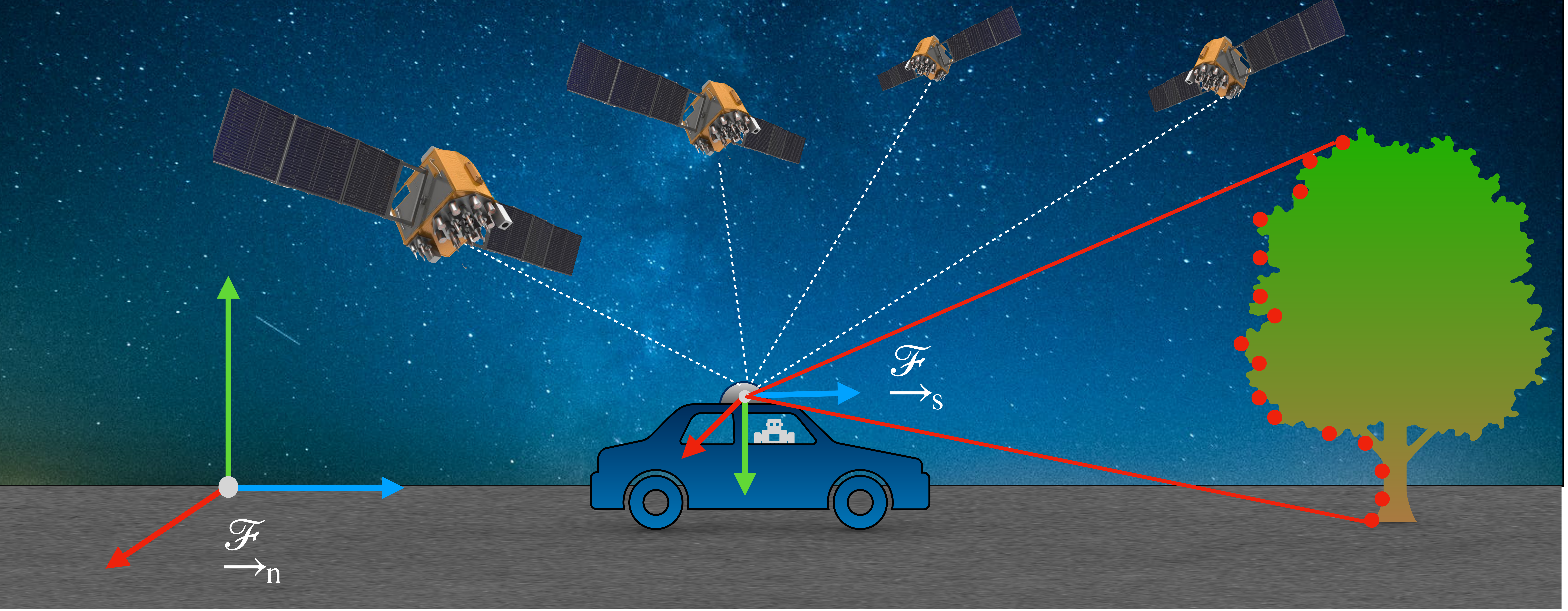
MODULE 5 LESSON 2

MULTISENSOR FUSION FOR STATE ESTIMATION

Multisensor Fusion for State Estimation



By the end of this video, you will be able to...

- Develop an error state extended Kalman Filter for estimating position, velocity and orientation using an IMU, GNSS sensor, and LIDAR.

Why use GNSS with IMU & LIDAR?

- Error dynamics are completely different and uncorrelated
- IMU provides 'smoothing' of GNSS, fill-in during outages due to jamming or maneuvering
  - *Wheel odometry is also possible (if only 2D position orientation is desired)*
- GNSS provides absolute positioning information to mitigate IMU drift
- LIDAR provides accurate local positioning within known maps

