

1 The Title

2 By

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4 *AN ESSAY PRESENTED TO AIMS RWANDA IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE AWARD OF*
5 *MASTER OF SCIENCE IN MATHEMATICAL SCIENCES*



DECLARATION

This work was carried out at AIMS Rwanda in partial fulfilment of the requirements for a Master of Science Degree.

I hereby declare that except where due acknowledgement is made, this work has never been presented wholly or in part for the award of a degree at AIMS Rwanda or any other University.

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Supervisor: Firstname Middlename Surname

ACKNOWLEDGEMENTS

- 14
- 15 This is optional and should be at most half a page. Thanks Ma, Thanks Pa. One paragraph in
16 normal language is the most respectful.
- 17 Do not use too much bold, any figures, or sign at the bottom.

¹⁸ DEDICATION

¹⁹ This is optional.

Abstract

A short, abstracted description of your essay goes here. It should be about 100 words long. But write it last.

An abstract is not a summary of your essay: it's an abstraction of that. It tells the readers why they should be interested in your essay but summarises all they need to know if they read no further.

The writing style used in an abstract is like the style used in the rest of your essay: concise, clear and direct. In the rest of the essay, however, you will introduce and use technical terms. In the abstract you should avoid them in order to make the result comprehensible to all.

You may like to repeat the abstract in your mother tongue.

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1. Introduction

Particle Physics or sometimes called High Energy Physics: [Jan: No need to put colon in math mode. Also: This colon (and the preceding clause) is not grammatical.] is the field of physics that pursues the ultimate structure of matter, this [Jan: is] possible in two ways, one is to look for elementary particles, the ultimate constituents of matter at their smallest scale and the other is to clarify what interactions are acting among them (*forces*) [Jan: Never use math mode to change font of text. Probably you want something like `\emph{}` here.] to construct matter as we see it. [Jan: There are a couple of grammatical mistakes in this sentence. Also, it is very long.]

There are four forces in nature, gravity, electromagnetism, weak nuclear force and strong nuclear force. In this essay our work will be on the strong nuclear force.

1.1 Strong Force

The gauge bosons that mediate this interactions [Jan: "this interaction" or "these interactions"] are eight gauge bosons called gluons, which come from the gauge group $SU(3)$ which has eight generators, gluons mediate the interaction between quarks.

[Jan: This is a thesis, not a research paper. Therefore, it needs to have an expository part. You shouldn't throw around terms like "gauge bosons", "gauge group" or even "gluons" and "quarks" without some introduction. Think of it like this: Dedan should be able to understand the first three pages of your thesis.]

[Jan: In particular, I think it would be helpful if you explained relations between all the particle names (hadrons, gluons, leptons, quarks, bosons etc.)]

Gluon is massless spin 1 particle, carrying charge called colour charges, glouns [Jan: s/gloun-s/gluons (here and elsewhere)] look like photons but photons do not carry electric charge, [Jan: Is colour charge the same as electric charge? It is not clear to me from this sentence.] because of this glouns can interact among themselves unlike photons. Normally the range of the force can be calculated by a simple argument of the uncertainty, [Jan: What is "argument of uncertainty"? Do you mean uncertainty principle or something else?] but this not the case for the strong force, the strong force is more complicated and involves a concept known as confinement.

The colour charge has strange property that it exerts a constant force that binds colour carrying particles together, this can be visualized using the analogy of a rubber band, the stronger you pull on the rubber band the tighter it feels. If you do not pull on it at all, it hang loose.

[Jan: s/hang/hangs] [Jan: It is not usual to address the reader in 2nd person in research texts. 3rd person or passive voice are more common. Personally I don't think it's a big deal, though.] The same thing happens for the particles, that means at a very short distance, the force is relaxed and the particles behave as a free particles, [Jan: s/a free particles/free particles] as the distance between them increases the force act [Jan: s/act/acts] like a rubber band, the force get [Jan: s/get/gets] them back in stronger pulling [Jan: You got lost in the grammar somewhere around this point.] and when the rubber band is stretched beyond its limits then it will cut into many pieces producing

more hadrons. [Jan: Future tense at the end is inconsistent with the rest of the sentence.] [Jan: Another long sentence. Consider dividing it into simpler parts.] This phenomenon is known as the colour confinement. In other words those particle [Jan: s/particle/particles] tend not be [Jan: s/not be/not to be] separated by a macroscopic distance. [Jan: What about when the rubber band "breaks"? Can the macroscopic distance be achieved then?] This limits the range of strong force, which is believed to be on order of $10^{-15}m$, [Jan: Google "siunitx" to see how to write units in Latex.] the dimension of a nuclear particle.

