The Title

By

Firstname Middlename Surname (email@aims.ac.rw)

June 2017

AN ESSAY PRESENTED TO AIMS RWANDA IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE AWARD OF MASTER OF SCIENCE IN MATHEMATICAL SCIENCES

DECLARATION

This work was carried out at AIMS Rwanda in partial fulfilment of the requirements for a Master of Science Degree.

I hereby declare that except where due acknowledgement is made, this work has never been presented wholly or in part for the award of a degree at AIMS Rwanda or any other University.

Scan your signature

Student: Firstname Middlename Surname

Scan your signature

Supervisor: Firstname Middlename Surname

ACKNOWLEDGEMENTS

This is optional and should be at most half a page. Thanks Ma, Thanks Pa. One paragraph in normal language is the most respectful.

Do not use too much bold, any figures, or sign at the bottom.

DEDICATION

This is optional.

Abstract

A short, abstracted description of your essay goes here. It should be about 100 words long. But write it last.

An abstract is not a summary of your essay: it's an abstraction of that. It tells the readers why they should be interested in your essay but summarises all they need to know if they read no further.

The writing style used in an abstract is like the style used in the rest of your essay: concise, clear and direct. In the rest of the essay, however, you will introduce and use technical terms. In the abstract you should avoid them in order to make the result comprehensible to all.

You may like to repeat the abstract in your mother tongue.

Contents

Declaration i

Acknowledgements ii

Dedication iii

Abstract iv

1 General Introduction 1

1.1 Introduction 1

1.2 Quarks 1

1.3 Strong Force 2

1.4 Large Hadron Collider 3

1.5 Monte Carlo Method 4

2 The Parton Shower 7

2.1 The Parton Shower 7

2.2 Hadronization 7

2.3 Parton Shower Simulation 7

3 Anti-k_t Jet Algorithm 12

3.1 Introduction 12

3.2 Jet Algorithms 12

4 Pythia 14

4.1 This is a section 14

References 15

1. General Introduction

1.1 Introduction

Particle Physics or sometimes called High Energy Physics, is the field of physics that pursues the ultimate structure of matter, this is possible in two ways, one is to look for elementary particles, the ultimate constituents of matter at their smallest scale and the other is to clarify what interactions are acting among them *(forces)* to construct matter as we see it.

In our current view, all matter is made of three kind of elementary particles, *leptons*, *quarks* and *mediators*.

Leptons have spin $\frac{1}{2}$, they naturally fall into three families, these are electron, muon, tau and their associated neutral neutrinos, those are described by lepton numbers for the example the electron has electron number 1 and the muon has electron number 0 and muon number 1 and so on. The three charged leptons have electric charge -1. There are also six anti leptons for example the positron which has electron number of -1 and electric charge $+1$, so all in all we have 12 leptons.

The quarks have spin $\frac{1}{2}$ too, there are six flavours of quarks, up, down, top, bottom, charm and strange, quarks have a colour charge, which is property that is related to the strong interactions, as the leptons there are also six anti quarks, we will talk about them in more details in following sections.

Every interaction has its mediators, the photon for electromagnetic interactions, the gluon for the strong interactions, the two W' and Z for the weak interactions, and presumably the graviton for gravity.

There are four forces in nature, gravity, electromagnetism, weak nuclear force and strong nuclear force. In this essay our work will be on the strong nuclear force.

1.2 Quarks

Quarks and leptons are the building blocks which build up matter. As mentioned above there are six "flavours" of quarks, these are up, down charm strange, top and bottom.

Quarks can successfully account for all known mesons and baryon which are known as hadrons, which are particles with spin $\frac{1}{2}$. Mesons consist of quark and anti quark for example the positive pion π^+ which consists of up and anti down quarks. Baryons consist of three quarks or three anti quarks for example the proton which consists of two up quarks and one down quark.

Quarks carry colour quantum number: red, green or blue, the colour name here is an analogy that is famously used among physicists to describe a three kind of generating force, we may call them by a number or any other index, also we can think of it as a three primary additive colours. Since all hadrons are colour charge neutral or colourless particles they have white charge.