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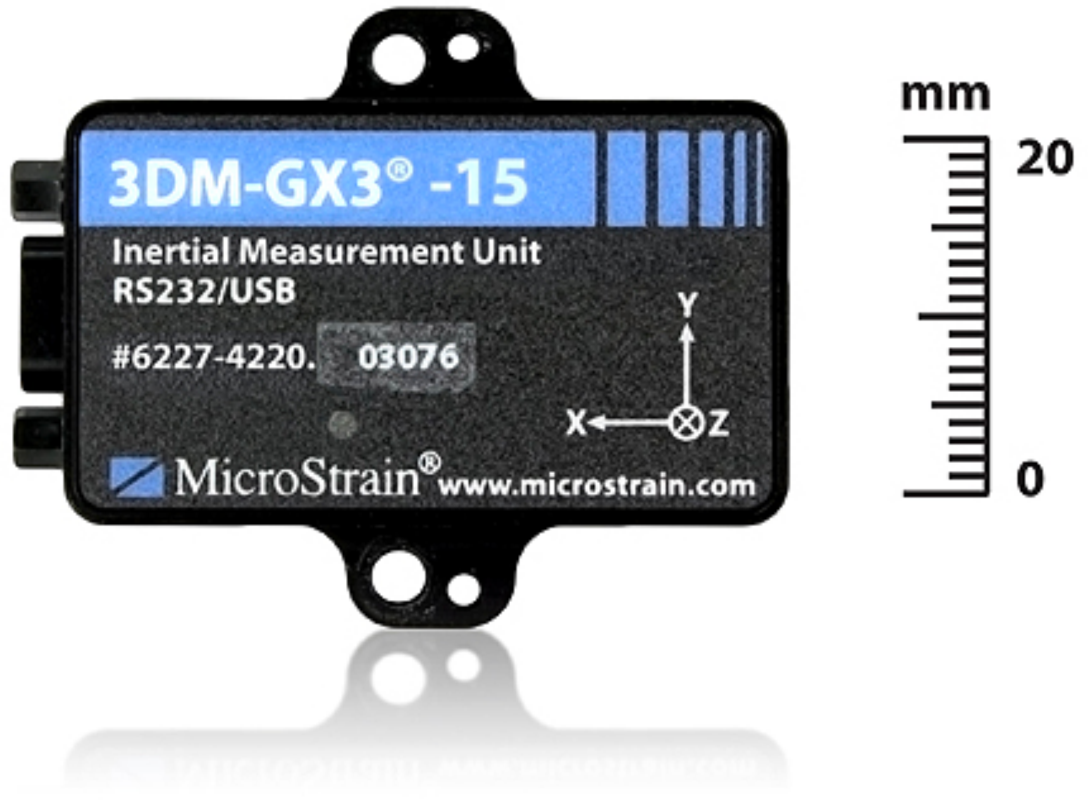

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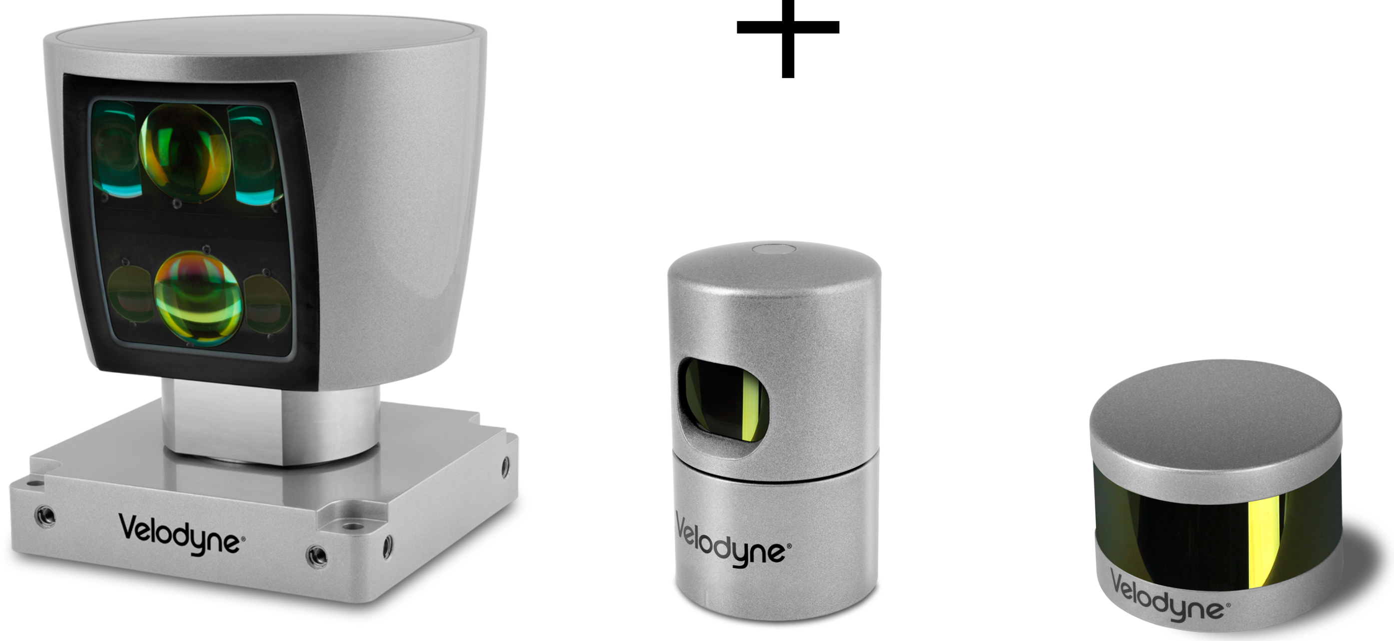
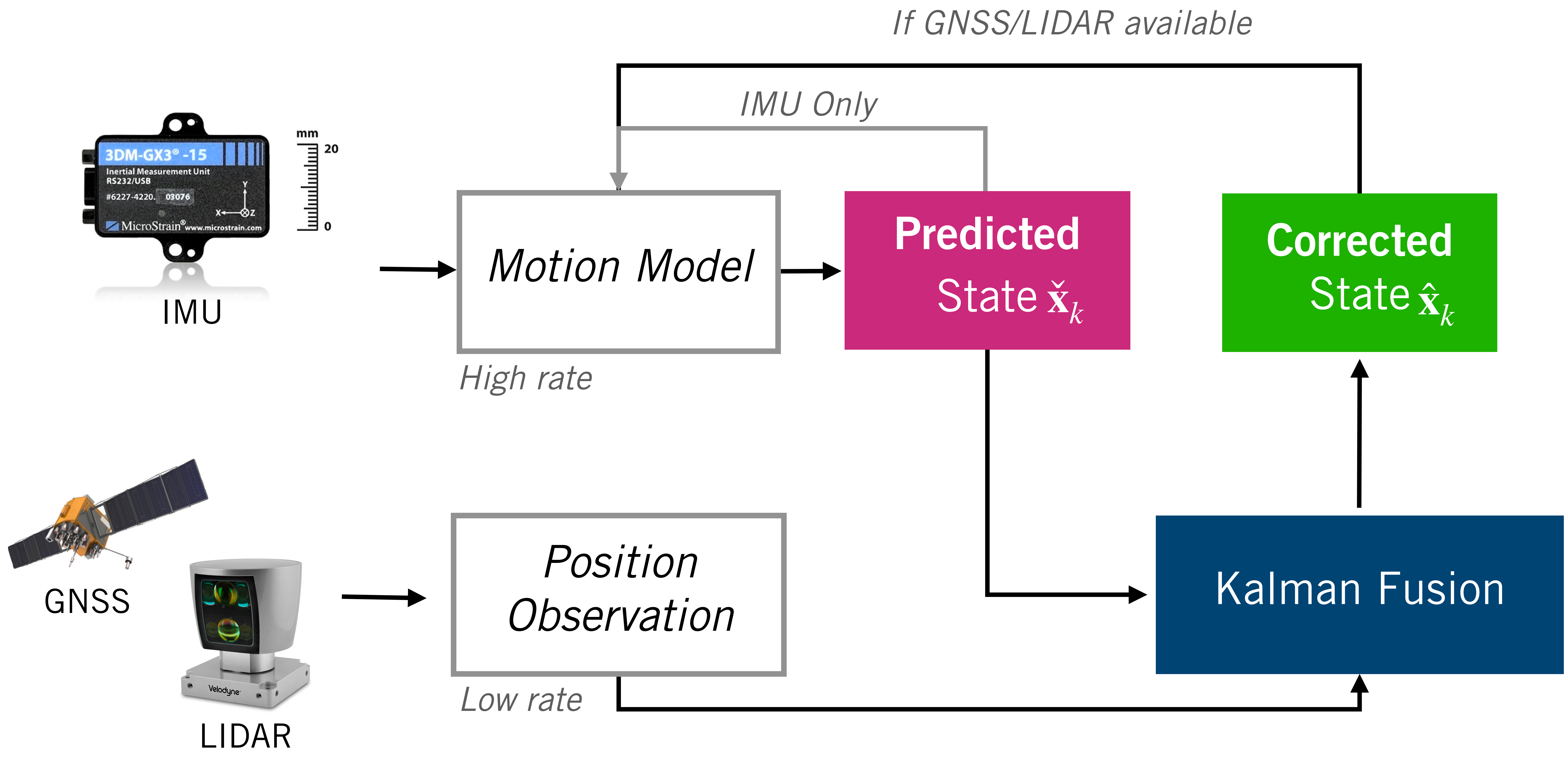


