Binary Search Tree

SWE2016-44

BST

Binary Search Tree is a node-based binary tree data structure.

Properties

- 1) The left subtree of a node contains nodes with keys less than the node's key and the right subtree with keys greater than node's key.
- 2) The left and right subtree each must also be a binary search tree and there must be no duplicate nodes.

BST

- 1) Insertion
- 2) Searching
- 3) Delete

While doing insertion in BST the new key is always inserted at leaf.

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Once a leaf node is found, the new node is added as a child of the leaf node.

```
struct node* insert(struct node* node, int key)
{
    /* If the tree is empty, return a new node */
    if (node == NULL) return newNode(key);

    /* Otherwise, recur down the tree */
    if (key < node->key)
        node->left = insert(node->left, key);
    else if (key > node->key)
        node->right = insert(node->right, key);

    /* return the (unchanged) node pointer */
    return node;
}
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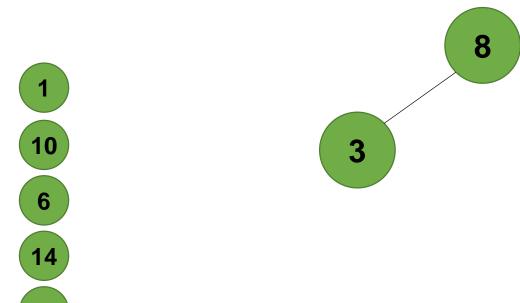


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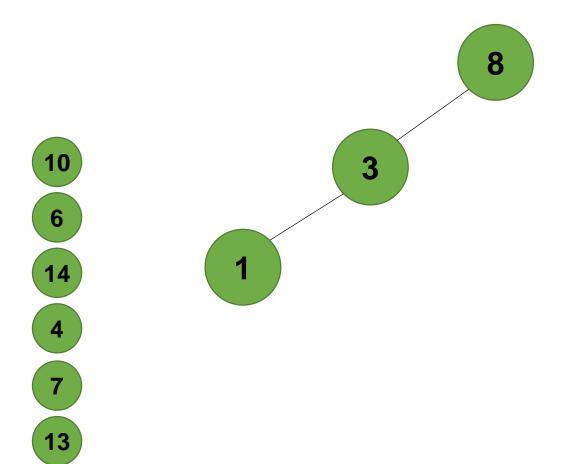
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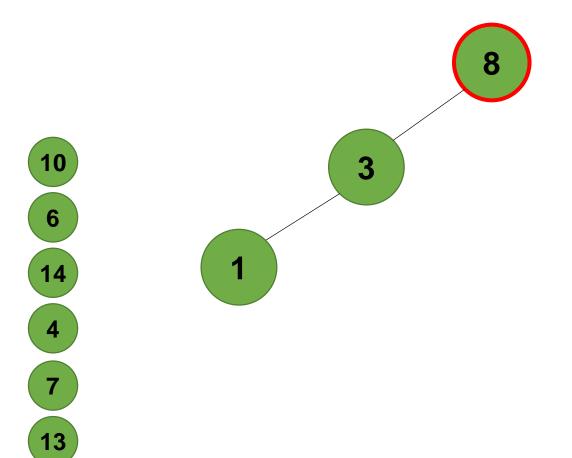
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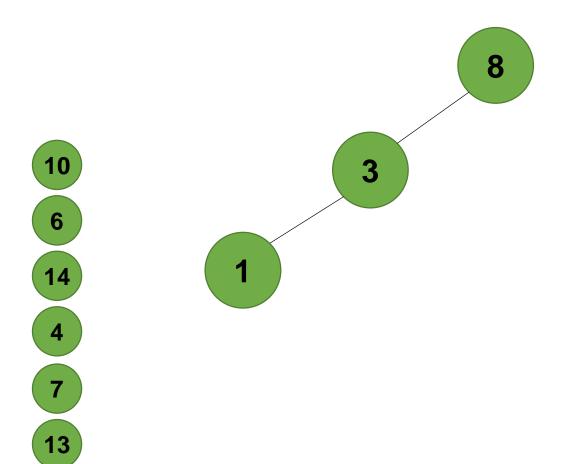
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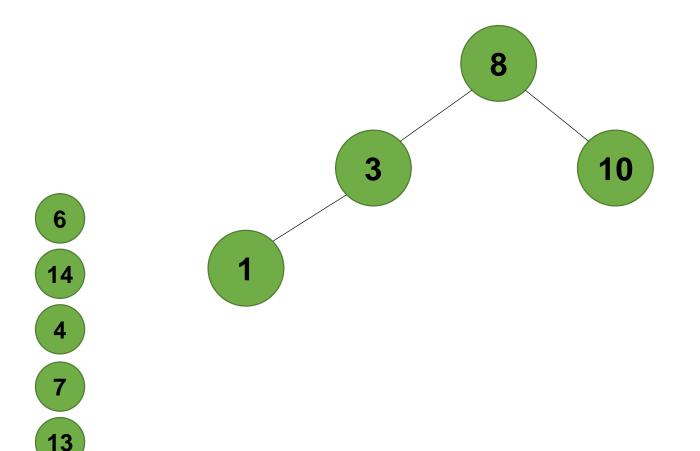
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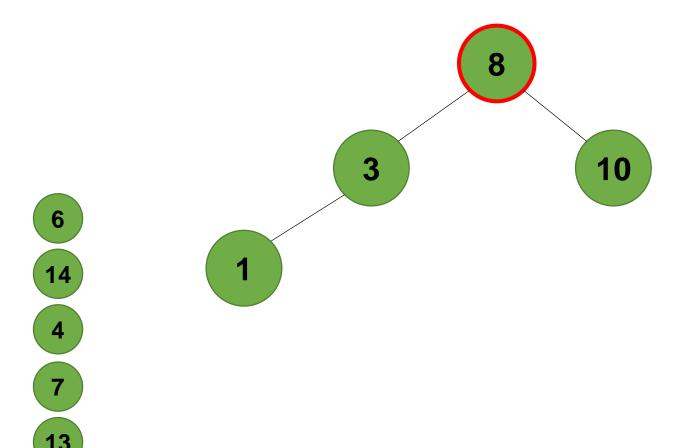
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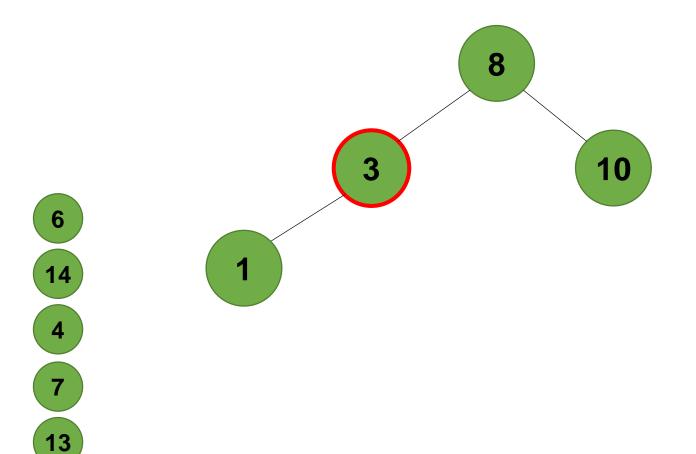
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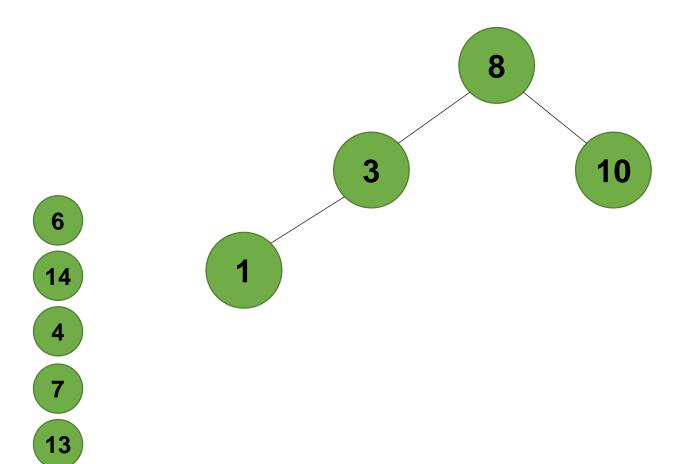
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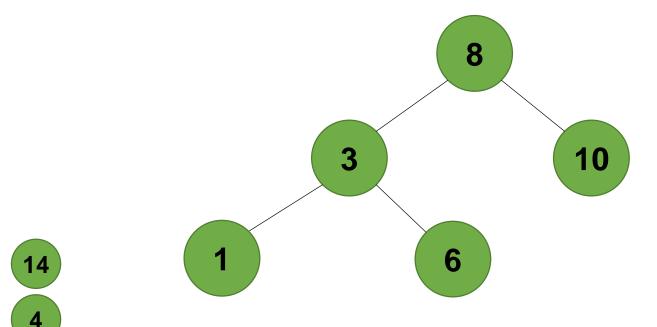
