Classifying 2-Stratifolds with Finite Fundamental Group

FLORIDA STATE UNIVERSITY
Mathematics

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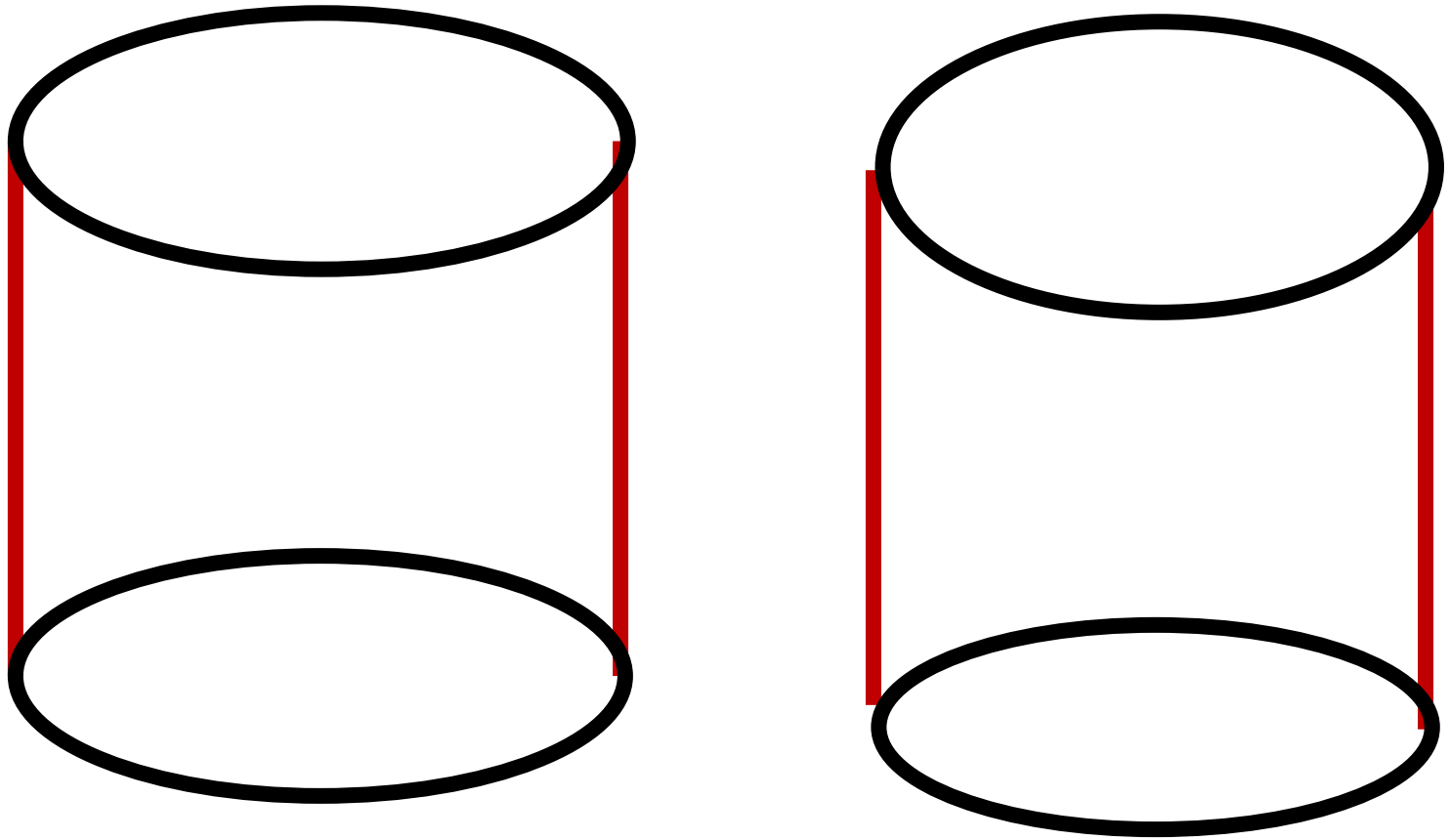
GSCAGT 2019

Seifert Fiber Space



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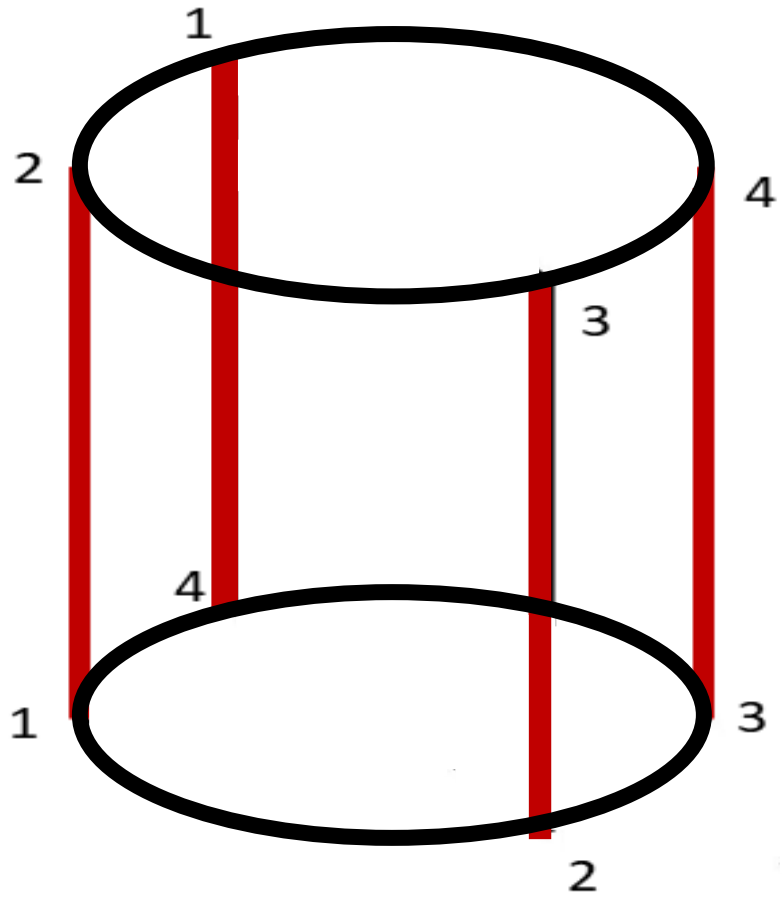

