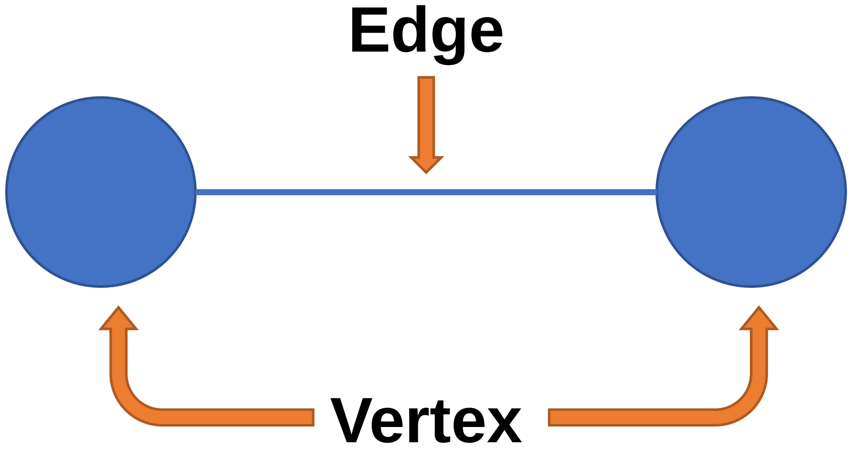
Graph Algorithm

SWE2016-44

Introduction

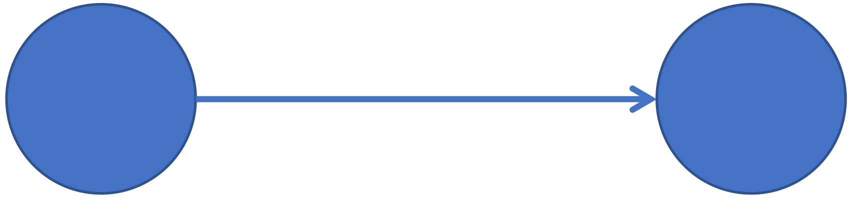
Graph: Represents pair-wise relationship between a set of objects

1. Vertex (nodes)
2. Edges (arc)

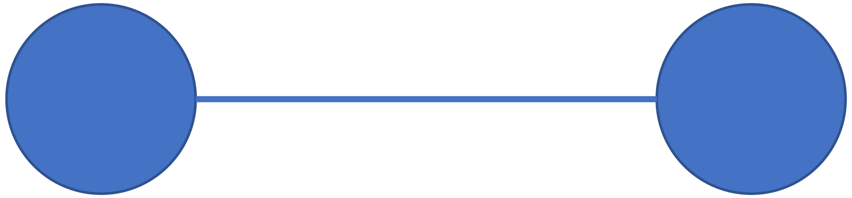


Introduction

Directed Graph (di-graph): have pair of ordered vertices (u,v)



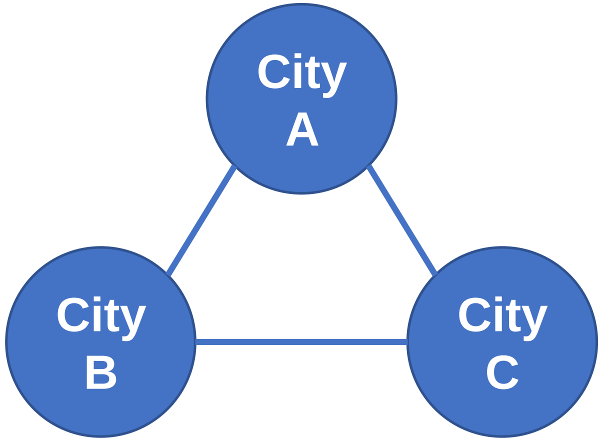
Un-Directed Graph: have pair of unordered vertices, (u,v) and (v,u) are same