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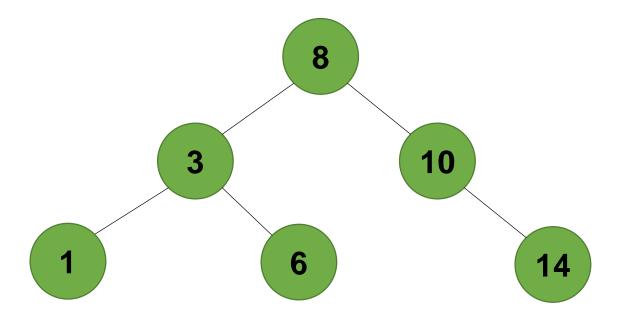
    /* return the (unchanged) node pointer */
    return node;
}
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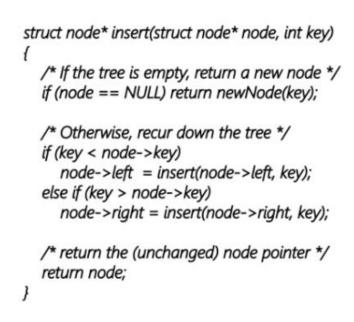


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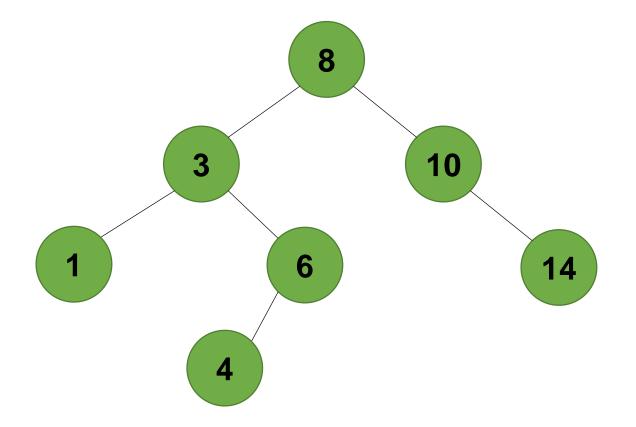




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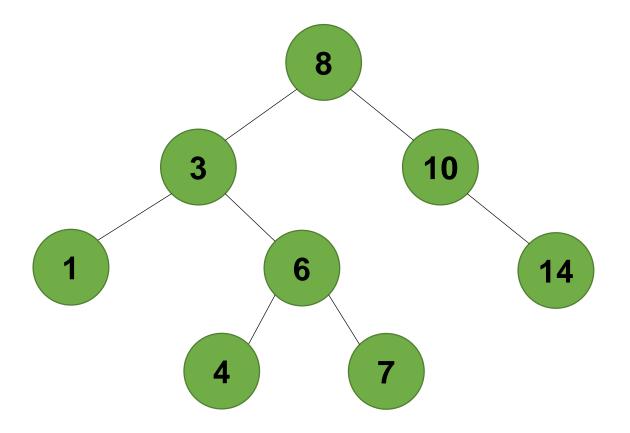
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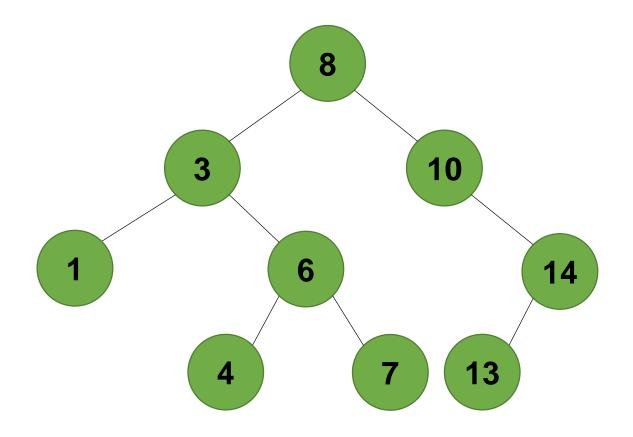


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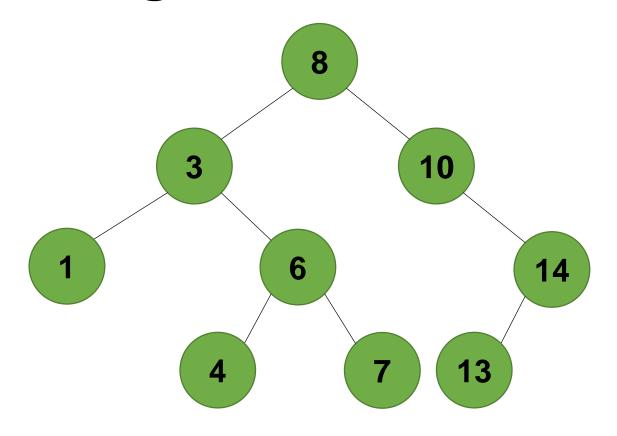
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To search a given key in Binary Search Tree, we first compare it with root.

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If the key is present at root, we return root. If key is greater than root's key, we recur for right subtree of root node.

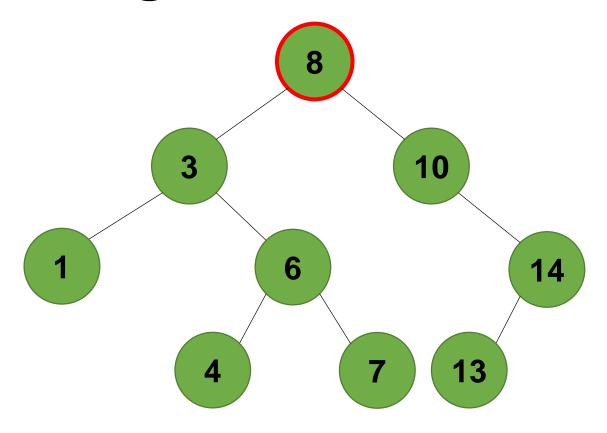