Tightly versus Loosely Coupled

	EKF Coupling	
	Tight	Loose
GNSS/LIDAR Measurement	Pseudo-ranges to satellites LIDAR point clouds	Position
Accuracy	(Potentially) Higher	(Potentially) Lower
Complexity	Higher	Lower

Extended Kalman Filter |
IMU + GNSS + LIDAR



Some Preliminaries

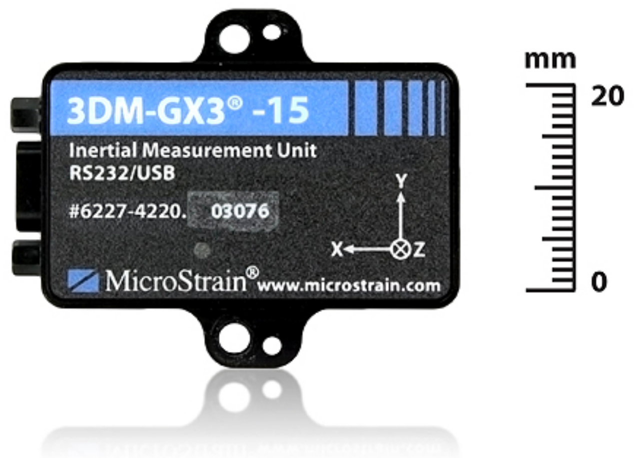
3D 3D 4D

Vehicle state consists of position, velocity and parametrization of orientation using a unit quaternion:

$$\mathbf{x}_k = \begin{bmatrix} \mathbf{p}_k \\ \mathbf{v}_k \\ \mathbf{q}_k \end{bmatrix} \in R^{10}$$