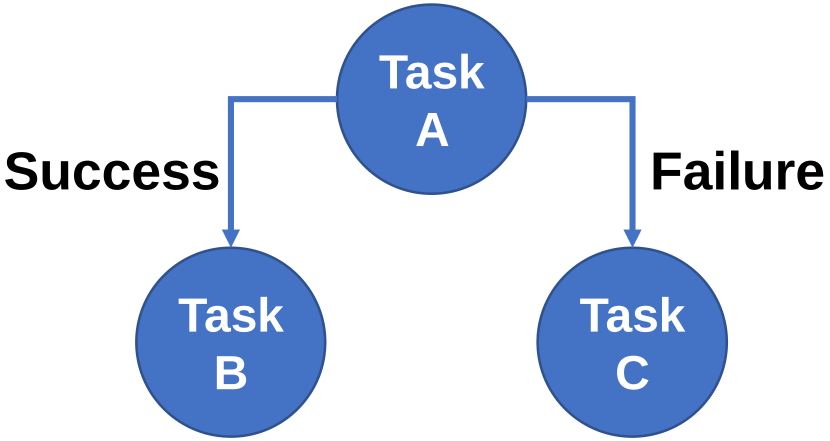
Graph Applications

1. Social Networks – Facebook, LinkedIn, etc

2. City-Road Network



3. Precedence Constraints



Graph Representation

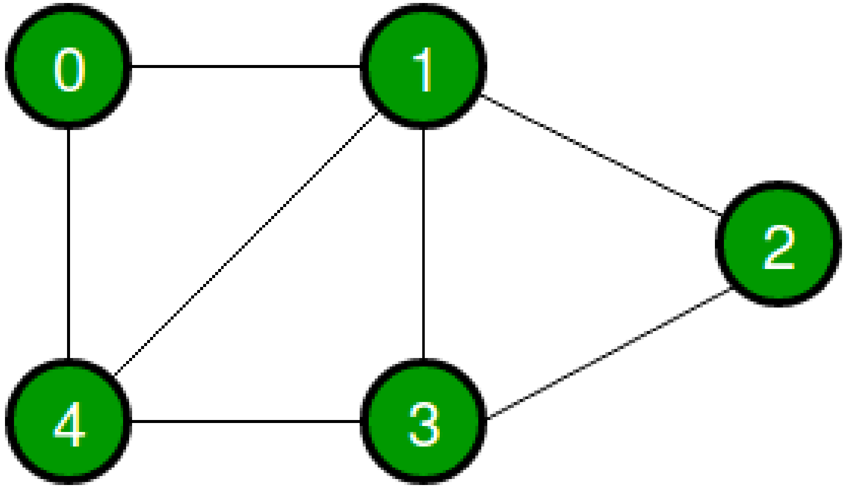
There are generally two ways to represent a graph data structure

- **Adjacency Matrix**
- **Adjacency List**

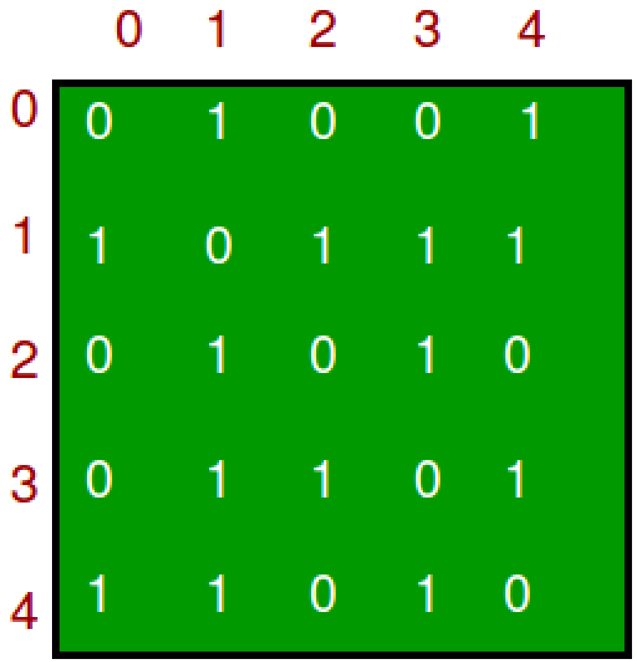
Graph Representation

Adjacency Matrix: Represents graph with ‘V’ nodes into an VxV 0-1 matrix where A_{ij}=1 means that vertex ‘i’ and ‘j’ are connected.

