

# Jets in Switzerland

By

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# DECLARATION

This work was carried out at AIMS Rwanda in partial fulfilment of the requirements for a Master of Science Degree.

I hereby declare that except where due acknowledgement is made, this work has never been presented wholly or in part for the award of a degree at AIMS Rwanda or any other University.

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# **ACKNOWLEDGEMENTS**

- 15 This is optional and should be at most half a page. Thanks Ma, Thanks Pa. One paragraph in  
16 normal language is the most respectful.
- 17 Do not use too much bold, any figures, or sign at the bottom.

# <sup>18</sup> DEDICATION

<sup>19</sup> This is optional.

20

# Abstract

21

At the Large Hadron Collider highly energetic protons are being collided in order to study the subatomic physics. As a result of this collision the proton constituents (quarks and gluons) are begin produced. [Jan: s/begin/being] However, in practical we can not see them. [Jan: Rephrase



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to sth like: "However, we cannot detect them directly."] The quarks and gluons undergo a showering process and then hadronize. In order to understand them a computational tools were built. These are the jet algorithms. [Jan: In order to understand what. I would add a sentence or two and rephrase.



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Last sentence not grammatical.]



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[Jan: Please summarize what your thesis does in a sentence or two.]



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# 1. General Introduction

[Jan: General comment: It feels to me you deleted a lot of text which either was good or needed changes, not deleting.]

[Jan: I understand that Sam brought up the issue of plagiarism, but please do not react in this way. You are allowed to write things that you learnt, both from literature and Sam's tutorial. The only thing that is (rightly) required is that you put them in your own words.]

[Jan: At the risk of repeating myself for 100th time: If you first read and then write "closed-book" (without looking at what you read while writing), you will be safe from plagiarism accusations.]

High energy particle physics is the field of physics that studies the fundamental structure of matter. This can be done in two ways, one is to look for elementary particles, the building blocks of matter at their smallest scale. The other is to define what interactions are acting among them (*forces*) to build the matter as we see it. In order to study the fundamental structure of the matter, we require tools. For this purpose scientists have built the particle accelerators such as the Large Hadron Collider.

## 1.1 Large Hadron Collider

[Jan: You could say a bit about previous colliders and their achievements. You could mention where CERN is.]

[Jan: You could mention how long ago Higgs boson was proposed and how big deal it was to confirm it (it was the last particle in SM to be verified, no?), citing appropriate papers.]

Hadron colliders are devices made to explore the world of particle physics, they work as theories testers [Jan: s/theories/theory] and also as discovery machines, an example of these hadron colliders is the Large Hadron Collider (LHC) at CERN (Evans and Bryant, 2008).

Among the main missions of the LHC is the investigation of the properties of the Higgs boson. The importance of the Higgs boson stems from investigating the validity of the standard model suggests that the masses of the elementary particles are generated through the Higgs mechanism (Nagashima and Nambu, 2010). In July 2012, the ATLAS and CMS experiments at CERN's Large Hadron Collider announced that they have had each observed new particle in the mass region around 126 GeV (Aad et al., 2012). This particle is consistent with the Higgs boson predicted by the standard model.

Inside the Large Hadron Collider,  $10^{11}$  protons are being accelerated to a high kinetic energy and then collided up 40 million times per second to provide 14 TeV proton-proton collision. The LHC also collides heavy ions. Two general purpose detectors, ATLAS (A Toroidal LHC ApparatuS) Aad et al. (2008) and CMS (Compact Muon Solenoid) Collaboration (2008) have been built for probing proton-proton and ion-ion collisions.

Most of the interesting physics at LHC involves studying the results of these interactions (colli-



sions). It has been observed that as a result of these collisions, stable partons are formed, partons consists of quarks and gluons. From experimental point of view, these collisions lead to the production of these partons along side the production of another particles like the (*Higgs*) boson. However, new particles are not detected. The study of quarks and gluons in LHC is challenging because of the production of a single quark or a gluon will actually appear in the detectors as many particles collimated in the same general direction, all arriving at once. The detection of a collimated flow of particles of this nature is called a **jet** clustering (Ellis et al., 2008).

## 1.2 Quarks and Gluons

[Jan: Say what are quarks first (you used to do that...)]

There are six "flavours" of quarks, these are up, down charm strange, top and bottom. Quarks can successfully account for all known mesons and baryons which are known as hadrons, hadrons are colour charge neutral particles. Quarks carry colour charge: red, green or blue, the colour name here is an analogy that is famously used among physicists to describe a three kind of generating force. The colour charges are called by their number or any other index. We can also we think of it as a three primary additive colours.

The other partons (gluons) mediate the interaction between quarks.

Gluon is a massless spin 1 particle, carrying charge called colour charges as well as the quarks, because of of the colour charge gluons can interact among themselves. [Jan: Grammar. "Similar to quarks, they carry color charge..."]

The theory that describes the colour charge of the quarks and gluons is called (*Quantum Chromodynamics*). The colour charge has strange property that it exerts a constant force that binds colour carrying particles together, this can be visualized using the analogy of a rubber band, the stronger you pull on the rubber band the tighter it feels. If we do not pull on it at all, it hangs loose. The same thing happens for the particles, that means at a very short distance, the force is relaxed and the particles behave as free particles. As the distance between them increases, the force acts like a rubber band and pulls them back stronger. When the rubber band is stretched beyond its limits, it cuts into many pieces producing more particles. This phenomenon is known as the colour confinement. In other words, these particles tend not to be separated by a macroscopic distance (Nagashima and Nambu, 2010).

Due to the complex nature of the event, event in this context refers to the result of the particle interaction (collision), at the Large Hadrons Collider, the description of the final state involves a multi-particle calculations. [Jan: Don't define what event is mid-sentence.] Monte Carlo event generators provide a good approximation of collisions. [Jan: Since "direct" calculations are computationally difficult/expensive, right?]

## 2. Monte Carlo Computational Techniques

The name Monte Carlo method is a set of mathematical tools that was first used by scientists working on nuclear projects in Los Alamos. [Jan: Better say that the name “Monte Carlo method” was first used. Your statement is arguable.] The essence of this is to generate numbers with probability that can be used to study physical phenomena. In our context, the definition of Monte Carlo method is the use of randomly generated numbers to imitate a physical behaviour that is considered to be random (quantum mechanical) (Kalos and Whitlock, 1986).

One of the most important components of the Monte Carlo method is generating samples of different probability distribution functions (pdf). This is essential since we are simulating various variables that have different numerical behaviour. For example if our pseudo-random numbers are uniformly<sup>1</sup> distributed in the interval  $[0, 1]$  and instead we need numbers that have normal distribution restricted to the same interval. [Jan: Grammar. Usual notation is  $U(0, 1)$ .] This part of the essay discusses two methods which are widely used. [Jan: ...for that purpose.]

### 2.1 The Inverse Transform Method

The inverse transform method is used for generating random numbers that are distributed according to a specific distribution.

[Jan: Make clear that you want to sample according to  $p(x)$  and that you assume you can make uniform samples.]

Let  $x$  be a random variable distributed with probability density function  $p(x)$ . Let  $u$  be a random variable that is uniformly distributed in  $[0, 1]$  and  $P(x)$  is the cumulative density function (CDF), [Jan: CDF of what?] then we set

$$x = P^{-1}(u) \quad (2.1.1)$$

(Weinzierl, 2000).

The following pseudo-code shows the inverse-transform method algorithm

Generate a uniform random  $u$  in  $[0, 1]$

**return**  $x = P^{-1}(u)$

[Jan:  $u$  uniform in  $[0, 1]$ ]

The inversion method is exact when an explicit form of the CDF is known. The CDF is a continuous and a strictly increasing function. It can be obtained either analytically through the integration of the PDF or numerically. Sometimes finding the CDF is computationally difficult, this causes the inverse transform method to become less efficient (Devroye, 1986).

