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HKBK College of Engineering

Department of Engineering Mathematics

Second Internal Assessment Test

AY: 2021-22 | Subject: CALCULUS AND DIFFERENTIAL EQUATIONS | Code:21MAT11

Semester: I | Date:28/02/2022 | Time: 9:00am – 10:15 am | Max Marks:40

Modules Included for Test: 2 & 3

Answer any two full Questions selecting at least ONE question from each part. Each question carries 20 marks

Q.n	Mod #	CO#	BL	Questions	Marks
1a	2	2	2	Examine the function $f(x,y) = xy(1-x-y)$ for extreme values	7
1b	2	2	5	If $U = f(2x-3y,\ 3y-4z,\ 4z-2x)$, show that $6u_x + 4u_y + 3u_z = 0$	7
1c	2	2	5	If $x = r\sin\theta\cos\phi,\ y = r\sin\theta\sin\phi,\ z = r\cos\theta$ show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta$	6
				OR	
2a	2	2	2	If $x+y+z = u,\ y+z = uv$ and $z = uvw$, find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$	7
2b	2	2	2	If $U = Tan^{-1}(\frac{y}{x}),\ x = e^t - e^{-t};\ y = e^t + e^{-t}$. Find $\frac{du}{dt}$ by using partial derivatives.	7
2c	2	2	2	Find the extreme values of $x^3 + y^3 - 3axy, a \geq 0$	6
3a	3	3	2	A copper ball originally at 80° C cools down to 60° C in 20 minutes, if the temperature of the air being 40° C, what will be the temperature of the ball after 40 minutes from the original?	7
3b	3	3	5	Solve $\frac{dy}{dx} + y\,Tan\,x = y^3 Sec\,x$	7
3c	3	3	5	Solve $(1+e^{\frac{x}{y}})dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$	6
				OR	
4a	3	3	5	Solve $y(x+y)dx + (x+2y-1)\,dy = 0$	7
4b	3	3	5	Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$	7
4c	3	3	2	A body in air at 25 °C cools from 100 °C to 75 °C in one minute. Find the temperature of the body at the end of 3 minutes?	6

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HKBK COLLEGE OF ENGINEERING, BANGALORE - 560045

DEPT. OF ENGINEERING MATHEMATICS

2nd IA SCHEME COMMON TO ALL BRANCHES

SCHEME & SOLUTIONS

Subject Name: CALCULUS & DIFFERENTIAL EQUATIONS

Subject Code: 21MAT11

Q #	Solution	Marks Allocated
1)a)	$f(x,y) = xy(1-x-y)$ $\Rightarrow f(x,y) = xy - x^2y - xy^2$ $f_x = y - 2xy - y^2$, $f_y = x - x^2 - 2xy$ $f_x = 0 \Rightarrow y[1-2x-y] = 0 \Rightarrow y = 0$ & $2x+y=1$ $f_y = 0 \Rightarrow x[1-x-2y] = 0 \Rightarrow x = 0$ & $x+2y=1$ on pairing the equations $\left.\begin{matrix} y=0 \\ x=0\end{matrix}\right\}$ $\left.\begin{matrix} y=0 \\ x+2y=1\end{matrix}\right\}$ $\left.\begin{matrix} 2x+y=1 \\ x=0\end{matrix}\right\}$ $\left.\begin{matrix} 2x+y=1 \\ x+2y=1\end{matrix}\right\}$ The stationary points are $(0,0)$, $(1,0)$, $(0,1)$, $\left(\frac{1}{3}, \frac{1}{3}\right)$	2
	$A = -2y$, $B = f_{xy} = 1-2x-2y$, $C = -2x$	

points	$(0,0)$	$(1,0)$	$(0,1)$	$\left(\frac{1}{3},\frac{1}{3}\right)$
$A = -2y$	0	0	-2	$-2/3$
$B = 1-2x-2y$	1	-1	-1	$-1/3$
$C = -2x$	0	-2	0	$-2/3$
$AC - B^2$	-1	-1	-1	$\frac{1}{3}$
conclusion	Saddle pt	Saddle pt	Saddle pt	Max pt

(4)

Max. value of $f(x,y)$ is $f(1/3, 1/3)$ (1)

$f(1/3,1/3) = \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(1 - \frac{1}{3} - \frac{1}{3}\right) = \frac{1}{27}$