Name (Flavour)	Symbol	Electric charge(in units of e)	Mass
Up	u	$+\frac{2}{3}$	$1.7 - 3.1\frac{Mev}{c^2}$
Down	d	$-\frac{1}{3}$	$4.1 - 5.7\frac{Mev}{c^2}$
charm	c	$+\frac{2}{3}$	$1.18 - 1.34\frac{Gev}{c^2}$
strange	s	$-\frac{1}{3}$	$80 - 130\frac{Mev}{c^2}$
top	t	$+\frac{2}{3}$	$172.3 - 173.5\frac{Gev}{c^2}$
bottom	b	$-\frac{1}{3}$	$4.13 - 4.37\frac{Gev}{c^2}$

Table 1.1: properties of Quarks

Unlike other elementary particles quarks carry fractional charge and possess new quantum numbers. The table 1.1 summarizes some properties of quarks. Each quark flavour is associated with its own quantum number[1](the capital letters), those quantum numbers describes the decay of the particles, those first were meant to explain the fact that some particles decay slower than other particles, for example the first mentioned below the "Strangeness", it has been noticed that the higher the mass the lower the strangeness, these numbers are conserved in strong and electromagnetic interactions but not in weak interaction, These are:

- Strangeness: $S = -1$ for s-quark.
- Charm: $C = +1$ for c-quark.
- Beauty: $\tilde{B} = -1$ for b-quark.
- t - quark has life time too short to form hadrons.
- up and down quarks have nameless flavour quantum numbers.

(Nagashima and Nambu, 2010)

[Jan: Citations: If section(s) are based on a textbook, cite it in the beginning, saying something like "the following presentation is based on \cite{...}"] !

1.3 Strong Force

As we mentioned the elementary particles that mediate these interactions are eight called gluons, which come from the gauge group $SU(3)$ which has eight generators, gluons mediate the interaction between quarks.

Gluon is massless spin 1 particle, carrying charge called colour charges, gluons look like photons but photons do not carry electric charge, because of that gluons can interact among themselves unlike photons. Normally the range of the force can be calculated by a simple argument of the

[1] These are the set of numbers that describes a conserved quantities in the dynamic of a quantum system, for example the set of numerical solutions of Schrödinger equation of the Hydrogen atom.

uncertainty[2], but this not the case for the strong force, the strong force is more complicated and involves a concept known as confinement.

The colour charge has strange property that it exerts a constant force that binds colour carrying particles together, this can be visualized using the analogy of a rubber band, the stronger you pull on the rubber band the tighter it feels. If we do not pull on it at all, it hangs loose. The same thing happens for the particles, that means at a very short distance, the force is relaxed and the particles behave as free particles, as the distance between them increases the force acts like a rubber band, the force gets them back in stronger pulling and when the rubber band is stretched beyond its limits then it will cut into many pieces producing more particles. This phenomenon is known as the colour confinement. In other words those particles tend not to be separated by a macroscopic distance. This limits the range of strong force, which is believed to be on order of 10^{-15}m, the dimension of a nuclear particle.

The theory which describes this force is called $Quantum\ Chromodynamics$. [Jan: Citation for this section would be nice.] !

1.4 Large Hadron Collider

Hadron colliders are devices made to explore the world of particle physics, they work as theories testers and also as a discovery machines, an example of these hadron Colliders is the LHC in CERN.

In the Large hadron Collider two beams of hadrons (protons) are being accelerated to a high kinetic energy and then collided with each other. It has started operation in 2009.

Most of the interesting physics at LHC involves investigating the results of these interactions(collisions), as a result of this collision stable partons are formed, partons consists of quarks and qluons. The detectors gather information about the particles, including their speed, mass, charge, from which we can identify the particles. The detectors consists of sub-detectors, each is designed to look for a particular properties. For example, the tracking devices reveal of the particle; calorimeters stop, absorb and the particle energy.