---

<sup>1</sup>Uniform means that every value in the range of the distribution is equally likely to occur. This distribution is widely used for generating random numbers for other distributions, it is denoted by  $U$ .

## 2.2 The Accept Reject Method

As the inverse transform method, the accept reject method also is used for generating numbers according to a specific distribution.

Let  $x$  be a uniformly distributed variable in the interval  $[0, 1]$ , which is the variable of interest. A different distribution of  $x$  is required, let  $p(x)$  denotes the pdf of the required distribution. Another random uniformly distributed variable  $y$  is generated, this will serve as the accept reject tool. First we calculate the maximum of  $p(x)$ , hence the  $y$  is generated in the interval  $[0, p(x)_{max}]$ . Now we check if  $y \leq p(x)$ . If this is the case, accept  $x$ , otherwise reject and start again.

```

 $x \leftarrow$  uniform in  $[0, 1]$ 
 $y \leftarrow$  uniform in  $[0, p_{max}(x)]$ 
if  $y \leq p(x)$  then
    accept  $x$ 
else
    Reject
end if
Begin again

```

[Jan: I think it's better if you use loops instead of "begin again". Make clear that "accept" means "return  $x$  as our sample" and "reject" means "do nothing".]

For example, if we have a number  $x$  that falls under uniform distribution and we want to reshape this so that we get a number that has the distribution  $\frac{1}{x}$ , then we find the maximum value of probability density function ( $p(x)$ ) for  $x$ . Here, we add small number to  $x$  so that we avoid the singular point when  $x = 0$ . After that we generate another sample  $y$  that is also uniformly distributed in the interval  $[0, p_{max}]$ . Now we check if  $y$  is less than  $p(x)$ . If so, we add  $x$  to our distribution, if not we start again (Weinzierl, 2000).

The histogram in figure 2.1 demonstrate the example above. [Jan: In the figure: What is  $\epsilon$ ?]

Beside the fact that the numbers which are generated from Monte Carlo method are not truly random, whereby the numbers are generated through a deterministic algorithms (Kalos and Whitlock, 1986). [Jan: Pseudorandomness has to do with computers, not Monte Carlo specifically. It is possible to run MC with "real" randomness.] Monte Carlo method also faces another challenge which it requires generating large samples to give sensible results. The histograms in figure 2.2 demonstrate that, in which  $\pi$  is calculated using numbers generated using Monte Carlo method. One sees that the error in the calculation when is decreasing as the number of the samples is increasing. [Jan: Decreasing how fast?]

[Jan: Why not have a separate section on  $\pi$ ? You used to write more about this.]

[Jan: Why not compare accept-reject and inverse transform? Without such discussion what is the point of presenting two methods to do the same thing?]

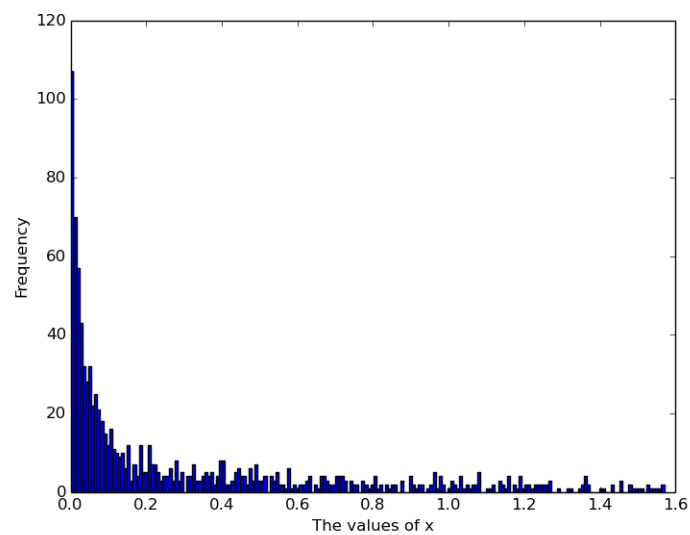
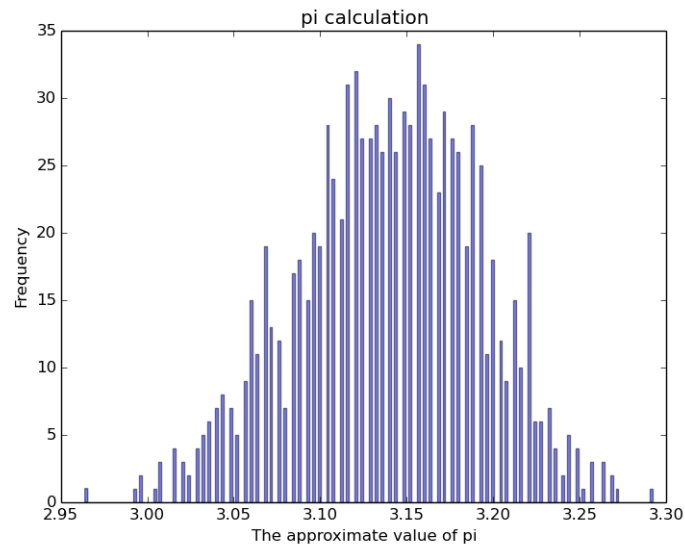
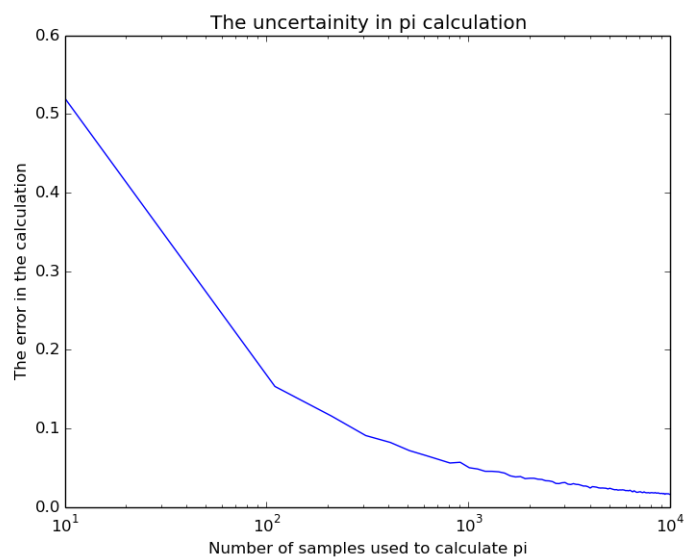


Figure 2.1: Sampling 1000 numbers with distribution  $\frac{1}{x+\epsilon}$  using accept-reject method, where  $\epsilon$  is a small number added to avoid the singularity.



(a) pi value



(b) The uncertainty

Figure 2.2: These histograms show the values of  $\pi$  (a) and the error that are generated from sampling 1000 numbers (b).

## 3. The Parton Shower

Parton showers are approximations of what is seen in the detectors (stable particles) to the hard scattering. The word "hard", means the process involves a transfer of large momentum, in which the partons are splitting and branching. They locally conserve flavour and four momentum, and also conserve the probability, in a sense that, the particle either splits into two or not. [Jan: Can you clarify "conserve probability" part? What are other choices for the particle?]

Since the parton showers are simulations of the branching and splitting processes, the quality of their predictions depend on how precise is the implementation. For example, the angular ordering, which accounts for knowing the prehistory of the splitting particles, *i.e.* [Jan: Don't write *i.e.*, in math mode. One way to write it is how I just did it, that is first full stop, then comma at the end.] knowing exactly from which partons are they coming from. (Höche, 2014). [Jan: I didn't understand this sentence.]

### 3.1 The Parton Shower Model

The parton shower model is built around the idea of successive one-to-two splittings, which are combined together forming a tree like sequence. [Jan: s/tree like/tree-like]

The Monte Carlo model of the parton shower can be described as a sequence of stochastic and deterministic processes.