```
struct node* search(struct node* root, int key)
{
    // Base Cases: root is null or key is present at
    root
    if (root == NULL || root->key == key)
        return root;

    // Key is greater than root's key
    if (root->key < key)
        return search(root->right, key);

    // Key is smaller than root's key
    return search(root->left, key);
}
```

Let's search the key '4' from the BST we made earlier.

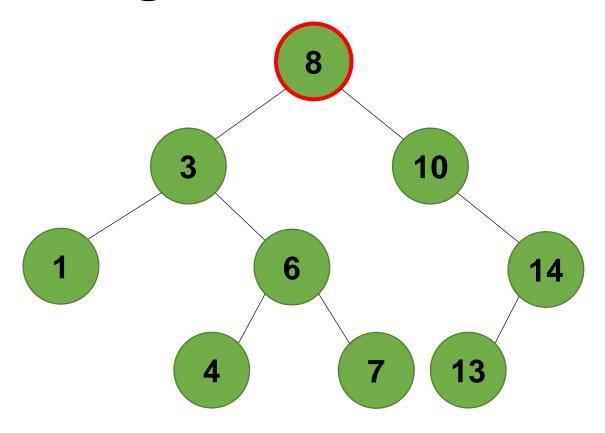


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```

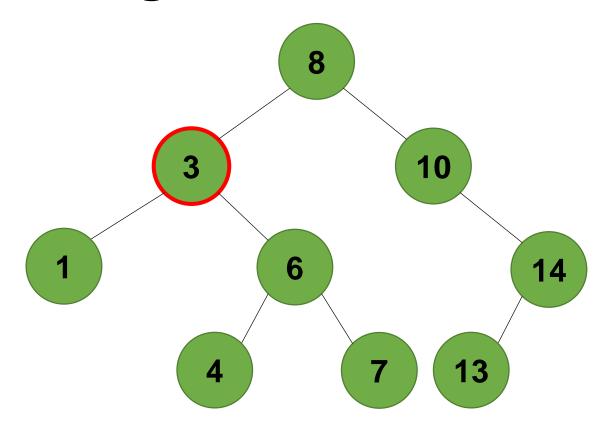
Starting from the root, keep moving downwards until you find the key required.



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    if (root->key < key)
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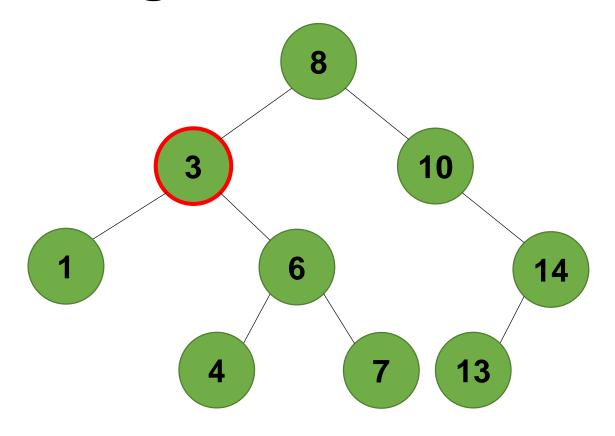
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    return search(root->left, key);
}
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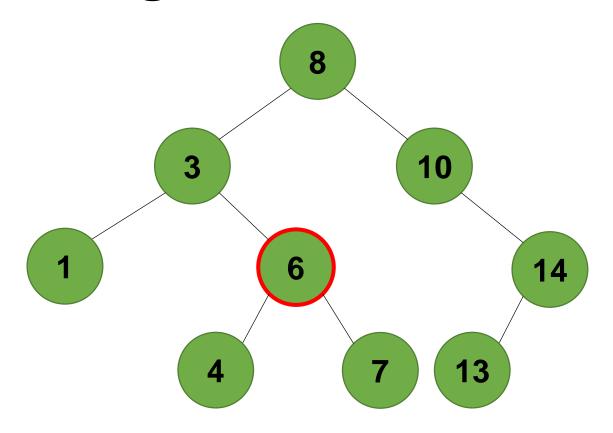