Because of the complex nature of the event at the hadrons colliders, the description of the final state involves a multi-particle calculations. The accurate prediction of the final state in hadron colliders is still one the hardest problems, this problem roots to the non-abelian nature of QCD, which leads to a colour confinement at a long distances. The two main problems are the description of the hadron formation and the evolution of QCD final states from short to long distances. Those problems can be tackled to a good approximation by Monte-Carlo event generators.

[2] $\Delta E \Delta t \approx mc^2 \Delta t > \frac{\hbar}{2} \Longrightarrow range = c\Delta t > \frac{\hbar}{2mc}$

1.5 Monte Carlo Method

The name Monte Carlo method is set of mathematical tools that first used by scientist working on the nuclear project in Los Alamos. The essence of the is to generate numbers with probability that can be used to study physical phenomena. In our context the definition of Monte Carlo method would be, that in which we use randomly generated numbers to imitate a physical behaviour that is not necessarily considered to be random (Kalos and Whitlock, 1986).

1.5.1 Pseudo-random numbers. In a computer these are generated using a deterministic algorithm that generates a set of numbers that exhibits statistical randomness, those numbers are called pseudo-random (Kalos and Whitlock, 1986).

1.5.2 Samples with different PDFs. Generating samples of different probability distribution function(pdf) is essential since we are simulating various variables that have different numerical behaviour. For example if our pseudo-random numbers are uniformly[3] distributed in the interval $[0, 1]$ and instead we need numbers that have normal distribution restricted to the same interval. I will discuss two methods, which are used in this essay.

1.5.3 The acceptance-rejection method. The acceptance-rejection method was developed by von Neumann (Weinzierl, 2000). Assume we have access to a sample which is distributed according to the pdf $f(x)$, and the let us denote the pdf of the required distribution by $p(x)$, we assume that for both $p(x)$ and $f(x)$ x varies over a finite interval.

In simple words, first we generate x according to the uniform distribution over a given interval assume $[0, 1]$, then we find the maximum of $f(x)$, and then calculates $p(x)$, then we generate another number y which is also uniformly distributed over the interval $[0, f_{max}(x)]$, and checks $y \times f(x) \leq p(x)$. If this the case then accept x, if not reject x and start again, example of that, if have a number x that is fall under uniform distribution and we want to reshape this so that we get a number that has the distribution $\frac{1}{x}$, then we find the maximum value of probability density function $(p(x))$ for x, here we add small number to x so that we avoid the singular point when $x = 0$, after that we generate another sample y that is also uniformly distributed in the interval $[0, pdf_{max}]$, now we check if y is less than $p(x)$ we add x to our distribution if not we start again. The histograms in figure 1.1 demonstrate the example above.

Another related example of this method is calculation of pi, assume we have a box of side length D and a circle of diameter D inside that box, now the probability that a point in the box and is also in the circle is approximately the area of the circle over the area of the box, which is π over 4 here, hence, from this we can approximate the value of π. The histograms in Figure 1.2 exhibits this calculation and also the error in the calculation(Weinzierl, 2000).

1.5.4 The module random in python. The samples above were generated using Python module `random`, here are few words about this module.

This module implements pseudo-random number generators for various distributions.

In a computer pseudo-random numbers are generated using a deterministic algorithm that gen-

[3] Uniform means that every value in the range of the distribution is equally likely to occur. This distribution is widely used for generating random numbers for other distributions, it is denoted by U.