The deterministic stage is where the particle is produced with some four momentum and travels for a finite time and distance. After the time distance travel, the particle will split into two, soft particle (radiated) with angle  $\theta$  taking  $Z$  fraction of the initial particle energy,  $\theta$  and  $Z$  represents the stochastic part of the model. [Jan: Rephrase: "... will split into two, with one of the particles (which we call "radiated" or "emitted") having angle  $\theta$  with respect to the original particle and taking  $Z$  fraction..."]

There is one QCD approximation that will be useful regarding  $\theta$  and  $Z$  distributions. This is the *soft* and *collinear* approximation. *Soft* implies that an emitted particle has a very little energy compare to the particle that emitted it. *Collinear* means that it is emitted with angle very small relative to another particle in the event (Salam, 2010b). With each splitting the particle loses energy, at some point the particles will have low energy and then begin to stabilize (hadronize) leading to a colour neutral particles (hadrons).

To reflect the colour neutrality of the particles in our model the partons will be transformed into a stable hadrons which are colour neutral. This process is called hadronization. The process in which this happens is not quantitatively well understood. For this model, a very simple approach is taken, where a direct translation between each parton and hadron is made. To achieve this, a threshold energy is defined. Below it, the partons will begin to hadronize or stop splitting (Salam, 2010b). [Jan: This is still too close to wording in Sam's tutorial.]

## 3.2 Parton Shower and Hadronization Simulation

The following is a simple simulation for a splitting of a single particle into two. The parton shower simulation describes the parton shower and hadronization. The general idea of the two processes, parton shower and the hadronization can be simplified by the following algorithm

```

Make list of particles  $L$ 
Add initial particle  $P_i$  to  $L$ 
Define stability limit (hadronization limit) to be  $S$ 
While  $L$  is not empty
  if energy of  $P_i \geq S$  then
    Split into two and add to list and check again
  else
    Begin hadronization
  end if

```

[Jan: Make proper loop, not "check again". "Begin hadronization" is not clear to me. I would say "return a hadron" or similar.]

This process can be simulated in two or three space dimensions, for the case of three dimensions, another quantity  $\phi$  is needed. [Jan: Another to what? Don't talk about  $\phi$  yet.]

## 3.3 Two Dimensions Parton Shower

[Jan: s/Two Dimensions/Two-Dimensional]

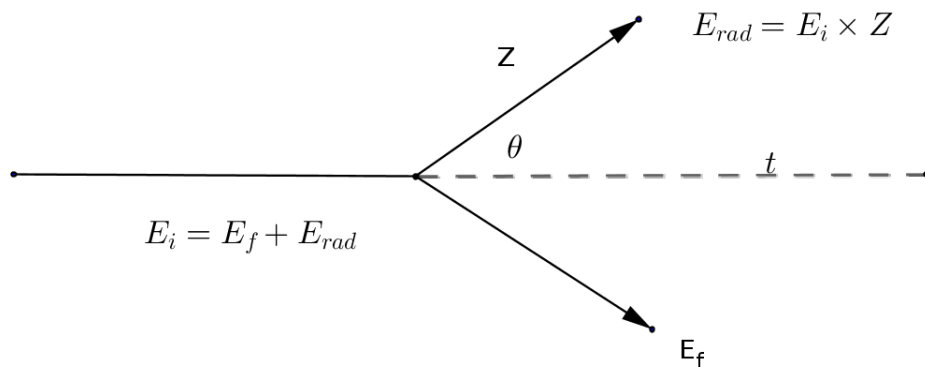


Figure 3.1: Illustration describing the splitting of a prticle into in two dimensions.

The two dimensional model accounts for one rotation with angle  $\theta$  (see figure 3.1). It evolves [Jan: s/evolves/requires (?)] generation of two random numbers, one of them represent the angle and the other represents the energy of the radiated particle. Here, both the energy fraction  $z$  and the angle  $\theta$  are following the distribution  $1/z$ , this comes from the QCD approximation that was mentioned in 1.2. [Jan: Are you sure this is correct ref? I would link two sections ago.] Where

the former lies in the interval  $[0, 1]$ . [Jan: Grammar.] To avoid the singularity at  $z = 0$ , a cutoff value of  $\epsilon$  is added to the denominator by setting  $p(z) = \frac{1}{\epsilon+z}$ . Considering the later in the interval  $[0, \pi/2]$ ,  $\theta$  is modified in a similar way as  $z$ . The variables  $\theta$  and  $z$  are generated by applying the accept-reject method on a set of numbers that are uniformly distributed. [Jan: Be consistent. Small or capital  $z$ ? Say what value of  $\epsilon$  you use.]

As for the kinematics description, the initial particle has four momentum  $(E, p_x, p_y, p_z)$ <sup>1</sup>, for simplicity we assume that the particle is travelling in  $x$  direction with energy  $E$ . Therefore, the four momentum which describes this particle is the following

$$P_i^\mu = \begin{pmatrix} E \\ p_x \\ 0 \\ 0 \end{pmatrix} \quad (3.3.1)$$

Where  $p_y$  and  $p_z$  are equal zero following our direction assumption. In a given splitting, with generated pair  $(\theta, z)$ , the kinematics of the radiated particle are determined.

To continue the showering process and to allow the model to proceed, it is necessary to determine the kinematics of the final state particle by applying the conservation of energy and momentum

$$P_i^\mu = P_f^\mu \quad (3.3.2)$$

And Since

$$P^\mu P_\mu = E^2 - (p_x^2 + p_y^2 + p_z^2) = E^2 - ||p|| = m_0^2 \quad (3.3.3)$$

[Jan: s/||p||/||p||<sup>2</sup>. Use \| for norm instead of ||.] Which is lorentz invariant quantity, *i.e.*, it does not depend on the frame. Here, we can make a simplifying assumption, in which we assume that the mass of the radiated particle is zero. This assumption is based on the fact that, in LHC the quark energy is  $\sim 1\text{MeV}$ , where [Jan: s/where/while] the hadron energy is  $\sim 1\text{TeV}$ . Now equation 3.3 will become

$$E^2 = ||p|| \quad (3.3.4)$$

(Salam, 2010b).

In describing the direction of the radiated particle after it is being produced, we use the rotation matrix in two dimensions which rotates the radiated particle with the angle  $\theta$

$$\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \quad (3.3.5)$$

From this we can determine the direction of the other particle. Here we will denote its four momentum by  $P_{part}^\mu$ . The final momentum  $P_f^\mu$  after the splitting is

$$P_{rad}^\mu + P_{part}^\mu \quad (3.3.6)$$

---

<sup>1</sup>The four momentum vector is usually written as  $(E/c, p_x, p_y, p_z)$ . Henceforth, we are considering the natural units in which  $c = 1$



275 and from the conservation equation 3.3 then

$$P_i^\mu = P_{rad}^\mu + P_{part}^\mu \quad (3.3.7)$$

276 Therefore,

$$P_{part}^\mu = P_i^\mu - P_{rad}^\mu \quad (3.3.8)$$

277 As in the algorithm 0, [Jan: I think you need captions for correct referencing of algorithms.] a  
 278 tunable parameter [Jan: s/parameter/energy threshold] is made to account for the hadronization.  
 279 The figure 3.2 shows a graphical representation of the two dimensions parton shower as explained  
 280 above. [Jan: Give value of your energy threshold so that the results can be reproduced.]

