

A CORRESPONDENCE BETWEEN PROJECTIVE GEOMETRY AND CIRCLE GEOMETRY

OPAL GRAHAM

FLASH TALK

SPRING 2019



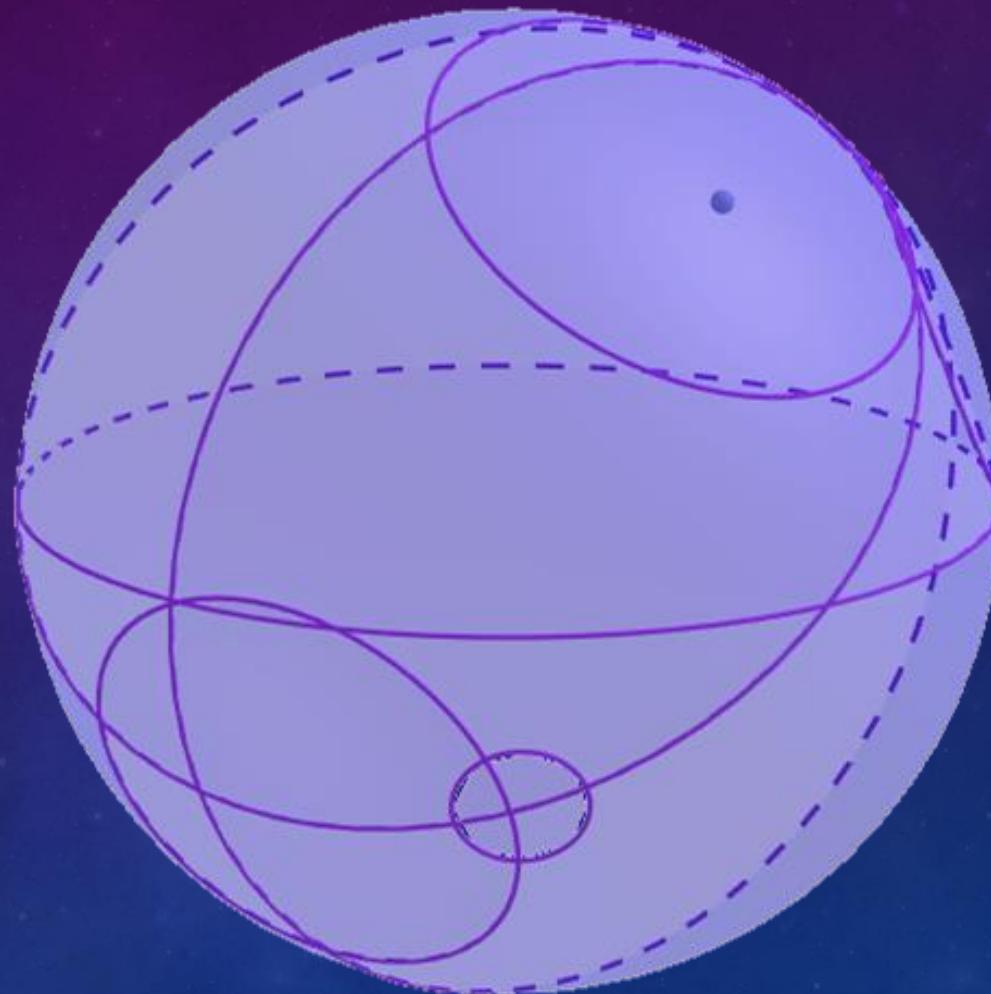
FLORIDA STATE UNIVERSITY
Mathematics

CIRCLE SPACE $\mathcal{C}(\mathbb{S}^2)$



CIRCLE SPACE $\mathcal{C}(\mathbb{S}^2)$

- Elements are circles of radius $0 \leq r \leq \pi$ in \mathbb{S}^2 .



PROJECTIVE SPACE \mathbb{RP}^3

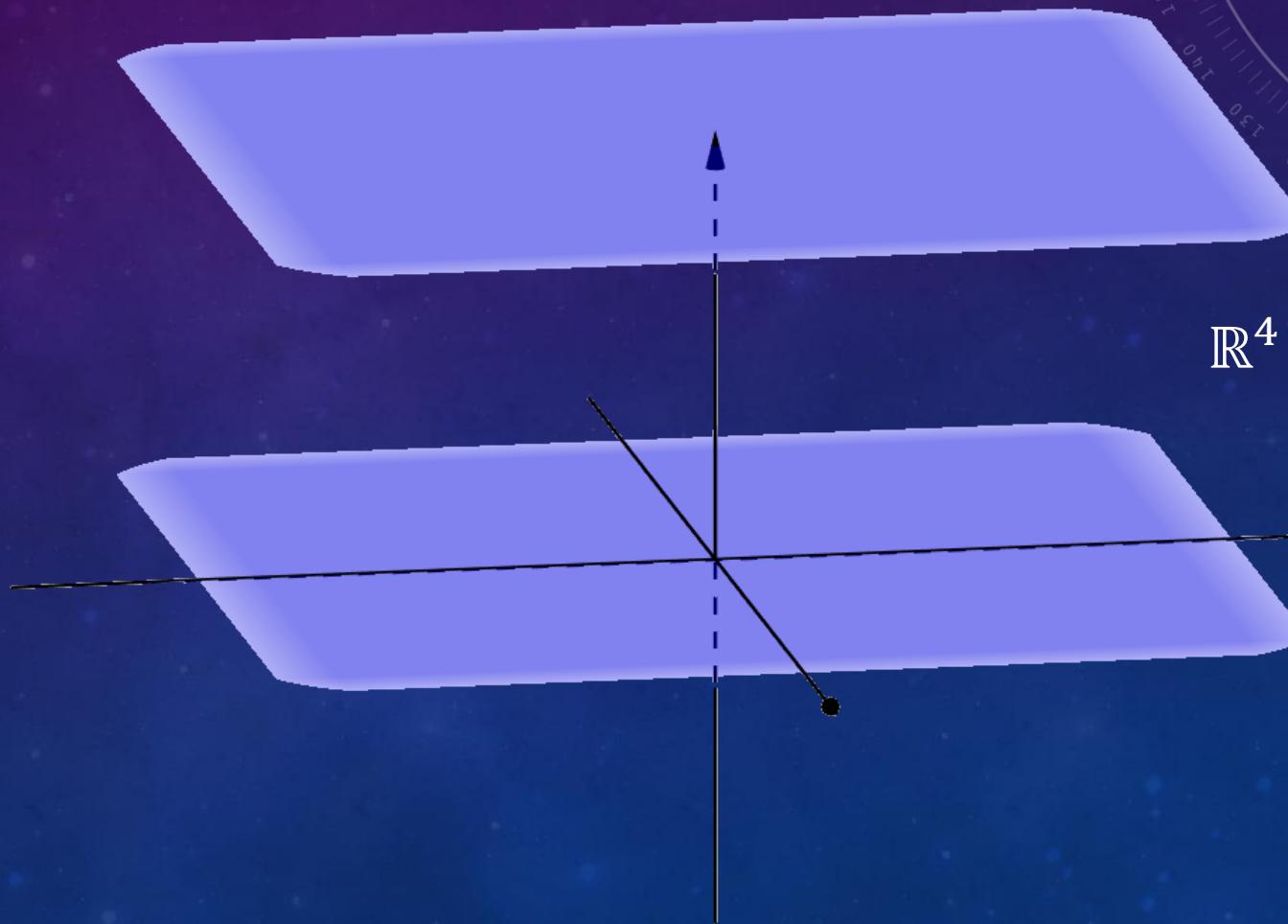


PROJECTIVE SPACE \mathbb{RP}^3

- Elements are points
in $\mathbb{E}^3 \cup \mathbb{RP}^2$.

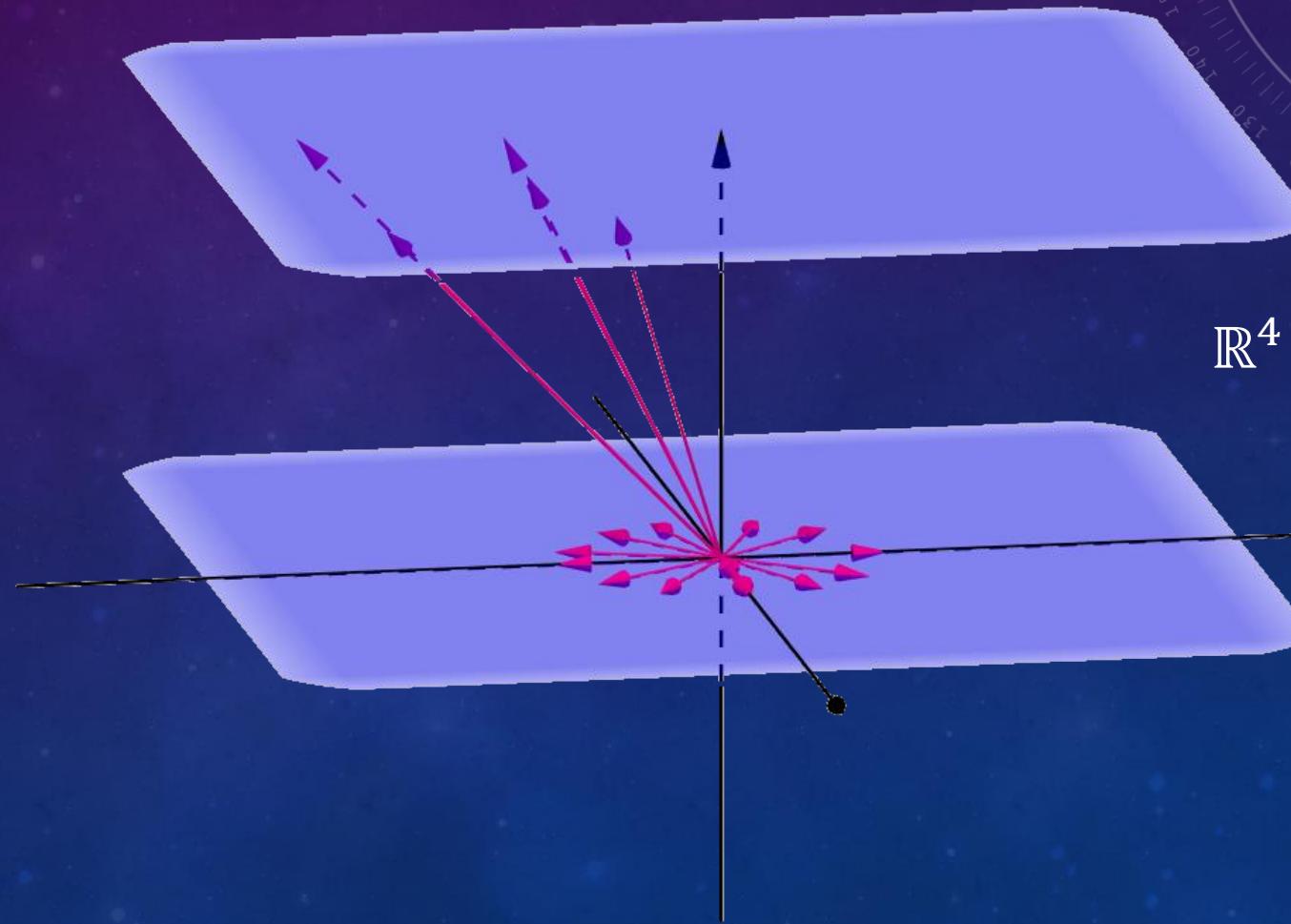
PROJECTIVE SPACE \mathbb{RP}^3

- Elements are points
in $\mathbb{E}^3 \cup \mathbb{RP}^2$.