Motion model input will consist of specific force and rotational rates from our IMU:

$$\mathbf{u}_k = \begin{bmatrix} \mathbf{f}_k \\ \boldsymbol{\omega}_k \end{bmatrix} \in R^6$$



Motion Model

Position $$\mathbf{p}_k = \mathbf{p}_{k-1} + \Delta t \mathbf{v}_{k-1} + \frac{\Delta t^2}{2}\left(\mathbf{C}_{ns}\mathbf{f}_{k-1} + \mathbf{g}\right)$$

Velocity $$\mathbf{v}_k = \mathbf{v}_{k-1} + \Delta t\left(\mathbf{C}_{ns}\mathbf{f}_{k-1} + \mathbf{g}\right)$$

Orientation $$\mathbf{q}_k = \mathbf{q}_{k-1} \otimes \mathbf{q}(\boldsymbol{\omega}_{k-1}\Delta t) = \boldsymbol{\Omega}\left(\mathbf{q}(\boldsymbol{\omega}_{k-1}\Delta t)\right)\mathbf{q}_{k-1}$$

where…

$$\mathbf{C}_{ns} = \mathbf{C}_{ns}(\mathbf{q}_{k-1}) \qquad \boldsymbol{\Omega}\left(\begin{bmatrix} q_w \\ \mathbf{q}_v \end{bmatrix}\right) = q_w\mathbf{1} + \begin{bmatrix} 0 & -\mathbf{q}_v^T \\ \mathbf{q}_v & -\{\mathbf{q}_v\}_\times \end{bmatrix} \qquad \mathbf{q}(\boldsymbol{\theta}) = \begin{bmatrix} \sin\frac{|\boldsymbol{\theta}|}{2} \\ \frac{\boldsymbol{\theta}}{|\boldsymbol{\theta}|}\cos\frac{|\boldsymbol{\theta}|}{2} \end{bmatrix}$$

sensor frame
navigation frame

Linearized **Motion Model**

Error State

$$\delta \mathbf{x}_k = \begin{bmatrix} \delta \mathbf{p}_k \\ \delta \mathbf{v}_k \\ \delta \boldsymbol{\phi}_k \end{bmatrix} \in R^9$$

3x1 rotation error

Error Dynamics

$$\delta \mathbf{x}_k = \mathbf{F}_{k-1} \delta \mathbf{x}_{k-1} + \mathbf{L}_{k-1} \mathbf{n}_{k-1}$$

measurement noise

where…

$$\mathbf{F}_{k-1} = \begin{bmatrix} \mathbf{1} & \mathbf{1}\Delta t & 0 \\ 0 & \mathbf{1} & -[\mathbf{C}_{ns} \mathbf{f}_{k-1}]_\times \Delta t \\ 0 & 0 & \mathbf{1} \end{bmatrix}$$

$$\mathbf{L}_{k-1} = \begin{bmatrix} 0 & 0 \\ \mathbf{1} & 0 \\ 0 & \mathbf{1} \end{bmatrix}$$

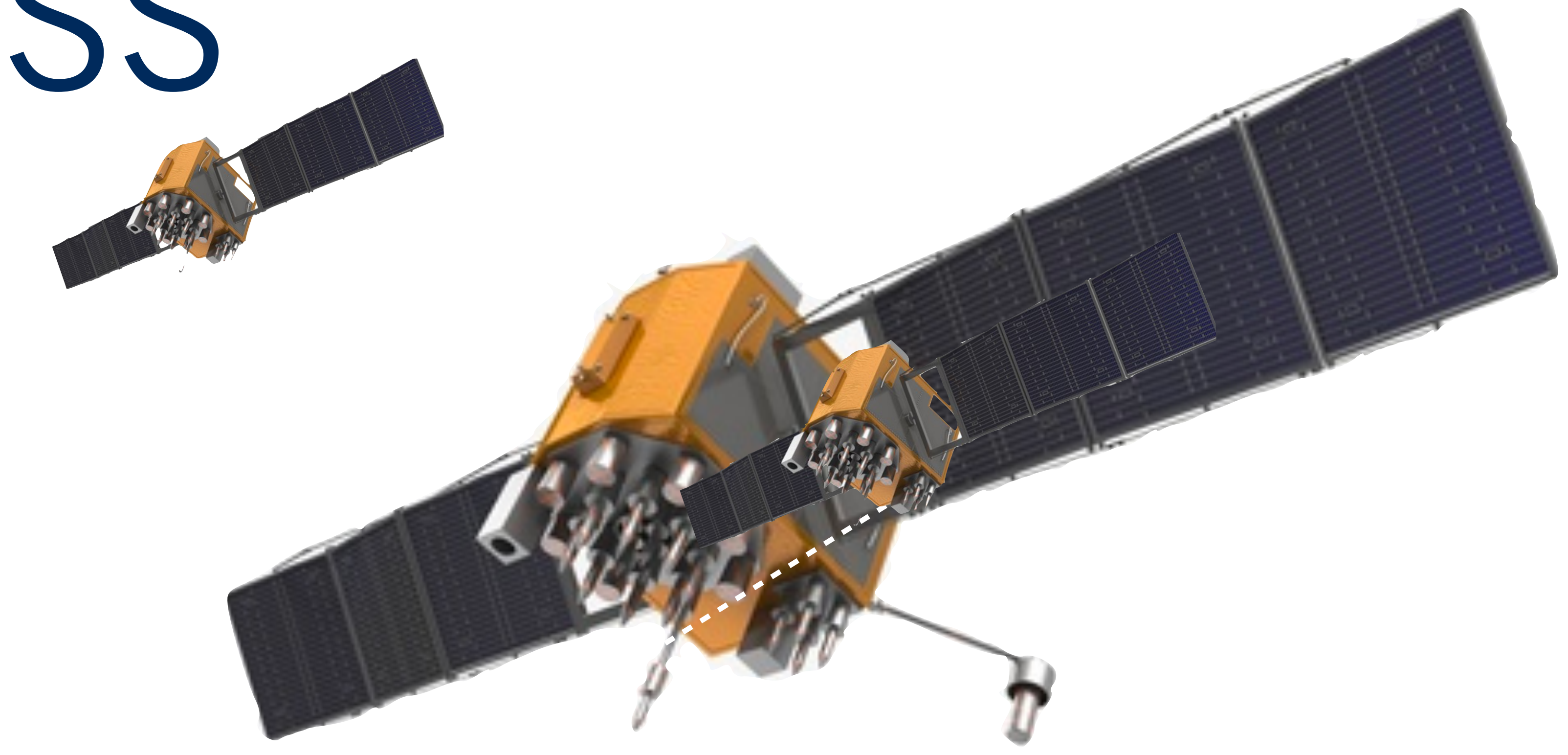
$$\mathbf{n}_k \sim \mathcal{N}\left(\mathbf{0}, \mathbf{Q}_k\right) \sim \mathcal{N}\left(\mathbf{0}, \Delta t^2 \begin{bmatrix} \sigma^2_{\text{acc}} & \\ & \sigma^2_{\text{gyro}} \end{bmatrix}\right)$$

