and the magnetizing force $H$ resulting from the current i is

$$H = \frac{Ni}{l} \qquad \text{(amp-turns)/m,}$$

where $l$ is the length of the magnetic path = $\pi D$. The units of $B$ in the mks system are webers/m$^2$ (1 weber-turn = 1 v-sec). The $B$-$H$ curve for steel as determined experimentally is shown in Figure 2-62.

(a) Determine the relationship between $\lambda$ and $i$ for the inductor.
(b) Compute the energy stored by the inductor when $i$ = 0.16 amp.
(c) Compute the energy stored by the inductor when $i$ = 0.5 amp.
(d) Assuming that the wire is No. 40 copper wire (resistance = 1070 ohms/1000 ft.), what is the power required to maintain the energy calculated in (b).
(e) How much energy is dissipated in 1 sec in (d)? Is this device an efficient energy storage design?

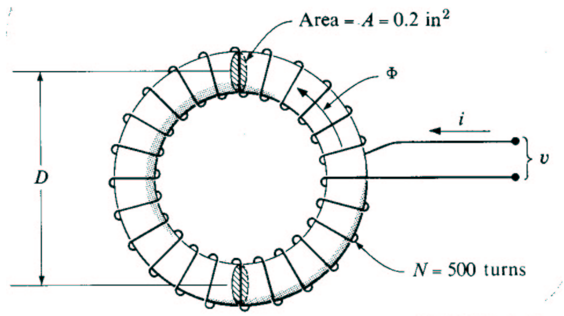



FIGURE 2–61

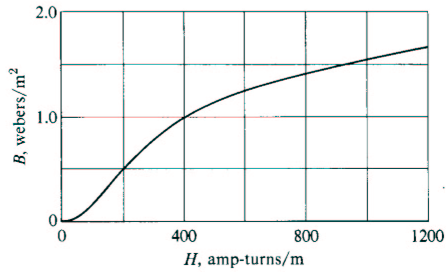
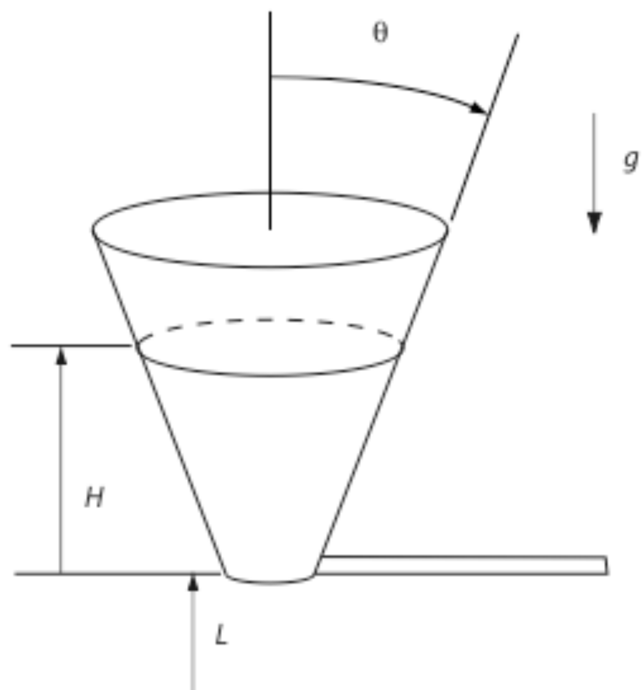



FIGURE 2–62

5. A conical tank is used to store water as shown below.




a. Verify that the volume of water stored in the tank, $V$, can be related to water height, $H$, as

$$V = \frac{\pi \tan^2 \theta}{3}\left(H^3 + 3H^2L + 3HL^2\right).$$

b. Obtain an expression for the potential co-energy $U_P$ as a function of $H$.
c. Use the Legendre relation $U_V = PV - U_P$ to calculate the energy $U_V$ stored in the tank as a function of $H$.
d. If $H = 1$ m, $\rho = 1000$ kg/m$^3$, $g = 9.8$ m/sec$^2$, $\theta = 15°$, and $L = .1$ m, how much energy is stored in this tank?

e. Assuming 100% efficiency, how long could you run a 100 W light bulb with this energy?