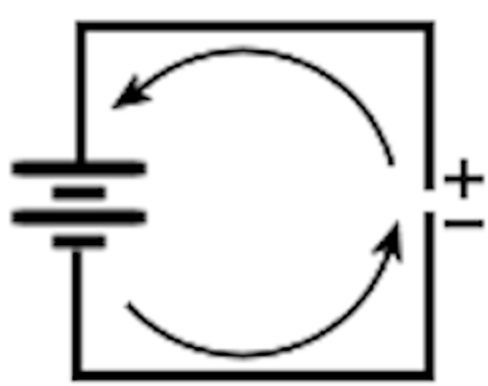
Capacitance

If you were to connect a wire across a DC power source, current would flow, with a flood of electrons rushing from the negative terminal of the power source, through the wire, to the positive terminal.

If you were then to sever the wire at some point, the current flow would cease because the circuit had been opened. If you were to measure the voltage at the cut ends of the wire, you would find that it was equal to that of the DC power source, because the applied voltage will always appear in full across an open circuit.

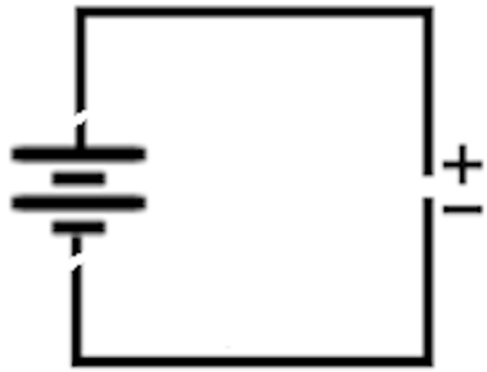


An electric field develops across the open, as electrons from the negative terminal of the power source are attracted to the severed end of that wire by the positive potential felt from the end of the wire connected to the positive side. That positive potential develops as electrons from that side are drawn to the power source.

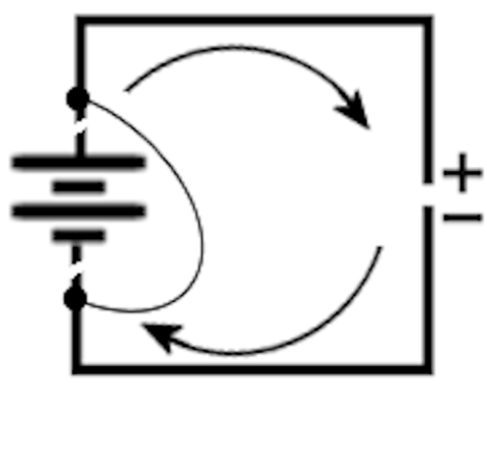
This ultimately results in a state of equilibrium, with an accumulation of negative charges on one side of the gap, and positive *ions* on the other side — “ion” referring to atoms with an imbalance of electrons and protons.

The field potential across the gap will then be equal and opposite to the electromotive potential of the power source.

If you were then to disconnect the wires from the power source and measure the voltage between those original ends of the wire, you would be measuring the potential of the residual electric field, which would be equal to the voltage of the power source. At this point, you would say that the severed wire had been *charged*.



If you were then to short those ends of the wire together, there would be a momentary rush of negative charges to the opposite side of the gap, neutralizing the positive ions on that side and restoring the electrically neutral condition of the wire.



This phenomenon is known as *capacitance* — the property of being able to collect a charge of electricity.

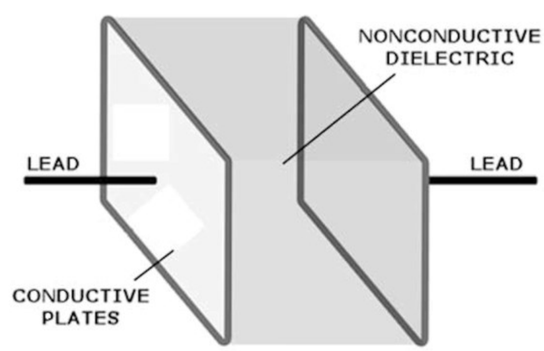
If the power applied to the circuit is changed to a signal of alternating polarity, the same process would occur, but in an alternating sequence. Current would flow one way, and then the other, the charge alternating in polarity accordingly. From a current point of view, this appears as if the circuit is actually continuous, rather than open. It therefore *appears* as if capacitance blocks direct current, but not alternating current.