The theory which describes this force is called *Quantum Chromodynamics*. [Jan: Citation for this section would be nice.]

1.2 Quarks

Quarks and leptons are the building blocks which build up matter. In the present [Jan: In the present?] there are six "flavours" of quarks, these are up, down charm strange, top and bottom, and can written as [Jan: s/can written/can be written]

$$\begin{pmatrix} \text{up} \\ \text{down} \end{pmatrix}; \quad \begin{pmatrix} \text{charm} \\ \text{strange} \end{pmatrix}; \quad \begin{pmatrix} \text{top} \\ \text{bottom} \end{pmatrix}$$

[Jan: What do you mean they can be written like that? What those matrices (vectors?) mean?] Quarks can successfully a count [Jan: s/a count/account] for all known mesons and baryon, where mesons are consist [Jan: s/are consist/consist] of quark and anti quark and baryons are consist of three quarks or three anti quarks. [Jan: Maybe give examples of mesons and baryons?]

Quarks carry colour quantum number: red, green or blue, since all hadrons are colour charge neutral, they must have white charge. [Jan: What is the meaning of white charge? How does it compare to other colors?]

Unlike other elementary particles quarks carry fractional charge and posses new quantum number. [Jan: s/posses/possess] [Jan: What is quantum number?] The next table summarize [Jan: s/summarize/summarizes] some properties of quarks. Each quark flavour is associated with an own quantum number, which is conserved in strong and electromagnetic interactions but not in weak interaction. These are:

-Strangeness: $S = -1$ for s-quark.

-Charm: $C = +1$ for c-quark.

-Beauty: $\tilde{B} = -1$ for b-quark -t-quark has life time too short to form hadrons

-up and down quarks have nameless flavor quantum numbers.(Nagashima and Nambu, 2010)

[Jan: What exactly is conserved here? Are these six values or just one?] [Jan: No manual lists, use itemize.]

[Jan: Please refer to the table from the text.]

[Jan: Maybe extend the table to explain other properties of quarks?]

Name (flavour)	symbol	charge(in units of e)	mass
Up	U	$+\frac{2}{3}$	$1.7 - 3.1 \frac{MeV}{c^2}$
Down	d	$-\frac{1}{3}$	$4.1 - 5.7 \frac{MeV}{c^2}$
charm	d	$+\frac{2}{3}$	$1.18 - 1.34 \frac{GeV}{c^2}$
strange	d	$-\frac{1}{3}$	$80 - 130 \frac{MeV}{c^2}$
top	d	$+\frac{2}{3}$	$172.3 - 173.5 \frac{GeV}{c^2}$
bottom	d	$-\frac{1}{3}$	$4.13 - 4.37 \frac{GeV}{c^2}$

Table 1.1: properties of Quarks

[Jan: Citations: If section(s) are based on a textbook, cite it in the beginning, saying something like "the following presentation is based on \cite{...}"]

[Jan: You can see you have a lot of language problems. You have to read what you wrote, possibly several times. You have to read every sentence, correcting typos, making sure it is grammatical and meaningful and that the meaning is the intended one. Pay special attention to -s endings in 3rd person.]

[Jan: From now on I will not be marking minor language mistakes, you have to find and correct them yourself.]

1.3 Large Hadron Collider

Hadron colliders are devices made to explore the world of particle physics, they work as theories testers and also as a discovery machines, an example of these hadron colliders is the LHC in CERN. In the Large hadron collider [Jan: Rule for capitalization: if given name, capitalize (almost) all words, otherwise don't capitalize. So, "hadron collider" (as a type of device), but "Large Hadron Collider" (the one at CERN).] two beams of hadrons(protons) [Jan: Put space before opening bracket.] are being accelerated to a high kinetic energy and then collided with each other. It has started operation in 2009 and in 2013 the Higgs particle has been discovered in LHC. Most of the interesting physics at LHC involves investigating the results of these interactions(collisions), as a result of this collision stable partons are formed, partons consists of quarks and gluons. Because of the complex nature of the event at the hadrons colliders, the description of the final state involves a multi-particle calculations. The accurate prediction of the final state in hadron colliders is still one the hardest problems, this problem roots to the non-abelian nature of QCD, which leads to a colour confinement at a long distances. The two main problems are the description of the hadron formation and the evolution of QCD final states from short to long distances. Those problems can be tackled to a good approximation by Monte-Carlo event generators.