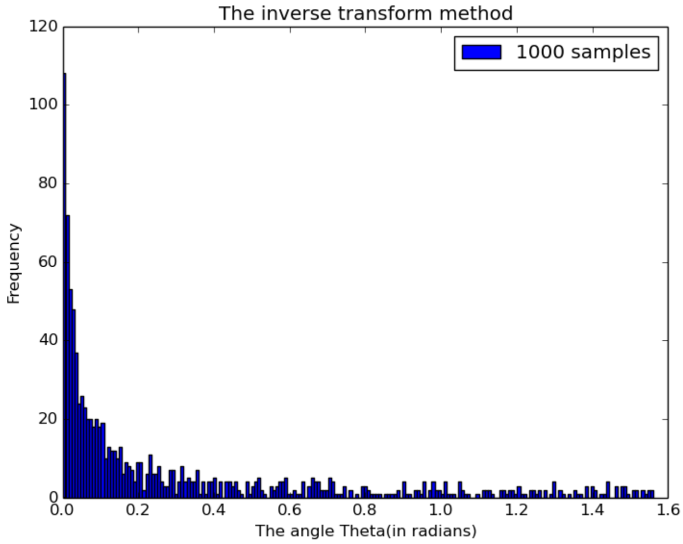


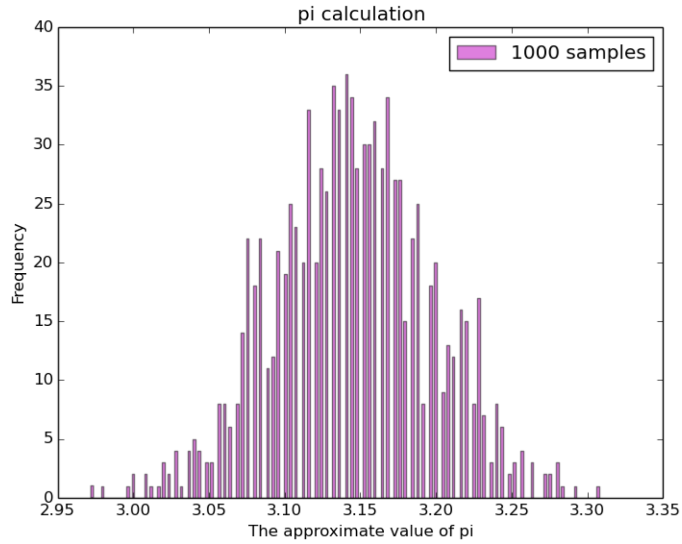
Figure 1.1: Generating values with distribution $\frac{1}{x}$

erates a set of numbers that exhibits statistical randomness, those numbers are called pseudo-random.

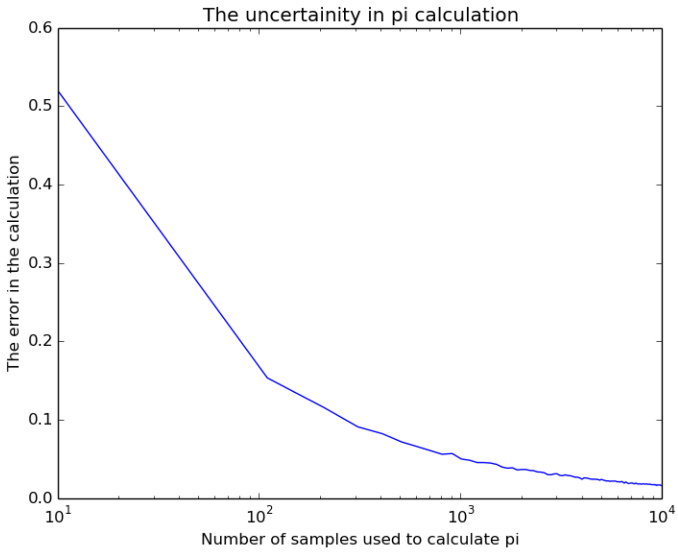
For integers, there is uniform selection from a range. For sequences, there is uniform selection of a random element, a function to generate a random permutation of a list in-place, and a function for random sampling without replacement.

On the real line, there are functions to compute uniform, normal (Gaussian), lognormal, negative exponential, gamma, and beta distributions. For generating distributions of angles, the von Mises distribution is available.

Almost all module functions depend on the basic function random(), which generates a random float uniformly in the semi-open range $[0.0, 1.0)$. Python uses the Mersenne Twister as the core generator. It produces 53-bit precision floats and has a period of $2^{19937} - 1$(Python foundation, 2007).