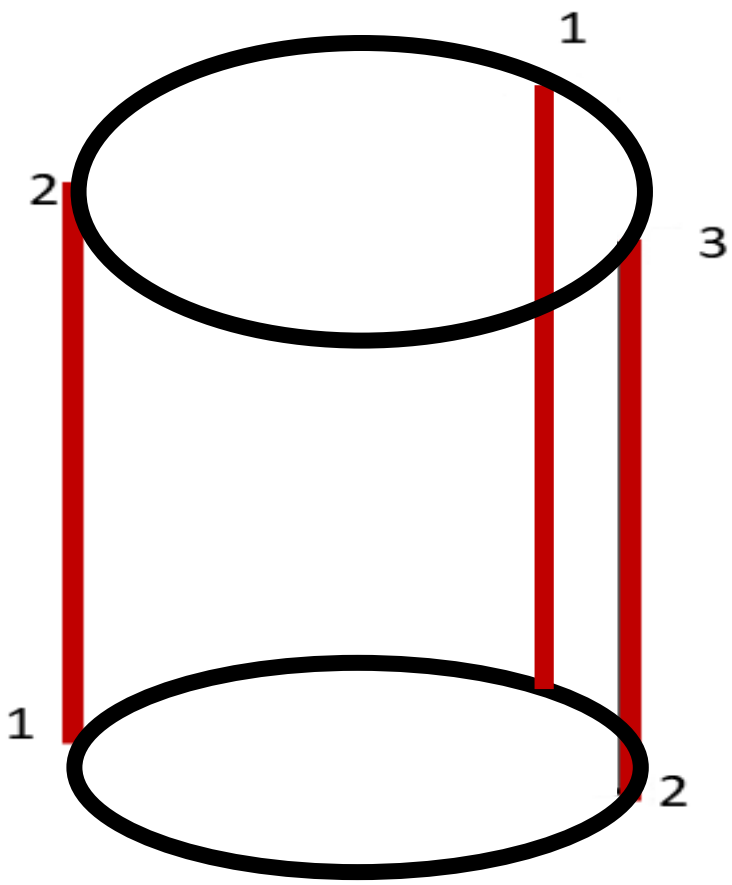

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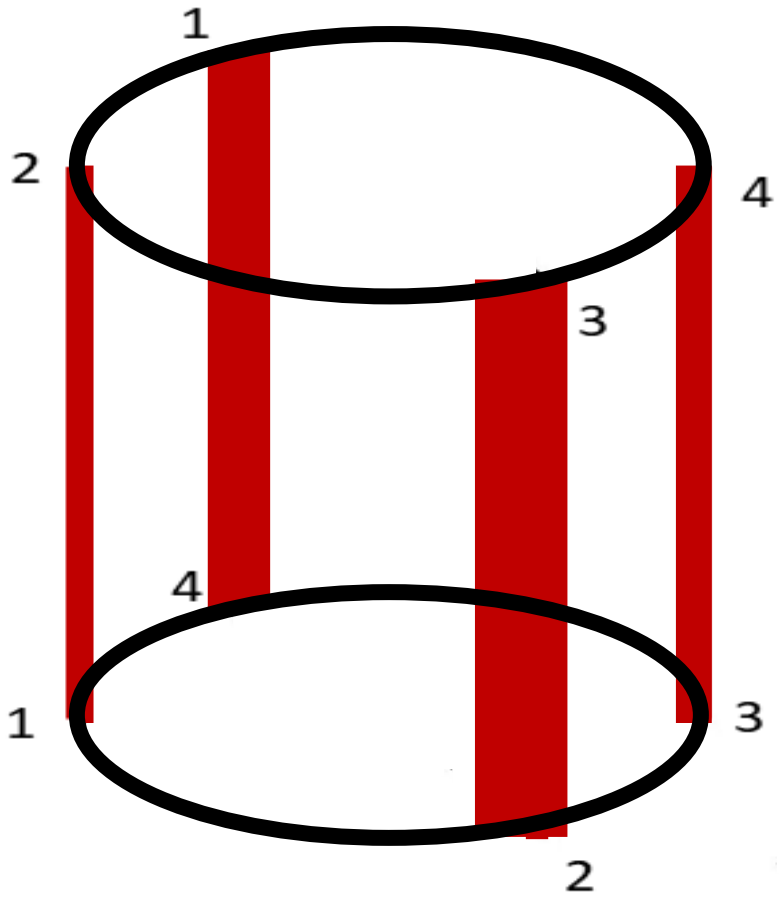