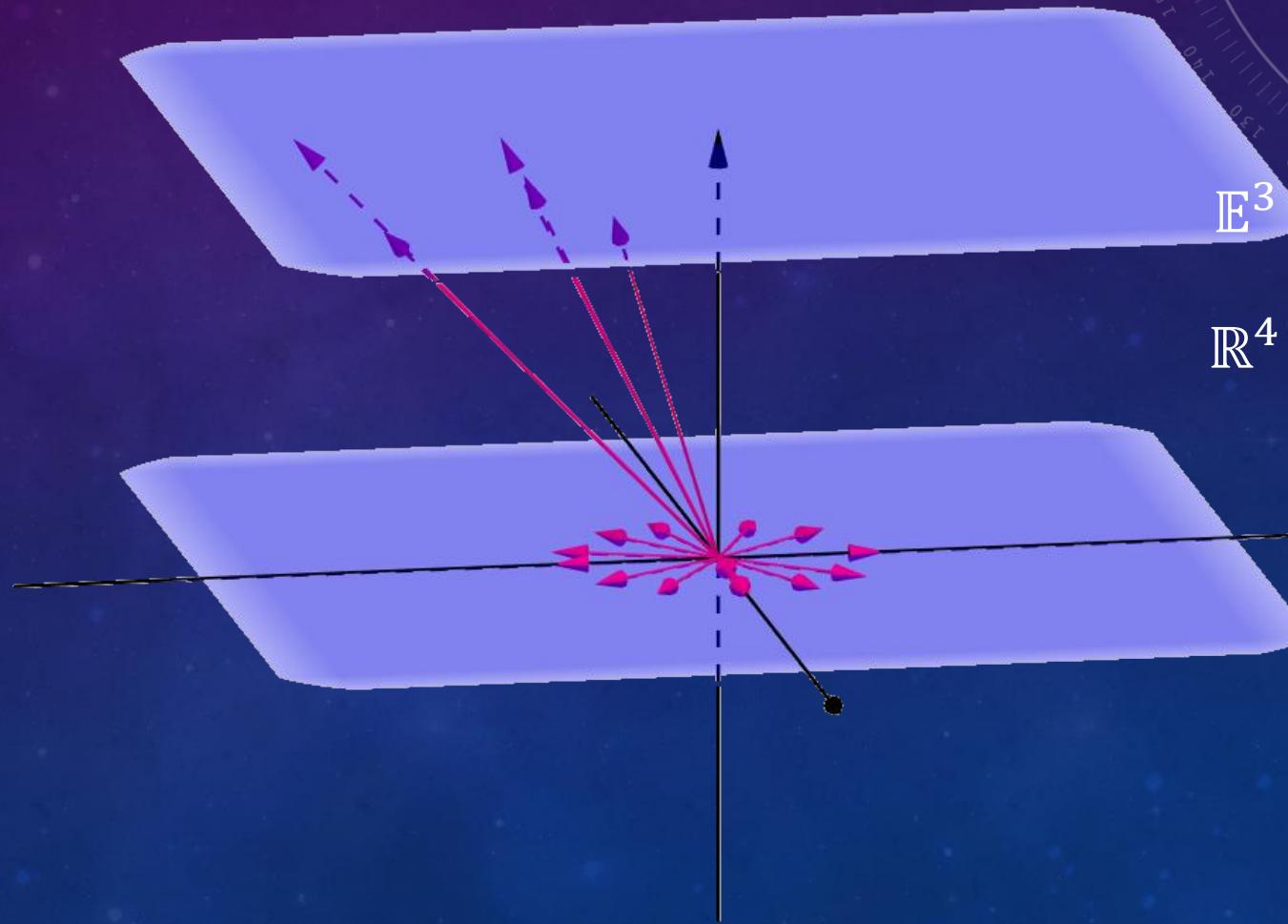
PROJECTIVE SPACE \mathbb{RP}^3

- Elements are points in $\mathbb{E}^3 \cup \mathbb{RP}^2$.



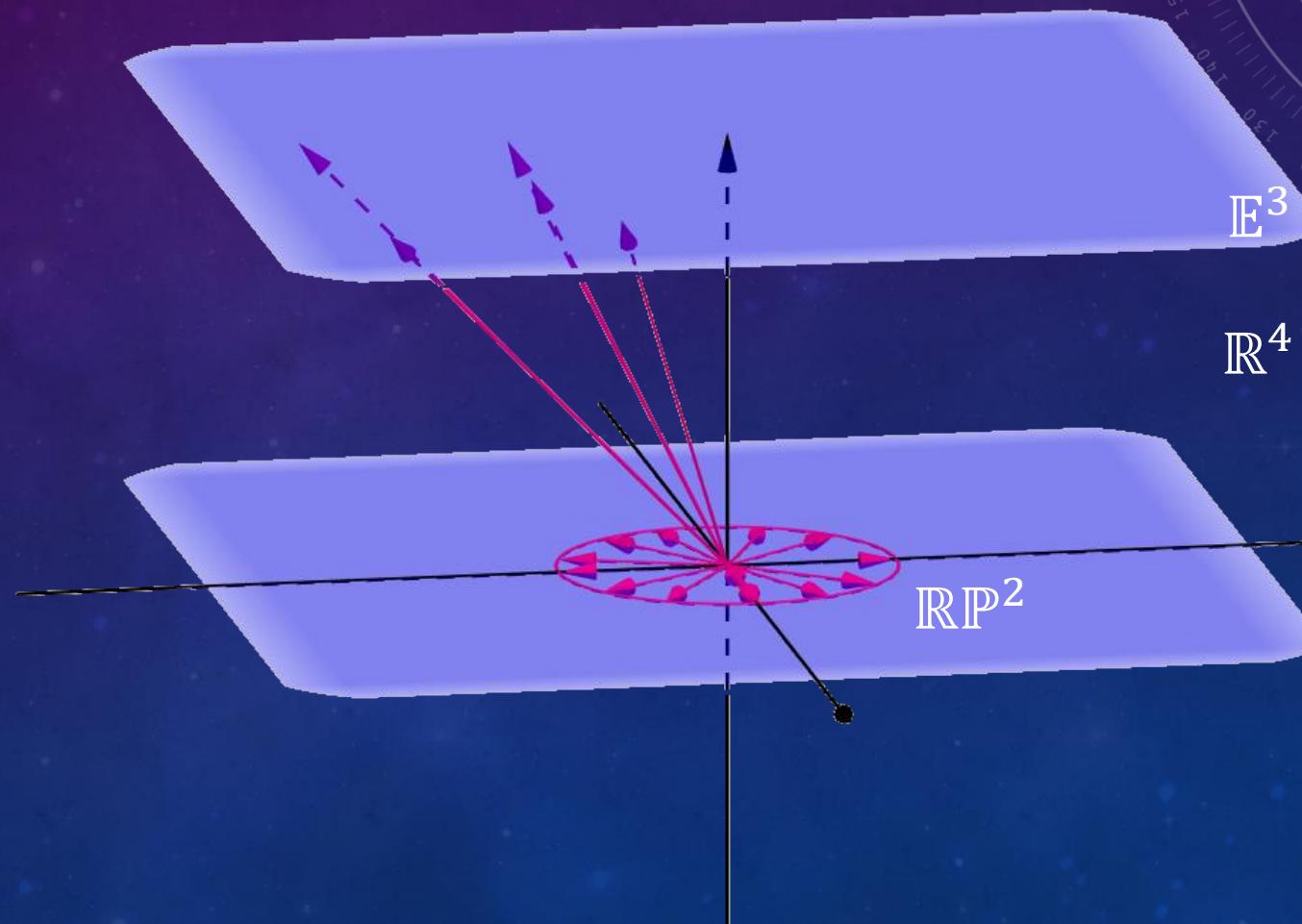
PROJECTIVE SPACE \mathbb{RP}^3

- Elements are points in $\mathbb{E}^3 \cup \mathbb{RP}^2$.



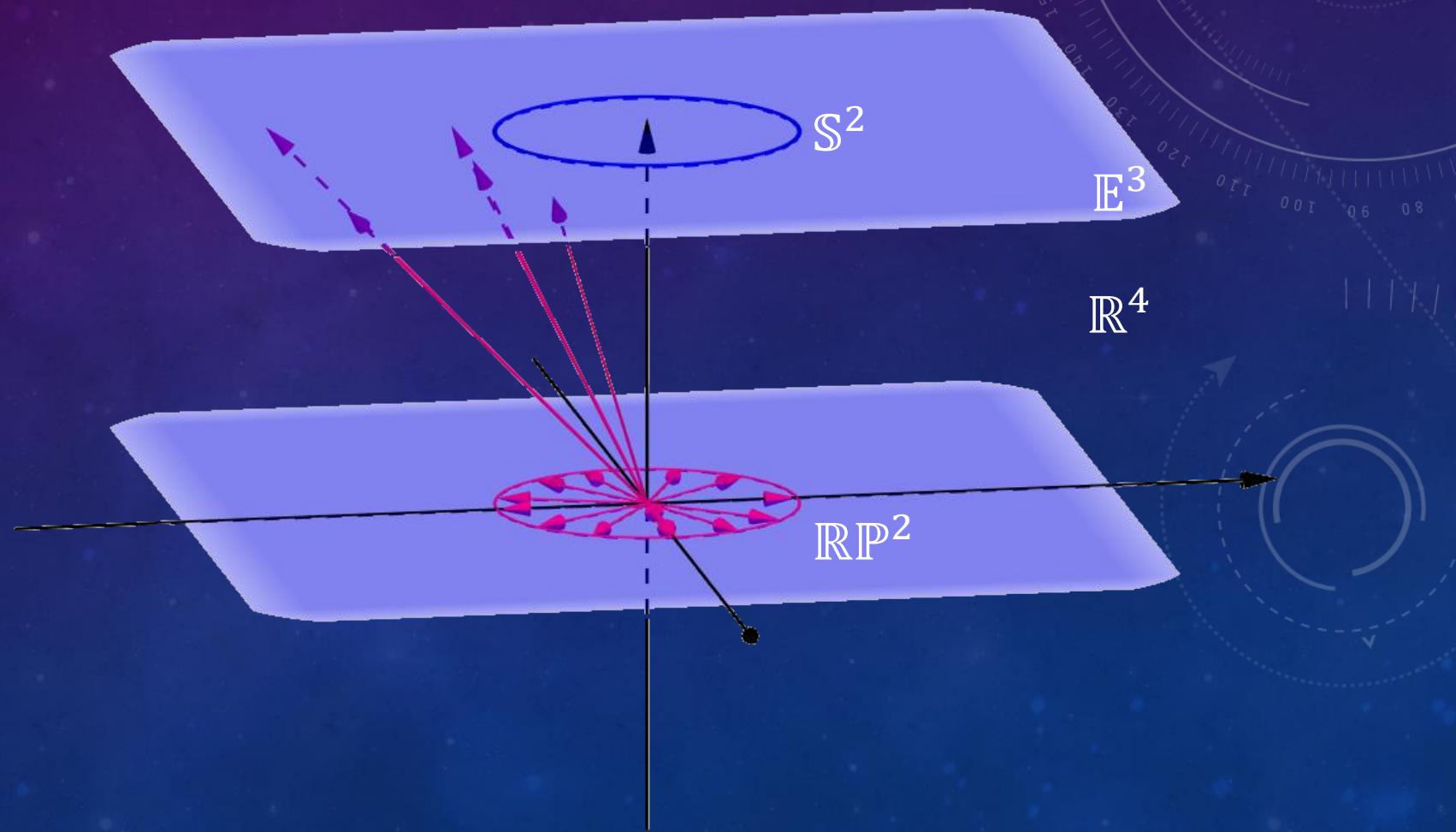
PROJECTIVE SPACE \mathbb{RP}^3

- Elements are points in $\mathbb{E}^3 \cup \mathbb{RP}^2$.