(a) pi value



(b) The uncertainty

Figure 1.2: At the top, this a histogram shows the values of π that are generated from sampling 1000 numbers. At the bottom the plot shows the error in the calculation when we use different number of samples

2. The Parton Shower

2.1 The Parton Shower

In general parton showers are approximations of the higher order real emission corrections(this refers to the stable hadrons) to the hard scattering. The word "hard" here means, the process involves a transfer of large momentum, either a violent scatter or creation of large mass. They locally conserve flavour and four momentum, and also they are consistent, which means, the particle either splits into two or not. Since the parton showers are simulate of the branching and splitting processes, the quality of their predictions depend on precise is the implementation, for example one can ensure the colour coherence through selecting an evolution variable representing the angular ordering, although, this in not the only choice to ensure the colour coherence (Höche, 2014). In the following we will at a simple implementation of a parton shower in python.

2.2 Hadronization

To reflect the colour neutrality of the particles in our model the partons will be transformed into a stable hadrons which are colour neutral, this process is called hadronization. The first implemented model and also follows Monte-Carlo event generators was Feynman-Field Model, which gives an idea of the formation of the mesons through iteratively from a single quark. However, this model is not collinear safe, which means the model can mix the short and long distance physics. Now a days two models are common, the string model and clustering model(Höche, 2014).

2.3 Parton Shower Simulation

The following is a simple simulation in python for a splitting of a single quark, the simulation accounts for the four momentum conservation of the soft particles. [Jan: Why only soft?]

2.3.1 Physical description. As a result of a hadrons collision, quarks will fly away. Since they are charged particles(colour charge), the moving quarks will radiate. The quark will lose part of its energy emitting a gluon. If at the beginning, the quark had energy E_i and radiates energy E_{rad}, then the qluon takes $\frac{E_{rad}}{E_i}$ of the particle's initial energy. [Jan: You mean it takes E_{rad}/E_i *fraction* of initial energy.] It is radiated at an angle of θ to the quark initial direction. The quark will fly on radiating another gluon and so on until it becomes stable(the hadronization starts). [Jan: Put space before opening parenthesis (here and elsewhere).] The radiated gluons will decay into two quarks, which will later radiates gluons, which will radiate gluons again and so on. The result is a shower of partons decaying into two partons.

[Jan: You need to make clear that in your model you do not make distinction between quarks and gluons.]

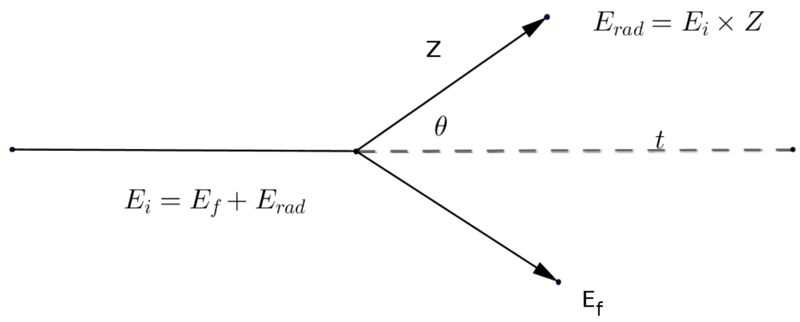


Figure 2.1: The splitting of a particle into two particles

[Jan: The picture with collision is way too small. Also caption is missing.]

Now we begin with 4- momentum vector

$$P^{\mu} = \begin{pmatrix} E/c \\ p_x \\ p_y \\ p_z \end{pmatrix} ; P_{\mu} = \begin{pmatrix} E/c \\ -p_x \\ -p_y \\ -p_z \end{pmatrix} \tag{2.3.1}$$

And the inner product is given by

$$P^{\mu} P_{\mu} = P^{\mu} \eta_{\mu\nu} P_{\nu} = (m_0 c)^2 \tag{2.3.2}$$

$$P^{\mu} P_{\mu} = \left(\frac{E}{c}\right)^2 - (p_x^2 + p_y^2 + p_z^2) = (m_0 c)^2 \tag{2.3.3}$$

Now considering the natural units $c = 1$ this can be written as

$$m_0^2 = E^2 - \|p\| \tag{2.3.4}$$

Which is lorentz invariant quantity i.e does not depend on the frame. In our case the quarks in (LHC), the mass of the quark $\sim$ 1 Mev and the energy of the hadrons is $\sim$ 1 Tev, hence, we can assume that the mass of the quark(2.3.4) is 0.

