

# The Vibrating Reed

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## Abstract

This lab course analyses the characteristics of a vibrating reed. Aim is to not only understand the physical principals leading to the obtained results, but also to get an understanding of the technical principals of data aquisition. Using LabView we communicate with the various instruments, manipulating settings and reading out measurements. In a first part we find the resonance frequency  $\omega_R$  of the vibrating reed and its first two harmonics, in a second we determine the temperature dependency of  $\omega_R(T)$ .

## 1 Introduction

## 2 Theory

To get a good understanding of the underlying processes that occur in the experiment we need to understand how a damped harmonic oscillator under an external force behaves. We will derive how to calculate the Eigenvalues of the frequencies of an harmonic oscillator to later estimate their actual values.

### 2.1 Driven and damped harmonic oscillator with the example of a vibrating reed

By applying the following differential equation we can describe a harmonic oscillator, driven by the force  $F = F_0 \cos(\omega t)$  and damped by the dampening factor  $\gamma$  and with its Eigen-frequency  $\omega_0$ :

$$\frac{\partial^2 z}{\partial t^2} + \gamma \frac{\partial z}{\partial t} + \omega_0^2 z = f_0 \cos(\omega t) \quad (1)$$

$\omega$  is the frequency and  $f_0 = F_0/m$  is the Amplitude of the driving force, the latter normed to its mass.

Some helpful equations can be derived from this: By assuming that  $z = \zeta e^{i(\omega t - \phi)}$  we find the stationary solution for the Amplitude  $\zeta(\gamma)$ :

$$\zeta = \frac{f_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}} \quad (2)$$

It has the shape of a Lorentzian function. It is also possible to find the phase difference of driving force and oscillator:

$$\tan(\phi) = \frac{\gamma \omega}{\omega_0^2 - \omega^2} \quad (3)$$

Equation 1 reaches its maximum at

$$\omega_R = \sqrt{\omega_0^2 - \frac{\gamma^2}{2}}, \quad (4)$$

the resonance frequency.

The attenuation of the system is described by the quality  $\mathcal{Q}$  of the oscillator:

$$\mathcal{Q} = 2\pi \frac{\omega_0}{\gamma} \quad (5)$$

As described in ?? the oscillation of a vibrating reed can be described as follows:

$$\frac{\partial^2 z}{\partial t^2} = -\frac{d^2}{12} v_Y^2 \frac{\partial^4 z}{\partial x^4} \quad (6)$$

Here  $x$  is the coordinate of a small portion along the axis of the reed,  $z$  is the transversal displacement,  $t$  is the time,  $d$  the thickness of the reed and  $v_Y$  is the speed of sound inside the material the reed is made of. For a better understanding see figure ??. As

$$v_Y = \sqrt{\frac{E}{\rho}} \quad (7)$$

holds, we see that  $v_Y$  depends on the mass density  $\rho$  and the modulus of elasticity  $E$ . This way we can determine the different Eigen-frequencies  $\nu_n$

$$\nu_n = \alpha_n (2n + 1)^2 \frac{\pi}{16\sqrt{3}} \frac{d}{l^2} v_Y \quad (8)$$

$\alpha_n$  is calculated numerically:

$$\alpha_0 = 1,424987, \alpha_1 = 0,992249, \quad (9)$$

$$\alpha_2 = 1,000198, \alpha_3 = 0,999994, \quad (10)$$

$$\alpha_n \approx 1 \text{ for all other } n. \quad (11)$$

Applying a force on the damped harmonic oscillator, that attacks only at the end of the reed ( $x = l$ ), we can formulate the following differential equation:

$$\frac{\partial^2 z}{\partial t^2} + \frac{d^2}{12} v_Y \frac{\partial^4 z}{\partial x^4} = f_0 e^{i\omega t} \delta(x - l) \quad (12)$$

With only a small damping factor we can approximate the vibration modes around the resonance frequencies with a Lorentzian. The relative amplitude can then be approximately described by

$$\left| \frac{\zeta(\omega)}{\zeta(\omega_{R,n})} \right| \approx \frac{\gamma \omega_{0,n}}{\sqrt{(\omega_{0,n}^2 - \omega^2)^2 + \gamma^2 \omega_{0,n}^2}} \quad (13)$$

$$n = 0, 1, 2, \dots \quad (14)$$

### 3 Setup

**Graphik** The core element of our experiment is the vibrating reed.

#### 3.1

#### 3.2 Parameter estimation

Prior to the experiment we need to estimate certain parameters to the reed, we use, be able to run the experiment. These are the maximum Voltage  $U_{d,max}$ , which shows, how small our expected signals are and the base Eigenfrequency  $\nu_0$ , where we have to look for the Lorentz-peaks, we want to observe.

The maximum

#### 3.3 Vibrating Reed

### 4 Methods

### 5 Results

### 6 Discussion