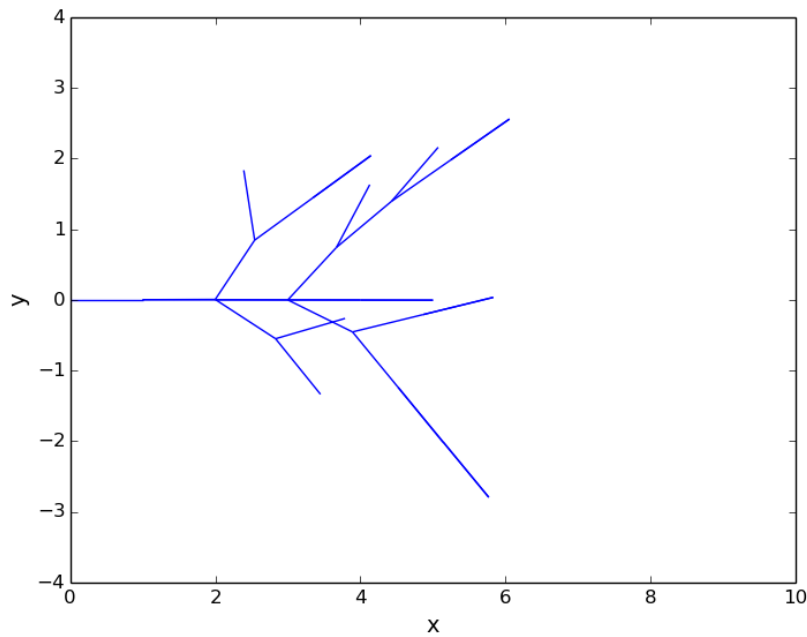


Figure 3.2: A two dimesions simulation of the spliting of a single parton.

## 281 3.4 Three Dimensions Parton Shower

282 Since the real physics we are modelling is in three dimension, we need to modify our model to  
 283 account for this. An additional splitting angle  $\phi$  will be included, which is the azimuthal angle of  
 284 the decay around the direction of travel of the initial particle, as shown in figure 3.3.

285 Unlike the angle  $\theta$ , there is no preference for the value of this angle and it should therefore be  
 286 chosen from a uniform distribution within the interval  $[0, 2\pi]$  (Salam, 2010b).

287 There are no major differences between the three and two dimensional models, except that in the  
 288 three dimensional model the particle will be rotated twice with the angles  $\theta$  and  $\phi$ . Given a unit

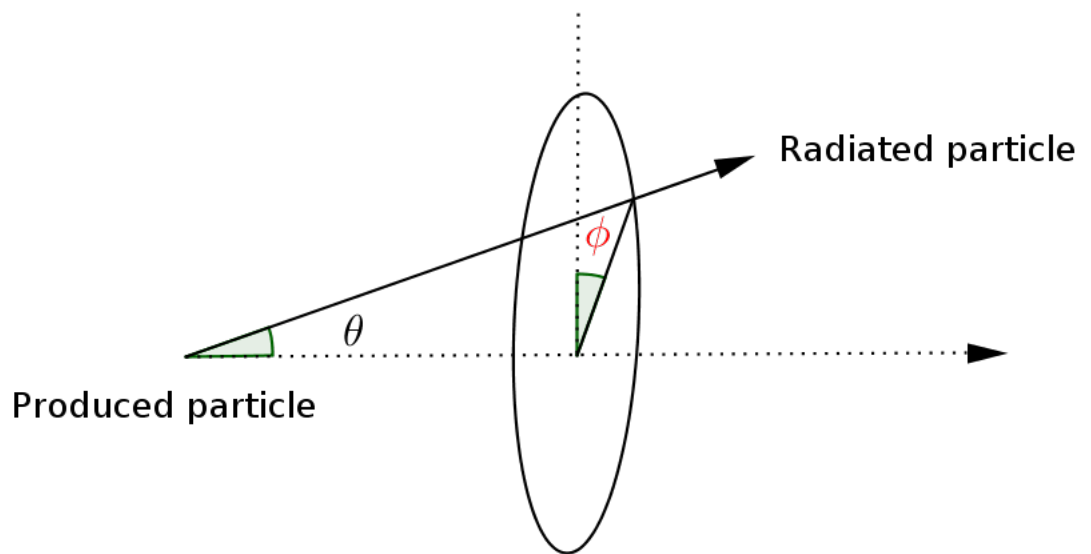


Figure 3.3: An illustration of the azimuthal angle  $\phi$  of a one-to-two splitting occurring in three dimensions.

289 vector  $\mathbf{u}$ , where  $\mathbf{u}$  is the vector [Jan: Delete "the vector".] in the direction of the axis of rotation, !  
 290 the rotation around the axis by angle  $\theta$  can be found by matrix

$$\begin{pmatrix} \cos \theta + u_x^2(1 - \cos \theta) & u_x u_y(1 - \cos \theta) - u_z \sin \theta & u_x u_z(1 - \cos \theta) + u_y \sin \theta \\ u_y u_x(1 - \cos \theta) + u_z \sin \theta & \cos \theta + u_y^2(1 - \cos \theta) & u_y u_z(1 - \cos \theta) - u_x \sin \theta \\ u_z u_x(1 - \cos \theta) - u_y \sin \theta & u_z u_y(1 - \cos \theta) + u_x \sin \theta & \cos \theta + u_z^2(1 - \cos \theta) \end{pmatrix} \quad (3.4.1)$$

291 [Jan: "In order to compute the momentum of radiated particle in three dimensions..."] First the !  
 292 momentum vector is rotated angle  $\theta$  around an arbitrary axis that is perpendicular to it, after  
 293 that the resulting vector is rotated around the original vector with the angle  $\phi$ . [Jan: Say that  
 294 the rest of the model is without changes.] The figure 3.4 shows a graphical representation of the !  
 295 three dimensional parton shower.

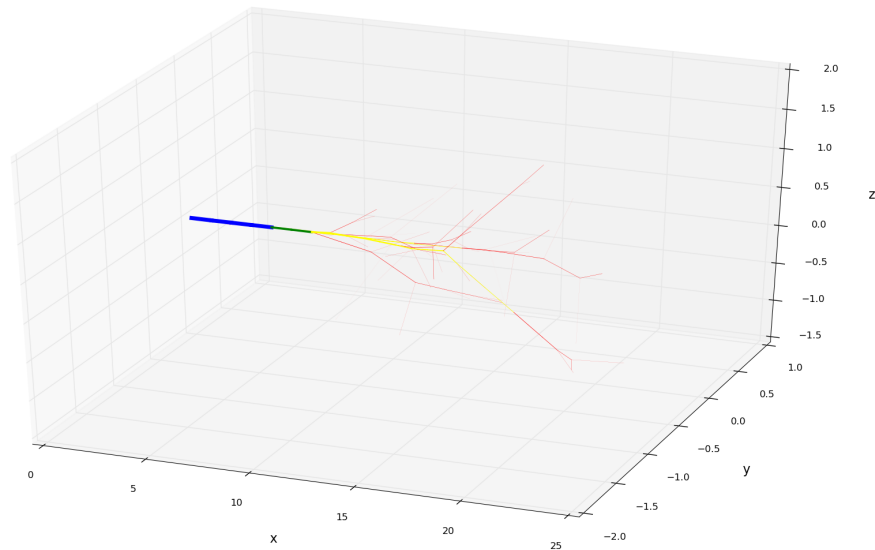


Figure 3.4: A three dimensions simulation of the splitting of a single parton. In this illustration the blue colour represents the particles with energy  $\geq 0.8$ , the green colour represents the particles with energy  $\geq 0.5$ , the yellow colour represents the particles with energy  $\leq 0.5$  and the red colour represents the particles with energy  $\leq 0.1$ .

## 296 3.5 An Improved Model

297 In order to build more realistic model, it is necessary to include more details, as we did with the  
 298 three dimensions model. Here we start by looking at the particle in the moment of production,  
 299 right after the collision of the two protons. For simplicity One can assume that the collision of

the protons conserves the momentum. Hence the produced particle also conserve momentum.