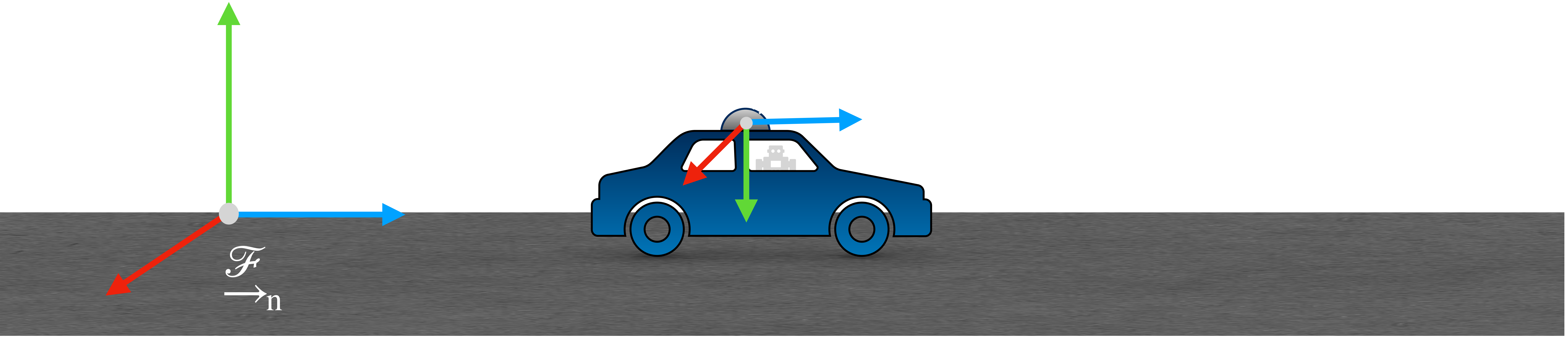
$\mathbf{1}$ is the 3×3 identity matrix

Measurement Model | GNSS

$$
\begin{aligned}
\mathbf{y}_k &= \mathbf{h}(\mathbf{x}_k) + \boldsymbol{\nu}_k \\
&= \mathbf{H}_k \mathbf{x}_k + \boldsymbol{\nu}_k = [1\ 0\ 0]\,\mathbf{x}_k + \boldsymbol{\nu}_k \\
&= \mathbf{p}_k + \boldsymbol{\nu}_k
\end{aligned}
$$

$$\boldsymbol{\nu}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_{\text{GNSS}})$$