[Jan: Consider splitting text above into paragraphs.]

1.4 The Parton Shower

In general parton showers are approximations of the higher order real emission corrections to the hard scattering. The word hard here means, [Jan: Put "hard" in quotes.] the process involves a transfer of large momentum, either a violent scatter or creation of large mass. They locally conserve flavour and four momentum, and also they are consistent, which means, the particle either splits into two or not. Since the parton showers are simulate of the branching and splitting processes, the quality of their predictions depend on precise is the implementation, for example one can ensure the colour coherence through selecting an evolution variable representing the angular ordering, although, this in not the only choice to ensure the colour coherence (Höche, 2014). [Jan: "higher order real emission corrections", "evolution variable representing the angular ordering", don't use lots of complicated words without explaining them.] In the following we will at a simple implementation of a parton shower in python.

1.5 Hadronization

To reflect the colour neutrality of the particles in our model the partons will be transformed into a stable hadrons which are colour neutral, this process is called hadronization. The first implemented model and also follows Monte-Carlo event generators was Feynman-Field Model, which gives an idea of the formation of the mesons through iteratively from a single quark. However, this model is not collinear safe, [Jan: Explain what "collinear safe" means.] which means the model can mix the short and long distance physics. Now a days two models are common, the string model and clustering model(Höche, 2014).

1.6 Monte Carlo Method

Monte Carlo method is set of mathematical tools that first used by scientist to generate game chance that its behaviour can be used to study interesting phenomena. In our context the definition of Monte Carlo method would be, that in which the usage of random numbers is deliberate in a calculation that involves a sequence of states whose evolution can be considered random(Höche, 2014). [Jan: I did not understand any of the two sentences above.]

1.6.1 Pseudo-random numbers. In a computer these are generated using a deterministic algorithm that generates a set of numbers that exhibits statistical randomness, those numbers are called pseudo-random.[the book]

The main properties of a good random generator are:

- **Long period:** the pseudo-random generators have a finite range after which they begin to repeat themselves, for a good random generator this range should much longer than the amount of numbers that are needed for the calculation.

- **Randomness:** The numbers should follow the uniform distribution and also should be independent of each other. [Jan: This is not correct. Pseudo-random numbers are, by definition, deterministic (so not random). What you want is that they “look like” i.i.d. uniform random numbers in all important respects (where “important” is not well-defined and depends on what exactly you are doing).]
- **Repeatability:** The same initial values should give the same sequence of a random numbers, this important for testing because one might need to repeat the calculation or in the coding terms debugging.
- **Portability:** One should be able to generate the same sequence in different machines.
- **Fast:** the generation of the pseudo-random numbers should not be time consuming (Weinzierl, 2000).

[Jan: I don't think it is necessary that you dwell on pseudorandomness, but once you mentioned it, please explain how it connects to Monte Carlo.]

1.6.2 Samples with different PDFs. Generating samples of different probability distribution function(pdf) is essential since we are simulating various variables that have different numerical behaviour. For example if our pseudo-random numbers are uniformly distributed in the interval $[0,1]$ and a sequence of specified distribution is needed. I will mention two methods, which are used in this essay. [Jan: Be clearer what you are trying to achieve. You want to sample from given distribution provided you can sample from another distribution (for example uniform over $[0,1]$).]

1.6.3 The acceptance-rejection method. The acceptance-rejection method was developed by von Neumann. [Jan: Citation please.] Assume we have a sample x which is uniformly distributed and the pdf is denoted by $f(x)$, [Jan: What do you mean that x is uniform?] and the unknown pdf is denoted by $p(x)$, [Jan: What do you mean $p(x)$ is unknown?] we assume that we can enclose $p(x)$ inside a shape that is $f(x)$. [Jan: Use math mode for mathematical symbols.]