Most electrical components display capacitance to some degree; even the spaces between components of a circuit have a natural capacitance. In the trade, this is usually called *stray capacitance*, and is sometimes a nuisance that can interfere with the proper operation of a circuit if not effectively dealt with.

What we are more interested in at this point is the intentional use of capacitance in order to provide a variety of very useful circuit functions. We do this by creating components that provide closely controlled values of capacitance, which are called *capacitors*.

Capacitors

Capacitors are basically nothing more than an expanded version of the open ends of the wire that we just talked about.



In the case of the severed wire, assuming a very small wire size, the cross-section of the cut ends is very small and the amount of capacitance would be miniscule. That fact is clearly evident from a basic formula for capacitance …

$$C = 0.2248(\frac{AK}{d})$$

… where:

- C = capacitance in picofarads
- A = the area in square inches of the *plates* (which in that case would be the faces of the cut ends of the wire)
- K = the nature of the insulating material separating the plates, called the *dielectric*
- d = the distance in inches between the plates

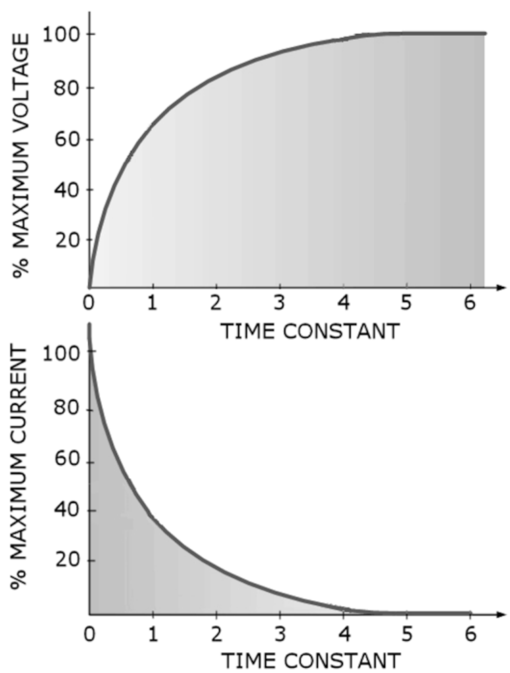
Capacitance is measured in *farads*, which are represented by the letter *F*. Continuing with our biographical allusions; this honors the renowned English experimenter and scientist Michael Faraday.

Practical values of capacitance are usually in the millionths or trillionths of farads, otherwise referred to as *microfarads* (μF) and *picofarads* (pF) …

$$.000001F = 1.0\mu F$$
$$.000000000001F = 1.0pF$$

Charge/Discharge Time Constant

When a DC voltage is connected to a capacitor, the current at the first instant will depend only on the resistance present in the circuit. And there will always be some resistance, even if only that of the wiring, capacitor leads, and its plates. As the capacitor charges, the current diminishes to zero at an exponential rate.



Meanwhile, the voltage rises in a complimentary manner — meaning exponentially at a rate equal and opposite to that of the current — until the full applied potential appears across the fully-charged capacitor.

When the DC voltage is removed and the source connections are shorted together, the capacitor's discharge characteristic is the same — current leading voltage.

The total time required for charge and discharge is the same, and is determined by the values of resistance and capacitance. The total amount of current involved, in coulombs (think 'jillions of electrons') is determined by the size of the capacitor.

Meanwhile, the rate at which that current can be supplied is determined by the amount of resistance in the circuit.

The charge/discharge characteristic is absolutely consistent, regardless of the values involved. The shape of the charge and discharge curves is always as shown above.

During one *time constant*, the capacitor will charge exactly 63% of the difference between the applied voltage and whatever charge it held at the beginning of that period.

For example, beginning at zero with 10v applied, at the end of the first time constant, the capacitor will have charged up to 6.3v.

Since the second time constant begins with 6.3v already on the capacitor, the difference is 3.7v. 63% of that is 2.33v, so by the end of the second time constant, the capacitor will have charged up to 8.63v.

Continuing with this exercise, the voltage on the capacitor at the end of the third, fourth, and fifth periods will be 9.49v, 9.81v and 9.93v, respectively. After five time constants the increases in charge are obviously infinitesimal, so the generally accepted convention is that the total charge time is five time constants.

The same situation occurs during discharge; the capacitor loosing 63% of its charge during the first time constant, and so on.