Example



Graph



$$\begin{array}{c|ccccc} & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 2 & 0 & 1 & 0 & 1 & 0 \\ 3 & 0 & 1 & 1 & 0 & 1 \\ 4 & 1 & 1 & 0 & 1 & 0 \end{array}$$

Adjacency Matrix

Graph Representation

Adjacency Matrix

Pros:

- **Easy to implement.**
- **Removing an edge takes O(1) time.**
- **Queries like whether there is an edge from vertex ‘u’ to vertex ‘v’ are efficient and can be done O(1).**

Graph Representation

Adjacency Matrix

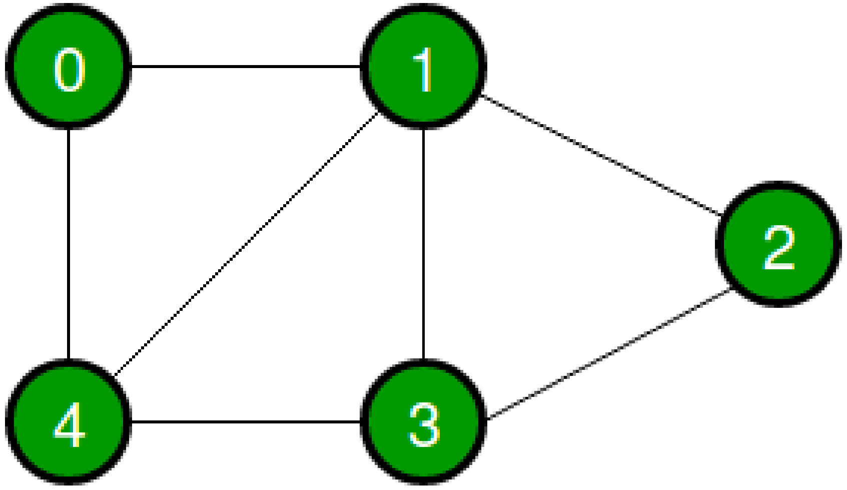
Cons:

- **Consumes more spaces O(V^2).**
- **Adding a vertex is O(V^2).**

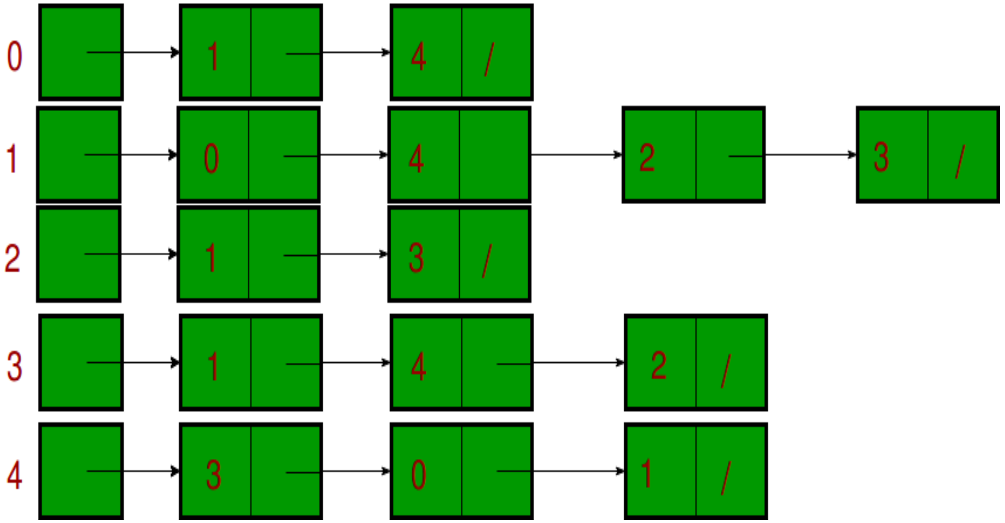
Graph Representation

Adjacency List: An array of linked lists is used. Size of the array is equal to number of vertices and each entry of array corresponds to a linked list of vertices adjacent to the index

Example



Graph



Adjacency List

Graph Representation

Adjacency List

Pros:

- **Saves space O(|V|+|E|), worst case O(V^2).**
- **Adding a vertex is easier.**