In simple words, one first generates x according to the uniform distribution [Jan: Uniform over what?] and then calculates $p(x)$ and $f(x)$, then generates another number which is also uniformly distributed, [Jan: How is this another number called? It is uniform over what?] and checks $y \times f(x) \leq p(x)$. If this the case then accept x , if not reject x and start again. [Jan: OK, but why does it work? Maybe give example when $f(x)$ is uniform over $[0,1]$?] [Jan: Split paragraphs here.] Example of this method is calculation of π , assume we have a box of side length D and a circle of diameter D inside that box, now the probability that a point in the box and is also in the circle is approximately the area of the circle over the area of the box, which is π over 4 here, hence, from this we can approximate the value of π . The histograms in Figure 1 exhibits this calculation and also the error in the calculation (Weinzierl, 2000). [Jan: I wouldn't call it “example” of what you described before. It is a related thing, but an example should give specific f and p .]

[Jan: The pi figure: 1) Caption would be nice. 2) Why uncertainty does not go to 0? Can you explain this?]

1.6.4 The inverse transform method. Let x be a random with pdf $p(x)$ that takes values in the interval $[0,1]$ and let y be a random variable that is uniformly distributed in the interval $[0,1]$.

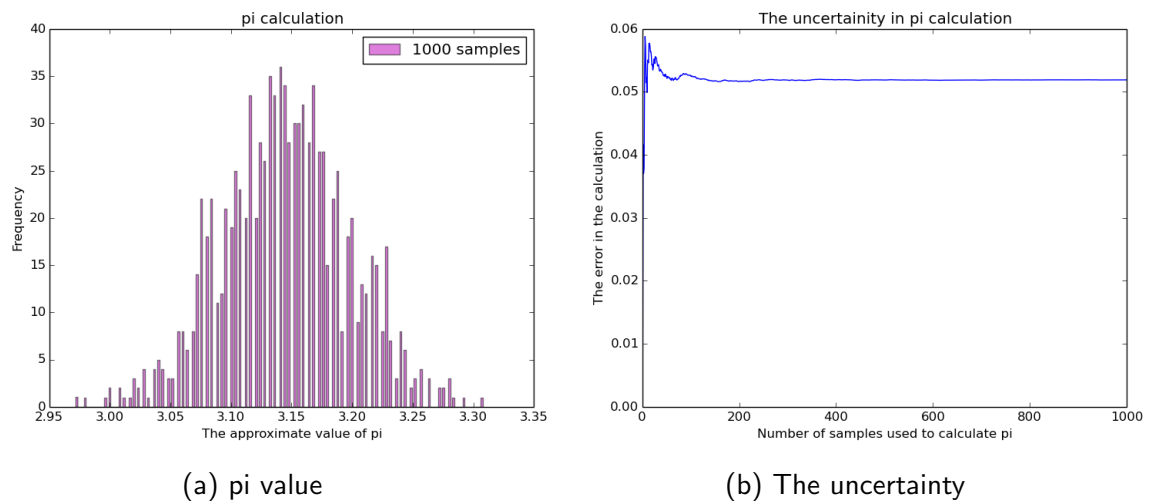


Figure 1.1

We set

$$x = p^{-1}(y) \quad (1.6.1)$$

[Jan: You need to put braces for superscripts, like this: p^{-1}] One application of this method is the importance sampling, in which certain values are preferred (Weinzierl, 2000). Example of this is generating an angle θ in the interval $[0, \pi/2]$ that is inversely distributed, [Jan: "Inversely distributed"? What does that mean?] where the small angles are desired. The histograms in figure 2 demonstrate this and the error in the calculation. [Jan: What do they demonstrate? What exactly is the distribution?]

[Jan: Why does this work? Consider writing formula for cdf to give the reader an idea. Also, in this method cdf is used, not pdf as you write.]

[Jan: Consider giving a file name (files will be attached as appendix) whenever you are presenting a calculation, like with the histograms here.]

1.6.5 The module random in python. The samples above were generated using *python* module random, here are few words about this module. [Jan: Don't use math mode to write Python. Use capital letter. For module, variable and file names you can use verbatim command like this: `\verb+my_file.py+`].

This module implements pseudo-random number generators for various distributions.

For integers, there is uniform selection from a range. For sequences, there is uniform selection of a random element, a function to generate a random permutation of a list in-place, and a function for random sampling without replacement.

