
Differential Equations, Part 1

Part I: First Order Differential Equations

This post will be a brief overview of first-order differential equations. I won't be focusing mainly on proofs here, but instead techniques for solving equations, as well as existence and uniqueness theorems

1.1 Integrating Factors

Integrating factors can be used to solve first order equations of degree 1, in other words equations of the form:

$$y' + P(t)y = Q(t)$$

The idea is to multiply through by a factor $\mu(t)$ suitably chosen so that the left hand side is an instance of the product rule. This integrating factor turns out to be:

$$\mu(t) = e^{\int P(t)dt}$$

Multiplying through, we are left with the product rule on the left hand side, yielding a new simpler differential equation:

$$(\mu(t)y(t))' = \mu(t)Q(t)$$

And this shouldn't be difficult to integrate in general.

1.2 Separable Equation

A separable differential equation can be rewritten in the form:

$$N(y)\frac{dy}{dx} = M(x)$$

Rewriting:

$$N(y)dy = M(x)dx$$

Integrating both sides gives a solution for y .