PROJECTIVE SPACE \mathbb{RP}^3

- Elements are points in $\mathbb{E}^3 \cup \mathbb{RP}^2$.
- Circle geometry is a subgeometry of projective geometry.



C-POINTS



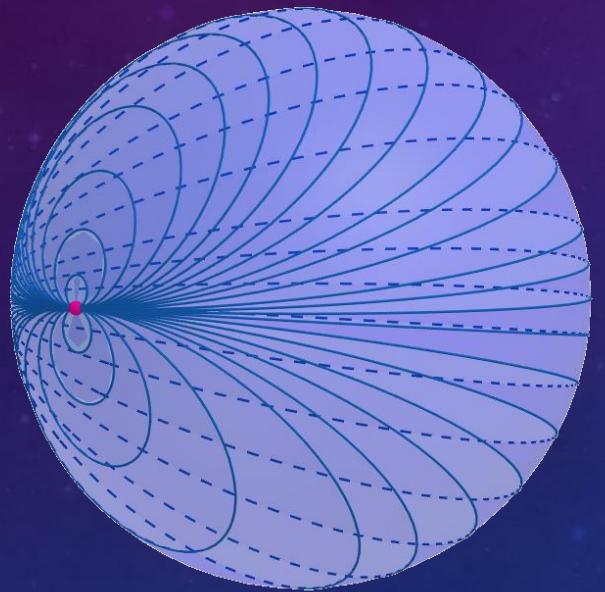
C-POINTS → PROJECTIVE PLANES



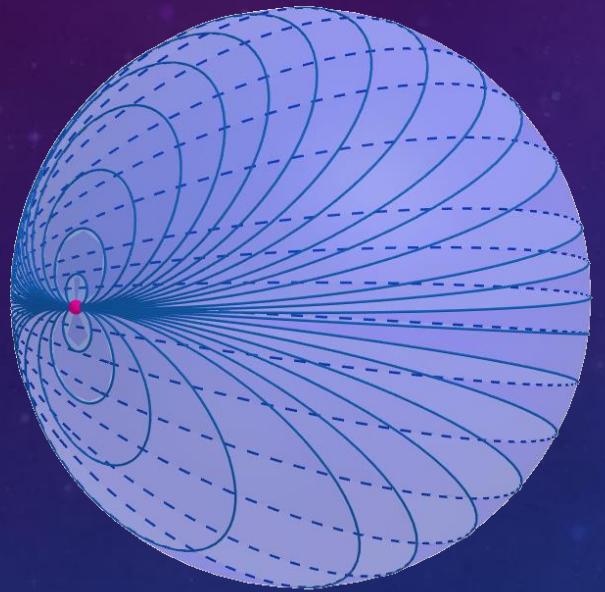
C-POINTS → PROJECTIVE PLANES



C-LINES



C-LINES

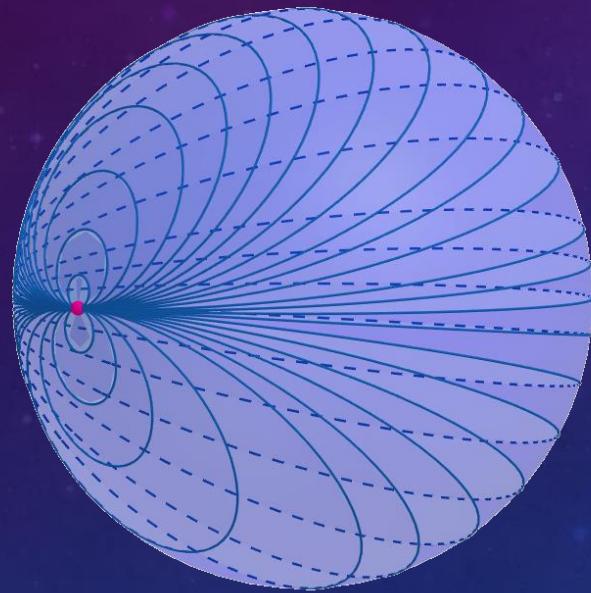


1. Intersect at one point

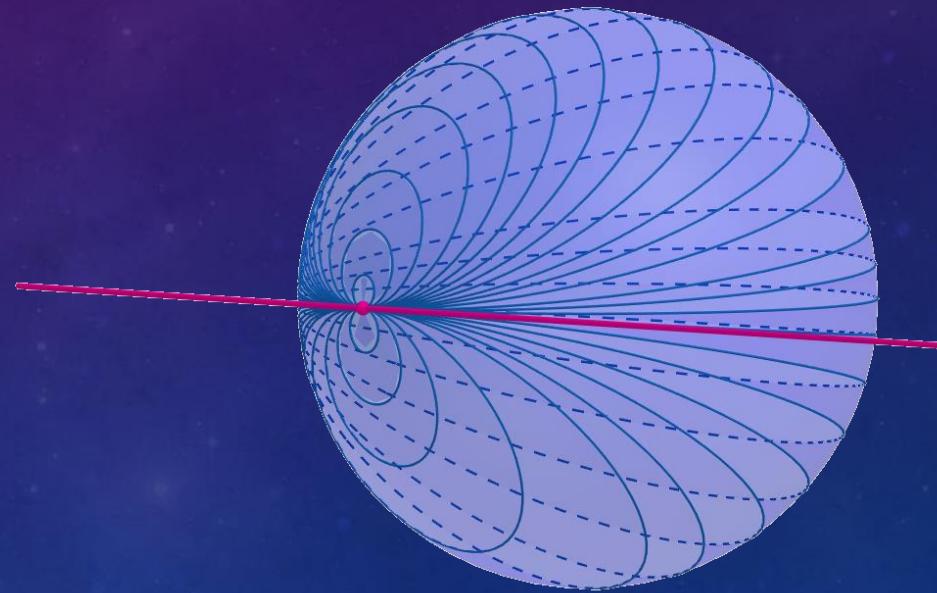
C-LINES



PROJECTIVE LINES



1. Intersect at one point

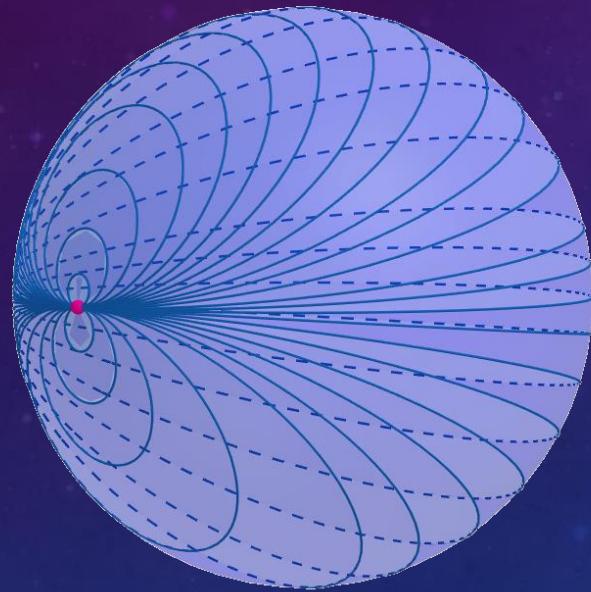


Tangent to \mathbb{S}^2 at point

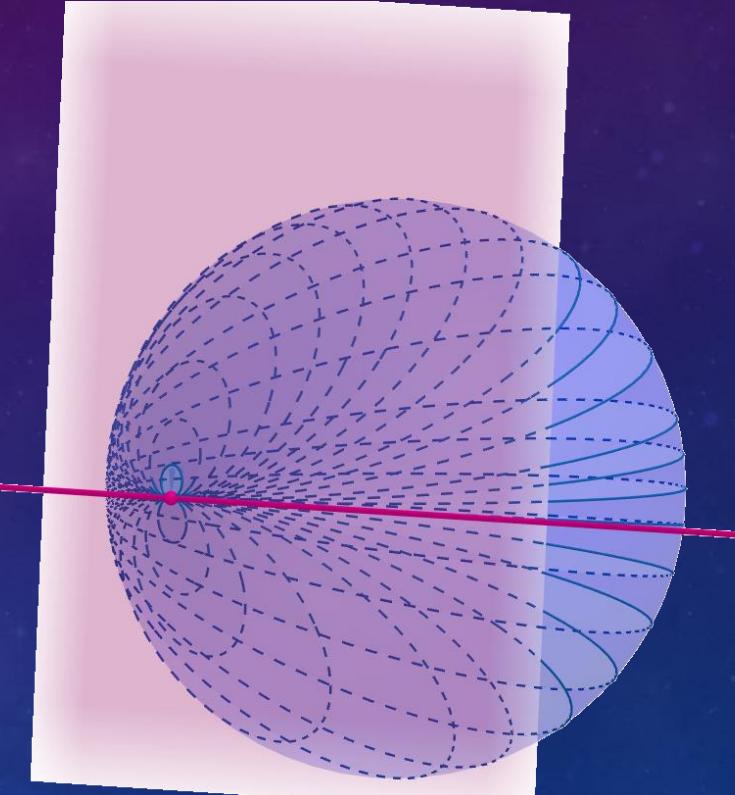
C-LINES



PROJECTIVE LINES



1. Intersect at one point

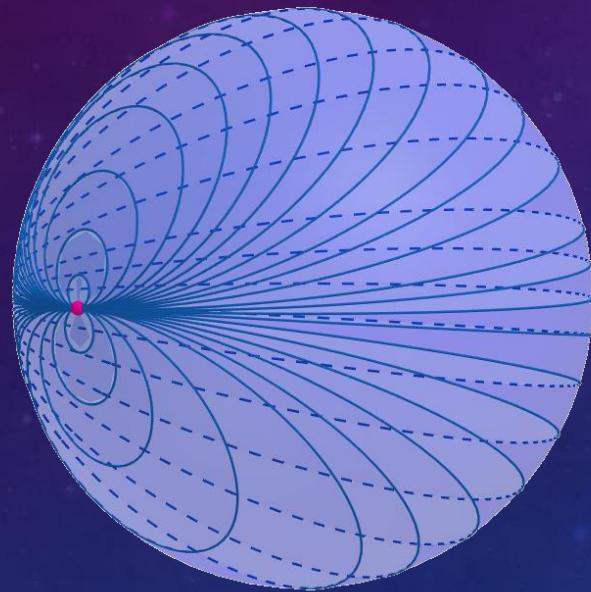