The time constant in seconds for any such circuit is determined by the resistance in ohms multiplied by the capacitance in farads …

$$\tau = RC$$

For example, a 1.0MΩ resistor in series with a 1.0μF capacitor provides a time constant of 1.0Sec. Regardless of the voltage applied, the capacitor will charge in five seconds, and discharge in five seconds.

Impedance

Since the current in a circuit containing capacitance is at any given moment not strictly a function of the applied voltage and the resistance in the circuit, there must be some other factor involved.

This situation is similar to that already discussed in connection with inductive circuits: it is called *impedance*. Recall that the unit of impedance is the ohm, and that an impedance of one ohm limits the flow of alternating current to one ampere when the applied signal potential is one volt.

In the case of capacitors, impedance is also referred to as *capacitive reactance*, the symbol for which is X_c.

If the AC current and voltage are known …

$$X_C = \frac{E}{I}$$

Otherwise, the capacitive reactance, in ohms, can be calculated using …

$$X_C = \frac{1}{2\pi f C}$$

… where f is the frequency of the alternating current in Hz, C is the capacitance in farads, and π, of course, is equal to 3.1416.

Since the reciprocal of 2π is 0.159, this is often simplified to …

$$X_C = \frac{0.159}{fC}$$

From this formula you can see that the impedance of a capacitor is inversely proportional to its capacitance, and the frequency of the signal applied to it.

Capacitors in Series and Parallel

The effect of connecting multiple capacitors in series or parallel is just the opposite that of resistors.

When two capacitors are connected in series, the total capacitance will be less than that of the smallest one. This is easy to understand since the situation is the same as if the distance between the plates in a single capacitor was drastically increased. The mathematical solution for capacitors connected in series is the same as that used for resistors in parallel …

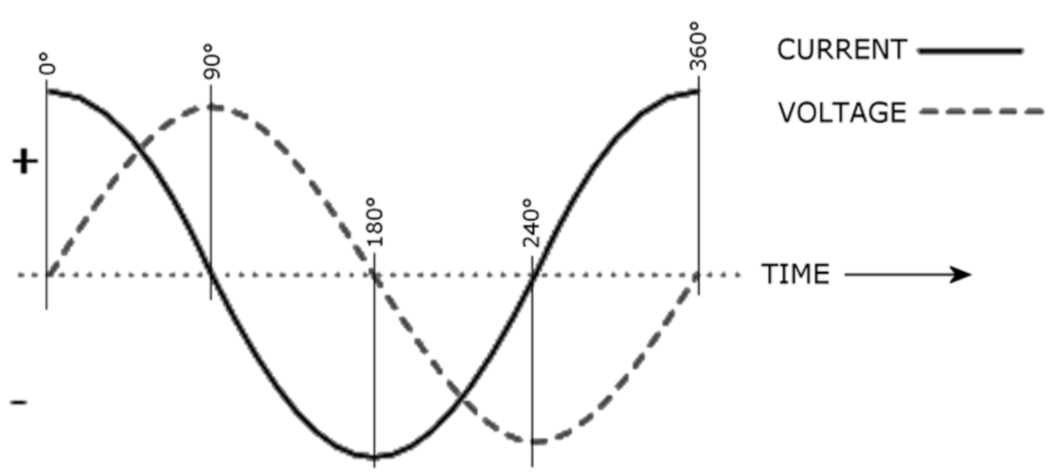
$$C_t = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + etc}$$

Capacitors connected in parallel simply add. This can be understood as being equivalent to increasing the size of the plates in a single capacitor …

$$C_t = C_1 + C_2 + C_3 + etc$$

Capacitive Phase Shift

As a result of a capacitor's nonlinear response to changes in voltage, in a capacitive circuit with an alternating current, the voltage across the capacitor lags the current by 90-degrees.



Thus, when connected in series with some sort of linear resistance, the signal across a capacitor will be 90° out of phase with the voltage drop across the resistance.

In this case, the current obviously leads the voltage build-up across the capacitor — current leads voltage. You will recall that the situation is just the reverse for inductors, which oppose changes in current — voltage leading current.

Way back near the middle of the last century, when I went through tech school, we were taught this memory crutch …

ELI – ICE
"Eli the Iceman"

… *E* leads *I* in a *"L"* (inductor), and *I* leads *E* in a *"C"* (capacitor).

So much for theory. Let's have a little fun.

