MATH1401

Fall 2021

Lecture 16

Assessing Models

Class Checklist

- **Homework 5 - Due Date** : Friday: 10/29 – 9 PM
  - Graded Questions:1.1-1.4,2.1-2.4,2.6,3.1,3.4,5.1-5.5

- **Quiz 12 - Probability** : Tuesday: 10/26

Summary – Sections 10.0-10.1

- **Random Samples** – Large Random Samples are indistinguishable from the population
- **Assessing Models** – General Framework for testing hypothesis

Discussion Question

A population has 100 people, including Rick and Morty.
We sample two people at random without replacement.

(a) P(both Rick and Morty are in the sample)

(b) P(neither Rick nor Morty is in the sample)

Discussion Question

A population has 100 people, including Rick and Morty.
We sample two people at random without replacement.

(a) P(both Rick and Morty are in the sample)
= P(first Rick, then Morty) + P(first Morty, then Rick)
= (1/100) * (1/99) + (1/100) * (1/99) = 0.0002

(b) P(neither Rick nor Morty is in the sample)
= (98/100) * (97/99) = 0.9602

Review: Distributions

- Any random quantity has a **probability distribution**:
  - All **possible** values it can take
  - The **probability** it takes each value

- After repeated draws, it has an **empirical distribution**:
  - All **observed** values it took
  - The **proportion of times** it took each value

- After **many independent draws**, the empirical distribution looks more and more like the probability distribution

(Demo)

Inference

Inference

- **Statistical Inference:**

  Making conclusions based on data in random samples

- **Example**:

  Use the data to guess the value of an unknown number

  fixed

  depends on the random sample

  Create an **estimate** of the unknown quantity

Terminology

- **Parameter**
  - A number associated with the population
- **Statistic**
  - A number calculated from the sample

A statistic can be used as an **estimate** of a parameter

(Demo)

Probability Distribution of a Statistic

- Values of a statistic vary because random samples vary
- “Sampling distribution” or “probability distribution” of the statistic:
  - All possible values of the statistic,
  - and all the corresponding probabilities
- Can be hard to calculate
  - Either have to do the math
  - Or have to generate all possible samples and calculate the statistic based on each sample

Empirical Distribution of a Statistic

- Empirical distribution of the statistic:
  - Based on simulated values of the statistic
  - Consists of all the observed values of the statistic,
  - and the proportion of times each value appeared

- Good approximation to the probability distribution of the statistic
  - if the number of repetitions in the simulation is large

(Demo)

Assessing Models

Models

- A model is a set of assumptions about the data
- In data science, many models involve assumptions about processes that involve randomness
  - “Chance models”
- **Key question:** does the model fit the data?

Approach to Assessment

- If we can simulate data according to the assumptions of the model, we can learn what the model predicts.
- We can then compare the predictions to the data that were observed.
- If the data and the model's predictions are not consistent, that is evidence against the model.

Jury Selection

Swain vs. Alabama, 1965

- Talladega County, Alabama
- Robert Swain, black man convicted of crime
- Appeal: one factor was all-white jury
- Only men 21 years or older were allowed to serve
- 26% of this population were black
- Swain’s jury panel consisted of 100 men
- 8 men on the panel were black

Supreme Court Ruling [in English]

- About disparities between the percentages in the eligible population and the jury panel, the Supreme Court wrote:

"... the overall percentage disparity has been small and reflects no studied attempt to include or exclude a specified number of Negroes"

- The Supreme Court denied Robert Swain's appeal

Supreme Court Ruling [in Data]

- **Paraphrase:** 8/100 is less than 26%, but not different enough to show Black men were systematically excluded
- **Question:** is 8/100 a realistic outcome if the jury panel selection process were truly unbiased?

Sampling from a Distribution

- Sample at random from a categorical distribution

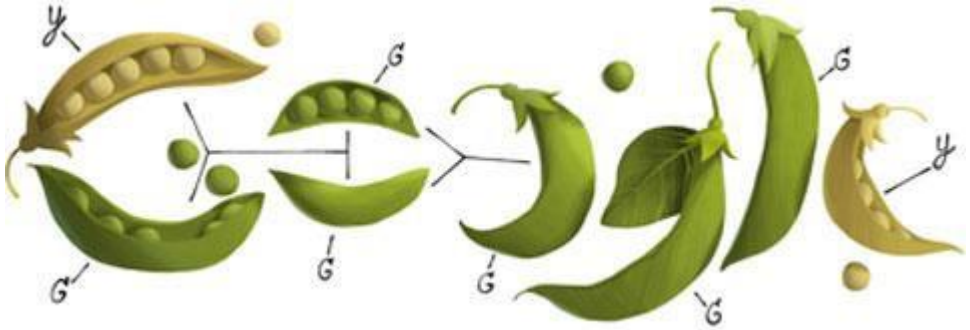
```
sample_proportions(sample_size, pop_distribution)
```

- Samples at random from the population
  - Returns an array containing the distribution of the categories in the sample

(Demo)

A Genetic Model

Gregor Mendel, 1822-1884



A Model

- Pea plants of a particular kind
- Each one has either purple flowers or white flowers

- Mendel's model:
  - Each plant is purple-flowering with chance 75%,
  - regardless of the colors of the other plants

- Question:
  - Is the model good, or not?

Choosing a Statistic

- Take a sample, see what percent are purple-flowering
- If that percent is much larger or much smaller than 75, that is evidence against the model
- ***Distance*** from 75 is the key

- Statistic:

  | sample percent of purple-flowering plants - 75 |

- If the statistic is large, that is evidence against the model

(Demo)

Two Viewpoints

Model and Alternative

- **Jury selection:**
  - **Model:** The people on the jury panels were selected at random from the eligible population
  - **Alternative viewpoint:** No, they weren't

- **Genetics:**
  - **Model:** Each plant has a 75% chance of having purple flowers
  - **Alternative viewpoint:** No, it doesn't

Steps in Assessing a Model

- **Choose a statistic** to measure discrepancy between model and data
- **Simulate the statistic** under the model's assumptions
- **Compare** the data to the model's predictions:
  - Draw a histogram of simulated values of the statistic
  - Compute the observed statistic from the real sample
- If the observed statistic is far from the histogram, that is evidence against the model