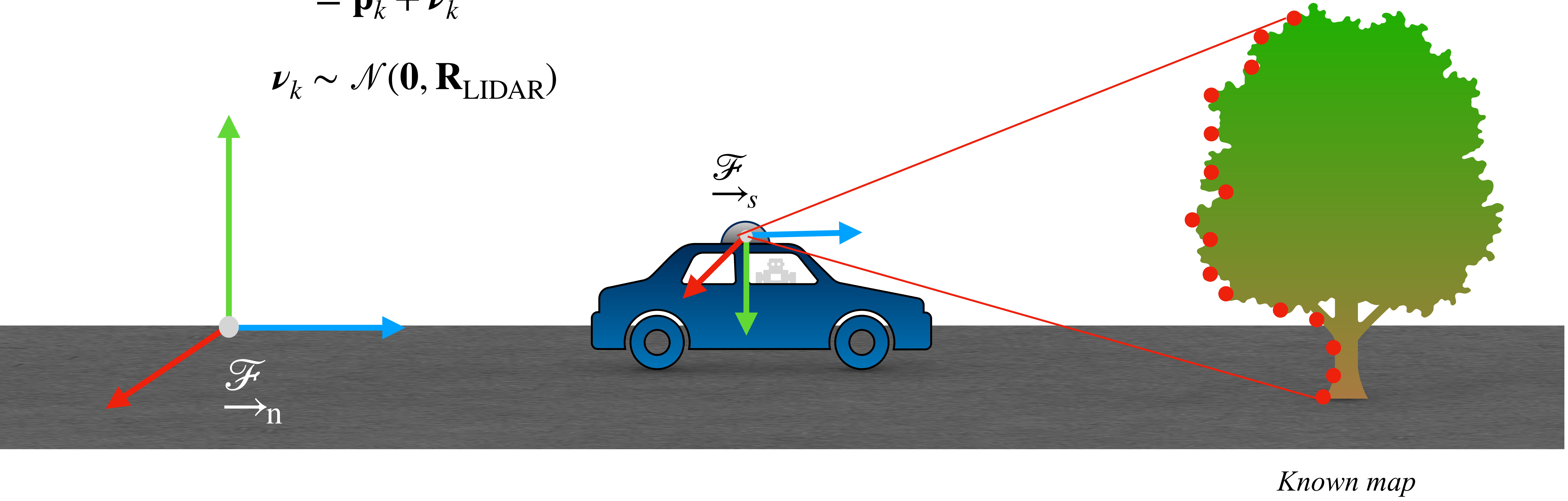




Measurement Model | LIDAR

$$\begin{aligned}\mathbf{y}_k &= \mathbf{h}(\mathbf{x}_k) + \boldsymbol{\nu}_k \\ &= \mathbf{H}_k \mathbf{x}_k + \boldsymbol{\nu}_k = [1 \ 0 \ 0]\, \mathbf{x}_k + \boldsymbol{\nu}_k \\ &= \mathbf{p}_k + \boldsymbol{\nu}_k\end{aligned}$$

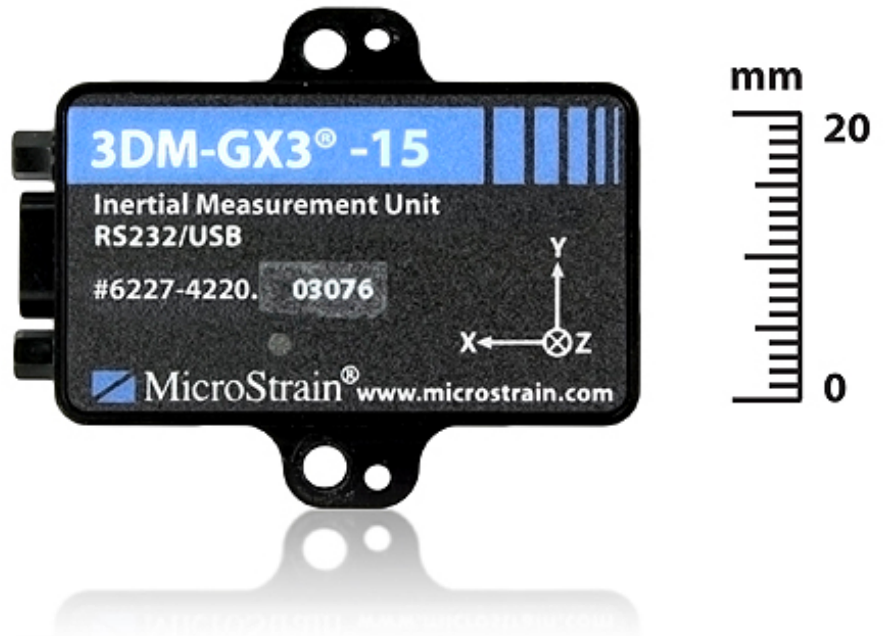
$$\boldsymbol{\nu}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_{\text{LIDAR}})$$



EKF | IMU + GNSS + LIDAR

Loop:

1. Update state with IMU inputs



$$\check{\mathbf{x}}_k = \begin{bmatrix} \check{\mathbf{p}}_k \\ \check{\mathbf{v}}_k \\ \check{\mathbf{q}}_k \end{bmatrix}$$

$$\check{\mathbf{p}}_k = \mathbf{p}_{k-1} + \Delta t \mathbf{v}_{k-1} + \frac{\Delta t^2}{2}\left(\mathbf{C}_{ns}\mathbf{f}_{k-1} + \mathbf{g}_n\right)$$