Tangent to \mathbb{S}^2 at point

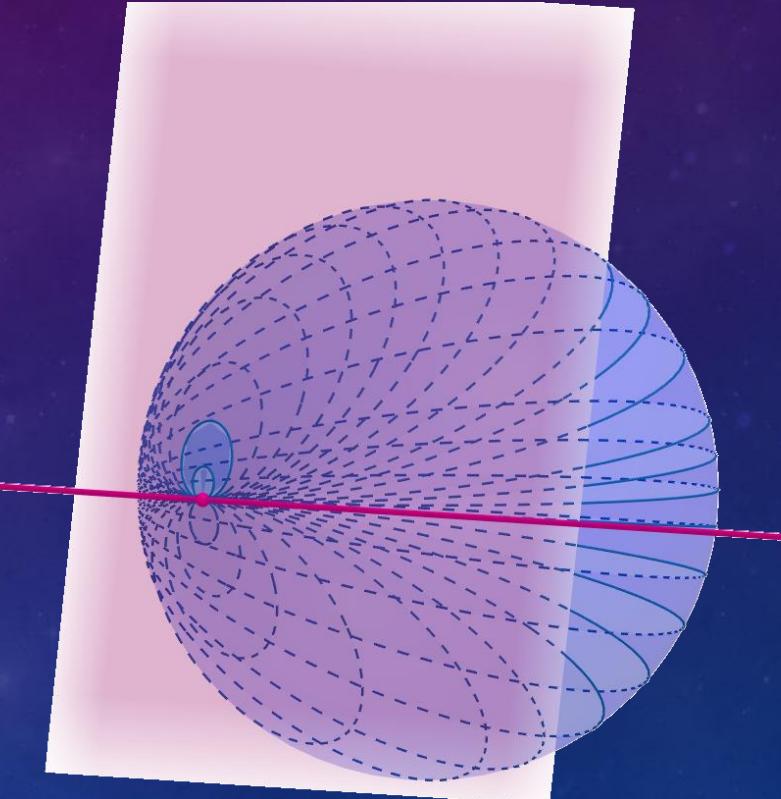
C-LINES



PROJECTIVE LINES



1. Intersect at one point

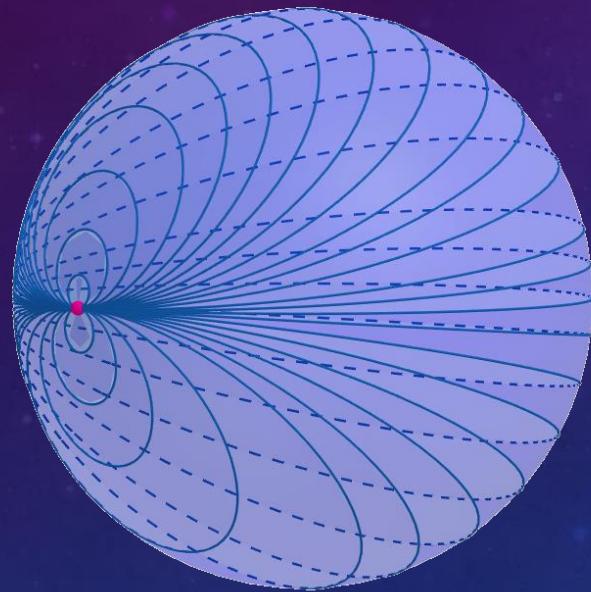


Tangent to \mathbb{S}^2 at point

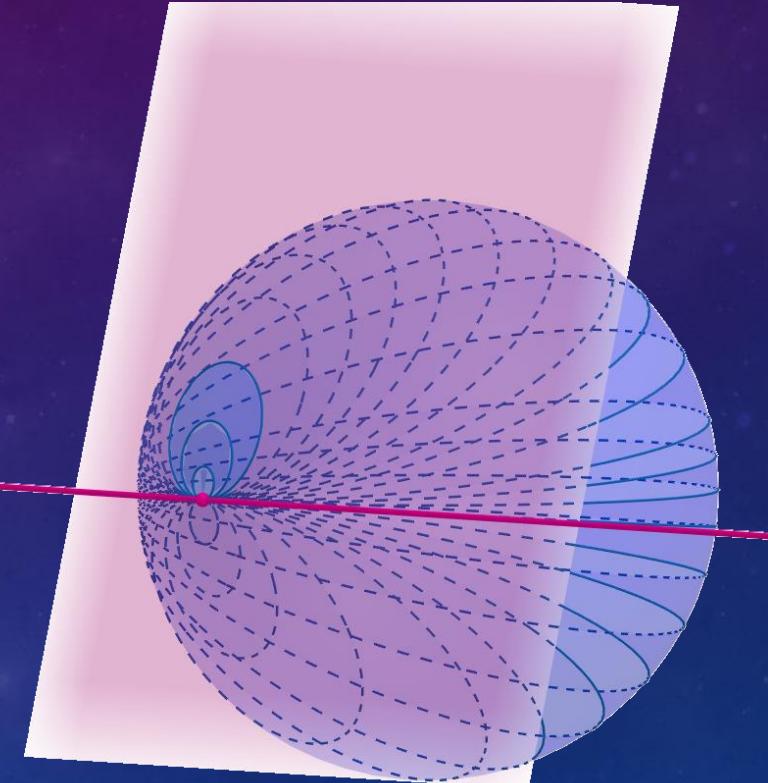
C-LINES



PROJECTIVE LINES



1. Intersect at one point

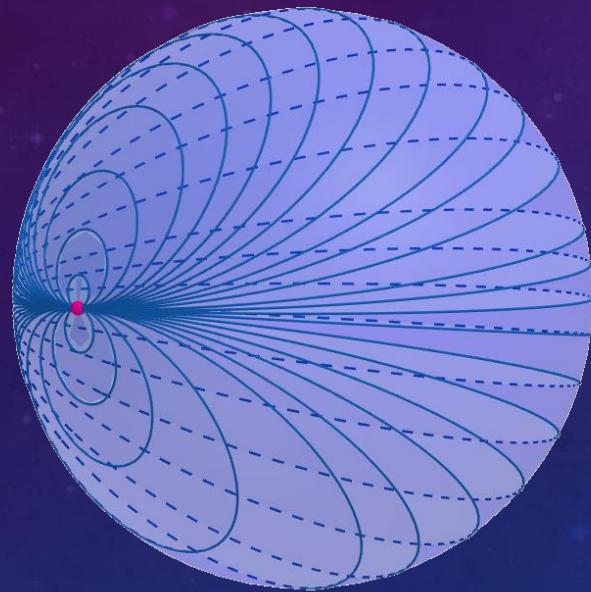


Tangent to \mathbb{S}^2 at point

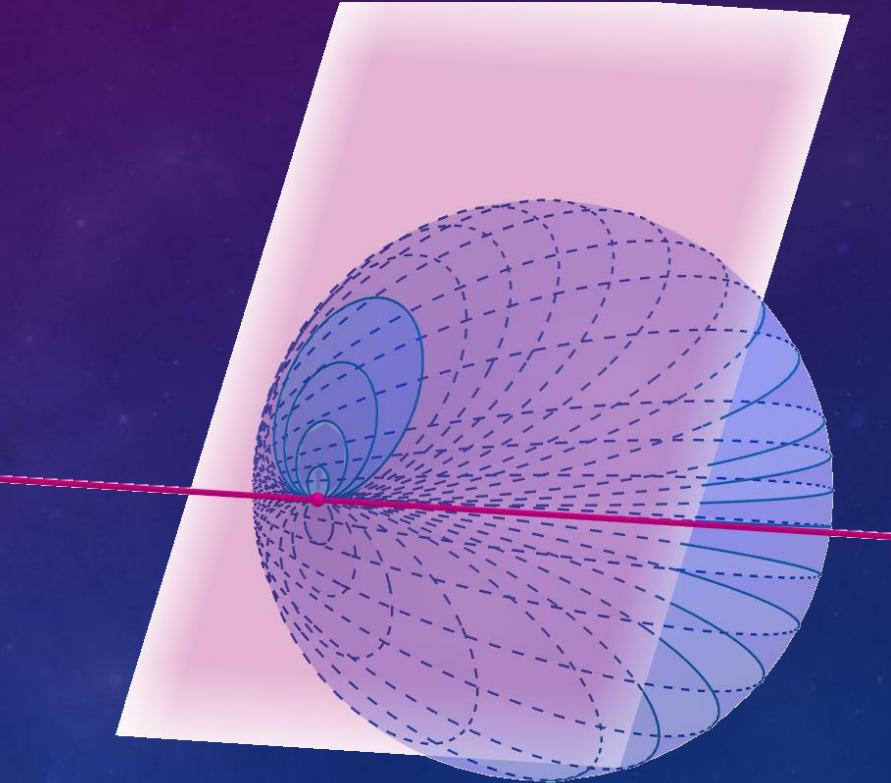
C-LINES



PROJECTIVE LINES



1. Intersect at one point

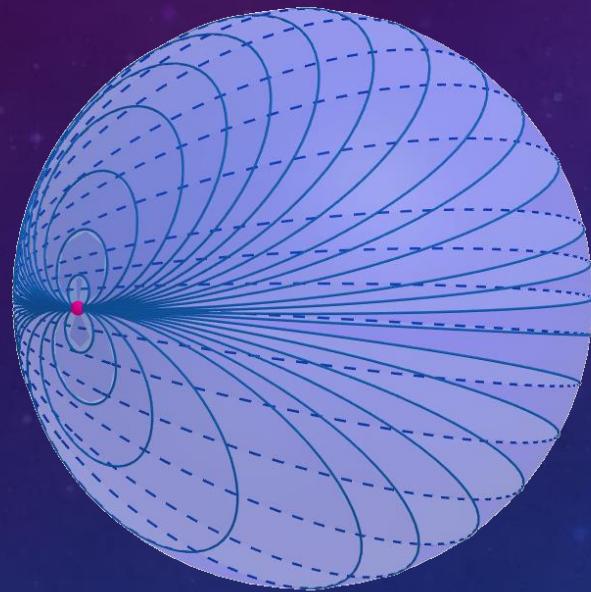


Tangent to \mathbb{S}^2 at point

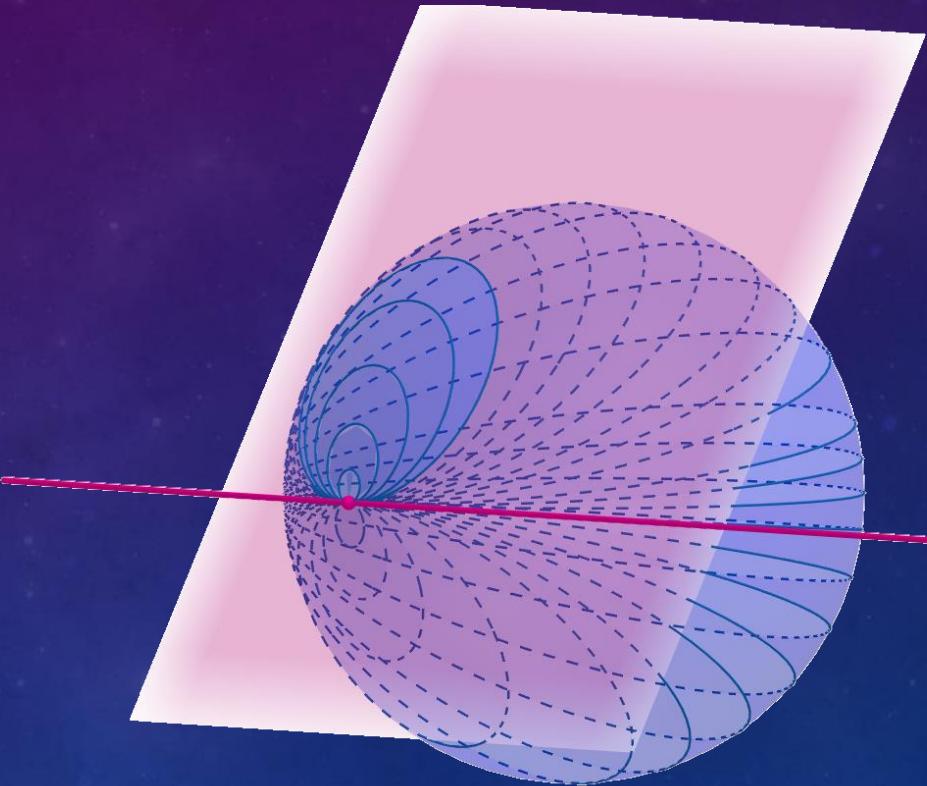
C-LINES



PROJECTIVE LINES



1. Intersect at one point

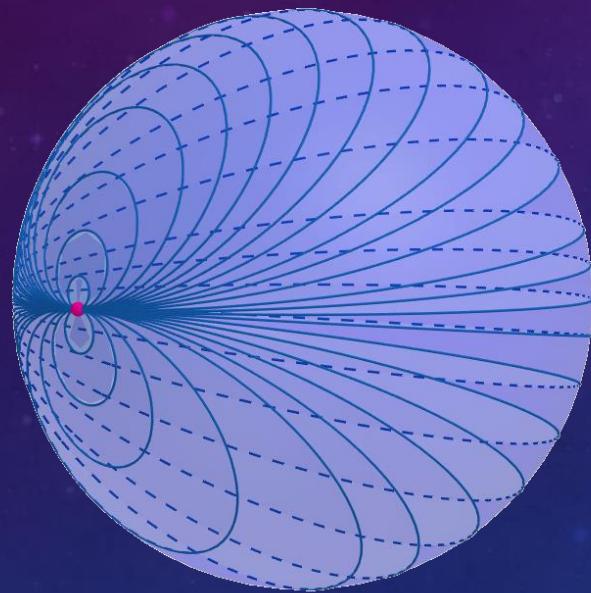


Tangent to \mathbb{S}^2 at point