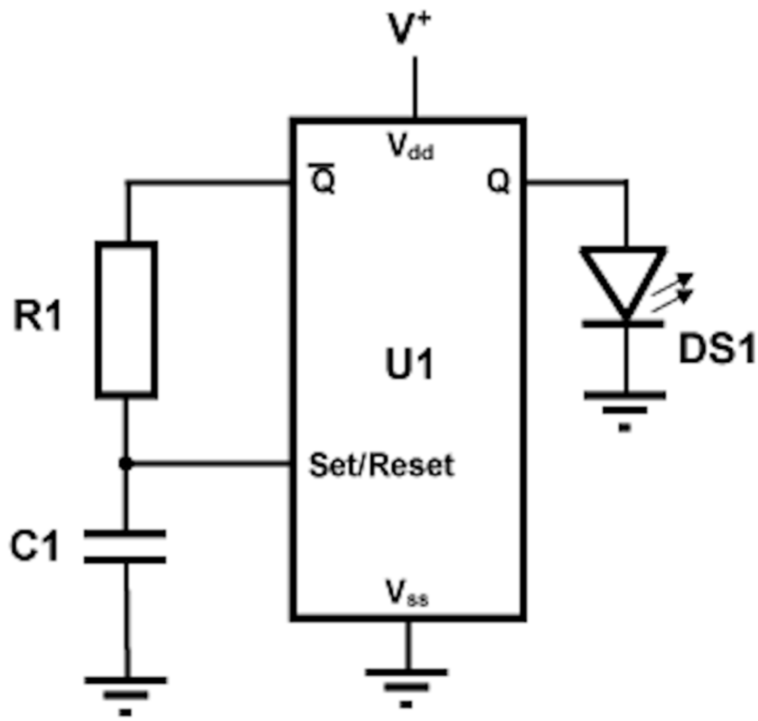
A Little Fun

Suppose that you're out on your yacht, in the middle of Spring Lake, on a dark, cloudy night. It's late, way past bedtime for most of your rich old neighbors, so most of the lights along the shoreline have long since gone dark. Oh, if only there was a beacon on your dock, to guide you home!

You decide to drop anchor and just wait for the breaking dawn, and meanwhile you set to work to create that little personal navigation aid — nothing fancy or expensive; just a simple little flashing light at the end of your dock; one second on, and one second off.

Thanks to this course … no problemo! Here's your bright idea …

DS_1 is an array of super-bright LED's taken out of a cheap little flashlight.



U_1 is a magical little “black box” sort of thing that will do the switching. It has two outputs; Q and $\overline{Q}$, which operate in a complimentary way — meaning that when Q goes high, $\overline{Q}$ will go low, and vice versa. This little chip will flash the LEDs on and off, as Q alternately switches between high and low.

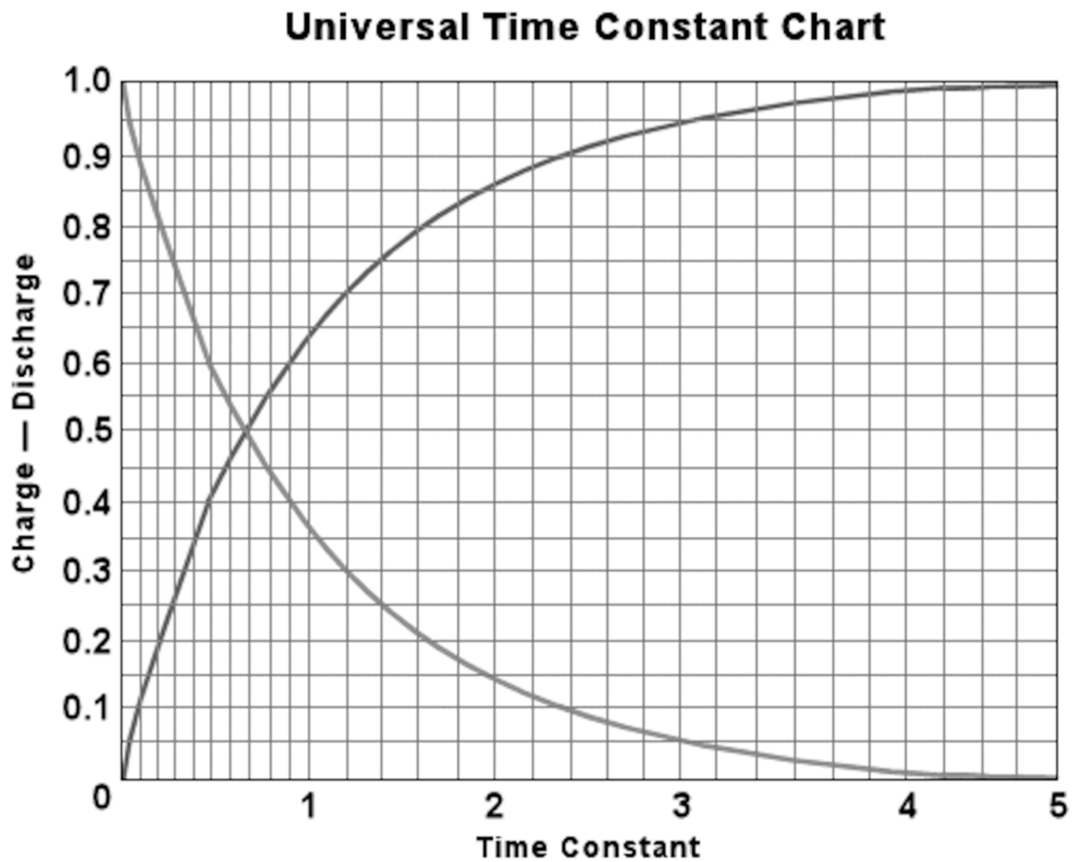
While Q is doing that, $\overline{Q}$ is doing just the opposite, going high when Q goes low to switch the LED’s off.