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$\frac{1}{7}$

Q#	Solution	Marks Allocated
1) b)	$u = f(2x-3y,\ 3y-4z,\ 4z-2x)$ $u \to (p, q, r) \to (x, y, z)$ $\frac{\partial u}{\partial x} = u_x = \frac{\partial u}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial u}{\partial q}\frac{\partial q}{\partial x} + \frac{\partial u}{\partial r}\frac{\partial r}{\partial x}$ $\frac{\partial u}{\partial y} = u_y = \frac{\partial u}{\partial p}\frac{\partial p}{\partial y} + \frac{\partial u}{\partial q}\frac{\partial q}{\partial y} + \frac{\partial u}{\partial r}\frac{\partial r}{\partial y}$ $\frac{\partial u}{\partial z} = u_z = \frac{\partial u}{\partial p}\frac{\partial p}{\partial z} + \frac{\partial u}{\partial q}\frac{\partial q}{\partial z} + \frac{\partial u}{\partial r}\frac{\partial r}{\partial z}$ $p = (2x-3y),\ q = 3y-4z,\ r = 4z-2x$ $u_x = \frac{\partial u}{\partial p}(2) + \frac{\partial u}{\partial q}(0) + \frac{\partial u}{\partial r}(-2)$ $6u_x = 12\frac{\partial u}{\partial p} - 12\frac{\partial u}{\partial r}$ $u_y = \frac{\partial u}{\partial p}(-3) + \frac{\partial u}{\partial q}(3) + \frac{\partial u}{\partial r}(0)$ $4u_y = -12\frac{\partial u}{\partial p} + 12\frac{\partial u}{\partial q}$ $u_z = \frac{\partial u}{\partial p}(0) + \frac{\partial u}{\partial q}(-4) + \frac{\partial u}{\partial r}(4)$ $3u_z = -12\frac{\partial u}{\partial q} + 12\frac{\partial u}{\partial r}$ $\therefore 6u_x + 4u_y + 3u_z = \cancel{12\frac{\partial u}{\partial p}} - \cancel{12\frac{\partial u}{\partial r}} - \cancel{12\frac{\partial u}{\partial p}} + \cancel{12\frac{\partial u}{\partial q}} - \cancel{12\frac{\partial u}{\partial q}} + \cancel{12\frac{\partial u}{\partial r}}$ $\therefore 6u_x + 4u_y + 3u_z = 0$	2 4 1
1) c)	$x = r\sin\theta\cos\phi;\ y = r\sin\theta\sin\phi$ $z = r\cos\theta.$	

Q#	Solution	Marks Allocated
	$$\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial \phi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial \phi} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial \phi} \end{vmatrix}$$ $$\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = \begin{vmatrix} \sin\theta\cos\phi & r\cos\theta\cos\phi & -r\sin\theta\sin\phi \\ \sin\theta\sin\phi & r\cos\theta\sin\phi & r\sin\theta\cos\phi \\ \cos\theta & -r\sin\theta & 0 \end{vmatrix}$$	4
	on expanding about the last row, $$J = \cos\theta[r^2\sin\theta\cos\theta\cos^2\phi + r^2\sin\theta\cos\theta\sin^2\phi) + r\sin\theta(r\sin^2\theta\cos^2\phi + r\sin^2\theta\sin^2\phi]$$ $$J = r^2\sin\theta(\cos^2\theta + \sin^2\theta) = r^2\sin\theta$$	2
2) a)	$u = x+y+z, \quad y+z = uv, \quad z = uvw$ $\Rightarrow x = u-y-z, \quad y = uv - z \quad z = uvw$ $\Rightarrow y = uv - uvw, \quad z = uvw$ $\therefore x = u - uv + uvw - uvw = u - uv$ $\therefore x = u - uv, \quad y = uv - uvw, \quad z = uvw$	3
	$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}$$	