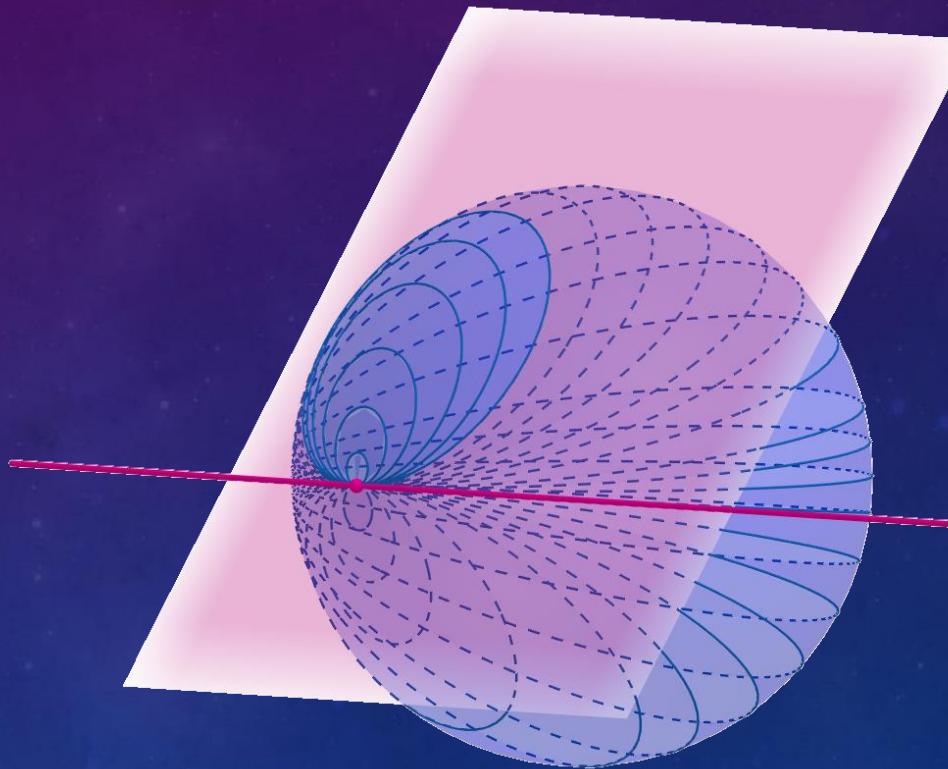
C-LINES



PROJECTIVE LINES



1. Intersect at one point

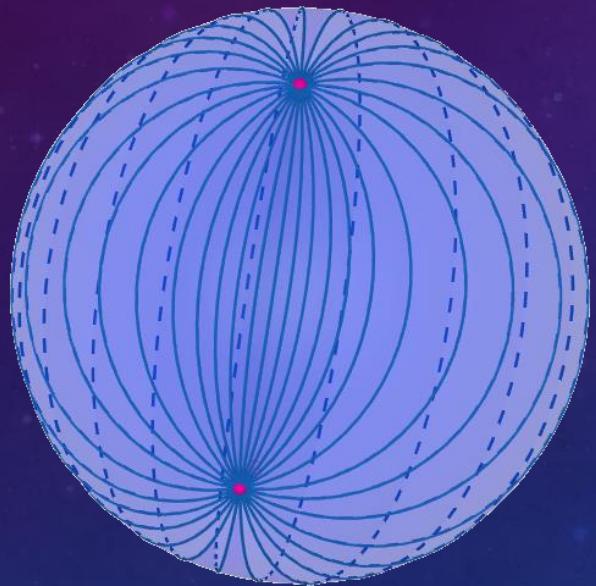


Tangent to \mathbb{S}^2 at point

C-LINES



PROJECTIVE LINES

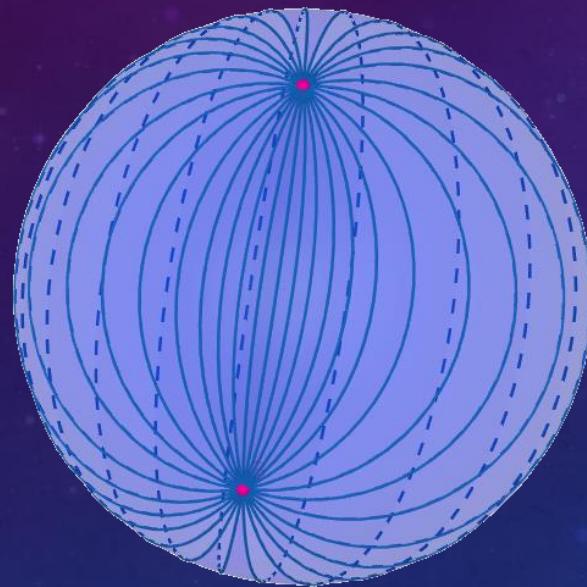


2. Intersect at two points

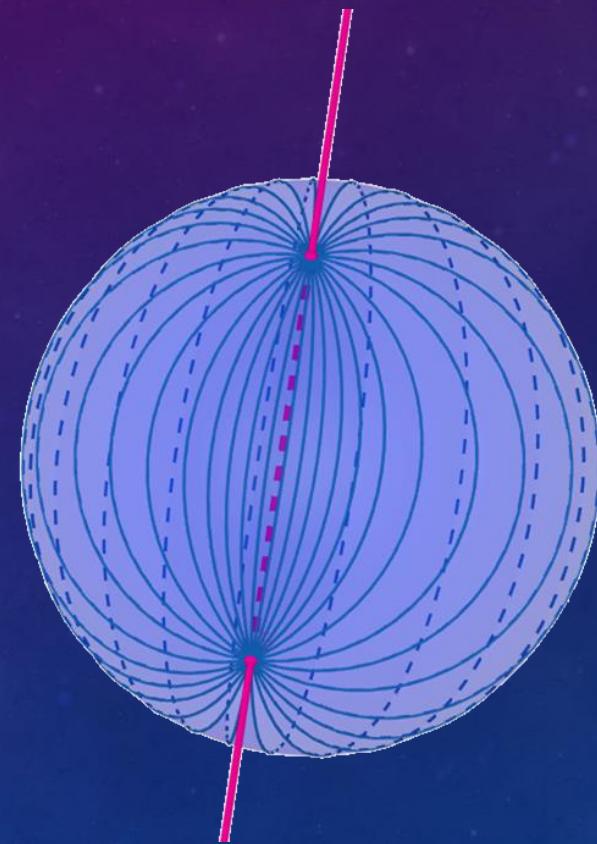
C-LINES



PROJECTIVE LINES



2. Intersect at two points

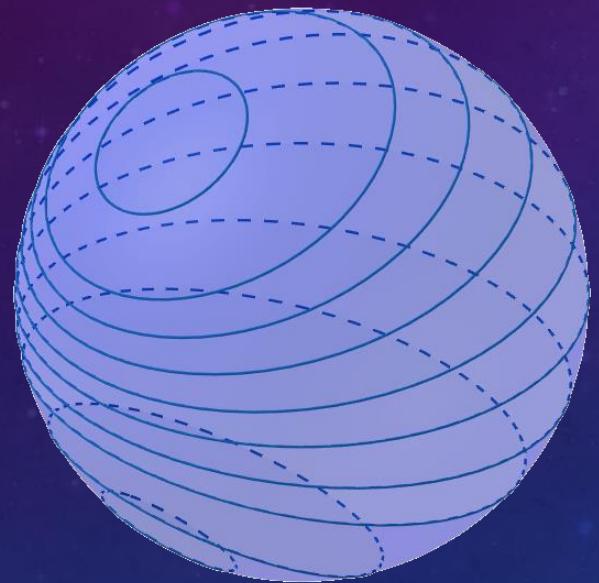


Intersects \mathbb{S}^2 at two points

C-LINES



PROJECTIVE LINES

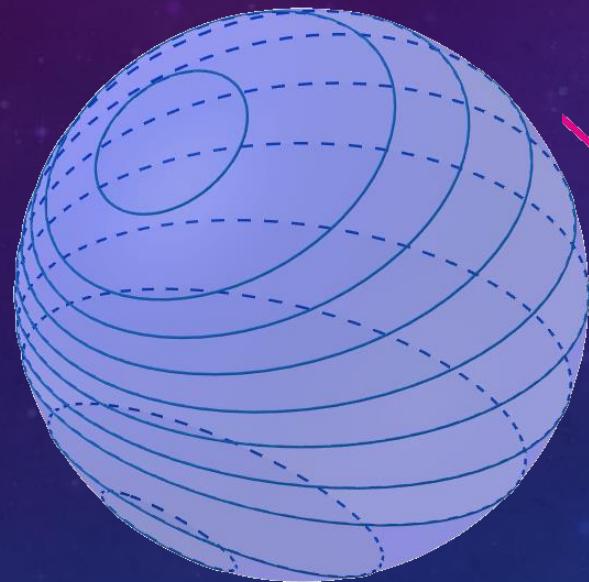


3. Don't intersect

C-LINES



PROJECTIVE LINES

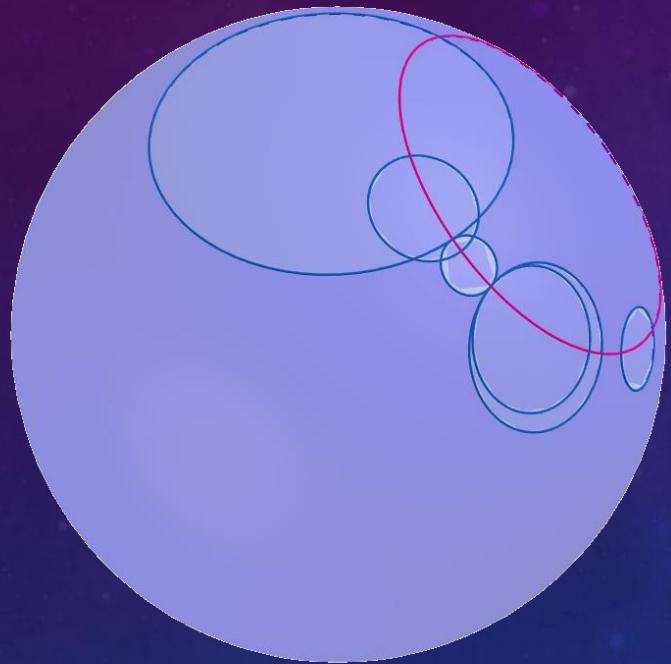


3. Don't intersect

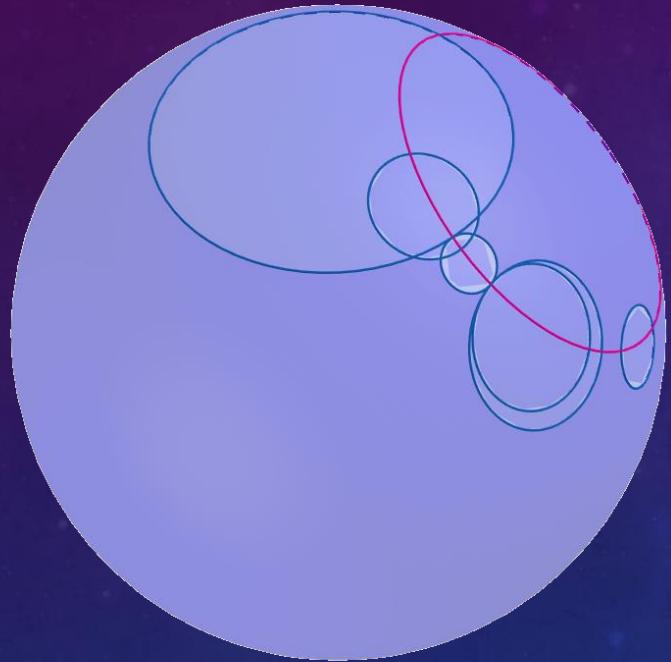


Doesn't intersect \mathbb{S}^2 .

