Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 2 (2010) 3237–3242

Procedia Engineering

www.elsevier.com/locate/procedia

8th Conference of the International Sports Engineering Association (ISEA)

Golf Ball Landing, Bounce and Roll on Turf

Woo-Jin Roh[a], Chong-Won Lee[b]*

[a, b]*Center for Noise and Vibration Control (NoViC)*
Department of Mechanical Engineering, KAIST, Daejeon, Korea

Received 31 January 2010; revised 7 March 2010; accepted 21 March 2010

Abstract

The aim of this work is to analyze the landing effect of a golf ball based on physics. The effect of the launch speed, impact angle, backspin, and green firmness on the run for a variety of golf shots is considered. We analyze green dynamics using stimpmeter and Werner and Greig's empirical equations. Reasonable empirical equations are suggested to satisfy mechanical principles. The resistance moment is also calculated to find rolling resistance of a golf ball on the green. Then, the rolling friction coefficient is deduced using previous results. Finally, the criteria for the various run conditions such as the backward bounce and the forward bounces before backward roll are discussed in detail based on the analysis.

Keywords: the run; landing; bounce; roll; green; backspin; stimpeter

1. Introduction

The game of golf has been around for over five hundred years. It can be assumed that even during the very early stages of the game, players, especially those with a scientific inclination, were curious as to the behaviour of the ball and the clubs, and experimented on ways to improve their performance. Over the years, changes to the equipment have been based more on trial and error than on any application of scientific principles. It has only been over the last several decades that science, physics in particular, has been used in a significant way to understand and improve the performance of golfers and their equipment. Therefore, the final goal of this scientific research is also to improve the performance of golfers.

One of the most impressive shots in the game of golf is when a golf ball lands on the green and the ball, after initially bouncing forward, rolls back towards the pin. Unfortunately, for most of us, this shot seems limited to the abilities of a skilled golfer. This paper will consider the physics behind this shot as well as others.

First, we consider green dynamics using the stimpmeter [1] and Werner and Greig's [2] empirical formulas. New equation of coefficient of kinetic friction is suggested to satisfy scientific principles. The resistance moment is

* Corresponding author. Tel.: +82-42-350-3016; fax:+82-42-350-8220.
E-mail address: cwlee@kaist.ac.kr

doi:10.1016/j.proeng.2010.04.138

calculated to find rolling resistance of a golf ball on the green. Then, the rolling friction coefficient is deduced using previous results.

Next, the run of a golf ball landing on fairway and green is modelled. Penner [4] considered the types of runs that would be expected for golf shots using various clubs. The effect of launch velocity, impact angle, backspin, and green firmness on the run for a variety of golf shots is considered. It is found that for high-lofted iron shots, where the golf ball is given sufficient backspin, the ball may, for firm enough greens, initially bounce forward before running backwards. It is also found that the bounce form of forward or backward is determined by the landing backspin and the coefficient of kinetic friction.

2. Green dynamics

2.1. Stimpmeter

The stimpmeter is a 36 inches long extruded aluminium bar with a V-shaped groove extending along its entire length [1]. It has a precisely milled ball-release notch 30" away from the tapered end (the end that rests on the ground). The underside of the tapered end is milled away to reduce bounce as a rolling ball makes contact with the green. The V-shaped groove has an included angle of 145˚, thereby supporting a golf ball at two points 0.5" apart. A ball rolling down the groove has a slight overspin, which is thoroughly consistent and has no deleterious effect on the ensuing measurements. The ball-release notch is designed so that a ball will always be released and start to roll when the stimpmeter is raised to an angle of approximately 20˚. This feature ensures that the velocity of the ball will always be the same when it reaches the tapered end. The end result from the ball accelerating down the stimpmeter ramp is a speed of 1.88 *m/s*. The distance in ft that the ball travels is the stimp reading, S, i.e. 10.5ft roll means a stimp reading of 10.5, or $S = 10.5$.

2.2. Indentation of the ball at rest on the green

Once the ball starts the roll motion without slip on the green with the initial velocity, the ball travels a certain distance without slip and then stops. In order to stop the roll of ball, we should introduce a resistance moment between the ball and the green surface. For that purpose, we first investigate the deformation of the green surface due to the ball weight, i.e. indentation of the ball at rest on the green.