On the real line, there are functions to compute uniform, normal (Gaussian), lognormal, negative exponential, gamma, and beta distributions. For generating distributions of angles, the von Mises distribution is available.

Almost all module functions depend on the basic function `random()`, which generates a random

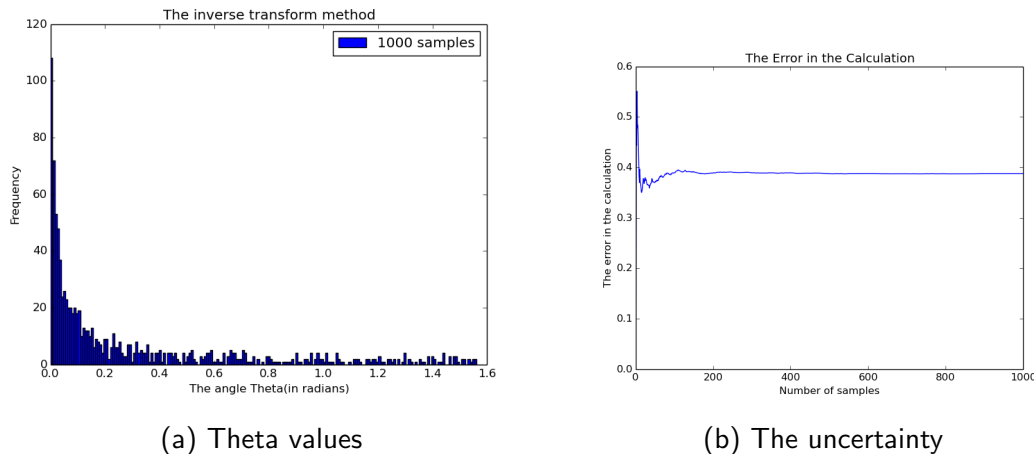


Figure 1.2

float uniformly in the semi-open range $[0.0, 1.0)$. Python uses the Mersenne Twister as the core generator. It produces 53-bit precision floats and has a period of $2^{19937} - 1$. [Jan: Please use mathematical notation instead of Python code.] The underlying implementation in C is both fast and threadsafe. [Jan: What does “threadsafe” mean? Explain or drop this.] The Mersenne Twister is one of the most extensively tested random number generators in existence.

[Jan: Where does this section come from? Citation please.]

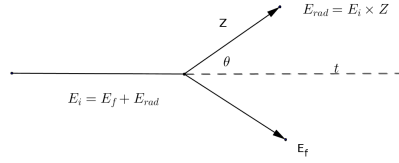
1.7 Parton Shower Simulation

The following is a simple simulation in python for a splitting of a single quark, the simulation accounts for the four momentum conservation of the soft particles. [Jan: Why only soft?]

1.7.1 Physical description. As a result of a hadrons collision, quarks will fly away. Since they are charged particles(colour charge), the moving quarks will radiate. The quark will lose part of its energy emitting a gluon. If at the beginning, the quark had energy E_i and radiates energy E_{rad} , then the gluon takes $\frac{E_{rad}}{E_i}$ of the particle's initial energy. [Jan: You mean it takes E_{rad}/E_i fraction of initial energy.] It is radiated at an angle of θ to the quark initial direction. The quark will fly on radiating another gluon and so on until it becomes stable(the hadronization starts). [Jan: Put space before opening parenthesis (here and elsewhere).] The radiated gluons will decay into two quarks, which will later radiates gluons, which will radiate gluons again and so on. The result is a shower of partons decaying into two partons.

[Jan: You need to make clear that in your model you do not make distinction between quarks and gluons.]

[Jan: The picture with collision is way too small. Also caption is missing.]



259 Now we begin with 4- momentum vector

$$P^\mu = \begin{pmatrix} E/c \\ p_x \\ p_y \\ p_z \end{pmatrix}; P_\mu = \begin{pmatrix} E/c \\ -p_x \\ -p_y \\ -p_z \end{pmatrix} \quad (1.7.1)$$

260 And the inner product is given by

$$P^\mu P_\mu = P^\mu \eta_{\mu\nu} P_\nu = (m_0 c)^2 \quad (1.7.2)$$

$$P^\mu P_\mu = \left(\frac{E}{c}\right)^2 - (p_x^2 + p_y^2 + p_z^2) = (m_0 c)^2 \quad (1.7.3)$$

262 Now considering the natural units $c = 1$ this can be written as

$$m_0^2 = E^2 - \|p\|^2 \quad (1.7.4)$$

263 Which is lorentz invariant quantity i.e does not depend on the frame. In our case the quarks in
 264 (LHC), the mass of the quark ~ 1 Mev and the energy of the hadrons is ~ 1 Tev, hence, we can
 265 assume that the mass of the quark(1.7.4) is 0.