Q#	Solution	Marks Allocated
	$J = \begin{vmatrix} 1-v & -u & 0 \\ v-vw & u-uw & -uv \\ vw & uw & uv \end{vmatrix}$	3
	$J = (1-v)[u^2v - \cancel{u^2vw} + \cancel{u^2vw}]$ $+ u[uv^2 - \cancel{uv^2w} + \cancel{uv^2w}]$	
	$J = u^2v - u^2v^2 + u^2v^2 = u^2v$	1
	$\therefore J = u^2v$	
2)b)	$u = \tan^{-1}\left[\frac{y}{x}\right]; \; x = e^t - e^{-t}, \; y = e^t + e^{-t}$	
	$u \to (x,y) \to t \Rightarrow u \to t$	
	$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$	1
	$\frac{\partial u}{\partial x} = \frac{1}{1+\frac{y^2}{x^2}}\left[-y/x^2\right] = \frac{-y}{x^2+y^2}$	
	$\frac{\partial u}{\partial y} = \frac{1}{1+\frac{y^2}{x^2}}\left[\frac{1}{x}\right] = \frac{x}{x^2+y^2}$	
	$\frac{dx}{dt} = e^t + e^{-t} = y; \; \frac{dy}{dt} = e^t - e^{-t} = x$	3
	$\frac{du}{dt} = \left[\frac{-y}{x^2+y^2}\right][y] + \left[\frac{x}{x^2+y^2}\right][x]$	
	$\frac{du}{dt} = \frac{x^2-y^2}{x^2+y^2} = \frac{(e^t-e^{-t})^2-(e^t+e^{-t})^2}{(e^t-e^{-t})^2+(e^t+e^{-t})^2}$	
	$\Rightarrow \frac{du}{dt} = \frac{-2}{e^{2t}+e^{-2t}}$	3

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HKBK COLLEGE OF ENGINEERING, BANGALORE - 560045
DEPT. OF ENGINEERING MATHEMATICS

SCHEME & SOLUTIONS

Subject Name: CALCULUS AND DIFFERENTIAL EQUATIONS Subject Code: 21MAT11

Q #	Solution	Marks Allocated
2) c)	$f(x,y) = x^3+y^3-3axy$; $a \geq 0$ $f_x = 3x^2-3ay$ and $f_y = 3y^2-3ax$	1
	We shall find the stationary points such that $f_x = 0$ and $f_y = 0$ $f_x = 0 \Rightarrow 3x^2-3ay=0 \Rightarrow x^2 = ay$ — ① $f_y = 0 \Rightarrow 3y^2-3ax=0 \Rightarrow y^2 = ax \Rightarrow x = \frac{y^2}{a}$ $\therefore$ ① $\Rightarrow \frac{y^4}{a^2} = ay \Rightarrow y^4 = a^3y$ $\Rightarrow y[y^3-a^3]=0$ $\Rightarrow y=0$ and $y=a$	2
	If $y=0$ then $x=0$ and if $y=a$ then $x=a$ $\therefore$ Stationary pt's are $(0,0)$ & (a,a) $A = f_{xx} = 6x$; $B = -3a$, $C = 6y$	
	points: $(0,0)$ / (a,a) $A = 6x$: 0 / $6a$ $B = -3a$: $-3a$ / $-3a$ $C = 6y$: 0 / $6a$ $AC-B^2$: $-9a^2$ / $27a^2$ Conclusion: Saddle pt. / Minimum pt.	2
	Minimum value of $f(x,y)$ is $f(a,a)$ $f(a,a) = a^3+a^3-3a^3 = -a^3$ at (a,a)	1

$\frac{5}{9}$

Q#	Solution	Marks Allocated
3) a)	we have $T = t_2 + (t_1 - t_2) e^{-kt}$ By data, $t_1 = 80^\circ c$, $t_2 = 40^\circ c$, $T = 60$ when $t = 20$ $60 = 40 + (80-40) e^{-20k}$ $\frac{20}{40} = 0.5 = e^{-20k} \Rightarrow \log_e 0.5 = -20k$ $\Rightarrow \cancel{-}0.6931 = \cancel{-}20k$ $\Rightarrow k = 0.0346$ $\therefore T = 40 + (80-40) e^{-0.0346(40)}$ $\Rightarrow T = \underline{50.02^\circ c}$	2 4 1
3) b)	$\frac{dy}{dx} + y \tan x = y^3 \sec x \quad \div y^3$ $\frac{1}{y^3}\frac{dy}{dx} + \frac{y}{y^3}\tan x = \sec x$ $\frac{1}{y^3}\frac{dy}{dx} + \frac{1}{y^2}\tan x = \sec x$ $t = \frac{1}{y^2} \Rightarrow \frac{dt}{dx} = -\frac{2}{y^3}\frac{dy}{dx} \Rightarrow -\frac{1}{2}\frac{dt}{dx} = \frac{1}{y^3}\frac{dy}{dx}$ $-\frac{1}{2}\frac{dt}{dx} + t\tan x = \sec x$ $\frac{dt}{dx} - 2t\tan x = -2\sec x$; $P = -2\tan x$, $Q = -2\sec x$ $e^{\int P dx} = e^{-2\int \tan x\, dx} = \frac{1}{\sec^2 x}$ Soln is $t\, e^{\int P dx} = \int Q\, e^{\int P dx} dx + C$ $\Rightarrow \frac{1}{y^2 \sec^2 x} = \int -2\sec x \cdot \frac{1}{\sec^2 x} dx + C$	2 2 2