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}
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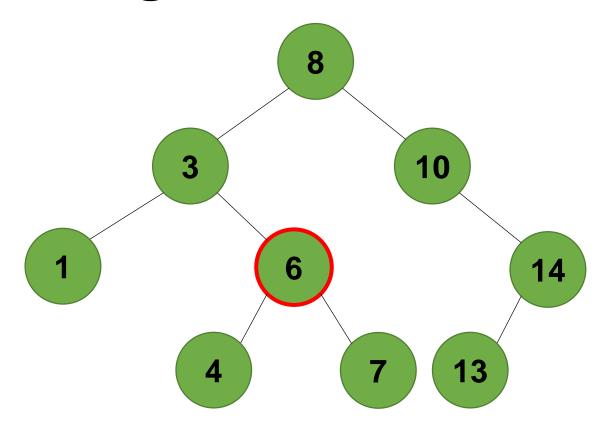
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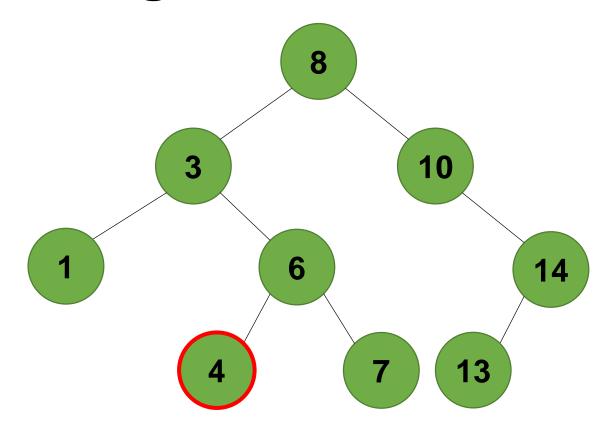
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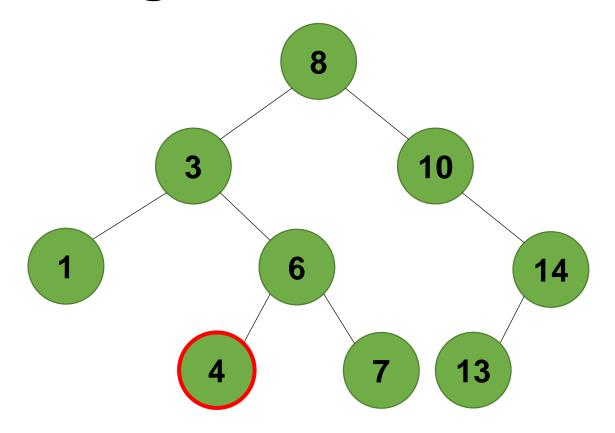
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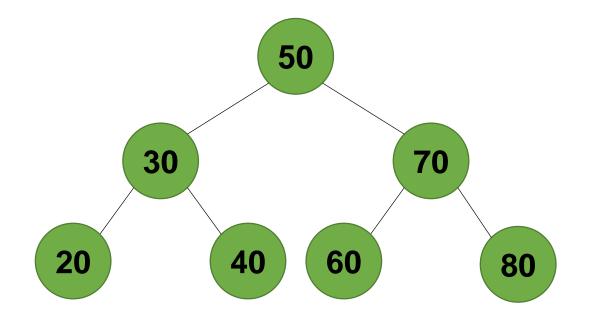
The worst case time complexity: O(h)

If the height of a skewed tree becomes n, the time complexity is O(n).

On average, $h \approx \log n \rightarrow O(\log n)$

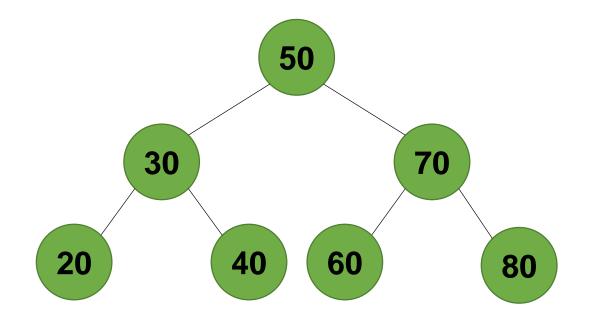
Deletion in BST has been divided into 3 cases.

- 1) Node to be deleted is leaf.
- 2) Node to be deleted has only one child.
- 3) Node to be deleted has two children.



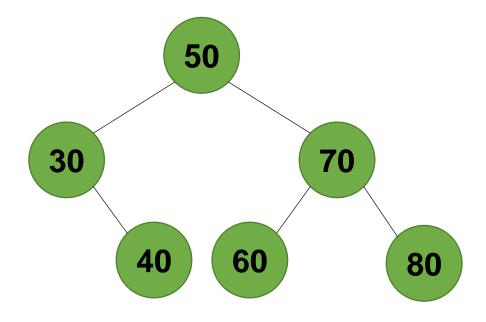
Case 1: Node to be deleted is leaf: Let's delete the node with value '20'