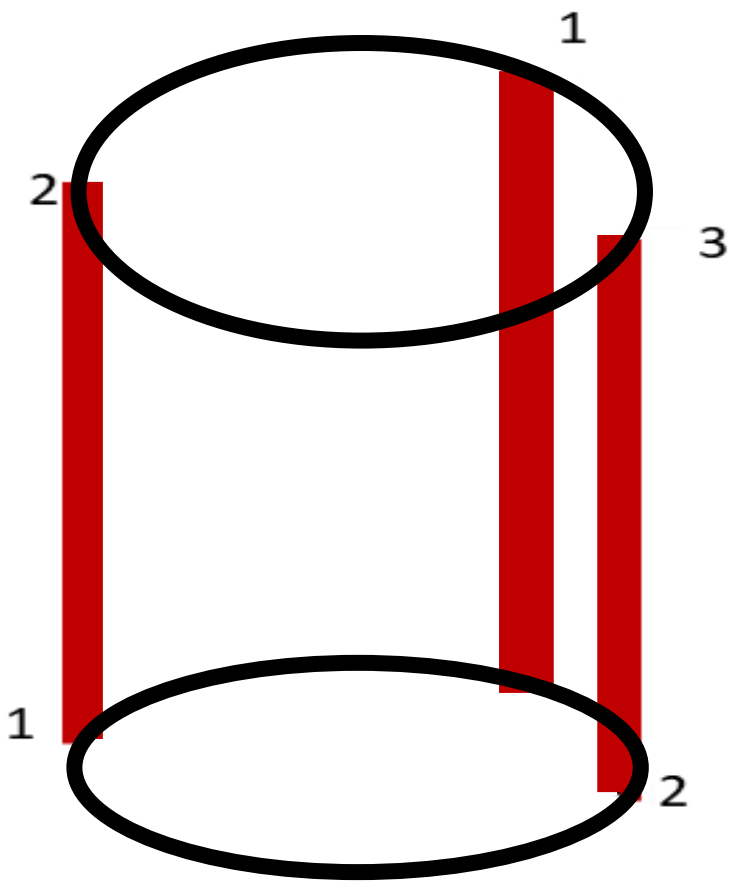

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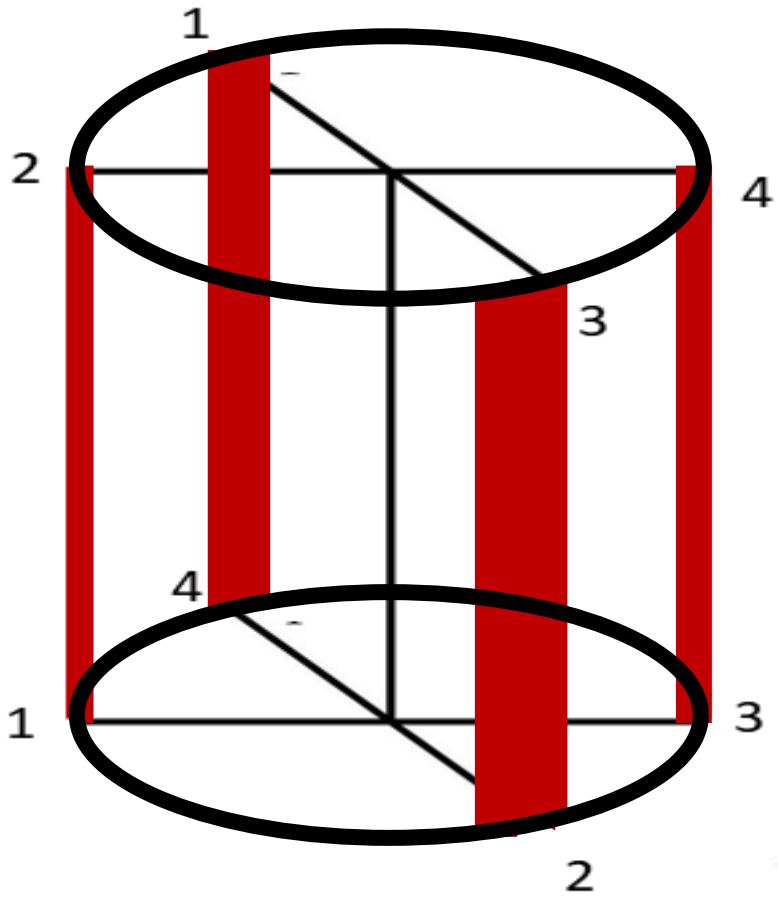