The switching is controlled by the *Set/Reset* input, which senses the charge on the capacitor C_1. When $\bar{Q}$ is high, C_1 will charge through the resistor R_1. When the charge on C_1 reaches maximum … the level of $V+$, in other words … Q will go high, switching DS_1 on.

At that same instant, $\bar{Q}$ will go low. The capacitor now begins to discharge through R_1. When the charge on the capacitor reaches zero, the outputs change states again.

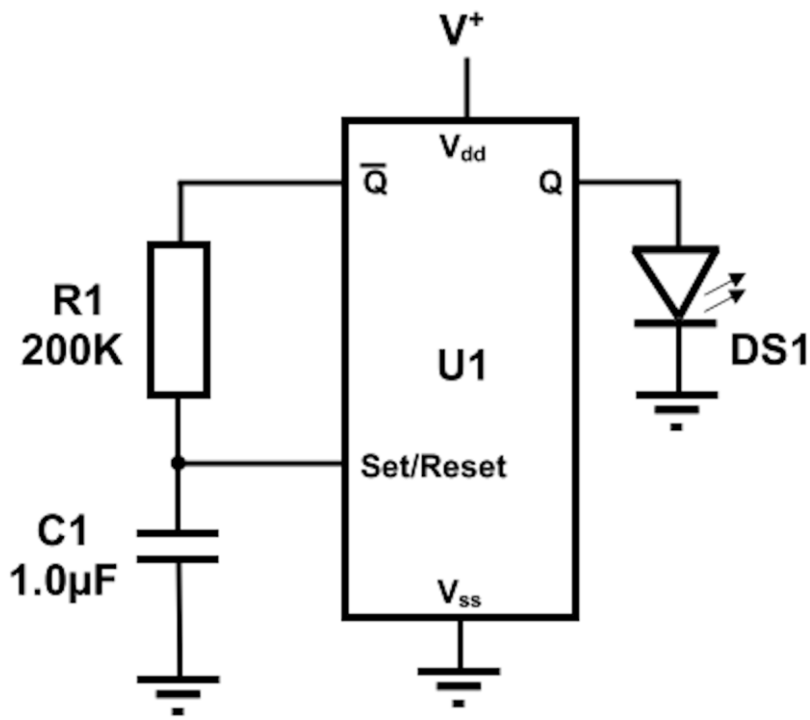
Your problem is to figure out what values to use for R_1 and C_1, such that DS_1 will be on for one second, and off for one second.

When figuring out RC timing problems, you'll usually find this chart helpful; the *Universal Time Constant Chart*. You probably won't need it this time, but it's provided here for future reference.



Have you come up with a solution?

Here's the answer: since the capacitor must charge to 100% of the applied voltage, and also discharge 100%, the charge and discharge cycles will take a full five time constants during each one-second interval.



Therefore, any combination of resistance and capacitance that has a time constant of 0.2-seconds will do. For example, 1.0µF and 200KΩ, or 0.2µF and 1.0MΩ, and so on.

In the next section, we'll talk about the nature of practical capacitors in typical applications.