```
struct node* deleteNode(struct node* root, int ke
  if (root == NULL) return root;
  if (key < root->key)
     root->left = deleteNode(root->left, key);
  else if (key > root->key)
     root->right = deleteNode(root->right, key);
  else
     // node with only one child or no child
     if (root -> left == NULL)
       struct node *temp = root->right;
       free(root);
       return temp;
     else\ if\ (root->right==NULL)
       struct node *temp = root->left;
       free(root);
       return temp;
    struct node* temp = minValueNode(root->right);
       root->key = temp->key;
    root->right = deleteNode(root->right, temp->key);
    return root;
```



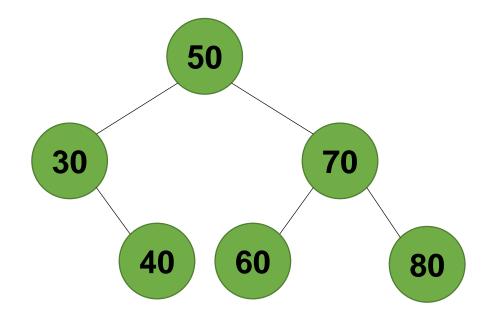
To delete a leaf node simply remove it from the tree.

```
struct node* deleteNode(struct node* root, int ke
  if (root == NULL) return root;
  if (key < root->key)
     root->left = deleteNode(root->left, key);
  else if (key > root->key)
     root->right = deleteNode(root->right, key);
  else
     // node with only one child or no child
     if (root->left == NULL)
       struct node *temp = root->right;
       free(root);
       return temp;
     else\ if\ (root->right==NULL)
       struct node *temp = root->left;
       free(root);
       return temp;
    struct node* temp = minValueNode(root->right);
       root->key = temp->key;
    root->right = deleteNode(root->right, temp->key);
    return root;
```



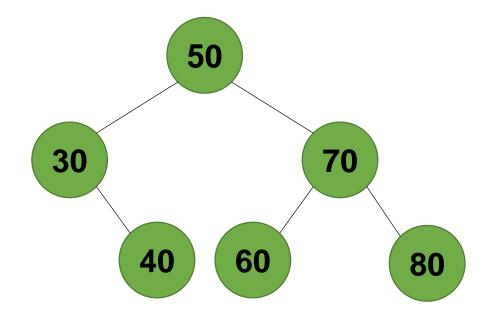
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  if (root == NULL) return root;
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     root->right = deleteNode(root->right, key);
  else
     // node with only one child or no child
     if (root->left == NULL)
       struct node *temp = root->right;
       free(root);
       return temp;
     else\ if\ (root->right==NULL)
       struct node *temp = root->left;
       free(root);
       return temp;
    struct node* temp = minValueNode(root->right);
       root->key = temp->key;
    root->right = deleteNode(root->right, temp->key);
    return root;
```



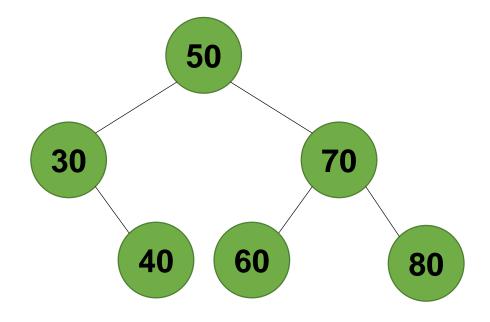
Since it was a leaf node, we deleted it from the tree without making any other changes.

```
struct node* deleteNode(struct node* root, int ke
  if (root == NULL) return root;
  if (key < root->key)
     root->left = deleteNode(root->left, key);
  else if (key > root->key)
     root->right = deleteNode(root->right, key);
  else
     // node with only one child or no child
     if (root -> left == NULL)
        struct node *temp = root->right;
        free(root);
        return temp;
     else\ if\ (root->right==NULL)
        struct node *temp = root->left;
        free(root);
        return temp;
    struct node* temp = minValueNode(root->right);
       root->key = temp->key;
    root->right = deleteNode(root->right, temp->key);
    return root;
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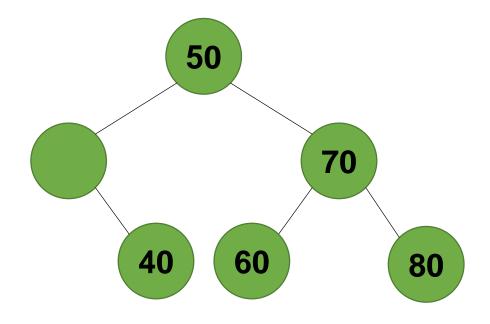
Case 2: Node to be deleted has only one child: Let's delete the node with value '30'

```
struct node* deleteNode(struct node* root, int ke
  if (root == NULL) return root;
  if (key < root->key)
     root->left = deleteNode(root->left, key);
  else if (key > root->key)
     root->right = deleteNode(root->right, key);
  else
     // node with only one child or no child
     if (root -> left == NULL)
        struct node *temp = root->right;
        free(root);
        return temp;
     else\ if\ (root->right==NULL)
        struct node *temp = root->left;
        free(root);
        return temp;
    struct node* temp = minValueNode(root->right);
       root->key = temp->key;
    root->right = deleteNode(root->right, temp->key);
    return root;
```



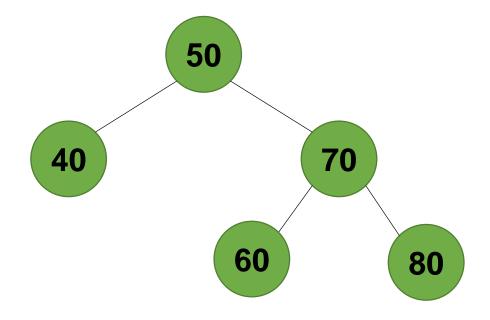
To delete this node, copy the child to the node and delete the child.