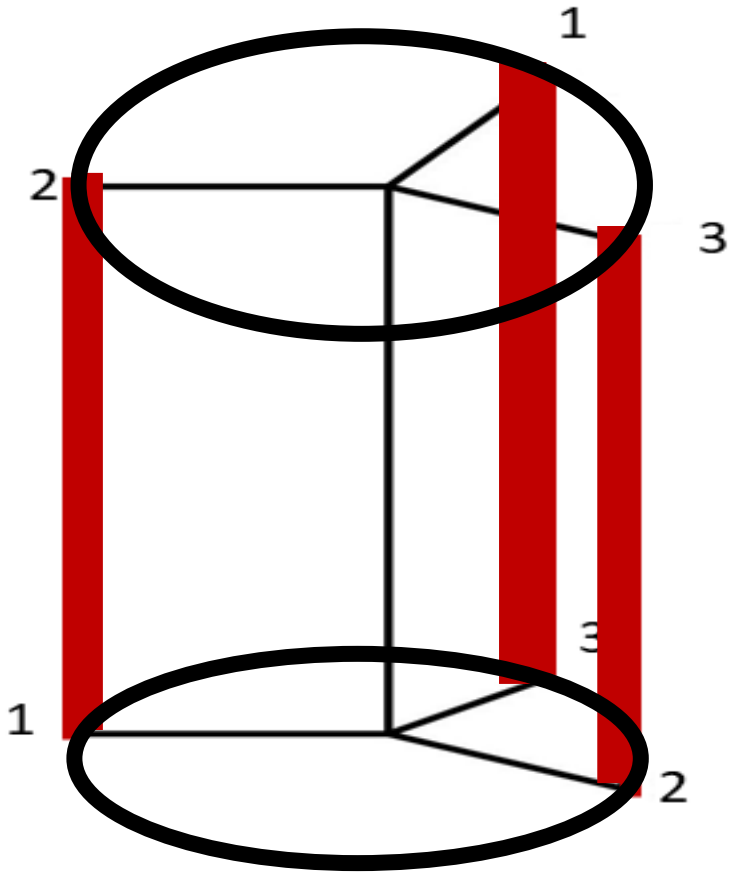

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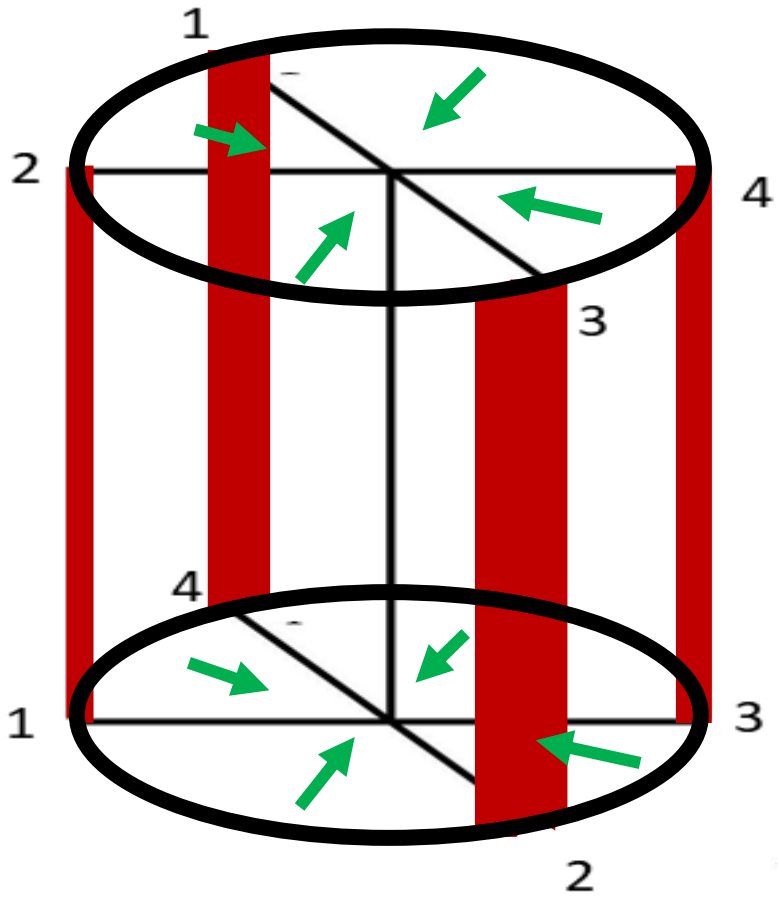

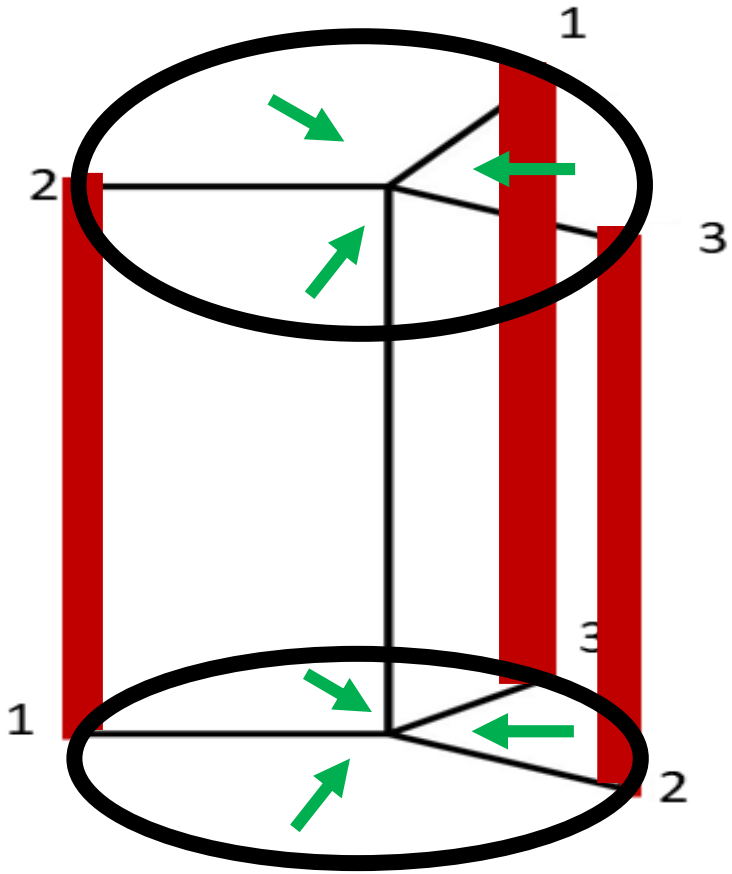

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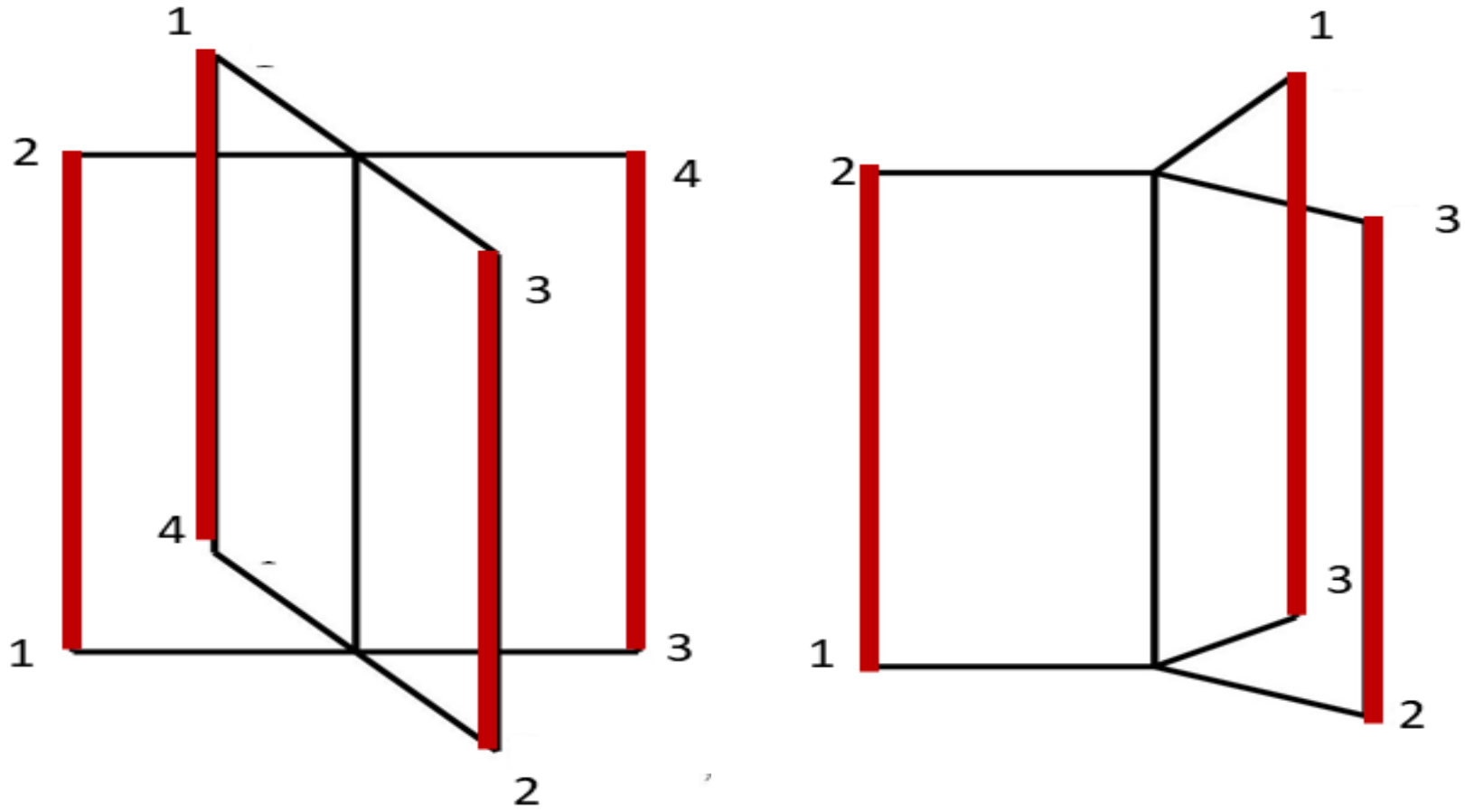