From the conservation of energy and momentum, [Jan: Don't put blank line before the equation here.]

$$P_i^{\mu} = P_f^{\mu}. \tag{2.3.5}$$

We assume that the intial patricle is moving in x -direction, we can write the initial 4-momentum as $P_i^{\mu} = (E, E, 0, 0)$, afterwards the particle will split. Therefore, the final momentum is given by $P_{rad} + P_{part}$, part here refers to the particle which has lost part of its energy, so $P_{rad} = (E_{rad},\ cos\theta \cdot E_{rad},\ sin\theta \cdot E_{rad},\ 0)$. From this, and since the particle is rotated with θ we can find the direction of the particle(part) with help of the rotation matrix [Jan: Please make clear that the other particle will have non-zero mass.]

$$R = \begin{pmatrix} \cos\theta & -sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{2.3.6}$$

The direction of the particle of the fraction enregy can be found using the following equation

$$P_{part} = P_i - P_{rad}. \tag{2.3.7}$$

In a three dimensional world, we have two rotation angles, the angular angle and the azimuthal angle. Given a unit vector $u = (u_x, u_y, u_z)$, where $u_x^2 + u_y^2 + u_z^2 = 1$,the matrix for rotating this particle by angle θ about an axis in the direction of u is

$$\begin{pmatrix} \cos\theta + u_x^2(1-\cos\theta) & u_x u_y(1-\cos\theta) - u_z\sin\theta & u_x u_z(1-\cos\theta) + u_y\sin\theta \\ u_y u_x(1-\cos\theta) + u_z\sin\theta & \cos\theta + u_y^2(1-\cos\theta) & u_y u_z(1-\cos\theta) - u_x\sin\theta \\ u_z u_x(1-\cos\theta) - u_y\sin\theta & u_z u_y(1-cos\theta) + u_x\sin\theta & \cos\theta + u_z^2(1-\cos\theta) \end{pmatrix} \tag{2.3.8}$$

[Jan: OK, but you have to explain 3D case in detail. Consider making separate sections for 2D and 3D. Picture showing what is angular (is this correct name?) and azimuthal angle would be helpful.] !

2.3.2 Implementation in python. First we will start with case of 2 dimensions model which accounts for one rotation matrix θ and it evolves generation of two random numbers, one them represent the angle and the other regards the energy of the radiated particle. Here both the energy fraction z and the angle $\theta$are following the distribution $1/x$, where the former lies in the interval [0.25, .75] [Jan: No, it does not lie in $[0.25, 0.75]$. Give the correct distribution. Also mention the cutoff values you are using (to avoid singularity at $x = 0$).] [Jan: Put all of numeric intervals in math mode.] and the later in the interval [0,$\pi/2$] and they are generated by applying the inverse transform method on a set of numbers that are uniformly distributed. ! !

As for the four momentum vector, the module *numpy* is used for this purpose, here we use the object *array*. `Numpy` is a scientific computing package in python which is widely used for these purposes, beside that it has powerful N-dimensional arrays it also has useful linear algebra tools and random number capabilities.

following the physical description a list contains the four momentum of the initial particle was defined, then we assumed that the particle has initial energy $=$ 1 energy unit, since the particle will split after certain distance an assumption of the distance before the decay was made is that the particle will move a distance of 1 unit and then it will decay, basically it is an iteration process, at the beginning we check the energy of the particle if it is a above the stability limit, which we assumed to 0.09, this particle will split, the direction of the radiated will follow the θ and its energy will be given from z, and then both new particles four momenta will be add to the list at the beginning and again those particles will be checked, now if the particle has energy that is equal or below the stability limit then the iteration process will be terminated.

