Applications of matrix computations Exercise 1 (November 3–4, 2014)

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1. Define the following matrices in Matlab. For example, the identity matrix of size 2×2can be given in the form $[[1,0];[0,1]]$.

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}.$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} 2 & 3 & 4 & 0 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 2 \end{bmatrix}.$$

   Check the sizes of the matrices you created using command `size`.

2. Let us construct some of the matrices in exercise 1 in a quicker way than writing every element explicitly. by using some of the following commands: ?

   (a) The command `zeros(m,n)` creates an all-zero matrix with m rows and n columns. Construct matrix B using `zeros`.

   (b) The command `ones(m,n)` creates an all-zero matrix with m rows and n columns. Construct intermediate matrices using `2*ones(3,3)` and `ones(1,3)`, and combine them to make C.

   (c) Use the command `eye` in such a way that the result is D.

   (d) Type `help repmat` into Matlab's command prompt to learn how the command `repmat` works. Use `repmat` to construct E, which consists of four 2×2 matrices that can be created using the command `eye`.

3. What does $A(:), B(:), \ldots$ do to the matrices in exercise 1? Please try it out and explain what you observe.

4. Consider the matrices

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 5 & 1 & 2 & 3 \\ 0 & 5 & 1 & 2 \\ 0 & 0 & 5 & 1 \end{bmatrix}.$$

   (a) Construct matrix G using the command `diag([1,2,3])`. What happens when you enter the command `diag([1,2,3],2)` or `diag([1,2,3],3)`?

   (b) Use the things you learned in (a) to construct H as a sum of four matrices each created using the command `diag`.

5. Consider a traveling zoo with elephants, ostriches and tarantulas. For some reason we only know that there are 46 legs, 12 heads and 2 trunks in the zoo. (Remark: of course, a spider's body consists of two parts: abdomen and cephalothorax. While the cephalothorax is technically not equal to a head, please work on the assumption of spiders having one head each.)

   (a) Collect the above information in the form of a matrix equation

   $$M \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 46 \\ 12 \\ 2 \end{bmatrix},$$

   where x is the number of elephants, y is the number of ostriches and z is the number of tarantulas. The matrix M has size 3×3.

   (b) Solve the matrix equation in (a) using Matlab and report the numbers of different animals in the traveling zoo.

6. Download the file `lego.mat` from the course website. Read it in Matlab using the command `load lego`. Then you will have the following image in your workspace as a matric called *im*:

   (a) Plot the image using the command `imagesc(im)`.

   (b) Improve the plot of (a) by using these commands: `colormap gray` and `axis equal`. Can you explain the meaning of the two commands?

   (c) Follow the instructions in this blog post:
   `blogs.helsinki.fi/smsiltan/2014/05/26/how-to-make-blackwhite-images-look-good/`
   and make the image brighter by applying `gam` with a suitable value between zero and one. You can choose the best value according to your own taste.