Empirical pressure-deflection relation for grass on the typical greens is suggested by Werner and Greig [2], which is given as

$$p_{wg}(\delta, S) = (-5.43 + 0.776S)\delta + (-6960 + 1144S)\delta^{(5.17-0.176S)} \quad [psi] \tag{1}$$

where p_{wg} is the pressure in psi on the grass and δ is the grass deflection in inch. The above empirical formula was derived from the measurement tests for the green with S=7.8 to 9.4 and $p_{wg}<5$ *psi*. On the other hand, the maximum grass deflection due to the ball weight provided by Werner and Greig [2] is

$$\delta_{\max} = 27.2S^{-2.54} \quad [in] = 691S^{-2.54} \quad [mm] \tag{2}$$

The empirical pressure-deflection relation (1) was derived from the measurement of grass deflection when a flat plate was loaded on the top of grass, so that the grass was loaded in the vertical direction only. However, the actual indentation of ball on the green is not flat but concaved so that the grass is subject to the horizontal as well as vertical load. To improve the noticeable discrepancy between the calculated and empirical results on the maximum indentation of ball at rest, we need to introduce a compensation factor for the empirical pressure-deflection relation of the turf, i.e.

$$p(\delta, S) = \sigma(S) p_{wg}(\delta, S)\ [psi] \tag{3}$$

where $\sigma(s)$ is the compensation factor that is the function of the stimp reading only. Note that the compensation factor reflects the bending as well as compression stiffness of the turf.

To check the compatibility of the above two empirical formulas, we compare the maximum deflection calculated using the compensated pressure-deflection relation (3) and the empirical relation (2). Referring to Fig. 1, we obtain the force balance relation, for computation of the maximum indentation of golf ball on the green, given as

$$\int_0^{\sin^{-1}\frac{a}{R}} p(\delta = R\cos\theta - \sqrt{R^2 - a^2}, S) 2\pi (R\sin\theta)(Rd\theta\cos\theta) = W = mg = 0.045 \times 9.8\ N \tag{4}$$

where θ is the angle coordinate shown in Fig.1. The numerical solution for the value of a, the radius of indentation, requires the iteration procedure. Fig. 2 compares the calculated maximum deflection from Eq.(4) and the corresponding empirical formula (2).

2.3. Rolling resistance of ball on the green

Now, using the compensated grass pressure-deflection relation (3), we can calculate the resistance of the turf to the ball, as the ball rolls on the turf. Using the $\delta_{\max} = R - \sqrt{R^2 - a^2}$ relation in Fig. 1, we first derive the relation for the contact angle as

$$\theta_a = \sin^{-1}\frac{a}{R} = \sin^{-1}\sqrt{1 - \left(1 - \frac{\delta_{\max}}{R}\right)^2} \tag{5}$$

where a is the indentation radius and δ_{max} is the maximum indentation of ball on the grass.

The resisting moment to the roll of ball is due to the forward compression of green by the ball. The resultant resisting moment can be derived as

$$M_g = 2\int_0^{\frac{\pi}{2}}\int_0^{\theta_a} p(\delta)(R\sin\theta\cos\phi)(Rd\theta\cos\theta)(Rd\phi\sin\theta) = 2R^3\int_0^{\theta_a} p(\delta)\sin^2\theta\cos\theta d\theta \int_0^{\frac{\pi}{2}}(\cos\phi)d\phi = 2R^3\int_0^{\theta_a} p(\delta)\sin^2\theta\cos\theta d\theta \tag{6a}$$

$$\delta = R\cos\theta - \sqrt{R^2 - a^2} \tag{6b}$$

Now we attempt to calculate the forward rolling resistance moment. The method is to compensate the Werner and Greig's empirical pressure-deflection relation so that the maximum indentation equation provided by Werner and Greig works out. This moment calculated based on the modified pressure-deflection relation is marked by solid curve in Fig. 3. The strange behaviour of the moment for S<7.5 is due to the inherent drawback with the Werner and Greig's equation, which is not physically sensible for below S=7. A dotted line in Fig. 3 is an approximate equation of the forward rolling resistance moment,

$$\hat{M}_g = \frac{0.006}{S}[N \cdot m] \tag{7}$$

The above equation fits well with Werner and Greig's empirical pressure-deflection relation from 8 to 11.