266 From the conservation of energy and momentum, [Jan: Don't put blank line before the equation
 267 here.] !

$$P_i^\mu = P_f^\mu. \quad (1.7.5)$$

268 We assume that the intial patricle is moving in x -direction, we can write the initial 4-momentum
 269 as $P_i^\mu = (E, E, 0, 0)$, afterwards the particle will split. Therefore, the final momentum is given
 270 by $P_{rad} + P_{part}$, part here refers to the particle which has lost part of its energy, so $P_{rad} =$
 271 $(E_{rad}, \cos\theta \cdot E_{rad}, \sin\theta \cdot E_{rad}, 0)$. From this, and since the particle is rotated with θ we can
 272 find the direction of the particle(part) with help of the rotation matrix [Jan: Please make clear
 273 that the other particle will have non-zero mass.] !


$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \quad (1.7.6)$$



274 The direction of the particle of the fraction enregy can be found using the following equation

$$P_{part} = P_i - P_{rad}. \quad (1.7.7)$$

275 In a three dimensional world, we have two rotation angles, the angular angle and the azimuthal
 276 angle. Given a unit vector $u = (u_x, u_y, u_z)$, where $u_x^2 + u_y^2 + u_z^2 = 1$, the matrix for rotating this
 277 particle by angle θ about an axis in the direction of u is

$$\begin{pmatrix} \cos \theta + u_x^2(1 - \cos \theta) & u_x u_y(1 - \cos \theta) - u_z \sin \theta & u_x u_z(1 - \cos \theta) + u_y \sin \theta \\ u_y u_x(1 - \cos \theta) + u_z \sin \theta & \cos \theta + u_y^2(1 - \cos \theta) & u_y u_z(1 - \cos \theta) - u_x \sin \theta \\ u_z u_x(1 - \cos \theta) - u_y \sin \theta & u_z u_y(1 - \cos \theta) + u_x \sin \theta & \cos \theta + u_z^2(1 - \cos \theta) \end{pmatrix} \quad (1.7.8)$$

278 [Jan: OK, but you have to explain 3D case in detail. Consider making separate sections for 2D and
 279 3D. Picture showing what is angular (is this correct name?) and azimuthal angle would be helpful.] 

280 **1.7.2 Implementation in python.** First we will start with case of 2 dimensions model which
 281 accounts for one rotation matrix θ and it evolves generation of two random numbers, one them
 282 represent the angle and the other regards the energy of the radiated particle. Here both the
 283 energy fraction z and the angle θ are following the distribution $1/x$, where the former lies in the
 284 interval $[0.25, .75]$ [Jan: No, it does not lie in $[0.25, 0.75]$. Give the correct distribution. Also mention
 285 the cutoff values you are using (to avoid singularity at $x = 0$).] [Jan: Put all of numeric intervals in
 286 math mode.] and the later in the interval $[0, \pi/2]$ and they are generated by applying the inverse
 287 transform method on a set of numbers that are uniformly distributed. 


288 As for the four momentum vector, the module *numpy* is used for this purpose, here we use the
 289 object *array*. Numpy is a scientific computing package in python which is widely used for these
 290 purposes, beside that it has powerful N-dimensional arrays it also has useful linear algebra tools
 291 and random number capabilities.

292 following the physical description a list contains the four momentum of the initial particle was
 293 defined, then we assumed that the particle has initial energy = 1 energy unit, since the particle
 294 will split after certain distance an assumption of the distance before the decay was made is that
 295 the particle will move a distance of 1 unit and then it will decay, basically it is an iteration process,
 296 at the beginning we check the energy of the particle if it is a above the stability limit, which
 297 we assumed to 0.09, this particle will split, the direction of the radiated will follow the θ and its
 298 energy will be given from z , and then both new particles four momenta will be add to the list
 299 at the beginning and again those particles will be checked, now if the particle has energy that is
 300 equal or below the stability limit then the iteration process will be terminated.