```
struct node* deleteNode(struct node* root, int ke
  if (root == NULL) return root;
  if (key < root->key)
     root->left = deleteNode(root->left, key);
  else if (key > root->key)
     root->right = deleteNode(root->right, key);
  else
     // node with only one child or no child
     if (root -> left == NULL)
       struct node *temp = root->right;
       free(root);
       return temp;
     else if (root->right == NULL)
       struct node *temp = root->left;
       free(root);
       return temp;
    struct node* temp = minValueNode(root->right);
       root->key = temp->key;
    root->right = deleteNode(root->right, temp->key);
    return root;
```



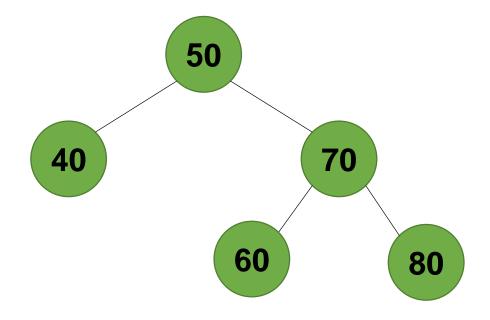
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     // node with only one child or no child
     if (root -> left == NULL)
       struct node *temp = root->right;
       free(root);
       return temp;
     else if (root->right == NULL)
       struct node *temp = root->left;
       free(root);
       return temp;
    struct node* temp = minValueNode(root->right);
       root->key = temp->key;
    root->right = deleteNode(root->right, temp->key);
    return root;
```



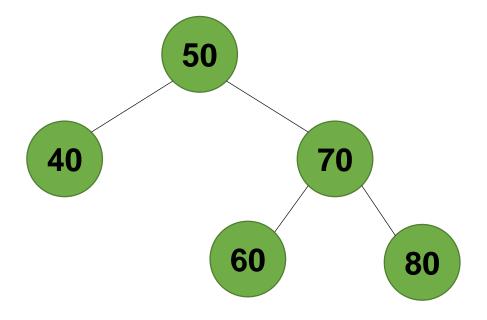
To delete this node, copy the child to the node and delete the child.

```
struct node* deleteNode(struct node* root, int ke
  if (root == NULL) return root;
  if (key < root->key)
     root->left = deleteNode(root->left, key);
  else if (key > root->key)
     root->right = deleteNode(root->right, key);
  else
     // node with only one child or no child
     if (root -> left == NULL)
       struct node *temp = root->right;
       free(root);
       return temp;
     else if (root->right == NULL)
       struct node *temp = root->left;
       free(root);
       return temp;
    struct node* temp = minValueNode(root->right);
       root->key = temp->key;
    root->right = deleteNode(root->right, temp->key);
    return root;
```



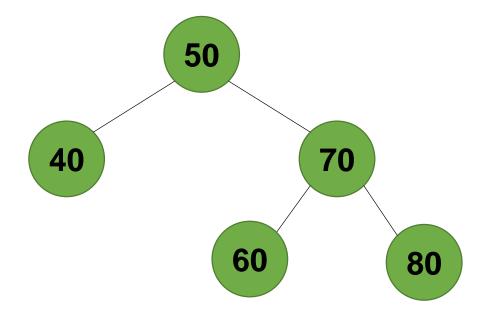
The node had only one child so we copied its child's value to it and deleted its child.

```
struct node* deleteNode(struct node* root, int ke
  if (root == NULL) return root;
  if (key < root->key)
     root->left = deleteNode(root->left, key);
  else if (key > root->key)
     root->right = deleteNode(root->right, key);
  else
     // node with only one child or no child
     if (root -> left == NULL)
        struct node *temp = root->right;
        free(root);
        return temp;
     else\ if\ (root->right==NULL)
        struct node *temp = root->left;
        free(root);
        return temp;
    struct node* temp = minValueNode(root->right);
       root->key = temp->key;
    root->right = deleteNode(root->right, temp->key);
    return root;
```



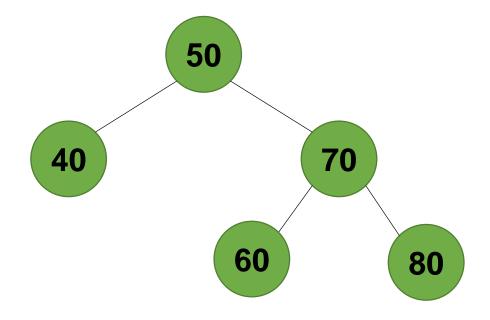
Case 3: Node to be deleted has two children: Let's delete the root node with value '50'

```
struct node* deleteNode(struct node* root, int ke
  if (root == NULL) return root;
  if (key < root->key)
     root->left = deleteNode(root->left, key);
  else if (key > root->key)
     root->right = deleteNode(root->right, key);
  else
     // node with only one child or no child
     if (root -> left == NULL)
        struct node *temp = root->right;
        free(root);
        return temp;
     else\ if\ (root->right==NULL)
        struct node *temp = root->left;
        free(root);
        return temp;
    struct node* temp = minValueNode(root->right);
       root->key = temp->key;
    root->right = deleteNode(root->right, temp->key);
    return root;
```



First, find inorder successor of the node.