Let us assume that the rearward recovery does not replace the whole forward compression work. It can be explained by energy dissipation due to the elastic hysteretic loss occurring in the complex straining of the material, which must occur during the rolling process. Then, the equations of motion on the green using the loss coefficient can be expressed as

$$J\ddot{\theta} = -\varepsilon M_g + FR', \quad m\ddot{x} = -F \ \Rightarrow\ m\ddot{x} = -F = -\frac{J\ddot{\theta}}{R'} - \frac{\varepsilon M_g}{R'} = -\frac{J}{RR'}\ddot{x} - \frac{\varepsilon M_g}{R'} \ \Rightarrow\ m\left(1 + \frac{2}{5r'}\right)\ddot{x} = -\frac{\varepsilon M_g}{R'} \tag{8}$$

where $0<\varepsilon\leq 1$ is the loss coefficient, F is the friction force, R' is the distance of the friction force from ball center, and $r' = \frac{R'}{R} < 1$. The above equation can be rewritten as

$$\ddot{x} = -\frac{\varepsilon M_g}{m\left(1 + \frac{2}{5r'}\right)R'},\ \dot{x} = v_{x0} - \frac{\varepsilon M_g}{m\left(1 + \frac{2}{5r'}\right)R'}t,\ x = v_{x0}t - \frac{\varepsilon M_g}{2m\left(1 + \frac{2}{5r'}\right)R'}t^2 \tag{9}$$

Thus, the time for the ball to stop and the corresponding ball travel distance become

$$t_f = \frac{m\left(1+\frac{2}{5r'}\right)R' v_{x0}}{\varepsilon M_g}, \quad x_f = v_{x0}t_f - \frac{\varepsilon M_g}{2m\left(1+\frac{2}{5r'}\right)R'}t_f^2 = v_{x0}^2 \frac{m\left(1+\frac{2}{5r'}\right)R'}{2\varepsilon M_g} \tag{10}$$

From substitution of $v_{x0}=v_{2x}=1.83m/s$, $x_f=S$, and $\hat{M}_g = \frac{0.006}{S}$ into equation (10), we obtain

$$x_f = S\,[ft] = 0.305\,S[m] = r'\left(1+\frac{2}{5r'}\right)\frac{v_{x0}^2 mR}{2\varepsilon \hat{M}_g} \tag{11a}$$

$$\varepsilon = \left(r'+0.4\right)\times 0.26 v_{x0}^2 \cong (0.9)\left(r'+0.4\right) \tag{11b}$$

From the condition that ε cannot be greater than 1, we obtain $r' \leq 0.7$. However, r' ranges approximately from 0.8 (for S=7) to 0.9 (for S=12), which also depends upon the stimp reading S. Note that the loss coefficient slightly increases as S increases, which is physically sensible. Since the exact value of r' is difficult to find, we assume that $\varepsilon \approx 1$ as a reasonable approximation, implying that the rearward recovery moment of grass is negligibly small compared with the forward compression moment. For the loss factor to be independent of S, the forward compression resistance moment should be inversely proportional to S.

The friction force reduce to

$$\frac{F}{mg} = -\frac{\ddot{x}}{g} = \frac{\hat{M}_g}{mg\left(r'+0.4\right)R} = \frac{0.006}{(0.045)(9.8)\left(r'+0.4\right)(0.0215)S} \quad \frac{0.48}{S[ft]} \quad \begin{cases} 0.07 \text{ for } S=7 \\ 0.04 \text{ for } S=12 \end{cases}. \tag{12}$$

As stated previously, the following equation for the acceleration of the golf ball on the green obtained from Penner [3],

$$\ddot{x} = -\frac{5}{7}\mu_r g \tag{13}$$

where μ_r is the rolling friction coefficient. From Eqs. (12) and (13), the rolling friction coefficient reduces to

$$\mu_r = \frac{7}{5}\frac{0.48}{S[ft]} \quad \begin{cases} 0.096 \text{ for } S=7 \\ 0.056 \text{ for } S=12 \end{cases} \tag{14}$$

implying that the roll friction coefficient on the green varies from 0.05 to 0.1.