[Jan: I don't think we assume momentum is conserved. We assume that the total momentum is zero.] Following this assumption, it can be said the produced particles come in pairs that have opposite directions. The produced particles have energy that is randomly distributed as well as the polar angle  $\theta$  and the azimuthal  $\phi$  angle. As for the energy, it has the exponential distribution  $e^{-\alpha E_{parton}}$ , where  $\alpha$  is a physically meaningful parameter that characterizes the collisions at the LHC and  $E_{parton}$  is randomly chosen energy of the parton. As for the polar and the azimuthal angles, the collision has no preferred angle in terms of the both angles  $\theta$  and  $\phi$  as shown in figure ??, [Jan: Delete figure ref.] hence, the former is uniformly distributed in the range  $[0, \pi]$  and the later is uniformly distributed in the range  $[0, 2\pi]$ . (Salam, 2010b).

There are three main programs in use at present for the generation of simulated collider events. They incorporate different combinations of the approaches of the model described above. These are HERWIG, PYTHIA and SHERPA (Buckley et al., 2011).

## 4. Jet Reconstruction

After the process of the splitting and branching, the quarks and gluons start to hadronize leading to a collimated spray of stable colourless hadrons. Hence in practical the only outcome of this process is hadrons [1.1](#). In order to study the quarks and the gluons, it is plausible to think of reversing this process. This process is called jet reconstruction, and the final outcome is called jets (quarks or gluons).

The jet reconstruction is essential in understanding the link between the observed physics or the long distance physics and the underlying physics (short distance physics) in the parton level.

The definition of the jet is central in comparing the data and the theoretical predictions. The definition is provided in the form of the jet algorithm, this means the jet algorithm and its corresponding parameters and recombination scheme.

### 4.1 Jet Algorithms

There are two broad classes of jet algorithms, the Cone algorithms and the Sequential algorithms. Both algorithms work on defining the jets by the idea of nearness.

It is important to recognize that jet algorithms involve two distinct steps. The first step is to identify the members of the jet, i.e., the partons that make-up the final stable jets. The second step is to construct the kinematic properties that will characterize the jet ([Berger et al., 2001](#)). For merging and combining the objects, we use the 4 -vector recombination scheme, whereby to combine two particles we add their four momenta ([Blazey et al., 2000](#)).

Here, we will focus on the second class the sequential algorithms.

### 4.2 Sequential Algorithms

These algorithms work by defining a distance between pairs of objects, performing a successive recombination of the pair of closest objects, and stopping when all objects are further apart.

One starts by first defining these distances,  $d_{ij}$  ([4.2.1](#)) and  $d_{iB}$  ([4.2.2](#)), where  $d_{ij}$  is the distance between objects (pseudo-jets) <sup>1</sup>  $i$  and  $j$ , and  $d_{iB}$  is the distance between the pseudo-jet  $i$  and the beam B. The clustering proceeds by identifying the smallest of the distances and if it is  $d_{ij}$  combine the pseudo-jets  $i$  and  $j$ . Otherwise, if it is  $d_{iB}$  calling  $i$  a jet and removing it from the list of objects. The distances are recalculated and the procedure repeated until no objects are left ([Cacciari et al., 2008](#)).

---

<sup>1</sup>Pseudo-jet since it is neither a particle, nor yet a full particle. It can be a single particle or a composition of particles.

342 The difference between the sequential algorithms lies in the definition of the distances measures:

$$d_{ij} = \min(k_{ti}^{2p}, k_{tj}^{2p}) \frac{\Delta_{ij}}{R^2}, \quad (4.2.1)$$

$$d_{iB} = k_{ti}^{2p}, \quad (4.2.2)$$

344 Where  $\Delta_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$  and  $k_{ti}$ ,  $\eta_i$  and  $\phi_i$  are the transverse momentum, rapidity  
 345 and the azimuth of the particle  $i$  (Cacciari et al., 2008). The exact formula for the transverse  
 346 momentum depends on the beam axis, which is  $z$  hence,  $k_t = \sqrt{p_x^2 + p_y^2}$ . The rapidity describes  
 347 the angle relative to the beam axis. The rapidity is defined as  $\frac{1}{2} \ln \frac{E_i + p_{zi}}{E_i - p_{zi}}$ , where  $E_i$  and  $p_{zi}$  are  
 348 the energy and component of the momentum along the beam axis of the particle  $i$  (Cacciari  
 349 et al., 2012). The parameter  $R$  describes the jet radius.

350 The inclusive  $k_t$  algorithm is defined with choice of  $p = 1$ , the case where  $p = 0$  corresponds  
 351 to the inclusive Cambridge/Aachen algorithm, where  $p = -1$  refers to anti- $k_t$  jet clustering  
 352 algorithm (Cacciari et al., 2008).

353 The implementation of anti- $k_t$  clustering algorithm can be illustrated by the following algorithm

```

354   Calculate all the  $d_{ij}$  and  $d_{iB}$ 
355   Find the minimum of  $d_{ij}$  and  $d_{iB}$ 
356   if minimum distance is  $d_{ij}$  then
357       Recombine  $i$  and  $j$  in a single object
358       return step one
359   else
360       the minimum is  $d_{iB}$ 
361       Declare  $i$  as a jet and remove from the list
362       Start from the beginning
363   end if
364   Repeat until no objects left
  
```

## 5. Jet Observables

The next step after performing the jet clustering on the generated parton shower, is to define final state observables, that can be used to extract specific properties of the underlying physics. Here, there are two kinds of observables, event observables and jet observables.

The most important feature of the observables is that they can be defined both for the Monte Carlo simulation and the real collision data, allowing for tuning the underlying parameters of the model used to generate the parton shower.

### 5.1 Single Jet Observables

As an example of the jet observables, the following association can be made  $(E, p_x, p_y, p_z)_{parton} \sim (E, p_x, p_y, p_z)_{jet}$ . Hence, the jets were formed by combining four momentum vectors together and the jet itself is composed of multiple particles. Focusing on the first entry  $E$ , one can recognize that the parton energy is represented as  $E_{parton} \sim E_{jet} = \sum_{i \in jetconstituents} E_i$ , where the jet constituents are the stable particles that the initially were in the event.

Another jet observable is the number of the constituents in the jet with highest energy. This gives an intuition about the formation of the jet, and allow as to tune the parameters of the parton shower.

Another single jet observable that we have worked on is *pseudo-mass* observable. This observable is described as follows

$$pseudo - mass(J) = E(j_1) \times E(j_2) \times \Delta_{ij}, \quad (5.1.1)$$

Where  $j_1$  and  $j_2$  are the two highest momenta of the jet  $J$  and  $\Delta_{ij}$  as given above. These calculations were performed on the jets with highest energy. Conditions where the jet with the highest energy has one constituent, this observable was not calculated.

The histograms in figures 5.1 and 5.2 illustrate the results of this calculation for 1000 events clustered with anti- $k_t$  algorithm with different values of  $R$ . Where the first shows the number of constituents in a jet and the second shows the pseudo-mass observable.

### 5.2 Event Observables

These observables associate the elements of initial event and final state particles (jets). An example of this is the number of jets in the event. This observable shows the number of parton that the event started with. The histogram in figure 5.3 shows the results of clustering 1000 events using anti- $k_t$  with different values of  $R$ . One sees the number of jets if it is affected by the choice of  $R$ . It worth noting that as  $R$  gets smaller, the number of jets in the event increases.