2-Stratifold

A closed 2-stratifold ***X*** is a 2-complex where

- ***X*** contains a collection ***S*** of finitely many s.c.c, such that ***X- S*** is a compact surface,

- and a neighborhood of each component in ***S*** consists of more than 2 sheets.

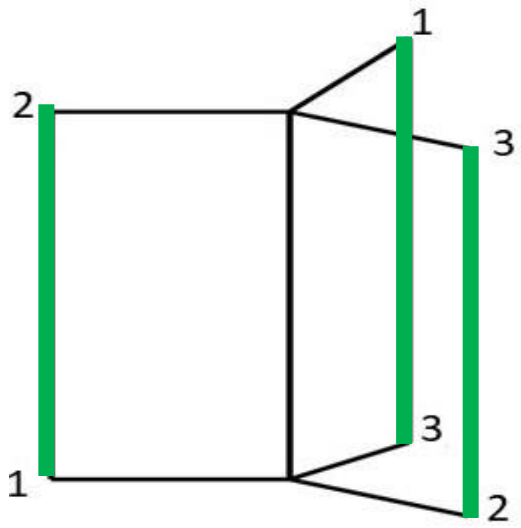
Branch Neighborhoods

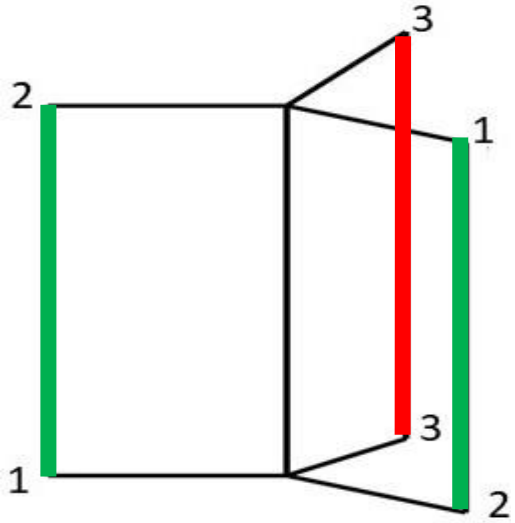
A simple closed curve in ***S***

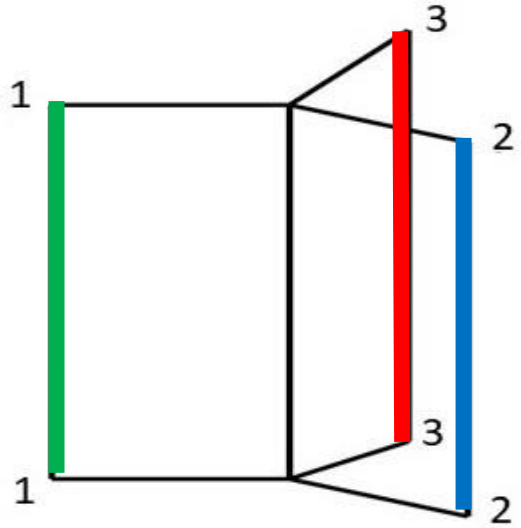
- is called a branch curve (or singular curve)

- and a regular neighborhood of a branch curve is called a branch neighborhood.