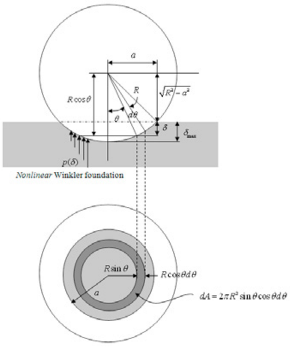

Fig. 1. Schematic diagram of maximum indentation of golf ball on the green

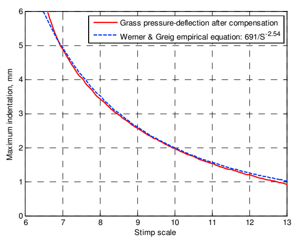

Fig. 2. Maximum indentation of golf ball at rest on the green

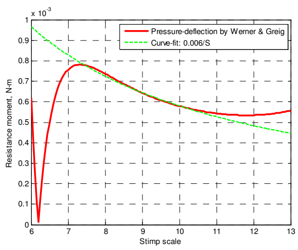

Fig. 3. Forward rolling resistance moment

3. The run of golf ball on the green[4]

The run of a golf ball landing on ground includes both the bounce and the roll. The ball landing angle and velocity, the orientation and magnitude of backspin and the firmness of the turf all are important to determine the bounce and roll distance. On impact, the golf ball starts penetrating and slipping across the turf. Fig. 4 shows the profile of the impact, with the golf ball impacting on the turf with a velocity of v_1 , an impact angle of θ_1 and a backspin of ω_1 and then rebounding with a velocity of v_2 , a bounce angle of θ_2 and a backspin of ω_2. The coordinates (x, y) indicate the horizontal and vertical directions, relative to the rigid ground. Following the Penner's run model [4], θc represents the average slope of the crater created by the ball impact on the turf, along which the resultant frictional force is assumed to act. If the frictional force $F(t)$ is great enough to put the ball into a state of pure rolling after the collision with the ground surface, the linear impulse-momentum and the torque impulse-angular momentum relations are given by

$$\int F(t)dt = m(v_{1x'} - v_{2x'}) = m\{v_1 \sin(\theta_1 - \theta_c) - v_{2x'}\},\ R\int F(t)dt = J(\omega_2 + \omega_1) = J\left(\frac{v_{2x'}}{R} + \omega_1\right) \tag{15}$$

Eliminating the unknown term, $\int F(t)dt$, from the above two equations, we obtain

$$v_{2x'} = \frac{5}{7}v_1 \sin(\theta_1 - \theta_c) - \frac{2R\omega_1}{7},\quad v_{2z'} = ev_1\cos(\theta_1 - \theta_c),\quad v_2 = \sqrt{{v_{2x'}}^2 + {v_{2z'}}^2},\quad \omega_2 = \frac{v_{2x'}}{R} = \frac{5v_1}{7R}\sin(\theta_1 - \theta_c) - \frac{2\omega_1}{7} \tag{16}$$

The coefficient of restitution, e, between a golf ball and turf and its dependence on impact speed has previously been measured by Penner [5]. It was found that the value of e decreased with increasing impact speed with the following function providing a good fit to the data,

$$\begin{aligned} &e = 0.510 - 0.0375v_{1z'} + 0.000903v_{1z'}^2 && v_{1z'} \le 20m/s \\ &e = 0.120 && v_{1z'} > 20m/s \end{aligned} \tag{17}$$

In general, it would be expected that the value of θ_c would increase with both the impact speed and the impact angle of the golf ball. Penner [4] assumed that the value of θ_c increases linearly with both the impact speed and the impact angle, i.e.

$$\theta_c = (15.4^o)\frac{v_1}{(18.6m/s)}\frac{\theta_1}{(44.4^o)} \tag{18}$$

The rebound angle of ball after collision with the ground can be expressed as

$$\tan(\theta_2 + \theta_c) = \frac{v_{2x'}}{v_{2z'}} = \frac{5}{7e}\left(\tan(\theta_1 - \theta_c) - \frac{2R\omega_1}{5v_1\cos(\theta_1 - \theta_c)}\right) \tag{19}$$

From the condition of backward bounce after impact, $\theta_2 < 0$,

$$\tan(\theta_2 + \theta_c) < \tan\theta_c \tag{20}$$

Thus, the minimum backspin for backward bounce is given by

$$\omega_1 > \left(\frac{v_1}{2R}\right)\{5\sin(\theta_1 - \theta_c) - 7e\tan\theta_c\cos(\theta_1 - \theta_c)\} \tag{21}$$

The above equation means that backward bounce is generated when the ball has a small impact angle, which is close to the vertical axis or a large backspin rate compared with the landing velocity.