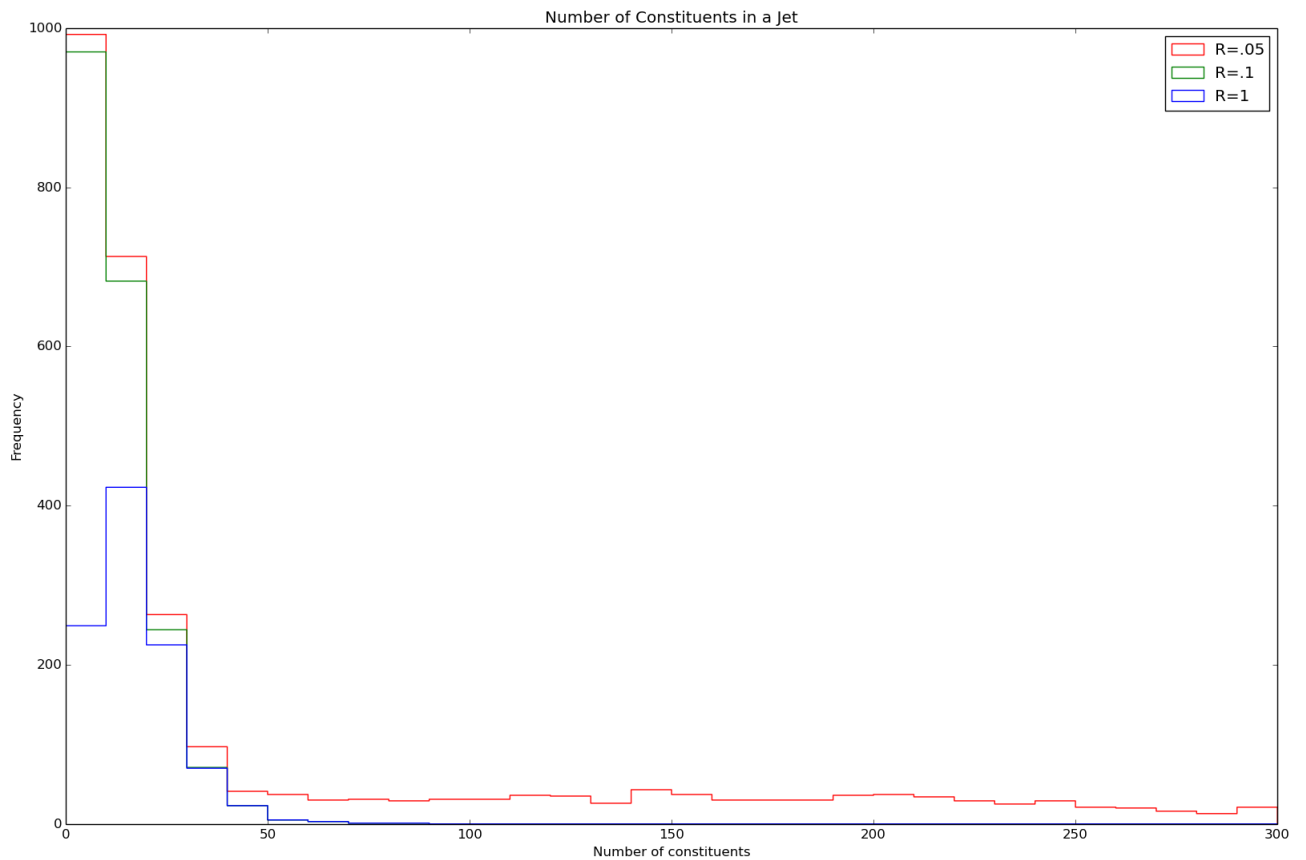


Figure 5.1: Number of constituents of a jet for different values of  $R$ . Note that here we are looking at the difference on the  $y$  axis. The values that correspond to  $R = .1$  are the difference between the blue line  $R = 1$  and maximum of  $R = .1$ .



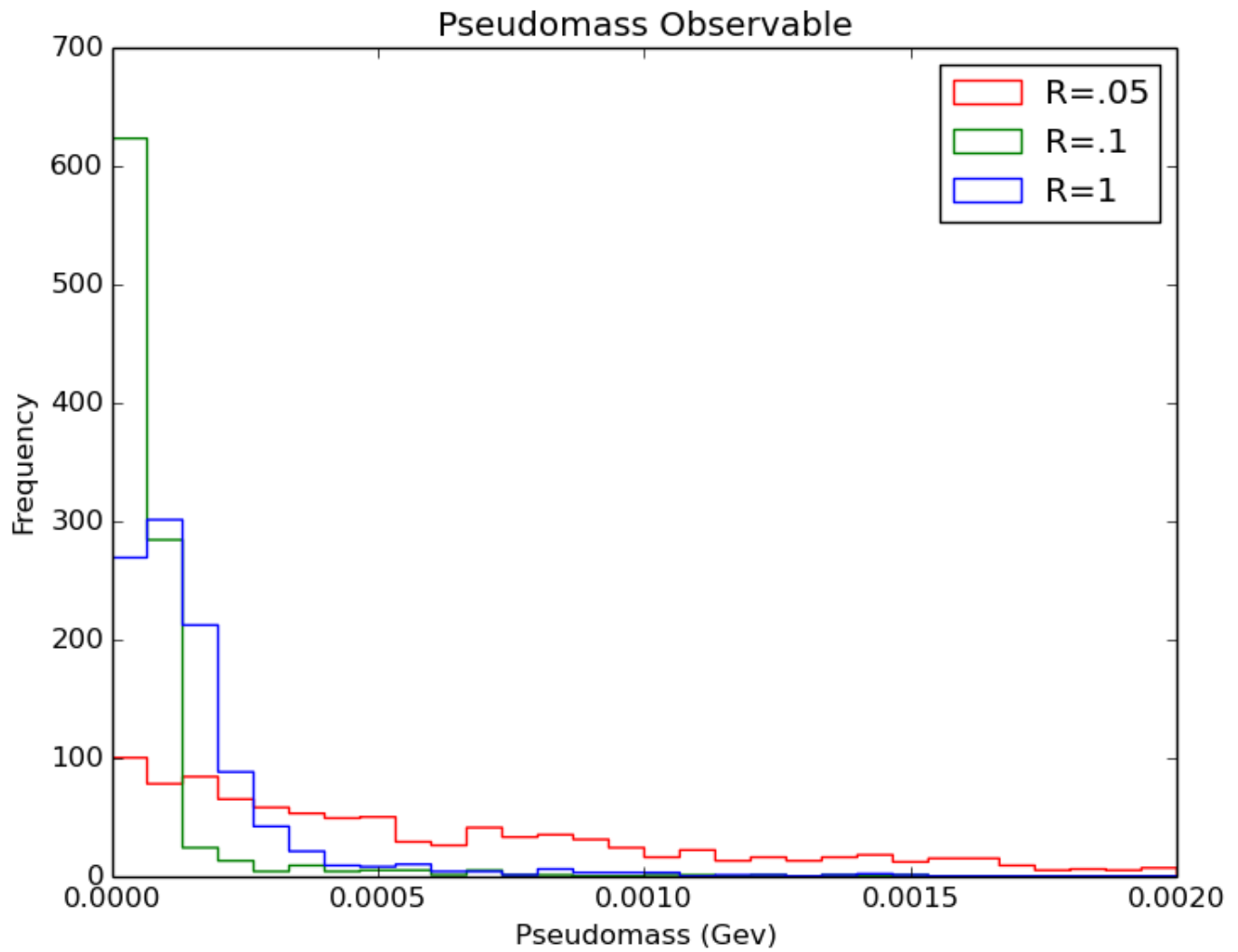


Figure 5.2: Pseudo-mass of jet when uses differnt values of  $R$ . Note that here we are looking at the difference on the  $y$  axis. The values that correspond to  $R = .1$  are the difference between the blue line  $R = 1$  and maximum of  $R = .1$ .