Trivalent Branch Neighborhoods

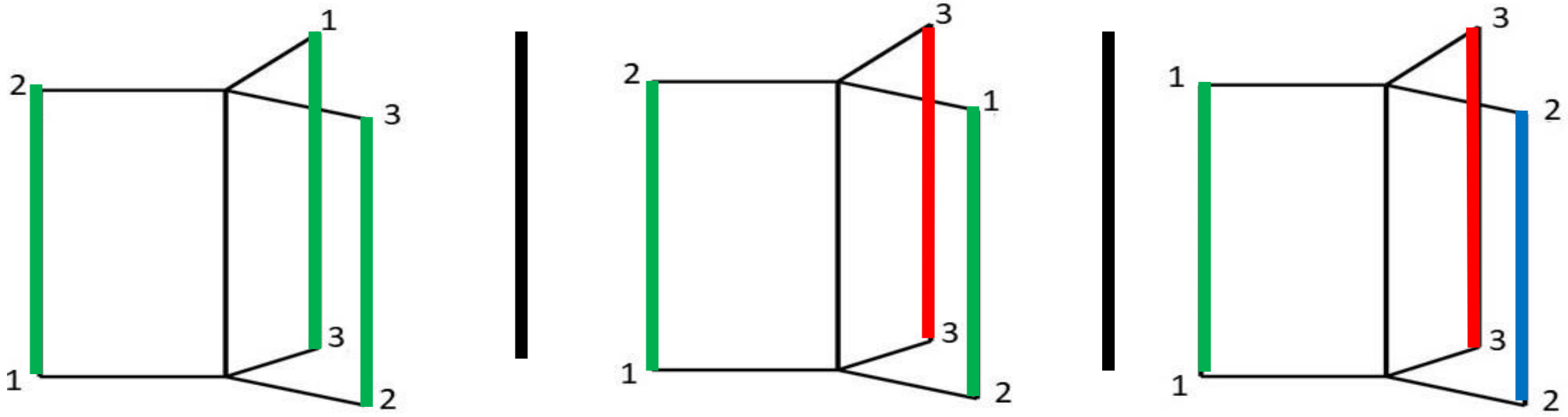




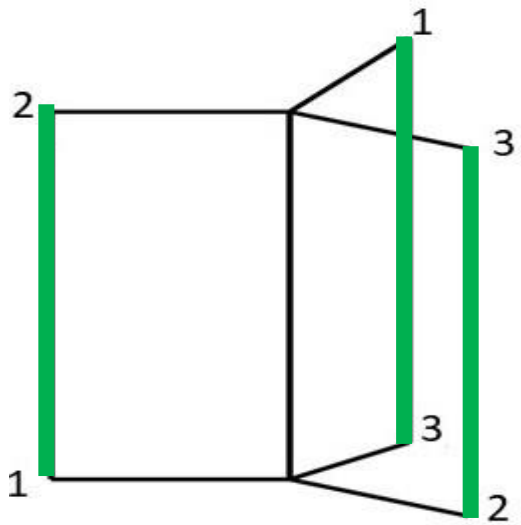


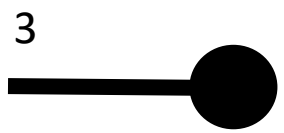
Gluing Action : (123)	Gluing Action : (12)(3)	Gluing Action : (1)(2)(3)
Boundary Components : 1	Boundary Components : 2	Boundary Components : 3
Boundary Words : 3a	Boundary Words : 2a, a	Boundary Words : a, a, a

