The Vibrating Reed

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Abstract This lab course analyses the characteristics of a vibrating reed. Aim is to not only understand the physical principals leading to the obtained results, but also to get an understanding of the technical principals of data acquisition. Using LabView we communicate with the various instruments, manipulating settings and reading out measurements. In a first part we find the resonance frequency ω_R of the vibrating reed and its first two harmonics, in a second we determine the temperature dependency of $\omega_R(T)$.

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Introduction

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Theory

To get a good understanding of the underlying processes that occur in the experiment we need to understand how a damped harmonic oscillator under an external force behaves. We will derive how to calculate the Eigenvalues of the frequencies of an harmonic oscillator to later estimate their actual values.

2.1 Driven and damped harmonic oscillator with the example of a vibrating reed

By applying the following differential equation we can describe a harmonic oscillator, driven by the force $F = F_0 cos(\omega t)$ and damped by the dampening factor γ and with its Eigen-frequency ω_0:

$$\frac{\partial^2 z}{\partial t^2} + \gamma \frac{\partial z}{\partial t} + \omega_0^2 = f_0 \cos(\omega t) \tag{2.1}$$

ω is the frequency and $f_0 = F_0/m$ is the Amplitude of the driving force, the latter normed to its mass.

Some helpful equations can be derived from this: By assuming that $z = \zeta e^{i(\omega t - \phi}$ we find the stationary solution for the Amplitude $\zeta(\gamma)$:

$$\zeta = \frac{f_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}} \tag{2.2}$$

It has the shape of a Lorentzian function. It is also possible to find the phase difference of driving force and oscillator:

$$\tan(\phi) = \frac{\gamma \omega}{\omega_0^2 - \omega^2} \tag{2.3}$$

Equation 2.1 reaches its maximum at

$$\omega_R = \sqrt{\omega_0^2 - \frac{\gamma^2}{2}}, \tag{2.4}$$

the resonance frequency.

The attenuation of the system is described by the quality $\mathcal{Q}$ of the oscillator:

$$\mathcal{Q} = 2\pi \frac{\omega_0}{\gamma} \tag{2.5}$$

As described in **??** the oscillation of a vibrating reed can be described as follows:

$$\frac{\partial^2 z}{\partial t^2} = -\frac{d^2}{12} v_Y^2 \frac{\partial^4 z}{\partial x^4} \tag{2.6}$$

Here x is the coordinate of a small portion along the axis of the reed, z is the transversal displacement, t is the time, d the thickness of the reed and v_Y is the speed of sound inside the material the reed is made of. For a better understanding see figure ??. As

$$v_Y = \sqrt{\frac{E}{\rho}} \tag{2.7}$$

holds, we see that v_Y depends on the mass density ρ and the modulus of elasticity E. This way we can determine the different Eigenfrequencies ν_n

$$\nu_n = \alpha_n (2n+1)^2 \frac{\pi}{16\sqrt{3}} \frac{d}{l^2} v_Y \tag{2.8}$$

α_n is calculated numerically:

$$\alpha_0 = 1,424987,\ \alpha_1 = 0,992249, \tag{2.9}$$
$$\alpha_2 = 1,000198,\ \alpha_3 = 0,999994, \tag{2.10}$$
$$\alpha_n \approx 1 \text{ for all other } n. \tag{2.11}$$

Applying a force on the damped harmonic oscillator, that attacks only at the end of the reed ($x = l$), we can formulate the following differential equation:

$$\frac{\partial^2 z}{\partial t^2} + \frac{d^2}{12} v_Y \frac{\partial^4 z}{\partial x^4} = f_0 e^{i\omega t} \delta(x - l) \tag{2.12}$$

With only a small damping factor we can approximate the vibration modes around the resonance frequencies with a Lorentzian. The relative amplitude can then be approximately described by

$$\left| \frac{\zeta(\omega)}{\zeta(\omega_{R,n})} \right| \approx \frac{\gamma \omega_{0,n)}}{\sqrt{(\omega_{0,n}^2 - \omega^2)^2 + \gamma^2 \omega_{0,n}^2}} \tag{2.13}$$

$$n = 0, 1, 2, \ldots\ . \tag{2.14}$$

Chapter 3

Setup

The core element of our experiment is the reed (1 in figure 3.1). It is a small metal plate that is fixed to the ground between two copper blocks. The free end is placed between two electrodes as can be seen in figure 3.1. Its dimensions are approximately 2 cm in length, 5 mm in width and 200 µm in depth.

3.1

3.2 Parameter estimation

Prior to the experiment we need to estimate certain parameters to the reed, we use, be abled to run the experiment. These are the maximum Voltage $U_{d,max}$, which shows, how small our expected signals are and the base Eigenfrequency ν_0, where we have to look for the Lorentz-peaks, we want to observe.

