Homework 1

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1 Binary addition algorithm correctness proof

The input number n can be denoted as $n = a_k ... a_0$ in binary, where $a_i = 0$, or $a_i = 1$ ($0 \leq i \leq k$, k is the most significant bit). The flipped number n' can be denoted as $n' = a'_k ... a'_0$ in binary. We have

$$n = \sum_{j=0}^{k} a_j \cdot 2^j \qquad \text{and} \qquad n' = \sum_{j=0}^{k} a'_j \cdot 2^j$$

Denote the position of first 0 in n from right to left to i, we have

$$n = \sum_{j=0}^{i-1} 1 \cdot 2^j + 0 \cdot 2^i + \sum_{j=i+1}^{k} a_j \cdot 2^j = \frac{2^i - 1}{2 - 1} + \sum_{j=i+1}^{k} a_j \cdot 2^j = 2^i - 1 + \sum_{j=i+1}^{k} a_j \cdot 2^j$$

and

$$n + 1 = 2^i + \sum_{j=i+1}^{k} a_j \cdot 2^j$$

After the flip in question, resulting number n' can be denoted as

$$n' = \sum_{j=0}^{k} a'_j \cdot 2^j = \sum_{j=0}^{i-1} 0 \cdot 2^j + 1 \cdot 2^i + \sum_{j=i+1}^{k} a'_j \cdot 2^j = 2^i + \sum_{j=i+1}^{k} a_j \cdot 2^j$$

Thus we have $n' = n + 1$, and the binary addition algorithm in question is correct.

2 Binary tree depth algorithm

Algorithm 1 Binary tree depth recursive

function MAXDEPTH(node)
    if $nodeisnil$ **then return** 0
    $leftDepth \leftarrow maxDepth(node.leftChild)$.
    $rightDepth \leftarrow maxDepth(node.rightChild)$.
    if $leftDepth > rightDepth$ **then return** $leftDepth + 1$
    else return $rightDepth + 1$

O(n) with the number of nodes in the tree, as each node will be visited exactly once.

3 Elementary-school-division algorithm

4 NIM game

(a)

(b)