301 As for plotting the results, the library *matplotlib* was used which is a library that is used to
 302 make 2 D plots in Python, as *matplotlib* has the ability to add many lines at once, here the
 303 Linecollection is used, which is a package in *matplotlib*, the diagram in figure 3 shows the 2 D
 304 simulation of the parton shower.

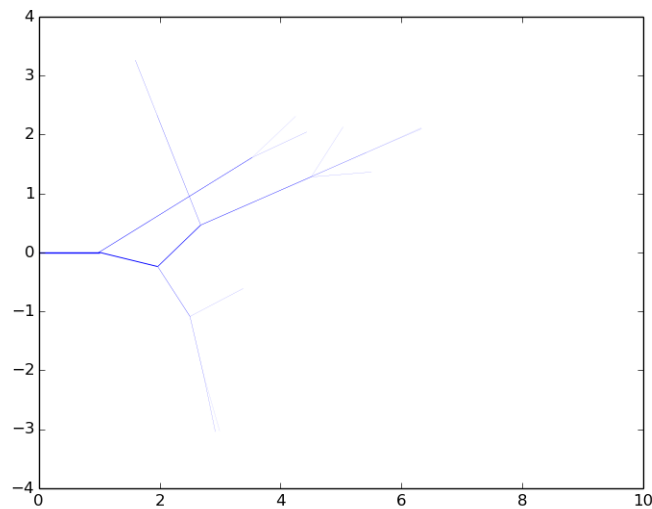


Figure 1.3: 2D simulation of the splitting of a single parton, here the colour fades as the energy decreases

The 3D simulation of the parton shower is essentially the same as for python code with few changes, in which now we have two rotation angles, θ and also ϕ which is the azimuthal angle which is uniformly distributed in the interval $[0, 2\pi]$. To simulate the rotation in three dimensions, the function `normv(u)` and function `rotation(v, angle)`, the former returns the axis of the rotation, the function input and the vector $[1, 1, 1]$ from a plane, from which we find a vector that is orthogonal to this plane, and the later is matrix of rotation, it takes the angle rotation and the axis of rotation as inputs.

Also here z (the energy fraction) now lies the interval $[0, 1]$ and the stability limit is 0.05. The digram in figure 4 shows the 3D simulation of single parton splitting.

[Jan: This description (both 2D and 3D case) should be way longer, more organized and more detailed. It should take several pages and you should explain what the code is doing precisely and in detail.]



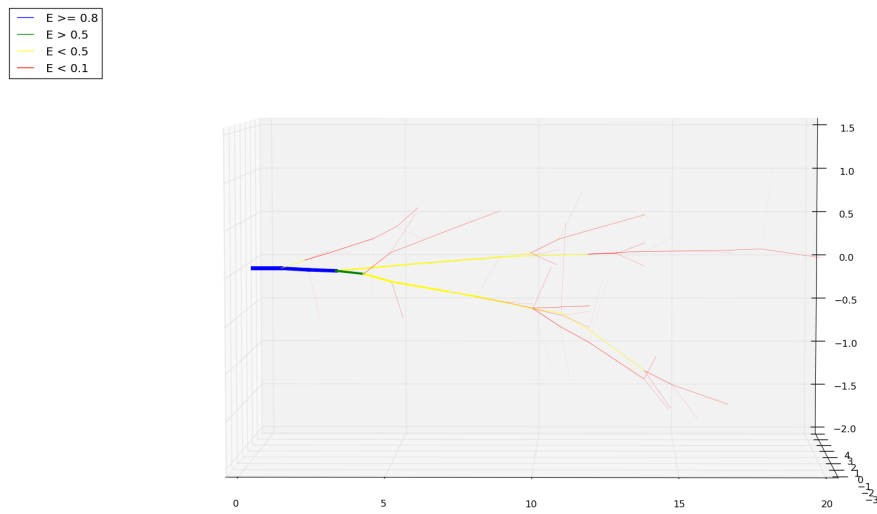


Figure 1.4: 3D simulation of the splitting of a single parton

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$$(3.1.1)$$

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3.3 This is a section

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5. Testing

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