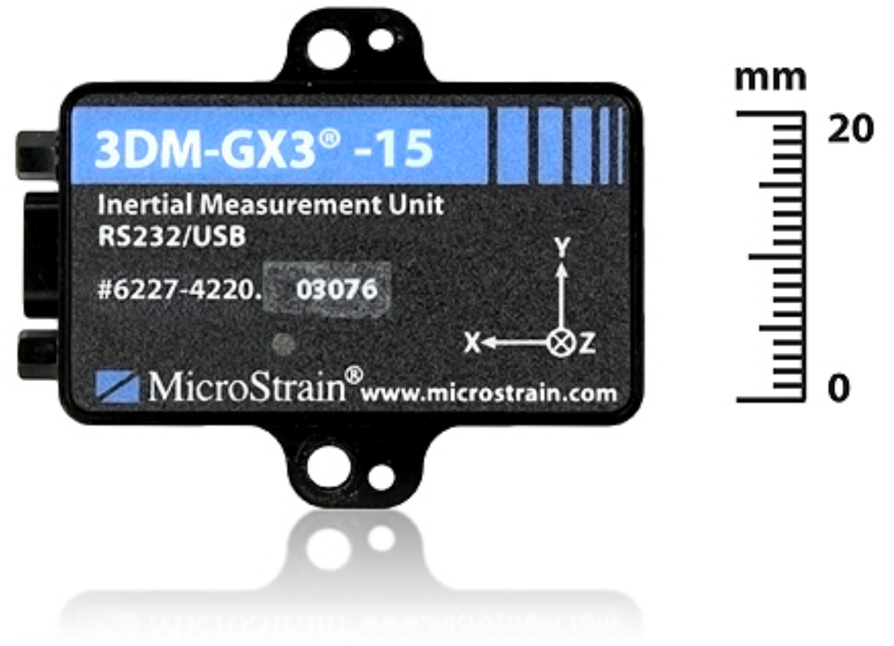
$$\check{\mathbf{v}}_k = \mathbf{v}_{k-1} + \Delta t\left(\mathbf{C}_{ns}\mathbf{f}_{k-1} + \mathbf{g}_n\right)$$

$$\check{\mathbf{q}}_k = \boldsymbol{\Omega}\left(\mathbf{q}(\boldsymbol{\omega}_{k-1}\Delta t)\right)\mathbf{q}_{k-1}$$

$\mathbf{p}_{k-1}, \mathbf{v}_{k-1}, \mathbf{q}_{k-1}$

can be either be corrected or uncorrected depending on whether or not there was a GNSS/LIDAR measurement at time step *k - 1*

EKF | IMU + GNSS + LIDAR



Loop:

1. Update state with IMU inputs
2. Propagate uncertainty

$$\check{\mathbf{P}}_k = \mathbf{F}_{k-1}\mathbf{P}_{k-1}\mathbf{F}_{k-1}^T + \mathbf{L}_{k-1}\mathbf{Q}_{k-1}\mathbf{L}_{k-1}^T$$

Can either be

$\hat{\mathbf{P}}_{k-1}$ or $\check{\mathbf{P}}_{k-1}$

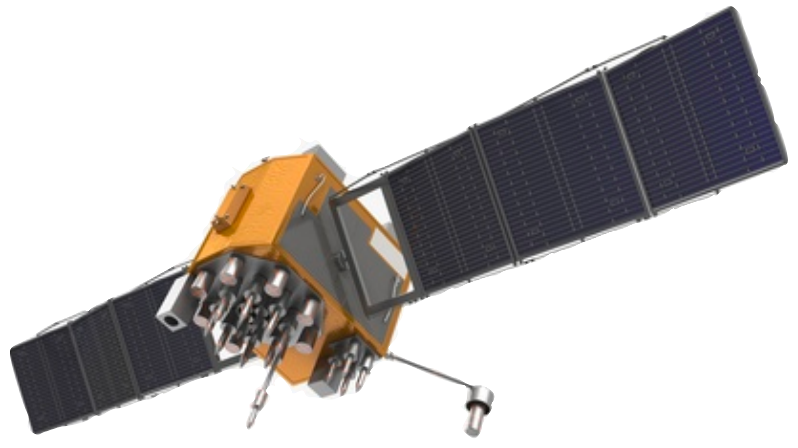
EKF | IMU + GNSS + LIDAR

Loop:

1. Update state with IMU inputs
2. Propagate uncertainty
3. If **GNSS** or **LIDAR** position available:
   1. Compute Kalman gain

$$\mathbf{K}_k = \check{\mathbf{P}}_k \mathbf{H}_k^T (\mathbf{H}_k \check{\mathbf{P}}_k \mathbf{H}_k^T + \mathbf{R})^{-1}$$