C-PLANES

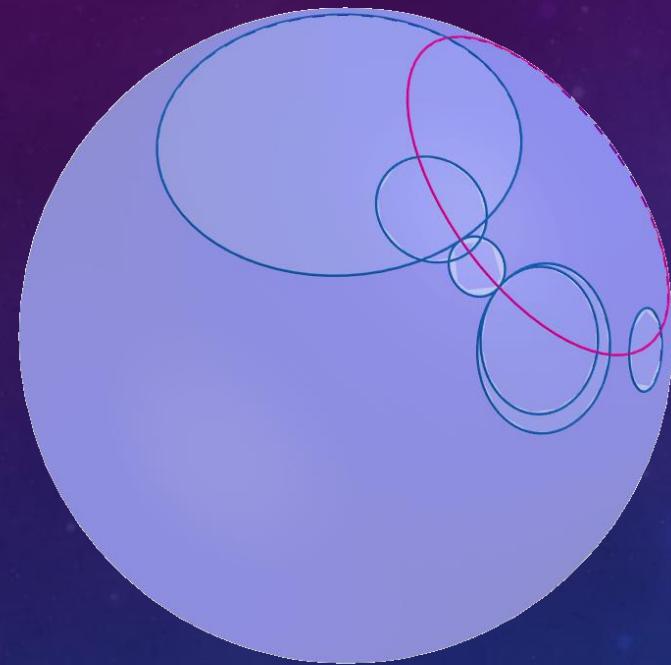


C-PLANES

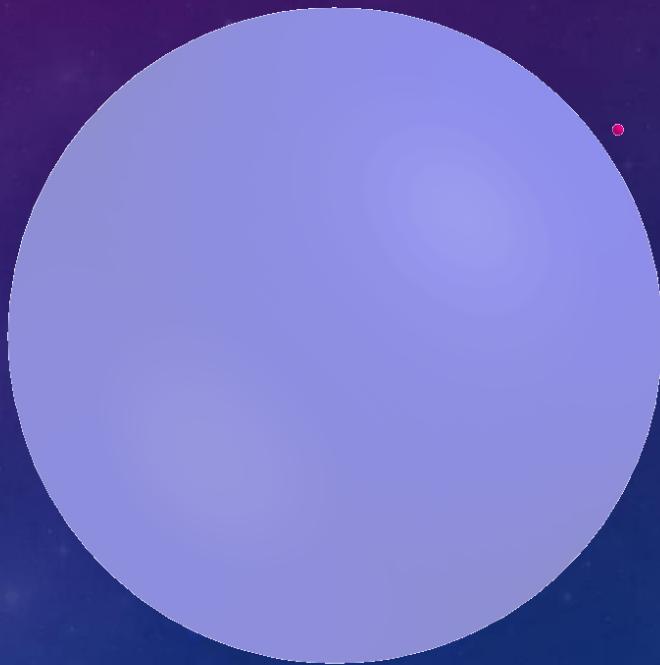


1. All circles
orthogonal to
generating circle

C-PLANES \leftrightarrow PROJECTIVE POINTS

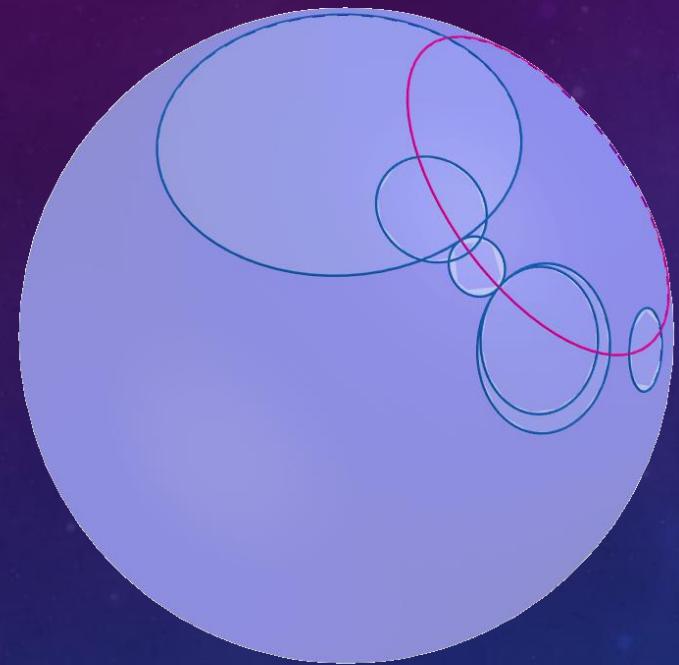


1. All circles
orthogonal to
generating circle

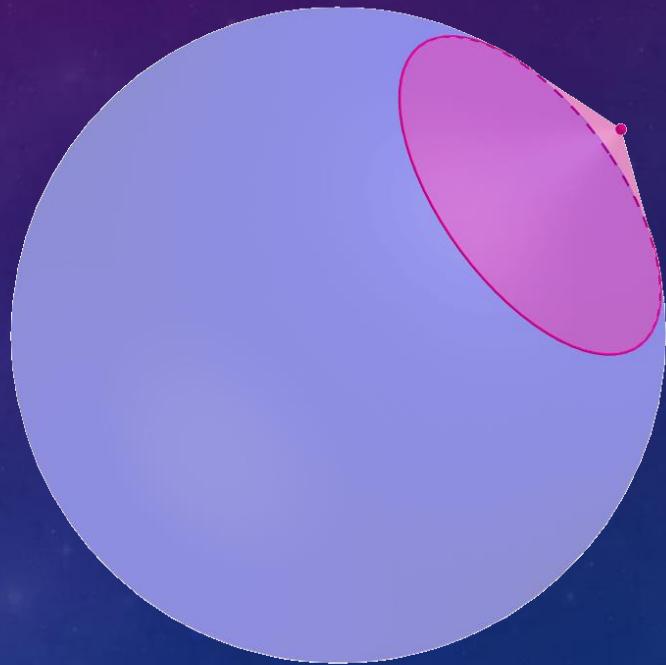


Point outside sphere

C-PLANES \leftrightarrow PROJECTIVE POINTS

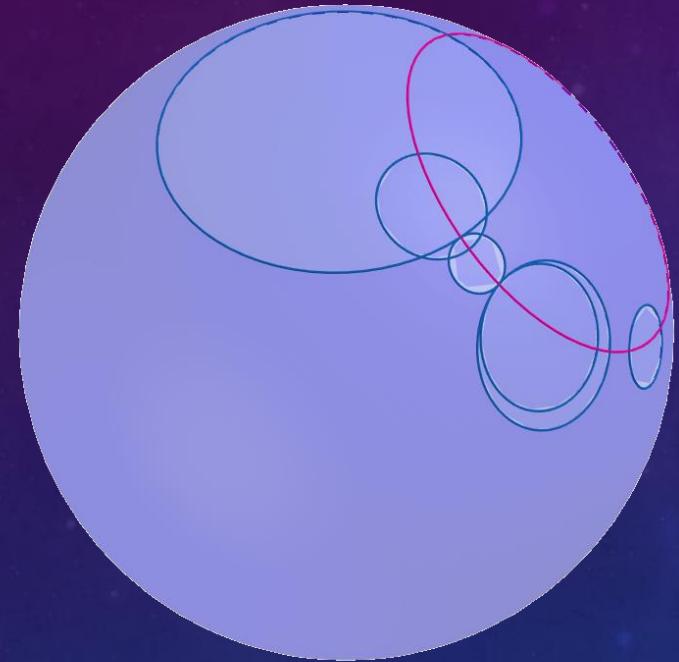


1. All circles
orthogonal to
generating circle

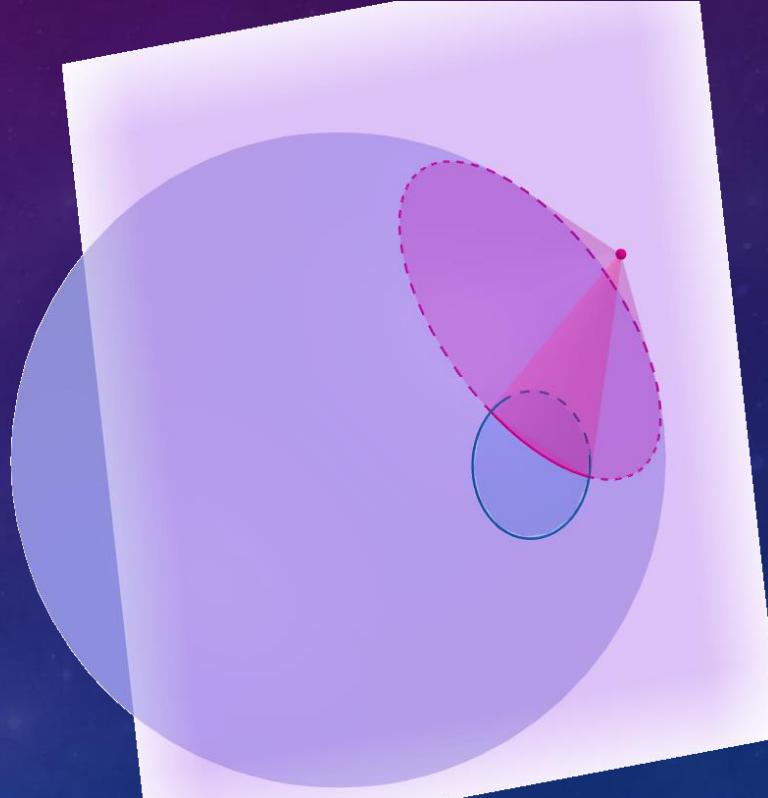


Point outside sphere

C-PLANES \leftrightarrow PROJECTIVE POINTS

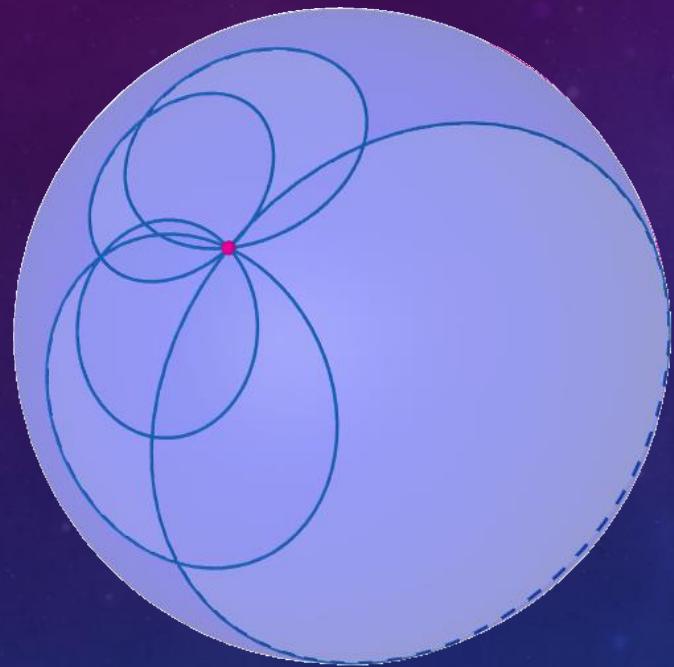


1. All circles
orthogonal to
generating circle



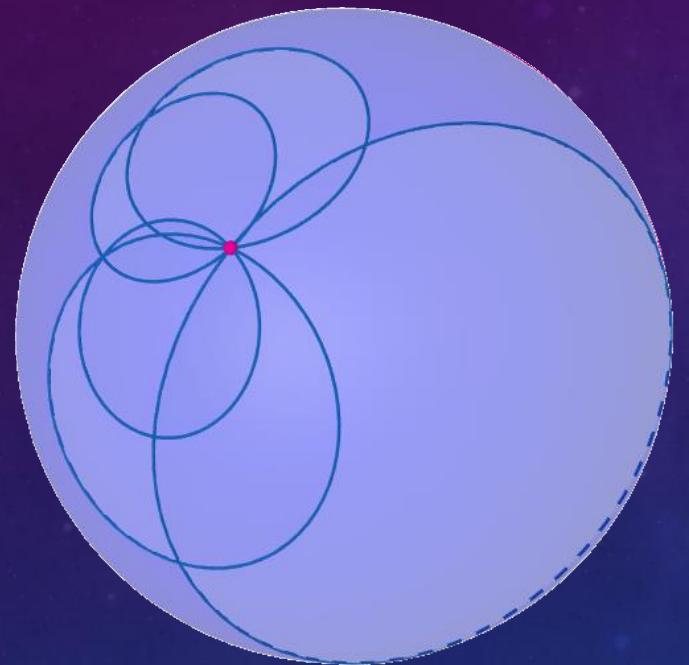
Point outside sphere

C-PLANES \leftrightarrow PROJECTIVE POINTS

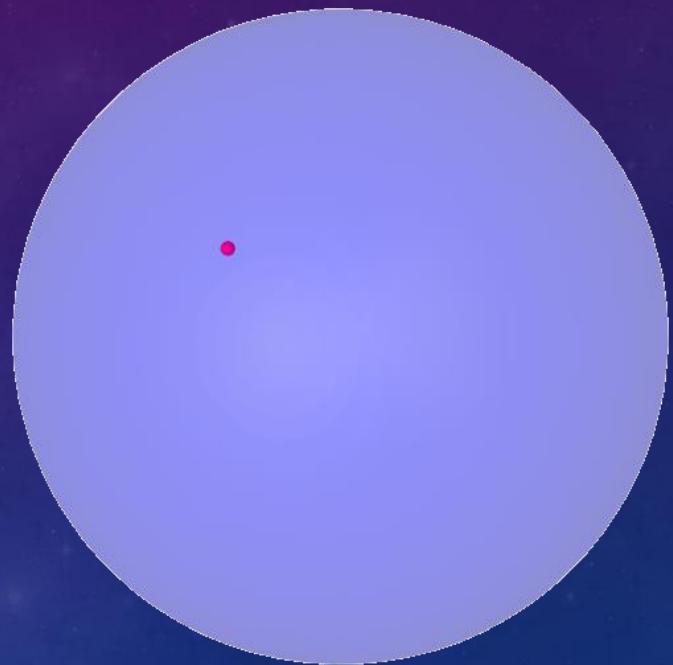


2. All circles through generating “circle”

C-PLANES \leftrightarrow PROJECTIVE POINTS

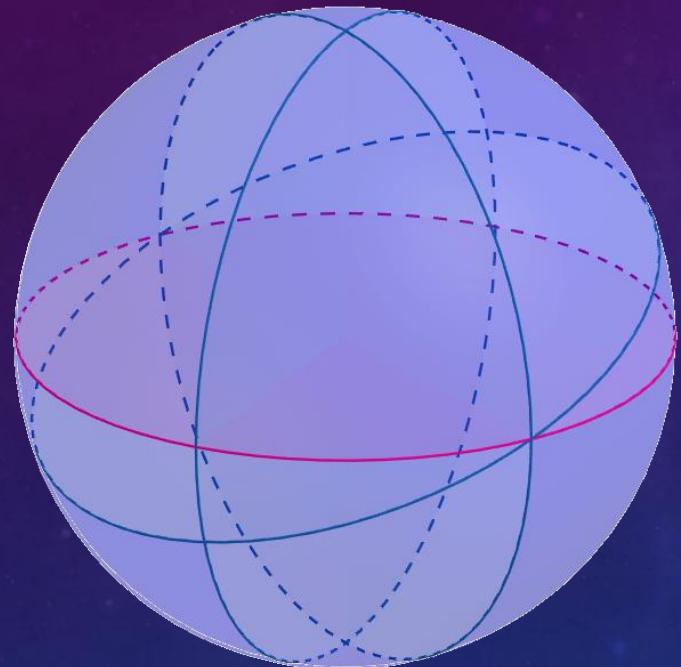


2. All circles