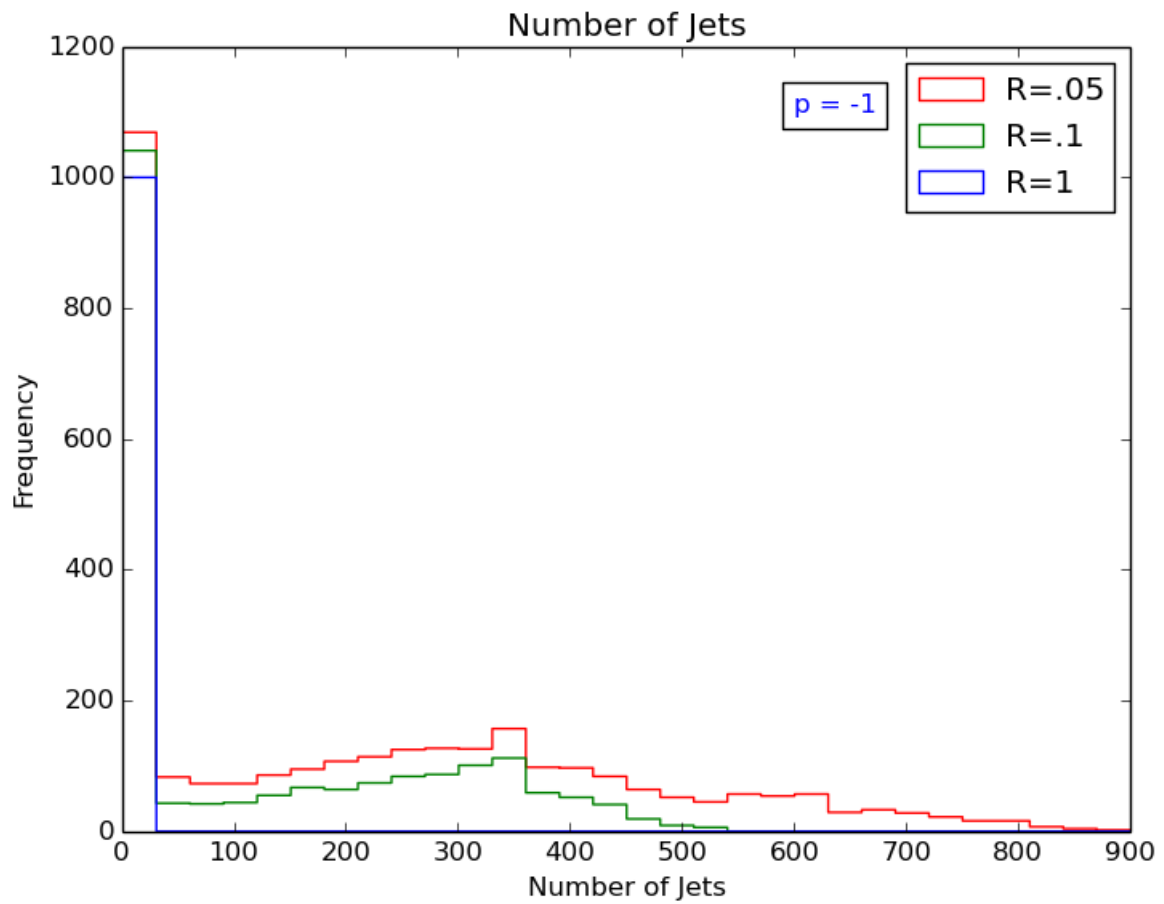


Figure 5.3: Number of jets for different values of  $R$ . Note that here we are looking at the difference on the  $y$  axis. The values that correspond to  $R = .1$  are the difference between the blue line  $R = 1$  and maximum of  $R = .1$ .

## 6. Pythia

Pythia is a general-purpose event generator. It is extensively used for studying physics at LHC. At the beginning the code was written in Fortran 77, recently it has been moved to C++. The latest version of Pythia (8.1) was released in 2007, which is written in c++ (Buckley et al., 2011).

In general Pythia performs on idea of the program designed here for the parton shower. However, it accounts for more of the underlying physics, that we have not covered in our model. Such as the quark flavour or in general the type of the particle which is splitting.

### 6.1 FastJet

FastJet is an analysis tools, it written in C++. It includes efficient implementation of all widely used sequential recombination jet algorithms. In the case of jet clustering, the working principle is the same as the algorithm that we implemented in chapter 4, but it is much faster (Buckley et al., 2011).

In our work we generated 1000 events using Pythia 8.1 and analysed them using FastJet using the configuration *di jet*, which is the a simulation of a collision that produces two particles.

We extracted the same observables as in 5.1, 5.3 and 5.2 the pseudo-mass, the number of constituents in the jet with highest energy and the number of jet observables. Here the value of the parameter  $R$  is set to 1.

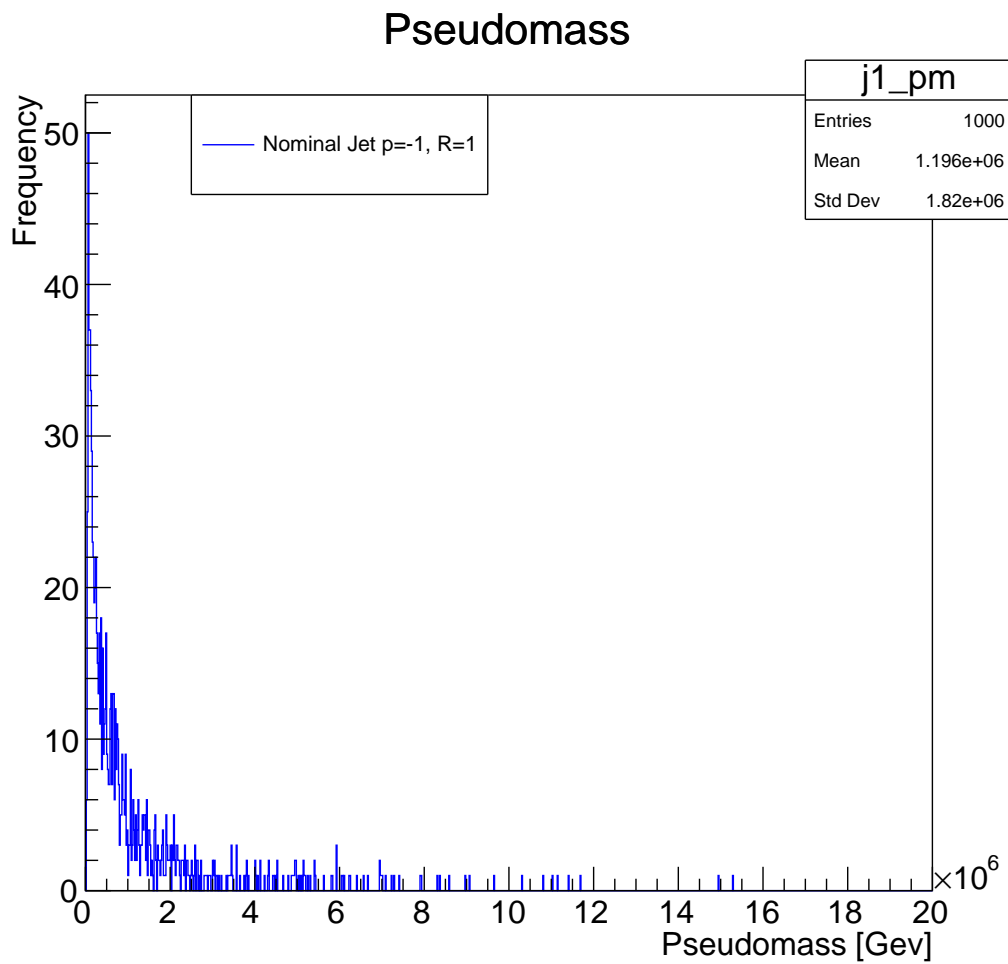


Figure 6.1: A histogram of the pseudo-mass of a jet observable obtained from analysing 1000 events.  $R$  is chosen to be 1.