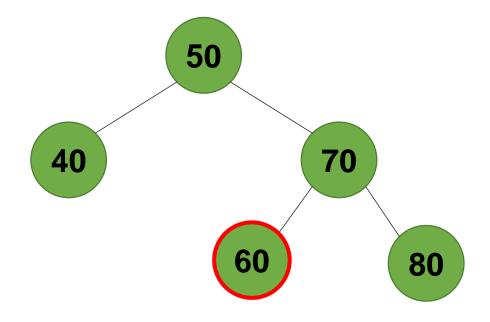
```
struct node* deleteNode(struct node* root, int ke
  if (root == NULL) return root;
  if (key < root->key)
     root->left = deleteNode(root->left, key);
  else if (key > root->key)
     root->right = deleteNode(root->right, key);
  else
     // node with only one child or no child
     if (root->left == NULL)
       struct node *temp = root->right;
       free(root);
       return temp;
     else if (root->right == NULL)
       struct node *temp = root->left;
       free(root);
       return temp;
    struct node* temp = minValueNode(root->right);
       root->key = temp->key;
    root->right = deleteNode(root->right, temp->key);
    return root;
```



40 50 60 70 80: Inorder Traversal

First, find inorder successor of the node.

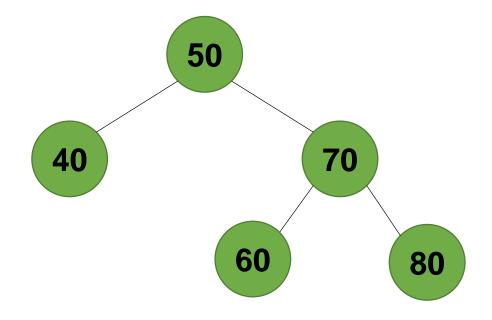
```
struct node* deleteNode(struct node* root, int ke
  if (root == NULL) return root;
  if (key < root->key)
     root->left = deleteNode(root->left, key);
  else if (key > root->key)
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  else
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    struct node* temp = minValueNode(root->right);
       root->key = temp->key;
    root->right = deleteNode(root->right, temp->key);
    return root;
```



40 50 60 70 80: Inorder Traversal

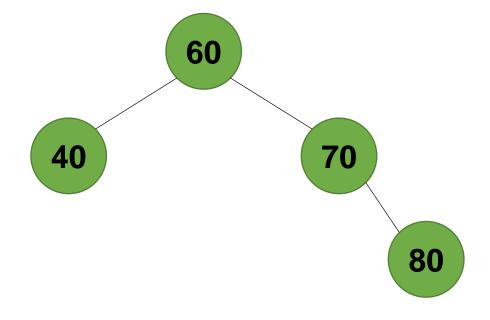
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       free(root);
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    struct node* temp = minValueNode(root->right);
       root->key = temp->key;
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    return root;
```



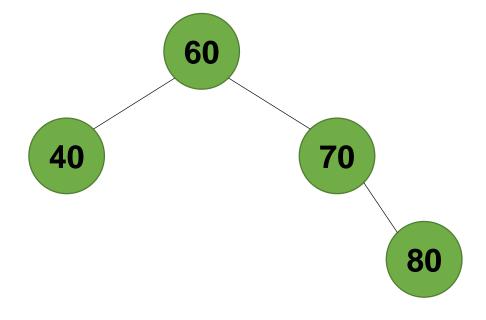
Now copy contents of the inorder successor to the node and delete the inorder successor.

```
struct node* deleteNode(struct node* root, int ke
  if (root == NULL) return root;
  if (key < root->key)
     root->left = deleteNode(root->left, key);
  else if (key > root->key)
     root->right = deleteNode(root->right, key);
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     else if (root->right == NULL)
        struct node *temp = root->left;
        free(root);
        return temp;
    struct node* temp = minValueNode(root->right);
       root->key = temp->key;
    root->right = deleteNode(root->right, temp->key);
    return root;
```



The inorder predecessor can also be used in the same manner.

```
struct node* deleteNode(struct node* root, int ke
  if (root == NULL) return root;
  if (key < root->key)
     root->left = deleteNode(root->left, key);
  else if (key > root->key)
     root->right = deleteNode(root->right, key);
  else
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     if (root->left == NULL)
       struct node *temp = root->right;
       free(root);
       return temp;
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       return temp;
    struct node* temp = minValueNode(root->right);
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    return root;
```

Complexity:

h is height of Binary Search Tree.

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The worst case time complexity: O(h)

If the height of a skewed tree becomes n, the time complexity is O(n).

On average, $h \approx \log n \rightarrow O(\log n)$

BST vs Hash Table

Hash Table supports following operations in $\Theta(1)$ time:

- 1. Search
- 2. Insert
- 3. Delete

For a <u>Self Balancing Binary Search Tree</u> the time complexity for these oprations is:

 $O(\log n)$

1. Can get all keys in sorted order by just doing Inorder Traversal of BST

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- 2. Doing order statistics, finding closest lower and greater elements, doing range queries are easy to do with BSTs

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- 2. Doing order statistics, finding closest lower and greater elements, doing range queries are easy to do with BSTs
- 3. BSTs are easy to implement compared to hashing
- 4. With Self Balancing BSTs, all operations are guaranteed to work in O(log n) time

Reference

• Charles Leiserson and Piotr Indyk, "Introduction to Algorithms", September 29, 2004

https://www.geeksforgeeks.org