through generating
“circle”



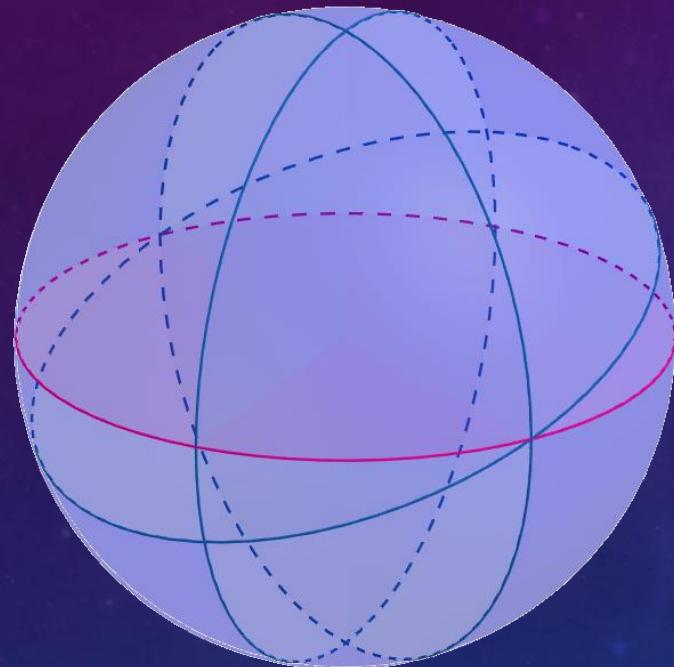
Point on sphere

C-PLANES \leftrightarrow PROJECTIVE POINTS

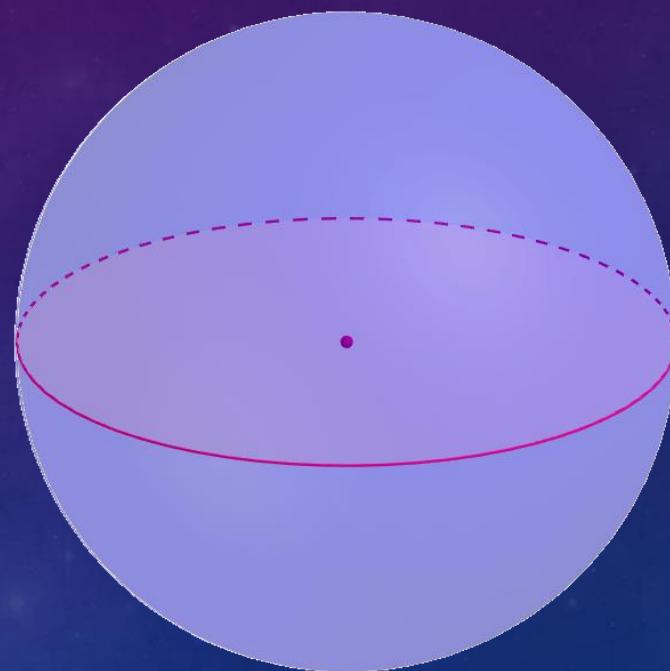


3. All circles
through antipodal
points on
generating circle

C-PLANES \leftrightarrow PROJECTIVE POINTS



3. All circles
through antipodal
points on
generating circle

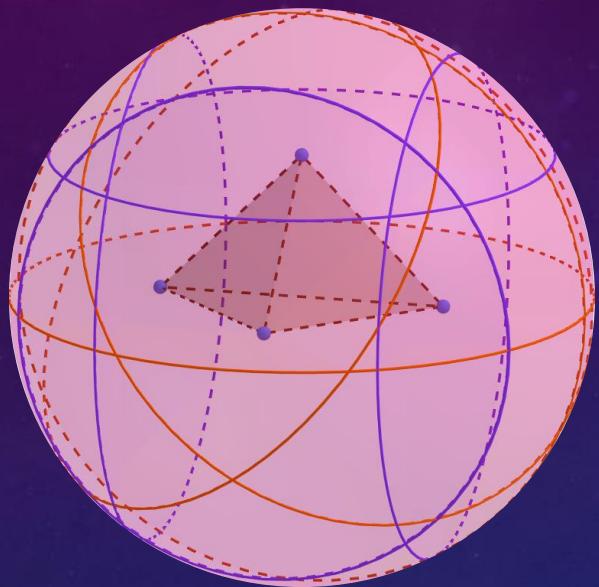


Point inside sphere

C-POLYHEDRA



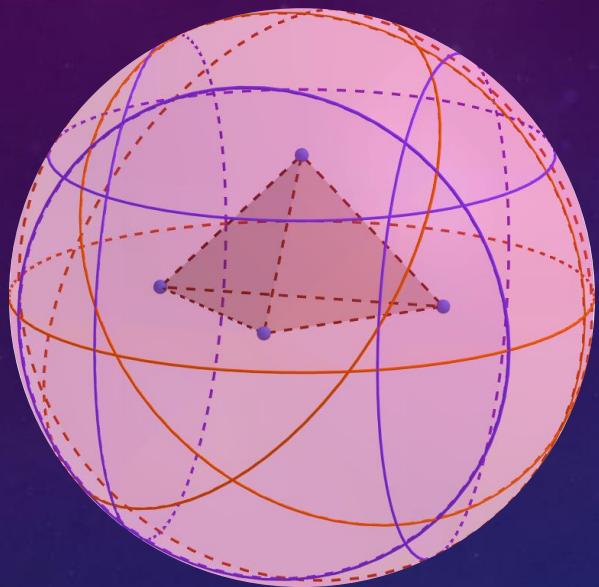
PROJECTIVE POLYHEDRA



C-POLYHEDRA



PROJECTIVE POLYHEDRA



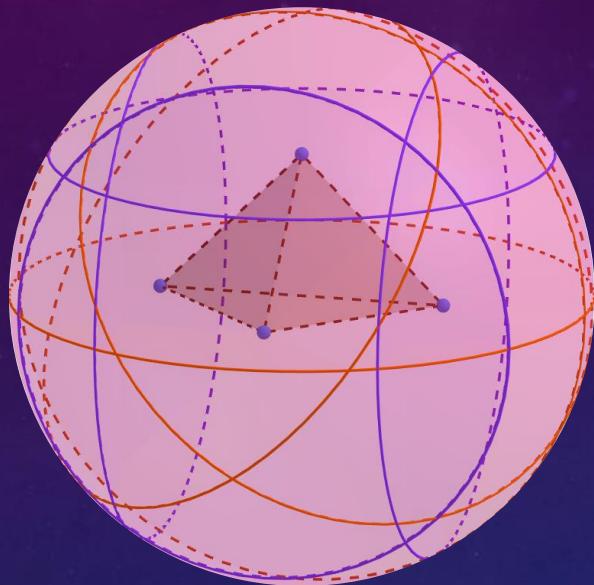
▲ Hyperbolic polyhedra :

- Andre'ev ('70)
- Rivin & Hodgson ('93)

C-POLYHEDRA

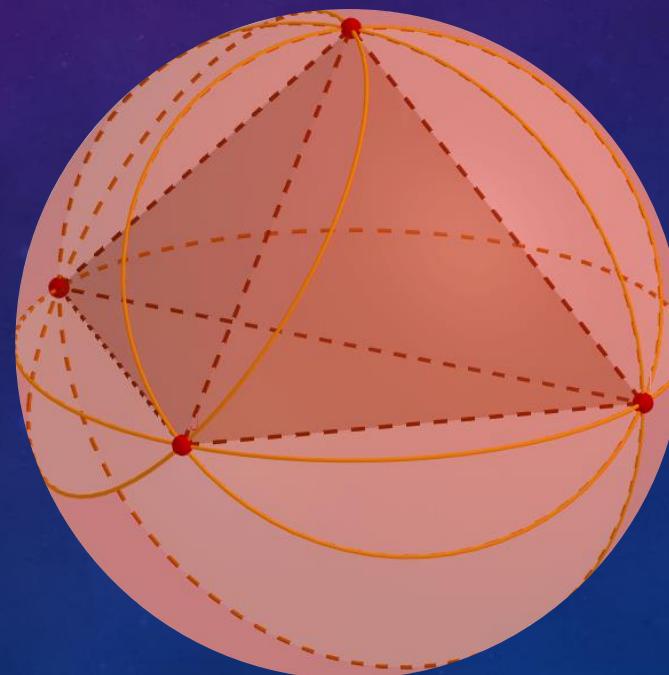


PROJECTIVE POLYHEDRA



- ▲ Hyperbolic polyhedra :
• Andre'ev ('70)
• Rivin & Hodgson ('93)

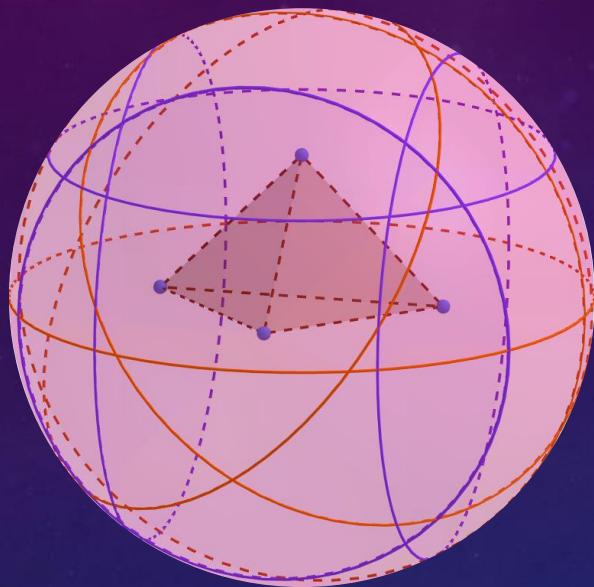
Ideal Polyhedra:
Rivin ('96) ▼



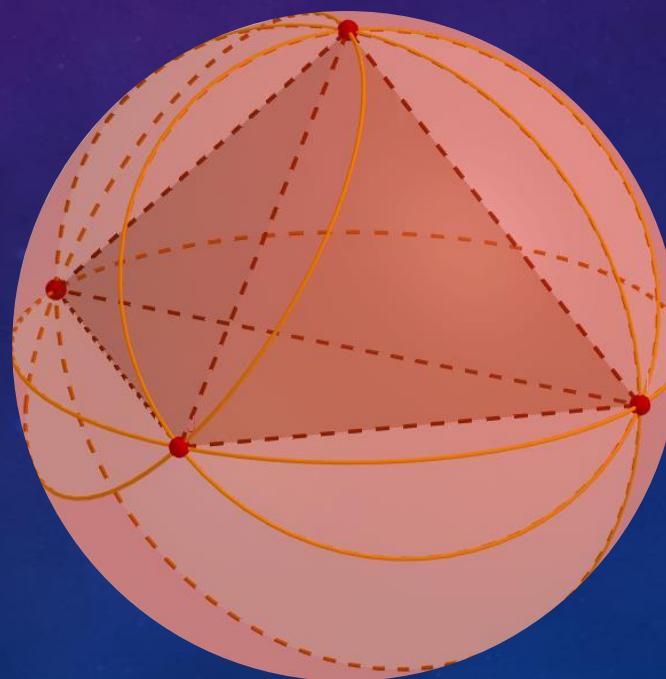
C-POLYHEDRA



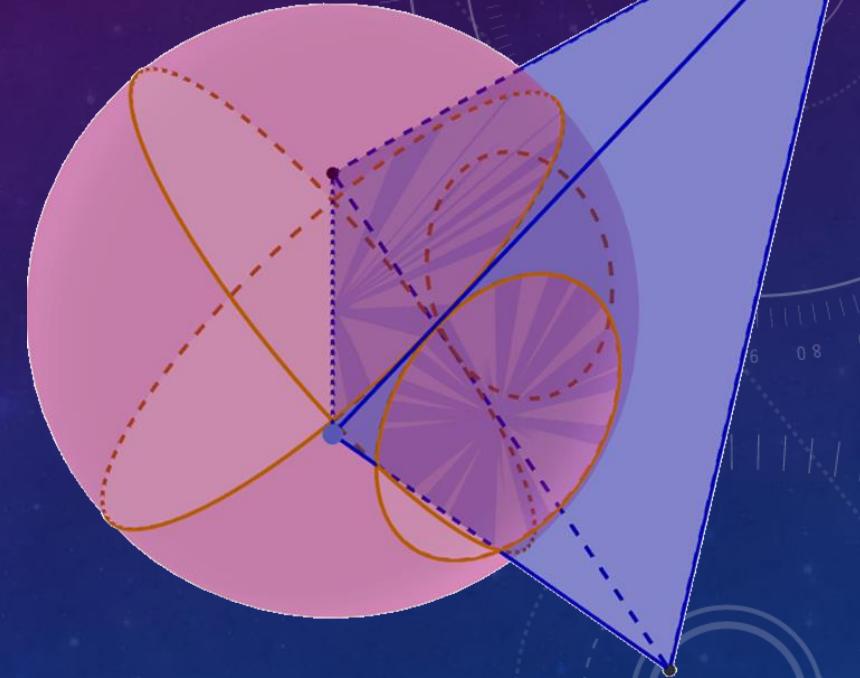
PROJECTIVE POLYHEDRA



- ▲ Hyperbolic polyhedra :
 - Andre'ev ('70)
 - Rivin & Hodgson ('93)



Ideal Polyhedra:
Rivin ('96) ▼



- ▲ Hyperideal polyhedra :
 - Bao & Bonahon ('02)
 - Bowers, Bowers, Pratt ('17)



“ALL GEOMETRY IS
PROJECTIVE GEOMETRY.”
ARTHUR CAYLEY

The background features several concentric circles of different sizes, all centered at the same point. Some circles are solid black, while others are dashed. Small black arrows point clockwise around each circle. In the upper right quadrant, there is a larger circle with a thick black border and a smaller circle inside it. A dashed arrow points from the center of this inner circle towards the text.

“ALL GEOMETRY IS
PROJECTIVE GEOMETRY.”
ARTHUR CAYLEY

THANK YOU!

DUALITY



DUALITY

- c-points \leftrightarrow projective points

DUALITY

- c-points \leftrightarrow projective points
- c-lines \leftrightarrow projective lines

DUALITY

- c-points \leftrightarrow projective points
- c-lines \leftrightarrow projective lines
- c-planes \rightarrow projective planes