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Q#	Solution	Marks Allocated
	$\Rightarrow \frac{\cos^2 x}{y^2} = -2\sin x + c$	1
3) i)	$(1+e^{x/y})dx + e^{x/y}(1-x/y)dy = 0$ $M = 1+e^{x/y}, \quad N = e^{x/y}(1-x/y)$ $\frac{\partial M}{\partial y} = e^{x/y}(-x/y^2);$ $\frac{\partial N}{\partial x} = e^{x/y}(-1/y) + (1-x/y)e^{x/y}(1/y)$ $\frac{\partial N}{\partial x} = \frac{-x e^{x/y}}{y^2} \quad \therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow$ Exact equation $\int M dx + \int N(y)\, dy = c$ $\int (1+e^{x/y})dx + \int 0\, dy = c$ $\Rightarrow x + \frac{e^{x/y}}{(1/y)} = c \Rightarrow x + y e^{x/y} = c$	4 2
4) a)	$y(x+y)dx + (x+2y-1)dy = 0$ $M = xy+y^2 \Rightarrow \frac{\partial M}{\partial y} = x+2y$ $N = x+2y-1 \Rightarrow \frac{\partial N}{\partial x} = 1$ $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = x+2y-1$ ... near to N $\frac{1}{N}\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right] = \frac{x+2y-1}{x+2y-1} = 1 = f(x)$	2 1

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Q#	Solution	Marks Allocated
	$e^{\int f(x)dx} = e^{\int 1 dx} = e^x$	
	multiply the given eq.n by e^x we have,	
	$M = e^x(xy+y^2)$ and $N = e^x(x+2y-1)$	
	$\frac{\partial M}{\partial y} = e^x \cdot x + e^x 2y$	
	$\frac{\partial N}{\partial x} = e^x(x+2y-1) + e^x$	
	$\frac{\partial N}{\partial x} = xe^x + 2 \cdot e^x y$	
	$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow$ Equation is exact	2
	Soln is $\int M dx + \int N(y) dy = c$	
	$\Rightarrow \int (xye^x + e^x y^2) dx + \int 0 dy = c$	
	$\Rightarrow y\int x e^x dx + y^2 \int e^x dx = c$	
	$\Rightarrow y[xe^x - e^x] + y^2 e^x = c$	
	$\Rightarrow xye^x - ye^x + y^2 e^x = c$	2
	$\Rightarrow e^x[xy - y + y^2] = c$	
(4) b)	$\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$	

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HKBK COLLEGE OF ENGINEERING, BANGALORE - 560045

DEPT. OF ENGINEERING MATHEMATICS
[redacted]

Subject Name: CALCULUS & DIFFERENTIAL EQUATIONS Subject Code: 21MAT11

Q #	Solution	Marks Allocated
	$(y\cos x + \sin y + y)dx + (\sin x + x\cos y + x)dy = 0$	1
	Let $M = y\cos x + \sin y + y$	
	$\frac{\partial M}{\partial y} = \cos x + \cos y + 1$	
	$N = \sin x + x\cos y + x$	
	$\frac{\partial N}{\partial x} = \cos x + \cos y + 1$ $\quad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow$ EXACT eqn	4
	$\int M dx + \int N(y) dy = c$	
	$\Rightarrow y\sin x + x\sin y + xy = c$	2
4)	c) we have $T = t_2 + (t_1 - t_2)e^{-kt}$	1
	By data, $t_1 = 100$, $t_2 = 25$, $T = 75$ when $t = 1$	
	$T = 25 + 75e^{-kt}$	
	$\Rightarrow 75 = 25 + 75e^{-k}$ or $e^{k} = \frac{3}{2} = 1.5$	
	$\Rightarrow k = \log_e(1.5) = 0.4055$	2
	we have to find T when $t = 3$	
	$\therefore T = 25 + 75e^{-0.4055t}$	
	$\therefore (T)_{t=3} = 25 + 75e^{-1.2165}$	3
	$\Rightarrow (T)_{t=3} = \underline{47.22}$	
	9/9	

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