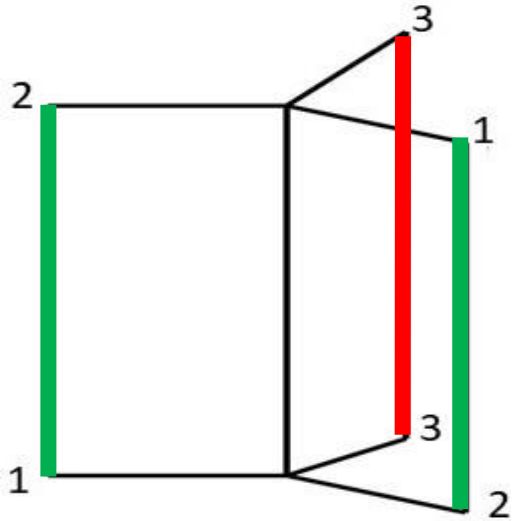
Trivalent Branch Neighborhoods

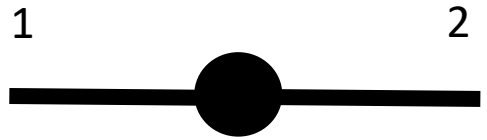


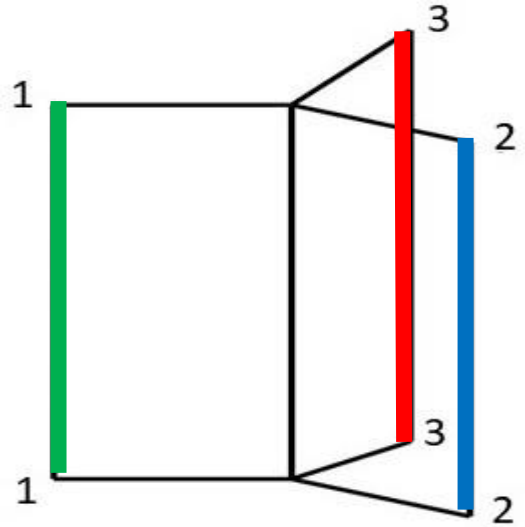
Trivalent Branch Neighborhoods

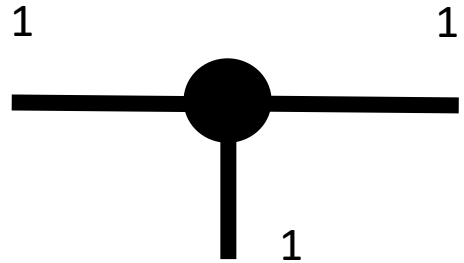




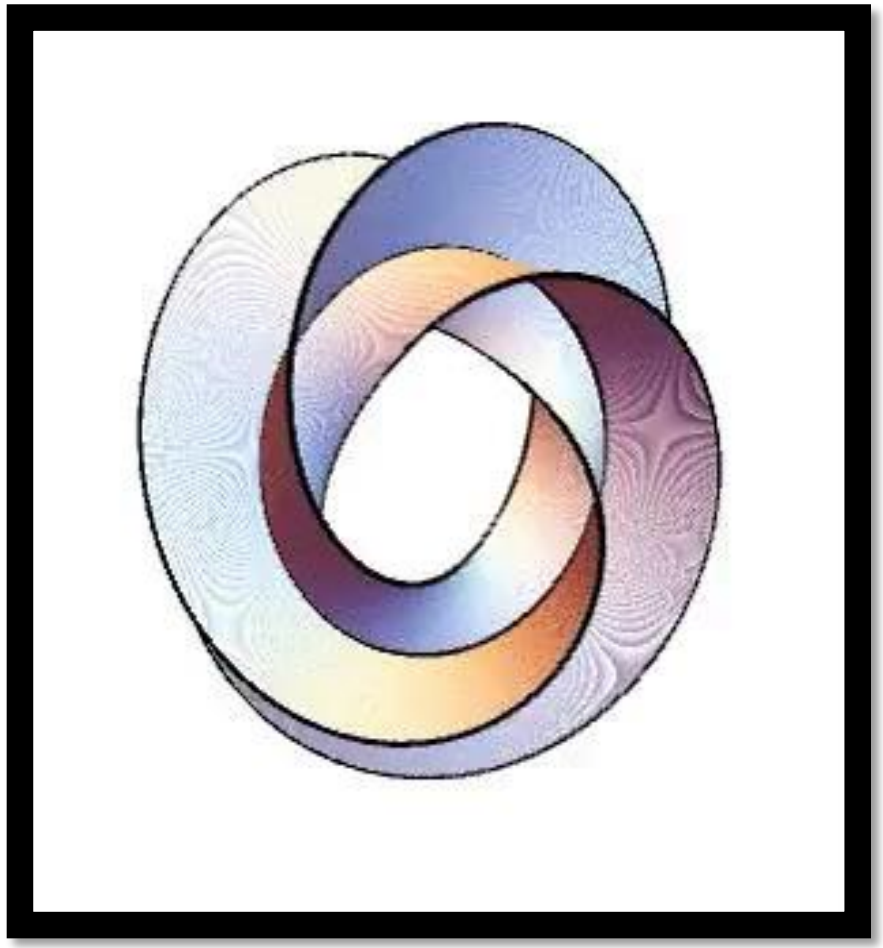


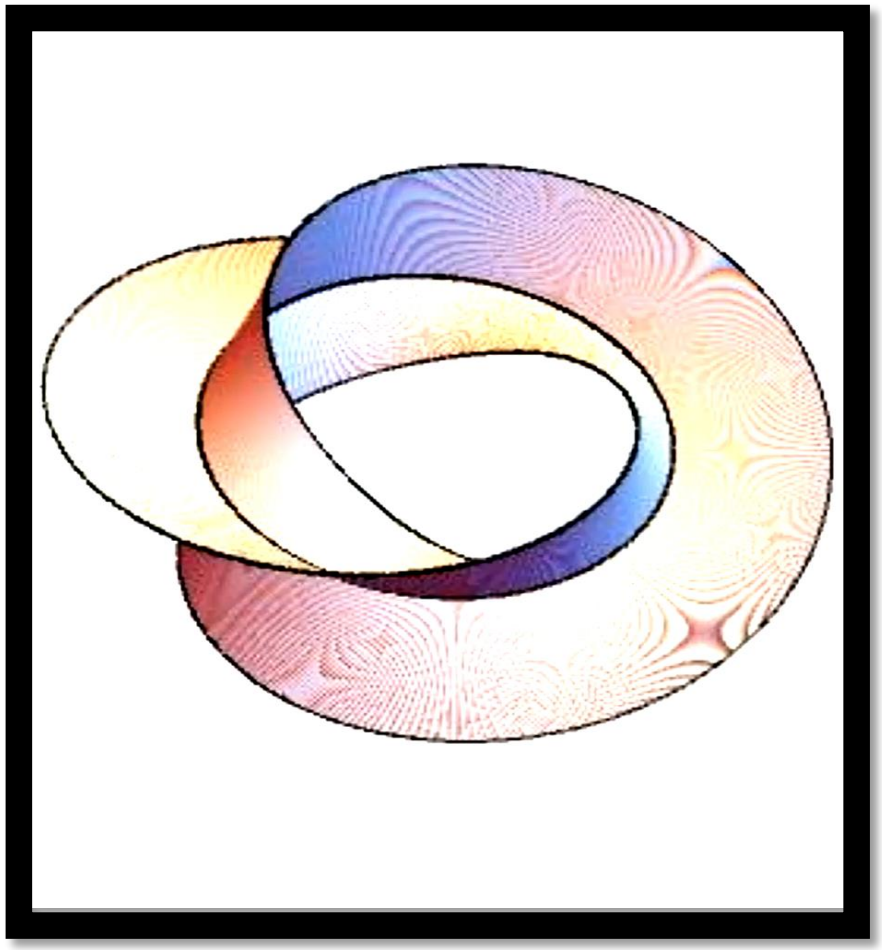




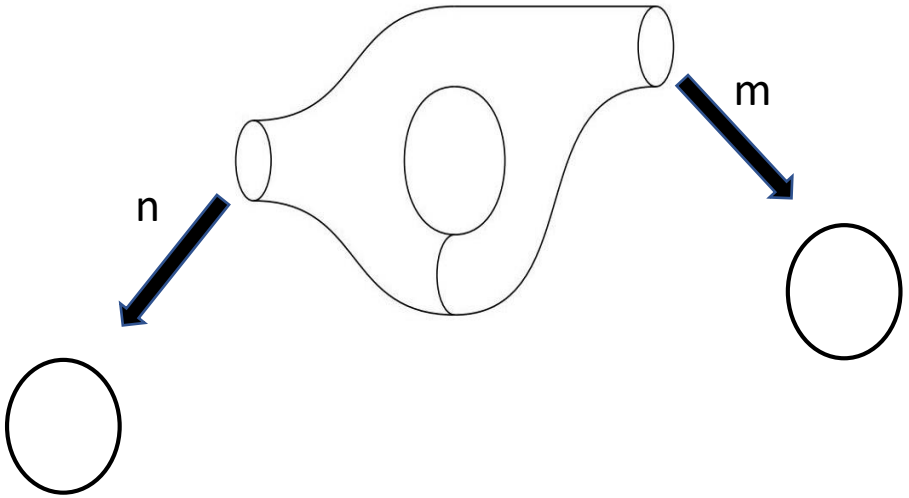


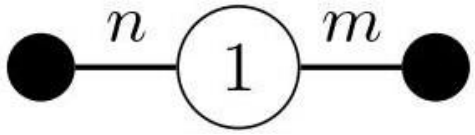
Embeddings of Branch Neighborhoods





Associated Graph





Generators

Generators from Branch curves

$$C, D, E, F, b_1, b_2$$

Generators from Torus

Relations

$$b_1^n = E, b_2^m = F, [C, D]EF = 1$$

Classifying 2-stratifolds

Classification of closed surfaces groups

Classification of closed surfaces

Classification of closed 2-stratifold groups

Classification of closed 2-stratifolds

End Goal

Main Goal. Classify all 2-stratifolds in terms of their labeled graph.