One of $\mathbf{R}_{\text{GNSS}}$ or $\mathbf{R}_{\text{LIDAR}}$

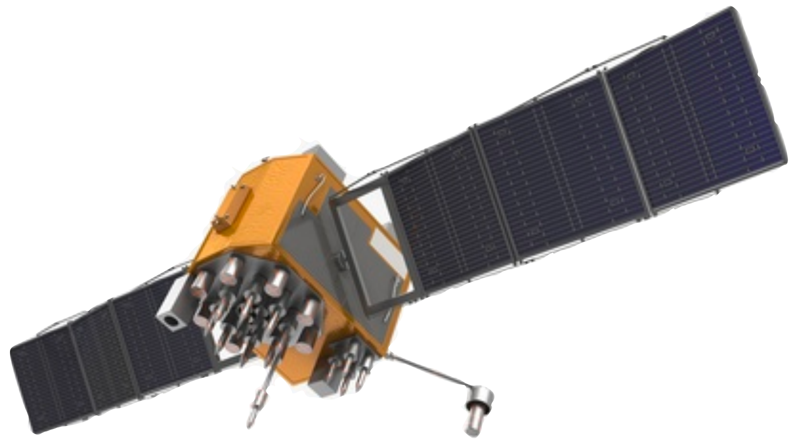


EKF | IMU + GNSS + LIDAR

Loop:

1. Update state with IMU inputs
2. Propagate uncertainty
3. If **GNSS** or **LIDAR** position available:
    1. Compute Kalman Gain
    2. Compute error state

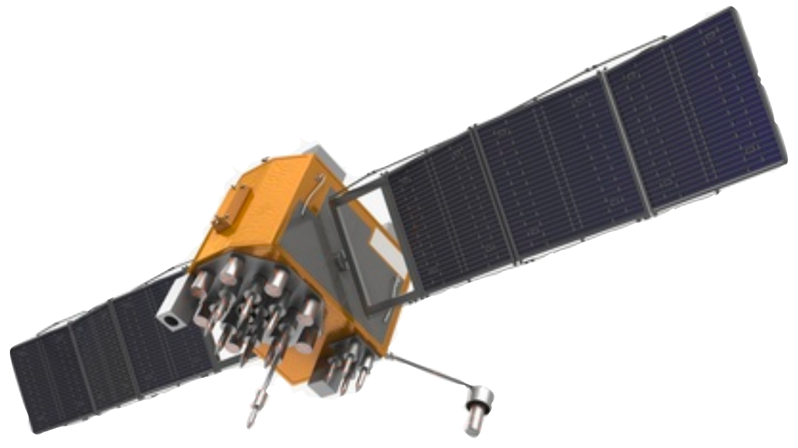
$$\delta \mathbf{x}_k = \mathbf{K}_k(\mathbf{y}_k - \check{\mathbf{p}}_k)$$



EKF | IMU + GNSS + LIDAR

Loop:

1. Update state with IMU inputs
2. Propagate uncertainty
3. If **GNSS** or **LIDAR** position available:
    1. Compute Kalman Gain
    2. Compute error state
    3. Correct predicted state



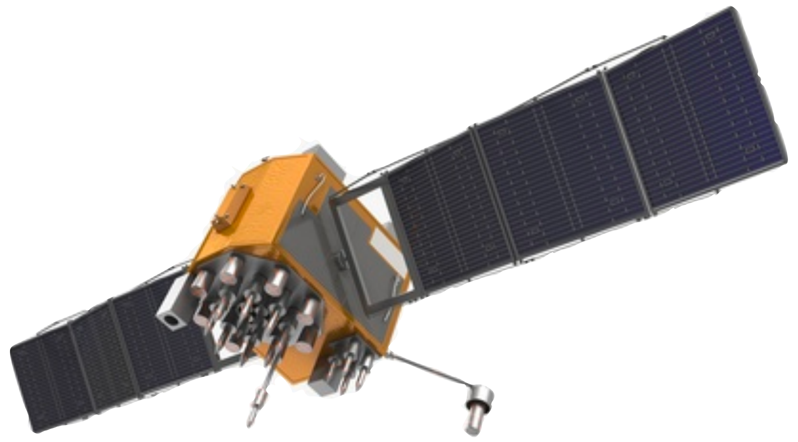
$$\hat{\mathbf{p}}_k = \check{\mathbf{p}}_k + \delta\mathbf{p}_k$$

$$\hat{\mathbf{v}}_k = \check{\mathbf{v}}_k + \delta\mathbf{v}_k$$

$$\hat{\mathbf{q}}_k = \mathbf{q}(\delta\boldsymbol{\phi}) \otimes \check{\mathbf{q}}_k \longleftarrow \text{global orientation error}$$

the orientation update involves multiplying by the error state quaternion on the left.

EKF | IMU + GNSS + LIDAR



Loop:

1. Update state with IMU inputs
2. Propagate uncertainty
3. If **GNSS** or **LIDAR** position available:
    1. Compute Kalman Gain
    2. Compute error state
    3. Correct predicted state
    4. Computed corrected covariance

$$\hat{\mathbf{P}}_k = (\mathbf{1} - \mathbf{K}_k \mathbf{H}_k)\check{\mathbf{P}}_k$$

Summary | EKF for Vehicle State Estimation

- We used a loosely coupled EKF to fuse GNSS with IMU and LIDAR measurements
- *Assumptions*:
    1. LIDAR provides positions in the same reference frame as GNSS (possible)
    2. IMU has no biases (not realistic!)
    3. State initialization is provided (realistic)
    4. Our sensors are spatially and temporally aligned (somewhat realistic)