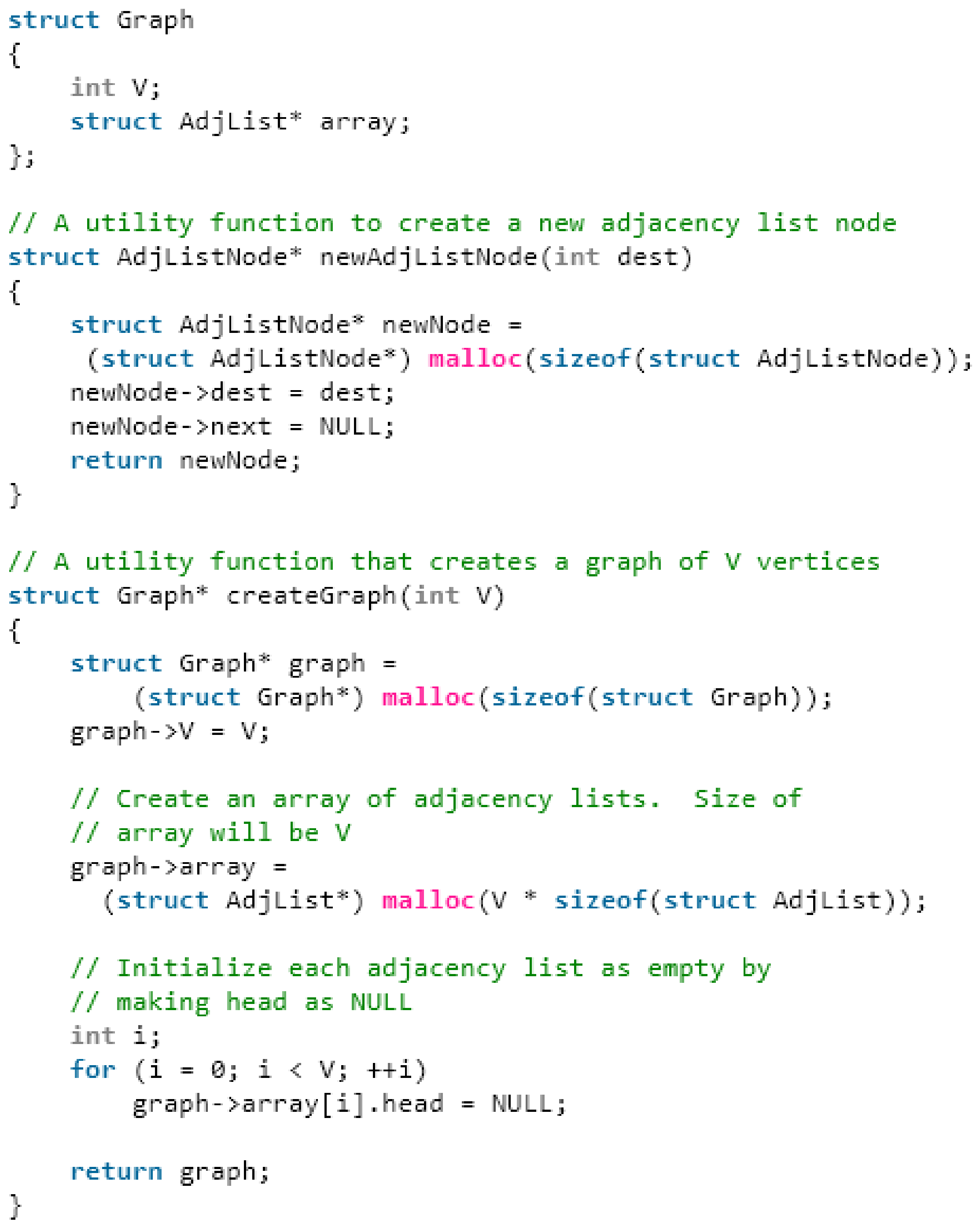
Graph Representation

Adjacency List

Cons:

- **Queries like whether there is an edge from vertex u to vertex v are not efficient and can be done O(V).**

Graph Implementation



```
struct Graph
{
    int V;
    struct AdjList* array;
};

// A utility function to create a new adjacency list node
struct AdjListNode* newAdjListNode(int dest)
{
    struct AdjListNode* newNode =
     (struct AdjListNode*) malloc(sizeof(struct AdjListNode));
    newNode->dest = dest;
    newNode->next = NULL;
    return newNode;
}

// A utility function that creates a graph of V vertices
struct Graph* createGraph(int V)
{
    struct Graph* graph =
        (struct Graph*) malloc(sizeof(struct Graph));
    graph->V = V;

    // Create an array of adjacency lists.  Size of
    // array will be V
    graph->array =
      (struct AdjList*) malloc(V * sizeof(struct AdjList));

    // Initialize each adjacency list as empty by
    // making head as NULL
    int i;
    for (i = 0; i < V; ++i)
        graph->array[i].head = NULL;

    return graph;
}
```

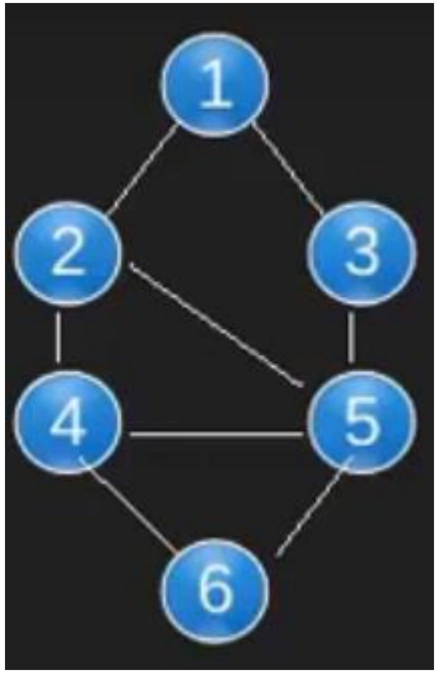
Breadth First Search (BFS)

Breadth First Search

Idea: Traverse nodes in layers

Problem: Since we have cycles, each node will be visited infinite times.

Solution: Use a Boolean visited array.