Secondary Goal. An efficient algorithm to decide if a 2-stratifold is of a given type.

Trivalent Classification

2-stratifolds X where at most 3 sheets meet are trivalent.

- Trivalent 2-stratifold with trivial or infinite cyclic fundamental group.
  J.C. Gómez-Larrañaga, F. González-Acuña, and W. Heil

- Trivalent 2-stratifolds with finite fundamental group.
  J.H.B.

Trivalent Algorithm

Given a trivalent 2-stratifold it can be determined if

- It is has trivial or infinite cyclic fundamental group.

  J.C. Gómez-Larrañaga, F. González-Acuña, and W. Heil

- It has finite fundamental group.

  J.H.B.

Initial Questions

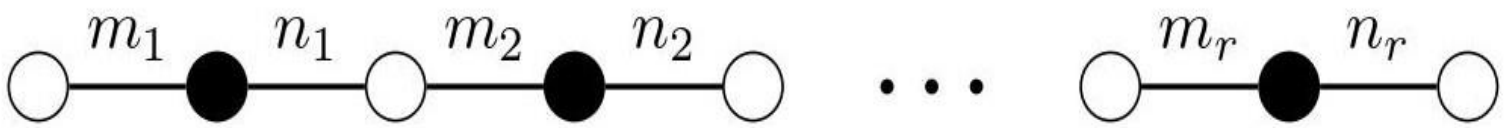
Question 1. What are the finite 2-stratifolds groups?

Answer 1. Finite Fuchsian Groups.

Question 2. What is the graph type of a 2-stratifold with finite fundamental group?

Answer 2. Tree where all white vertices have genus zero and at most one black terminal vertex.

Linear Stratifolds



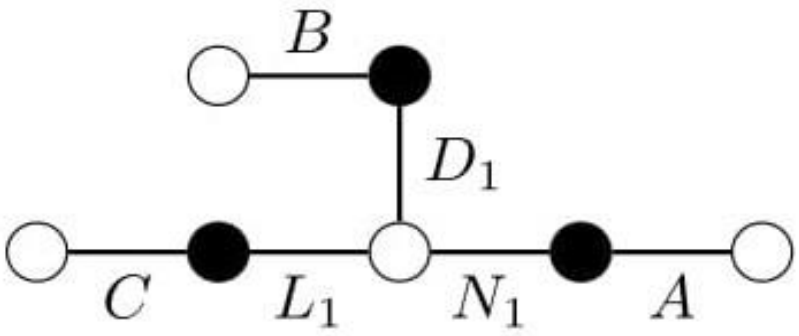
$$X_1^{n_1} = X_2^{m_2} \cdots X_{r-1}^{n_{r-1}} = X_r^{m\pi_r}$$

$$X_1^{m_1} = 1 \qquad 1 = X_r^{n_r}$$

Finite π_1

Finite Cyclic

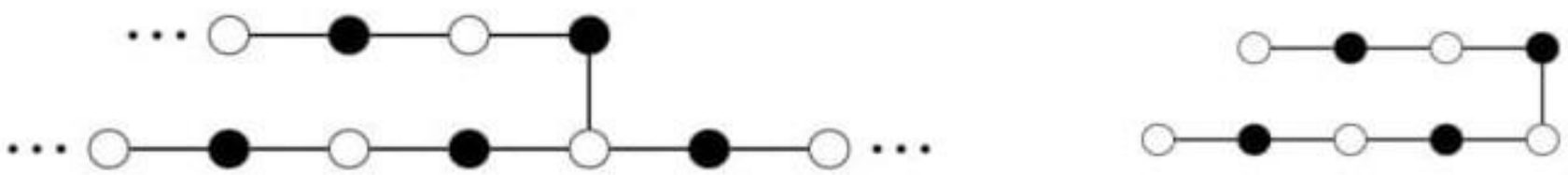
Star Stratifolds



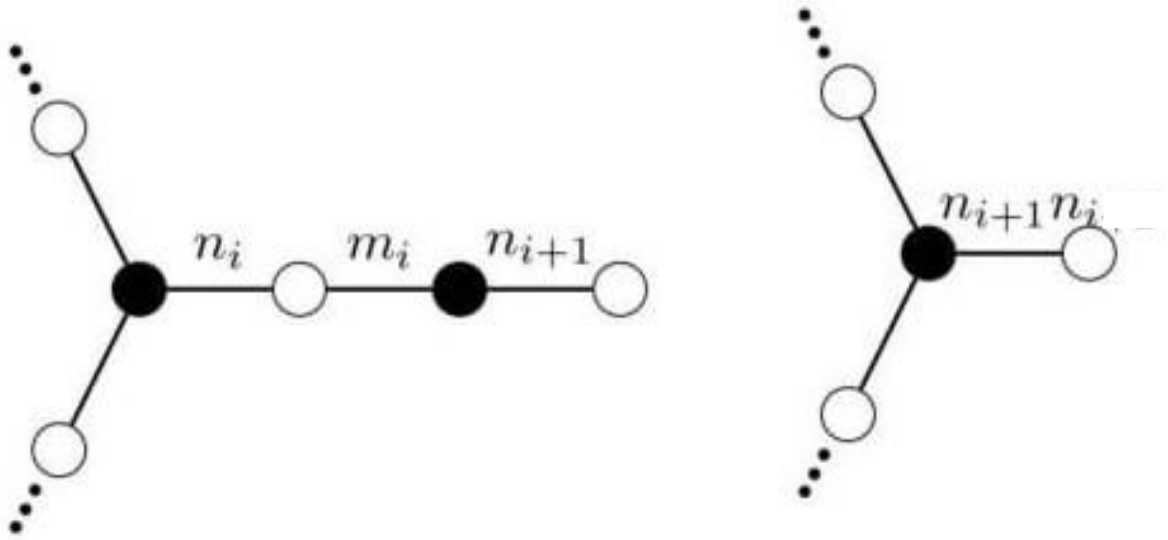
Finite π_1

Dihedral,
Octahedral, or
Dodecahedral

Reducing Graph Type: Pruning



Reducing Graph Type: Absorbing



Models of Trivalent 2-stratifolds

Thanks for listening !

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