As for plotting the results, the library *matplotlib* was used which is a library that is used to make 2 D plots in Python, as *matplotlib* has the ability to add many lines at once, here the Linecollection is used, which is a package in *matplotlib*, the diagram in figure 3 shows the 2 D simulation of the parton shower.

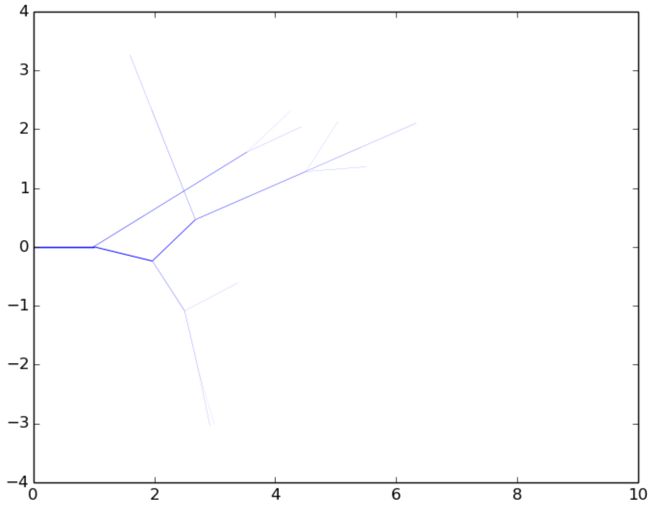


Figure 2.2: 2D simulation of the spliting of a single parton, here the colour fades as the energy decreases

The 3D simulation of the parton shower is essentially the same as for python code with few changes, in which now we have two rotation angles, θ and also ϕ which is the azimuthal angle which is uniformly distributed in the interval [0,2π]. To simulate the rotation in three dimensions, the function `normv(u)` and function `rotation(v,angle)`, the former returns the axis of the rotation, the function input and the vector [1,1,1] from a plane, from which we find a vector that is orthogonal to this plane, and the later is matrix of rotation, it takes the angle rotation and the axis of rotation as inputs.

Also here z (the energy fraction) now is lies the interval[0,1] and the stability limit is 0.05. The digram in figure 4 shows the 3D simulation of single parton splitting.

[Jan: This description (both 2D and 3D case) should be way longer, more organized and more detailed. It should take several pages and you should explain what the code is doing precisely and in detail.]

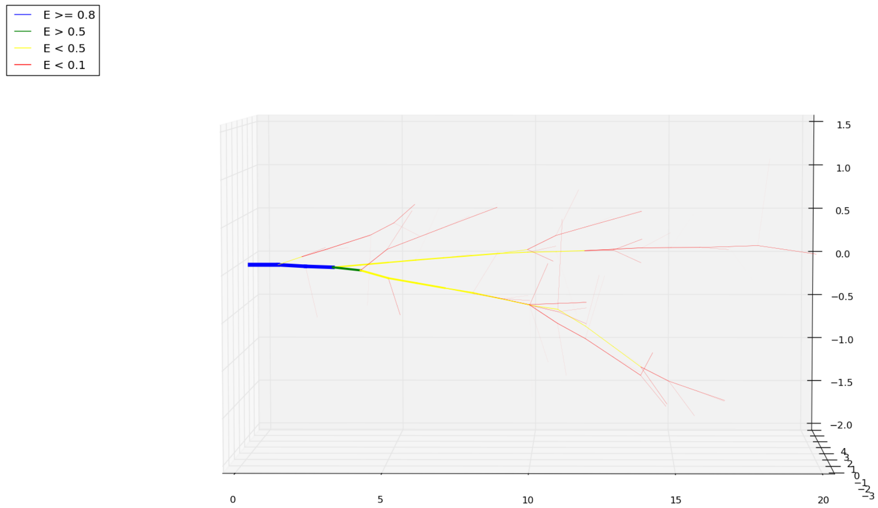


Figure 2.3: 3D simulation of the spliting of a single parton

3. Anti-k_t Jet Algorithm

3.1 Introduction

After the process of the splitting and branching, the quarks and gluons start to hadronize leading to a collimated spray of stable colourless hadrons called jets.