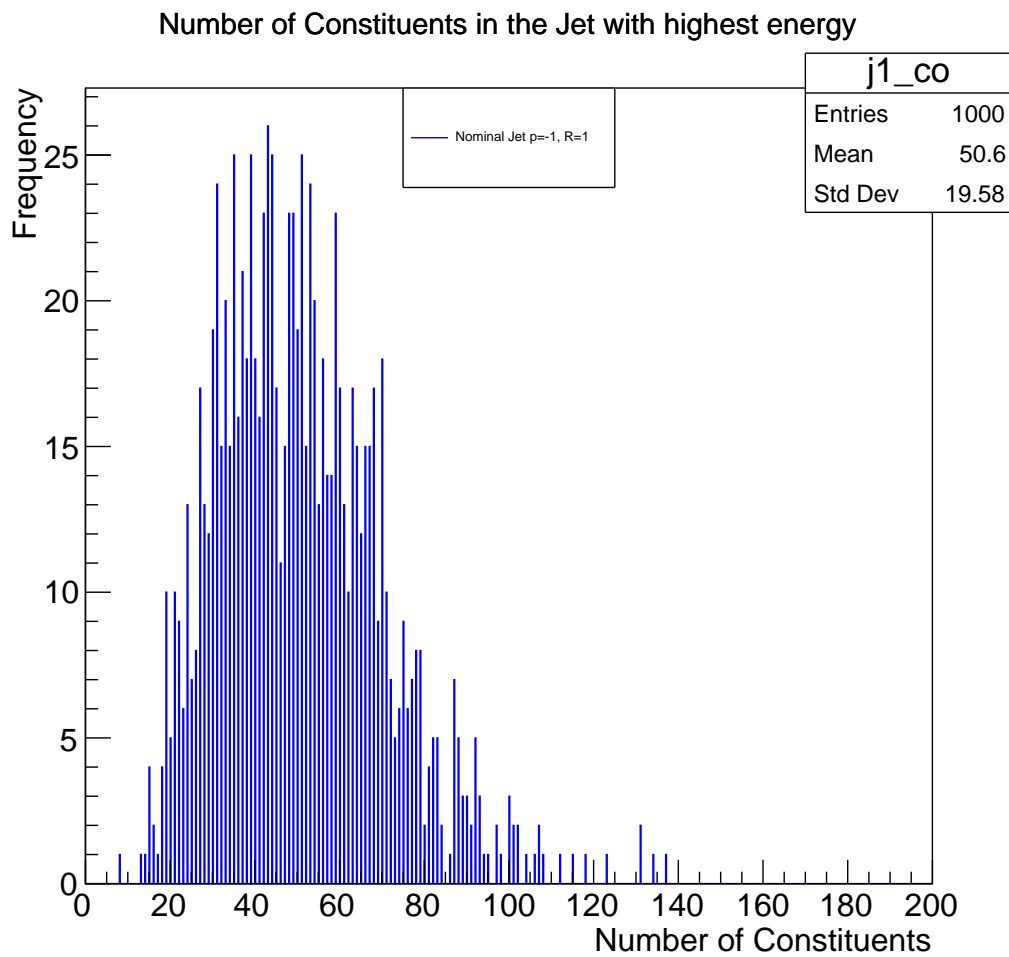


Figure 6.2: A histogram of the number of constituents in the jet with highest energy obtained from analysing 1000 events.

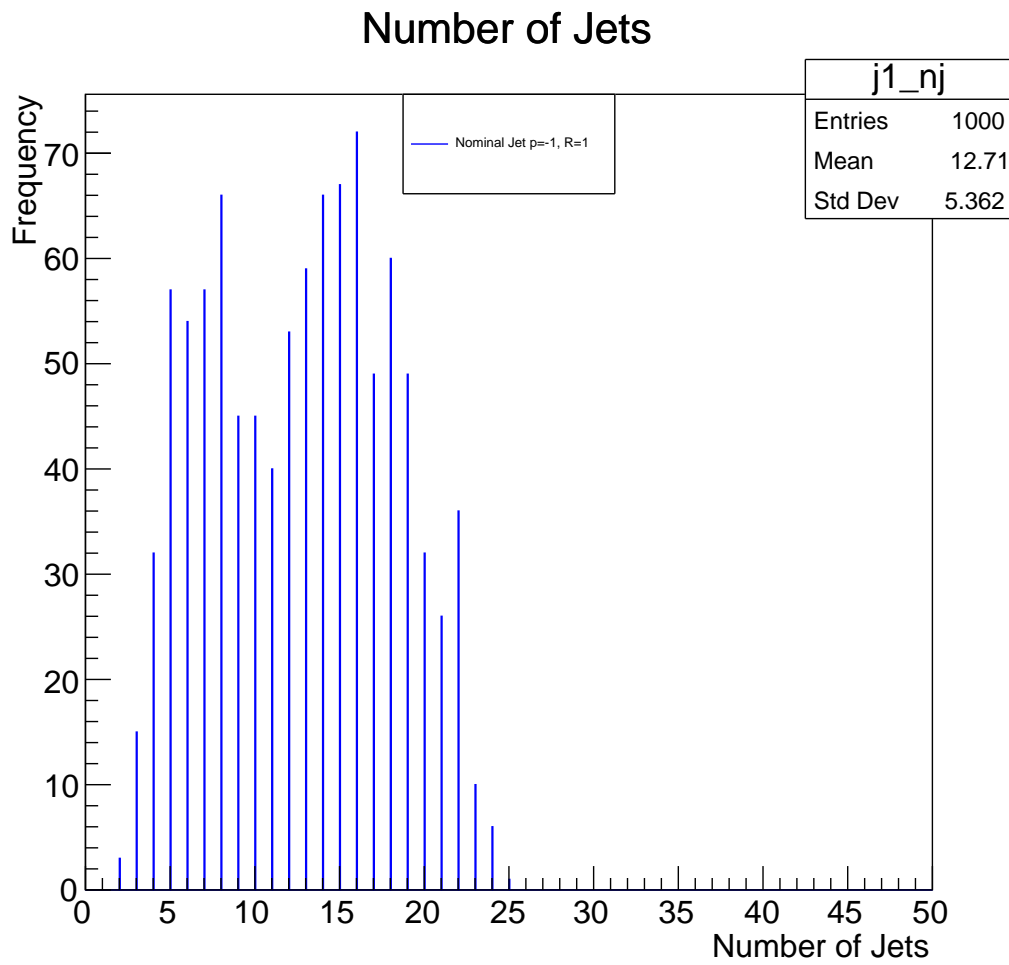


Figure 6.3: A histogram of the number of jet obtained analysing from 1000 events.

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# Appendix

The codes that are used in this essay are available on <https://github.com/KhalidOmer/MonteCarloTutorial>.

The file `README.md` contains a brief description of the codes and what they do as well as how to run them.

Note that all codes are written in python, except for chapter six the codes are in c++.

The codes are:

Chapter two:

- Obtaining a sample with  $p(x) = \frac{1}{x+\epsilon}$ , the file name is `accept-reject.py`.
- For the calculation of  $\pi$  and the error calculation see `Calculation_of_pi.y` and `uncertainty_in_pi_calculation.py`.

Chapter three:

- The two dimensions parton shower: `two_D_partonshower.py`.
- The three dimensions parton shower: `partonshower3d.py`

Chapter four:

- Anti- $k_t$  algorithm: `antikT_algorithm.py`.

Chapter five:

In this chapter the codes are divided into two files, one file for running the code and the other is for producing the histogram.

- The Pseudo-mass observable:  
`Pseudomass_observable.py`  
and `hist_pseudomass.py`.
- The number of constituents in the jet with highest energy observable:  
`number_of_constituents_observable.py`  
and `hist_of_n_of_constituents.py`.
- The number of jets observable:  
`number_of_Jets_observable.py`  
`hist_n_of_Jets.py`.

481 Chapter six:

482 The codes are available in the folder named PythiaSimulatorAnalysis. A file called README.md  
483 inside this folder contains the instruction on how to run these codes.