The maximum

3.3 Vibrating Reed

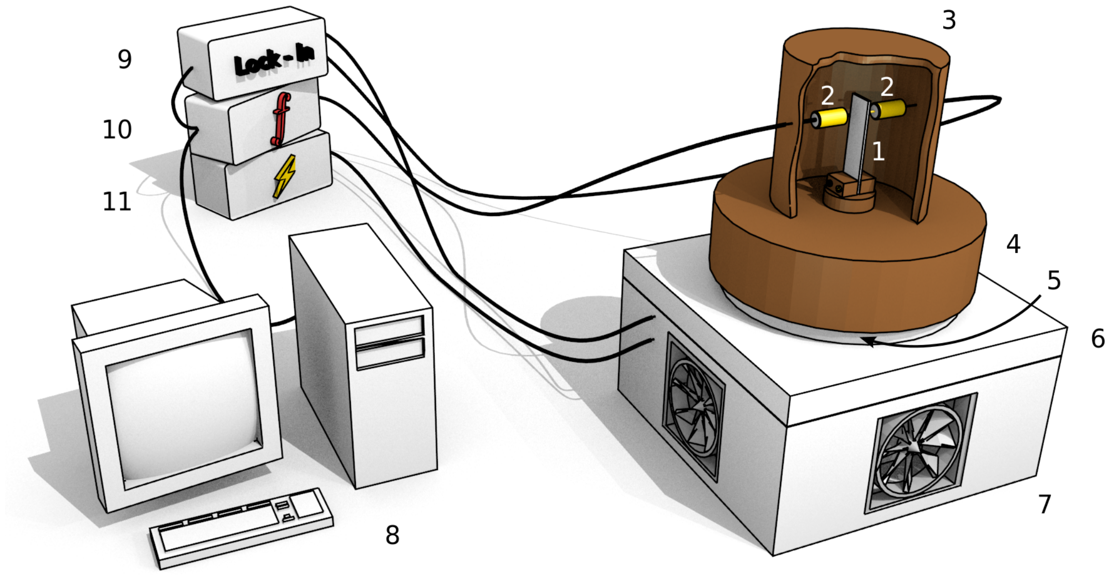

Figure 3.1: Schematic Setup of the vibrating reed experiment

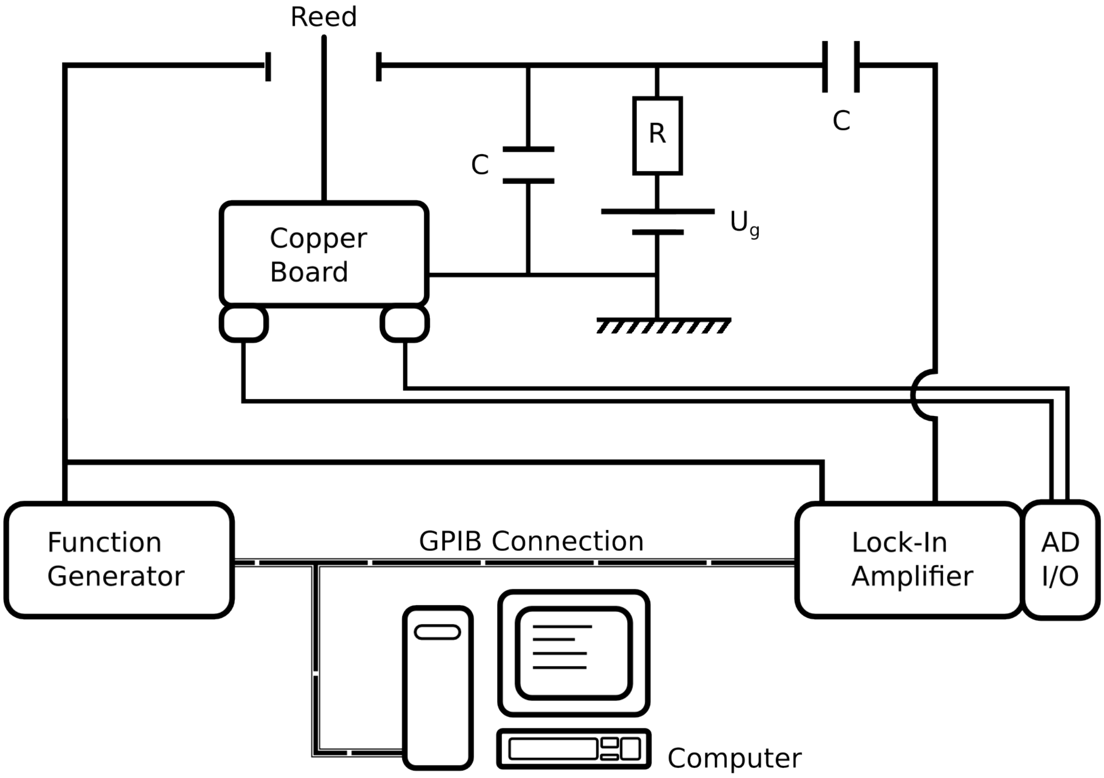

Figure 3.2: Electrical configuration of the vibrating reed experiment

Chapter 4

Methods

Chapter 5

Results

Chapter 6

Discussion