The QCD calculations provide a description of a final state in terms of the quark and gluons, however, in practical and due to confinement phenomenon those quarks and gluons can not be observed, the detectors can only observe those stable colourless hadrons.

The jet reconstruction is essential in understanding the link between the observed physics or the long distance physics and the underlying physics(short distance physics) in the parton level. Also, the accurate reconstruction is very important in comparing the theoretical predictions and the data.

This mapping between the stable hadrons(observed physics) and the partons(unobserved) from the hard scattering is done by the means of the jet algorithms.

Jet algorithms basically rely on merging, means that they merge objects that are somehow nearby each other. As for the accuracy of the algorithm we assume that the kinematics of the clustered jet provide a useful measure of the kinematics of the underlying short distance physics, in particular we assume the basic mismatch between the long observed hadrons and short distance unobserved partons does present any numerical limitations (Ellis et al., 2008).

The definition of the jet is central in comparing the data and the theoretical predictions, the definition is provided in the form of the jet algorithm, this means the jet algorithm and its corresponding parameters and recombination scheme, we will talk more about this in the following. In this part of the essay we will talk a bout the general idea of the jet algorithms and some of their properties and we will focus on one of them, the $anti - k_t$ and it is implementation in python.

3.2 Jet Algorithms

There are two broad classes of jet algorithms, the **Cone** algorithms and the **Sequential** algorithms. Where both of them work on defining the jets by the idea of the nearness.

It is important to recognize that jet algorithms involve two distinct steps. The first step is to identify the members of the jet, *i.e*, the partons that make-up the final stable jets. The second step is to construct the kinematic properties that will characterize the jet (Berger et al., 2001).

3.2.1 The Cone Algorithms. The cone algorithms associate hadrons into jets by identifying those are nearby in angle, $i.e$, they follow the geometrical intuition in defining the jets, where the jet is composed of hadrons and partons, whose momenta lie within a cone defined by a circle in $\eta - \phi$ plane. Where η is the pseudo-rapidity and can be calculated by $\ln(\cot \frac{\phi}{2})$, and ϕ is the azimuthal angle (Berger et al., 2001).

As for the first step in the algorithm, *i.e*, identifying the members of the jet, it is a simple sum over the all (short and long distances) within a cone centred at $\eta - \phi$ plane. Here one introduces the concept of *stable* cones as a circle of fixed radius R in the plane $\eta - \phi$. Such that the sum of all four momenta within the cone points to the same direction as the centre of the circle. The cone algorithms attempts to identify the stable cones. Thus, at least in principle one can think in terms of placing trial cones randomly in the plane $\eta - \phi$ and allowing them to follow until a stable cone or a jet is found (Ellis et al., 2008).

4. Pythia

4.1 This is a section

Text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text text.

References

C. F. Berger et al. Snowmass 2001: Jet energy flow project. *eConf*, C010630:P512, 2001.

S. D. Ellis, J. Huston, K. Hatakeyama, P. Loch, and M. Tonnesmann. Jets in hadron-hadron collisions. *Prog. Part. Nucl. Phys.*, 60:484–551, 2008. doi: 10.1016/j.ppnp.2007.12.002.

Stefan Höche. Introduction to parton-shower event generators. 2014. URL https://inspirehep.net/record/1328513/files/arXiv:1411.4085.pdf.

M.H. Kalos and P.A. Whitlock. *Monte Carlo Methods*. Monte Carlo Methods. Wiley, 1986. ISBN 9780471898399. URL https://books.google.rw/books?id=GyfvAAAAMAAJ.

Y. Nagashima and Y. Nambu. *Elementary Particle Physics: Quantum Field Theory and Particles*. Number v. 1. Wiley, 2010. ISBN 9783527630103. URL https://books.google.rw/books?id=J0l8s3pdOksC.

www.python.org Python foundation. Python reference manual. Technical report, The Us, 2007.

Stefan Weinzierl. Introduction to Monte Carlo methods. 2000.