Tensor network hands on

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We investigate the quantum Ising model in a transverse magnetic field described by the Hamiltonian

$$\hat{H} = -J \sum_{\langle i,j \rangle} \hat{\sigma}_i^z \hat{\sigma}_j^z - g \sum_i \hat{\sigma}_i^x, \tag{0.1}$$

where $J = 1$ is the coupling constant between neighboring spins i and j, and g is the transverse magnetic field pointing in x direction. σ_i^z and σ_i^x are Pauli matrices.

1 One dimension

The ising model has a quantum phase transition in one dimension as a function of the transverse field. Using the example code `ising1d_itebd.py`, you can carry out an evolution of an initial wave function written as an iMPS in imaginary time, which amounts to calculating

$$\lim_{\beta \to \infty} \mathrm{e}^{-\beta \hat{H}} |\psi\rangle \propto |\psi_0\rangle, \tag{1.1}$$

where $|\psi_0\rangle$ is the groundstate of the Hamiltonian.

- Using the example code, calculate the groundstate energy $E_0 = \langle \psi_0 | \hat{H} | \psi_0 \rangle / \langle \psi_0 | \psi_0 \rangle$ of the Hamiltonian on an infinite one dimensional spin chain. You can compare to the exact result[2] which is given by

$$E_0^{\text{exact}} = \int_0^\pi \frac{\sqrt{1 + g^2 - 2g\cos k}}{\pi} \mathrm{d}k. \tag{1.2}$$

  Check the convergence of your result as a function of β.

- Check how the error of the ground state energy ($E_0 - E_0^{\text{exact}}$) depends on the bond dimension at $g = 0.1$ and at $g = 1.0$.
- Calculate the ground state magnetization per site along the z direction $\langle \psi_0 | \hat{\sigma}_z^i | \psi_0 \rangle$ as a function of the transverse field. Where is the critical point?
- Now switch on a small field $h_z = 0.01$ in z direction by adding the term $-h_z \sum_i \hat{\sigma}_i^z$ to the Hamlitonian. How does the groundstate magnetization $\langle \psi_0 | \hat{\sigma}_z^i | \psi_0 \rangle$ as a function of the field g in x direction?

2 Honeycomb lattice

In two dimensions, we represent the wave function as an iPEPS on the honeycomb lattice, which has a coordination number of $z = 3$. Therefore, all involved tensors will have 4 indices: one physical with dimension 2 and 3 virtual with dimension D. The example code `ising_honeycomp.py` performs imaginary time evolution of the iPEPS using the simplified update. You can control the bond dimension of the iPEPS using the parameter `-D`.

- Using the example code, calculate the groundstate energy $E_0 = \langle \psi_0 | \hat{H} | \psi_0 \rangle / \langle \psi_0 | \psi_0 \rangle$ of the Hamiltonian on an infinite honeycomb lattice for different bond dimensions $D = 2, 3, 4 \ldots$. Is the groundstate energy converged?
- Calculate the ground state magnetization along the z direction $\langle \psi_0 | \hat{\sigma}_z^i | \psi_0 \rangle$ as a function of the transverse field. Where is the critical point? Compare your finding to the Monte Carlo result from Ref. [1].

References

[1] H. W. J. Blöte and Y. Deng. Cluster monte carlo simulation of the transverse ising model. *Phys. Rev. E*, 66:066110, Dec 2002.

[2] P. Pfeuty. The one-dimensional ising model with a transverse field. *Annals of Physics*, 57(1):79 – 90, 1970.