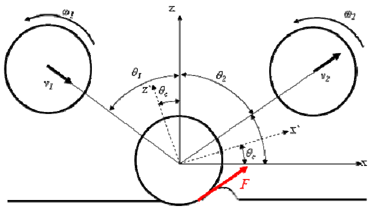

Fig. 4. The impact between a golf ball and compliant turf

As stated previously, μ_k of 0.4 means the fairway or rough and the values of the green are usually 0.2-0.25. If $\mu_c > \mu_k$ on the green, the golf balls will slip of the first bounce. Then, the quite complex phenomenon is caused about forward bounce or backward bounce. Fig. 5 expresses the bounce form due to the coefficient of kinetic friction and landing backspin when the impact velocity is 40 *m/s* with the loft angle of 46°. As is seen, the critical point to conclude forward or backward bounce is 7040 *rpm* when μ_k is larger than 0.232 of the fairway or the slow green. If μ_k is smaller than 0.232 of the slippery green, the bounce form is changed sensitively by the landing conditions. The point A, which is the landing backspin of 7200 *rpm* and the value of μ_k of 0.22, means the only forward bounce. Similarly, the point C, which is the landing backspin of 7800 *rpm* and the value of μ_k of 0.25, means the only backward bounce. On the other hand, on the point B, which is the landing backspin of 7800 *rpm* and the value of μ_k of 0.22, means that the first bounce is forward and second bounce is backward. This phenomenon is one of the most impressive shots in the game of golf. Similarly, on the point D, which is the landing backspin of 7800 *rpm* and the value of μ_k of 0.185, means that the first and second bounces are forward and third bounce is backward. These trajectories of the runs are shown in Fig. 6.

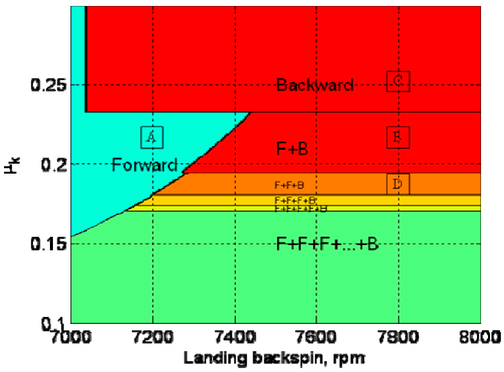

Fig. 5. The detailed bounce form due to landing backspin and the coefficient of kinetic friction

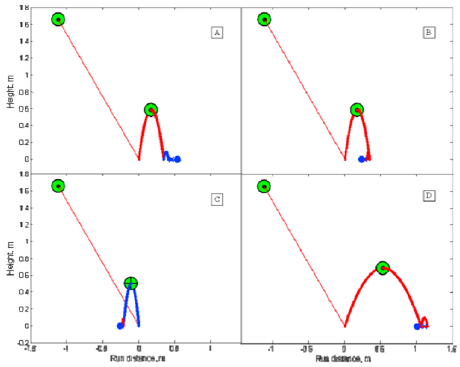

Fig. 6. The change of the bounce form by the landing conditions

4. Conclusion

This paper reviews and develops green dynamics and the run of a golf ball. We analyze green dynamics using the stimpmeter and Werner and Greig's empirical formulas. Reasonable empirical formulas are suggested to satisfy mechanical principles. The resistance moment is calculated to find rolling resistance of a golf ball on the green. Then, the rolling friction coefficient is deduced using previous results.

Treating the impact of a golf ball with compliant turf as being equivalent to an impact with a sloped rigid surface allowed for the analysis, as given by Daish, to be used. Results for the run are obtained from the model, which agree well with the actual behaviour of a golf ball during a game. For example, the modelled dependence of the run of a golf ball, in the case of drives, on the launch speed of the golf ball was found to agree, in general, with empirical results. It was also found that increasing the amount of backspin, for the higher lofted iron shots, increased the amount the ball runs backwards. In addition, it was found that for firmer greens, or smaller values of μ_k, golf balls with large amounts of backspin will initially bounce forwards before running backwards. Finally, it is found that the bounce form of forward or backward is caused by landing backspin and the coefficient of kinetic friction.

To further improve the rolling resistance model that has been presented, additional studies about the loss coefficient of ε<1 would be required. This would allow for a more accurate determination of the resistance moment of M_g and the maximum indentation of δ_{max} as well as the rolling coefficient of μ_r.

References

[1] USGA, *Stimpmeter Instruction Booklet*; 2004.
[2] F. D. Werner and R. C. Greig, *How Golf Clubs Really Work and How to Optimize Their Design*; 2000.
[3] A.R. Penner. The physics of putting. *Can. J. Phys* 2002; **80**; 83-96.
[4] A.R. Penner. The run of a golf ball. *Can. J. Phys* 2002; **80**; 931-940.
[5] A. R. Penner. The physics of golf: The optimum loft of a driver. *Am. J. Phys* 2001; **69**(5); 563-568.