

CS180 Homework 1

Due: 4:00pm, 04/06/2016

1. Prove the correctness of the following algorithm of binary addition.

Input: n (in binary representation)
Output: $n + 1$ (in binary representation)
find the first digit that is 0 starting from right to left. Denote the position of first “0” to i .
flip the i^{th} digit to 1 and flip the $1, 2, \dots, (i - 1)^{th}$ digit to 0
return the flipped number n'

2. A binary tree is a tree that every node has at most 2 children. The depth of a tree is the longest path that starts from the root and ends at a leaf. Give a recursive and iterative algorithm that compute the depth of a binary tree. Justify the correctness and the complexity of your algorithms.
3. You are given integers n and d in decimal representation. Describe a recursive and iterative version of elementary-school long division to divide n by d . Your primitives are addition, subtraction, multiplication of integers a and b , and integer division of a by b if $a < 10b$. Analyze the complexity (number of operations) of your algorithm. An operation is using any of the primitives between a and b .
4. Suppose you are playing the game of NIM. This game begins with a placement of n rows of matches on a table. Each row i has m_i matches. Players take turns selecting a row of matches and removing any or all of the matches in the row. Whoever claims the final match from the table wins the game. This game has a winning strategy based on writing the count for each row in binary and lining up the binary numbers (by place value) in columns. We note that a table is *favorable* if there is a column with an odd number of ones in it, and the table is *unfavorable* if all columns have an even number of ones.

Example: Suppose we start off with three rows of matches of 7, 8 and 9. The binary representations of number of matches are $7 = 0111, 8 = 0111, 9 = 1001$. Therefore, the board will look like this

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Since the third row from the right and the second row from the right, each have odd numbers of ones, the table is favorable.

- (a) Prove that, for any favorable table, there exists a move that makes the table unfavorable. Prove also that, for any unfavorable table, any move makes the table favorable for one's opponent. Write the algorithm that on an input of favorable table outputs the row and the number of matches to remove from that row, to make the table unfavorable.
- (b) Give a winning strategy for this game.

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- ★ Homework assignments are **strictly** due on the exact time indicated. Please submit a hard copy of your homework solution with your **Name, Bruin ID, Discussion Number**, clearly indicated on the first page. If your homework consists of multiple pages, please staple them together. Email attachments or other electronic delivery methods are not acceptable.
 - ★ We recommend to use \LaTeX , \LyX or other word processing software for submitting the homework. This is not a requirement but it helps us to grade the homework and give feedback. For grading, we will take into account both the correctness and the clarity. Your answer are supposed to be in a simple and understandable manner. Sloppy answers are expected to receive fewer points. Unless specified, you should justify your algorithm with complete proof of correctness and complexity.