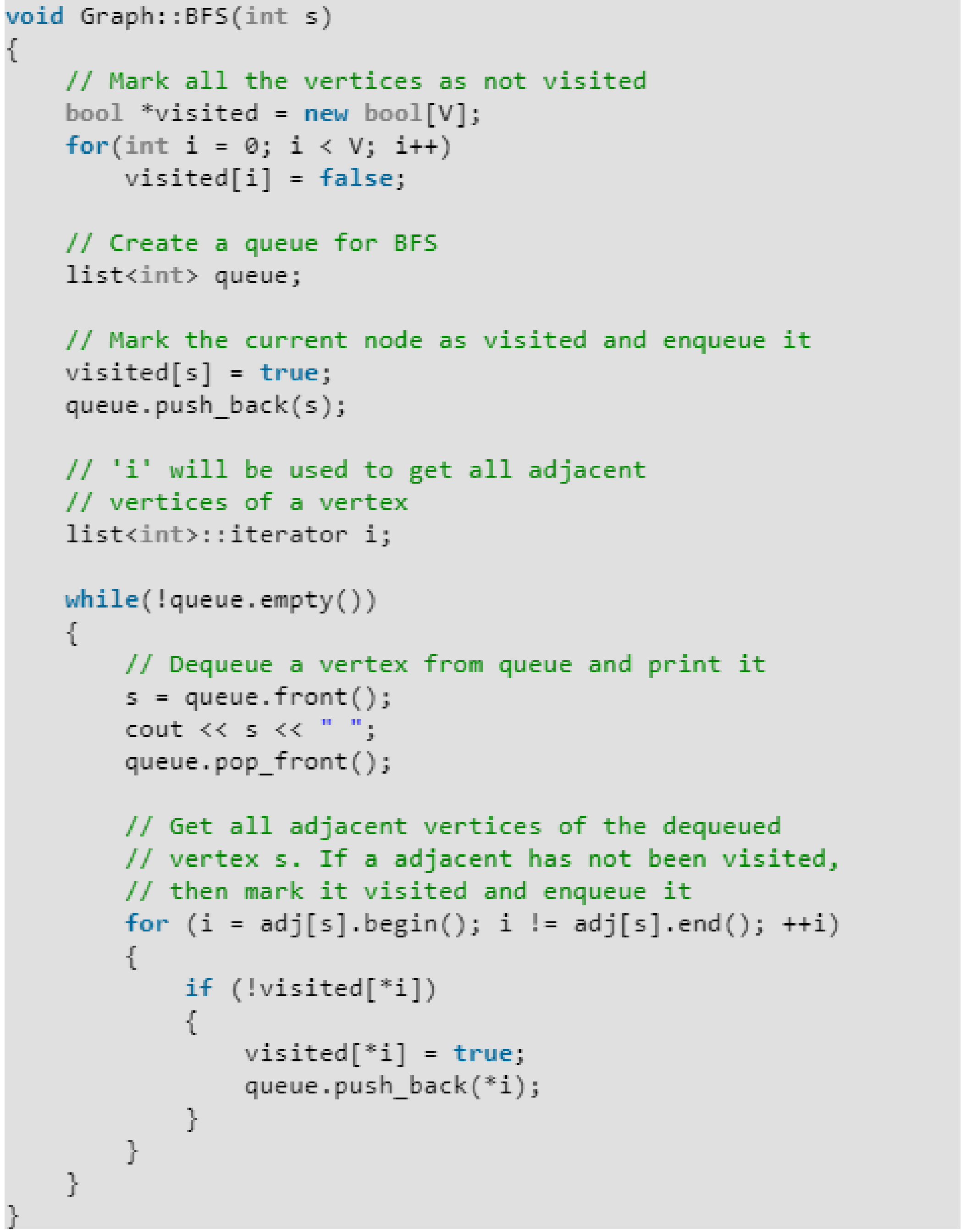
L1

L2

L3

L4

Implementation



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void Graph::BFS(int s)
{
    // Mark all the vertices as not visited
    bool *visited = new bool[V];
    for(int i = 0; i < V; i++)
        visited[i] = false;

    // Create a queue for BFS
    list<int> queue;

    // Mark the current node as visited and enqueue it
    visited[s] = true;
    queue.push_back(s);

    // 'i' will be used to get all adjacent
    // vertices of a vertex
    list<int>::iterator i;

    while(!queue.empty())
    {
        // Dequeue a vertex from queue and print it
        s = queue.front();
        cout << s << " ";
        queue.pop_front();

        // Get all adjacent vertices of the dequeued
        // vertex s. If a adjacent has not been visited,
        // then mark it visited and enqueue it
        for (i = adj[s].begin(); i != adj[s].end(); ++i)
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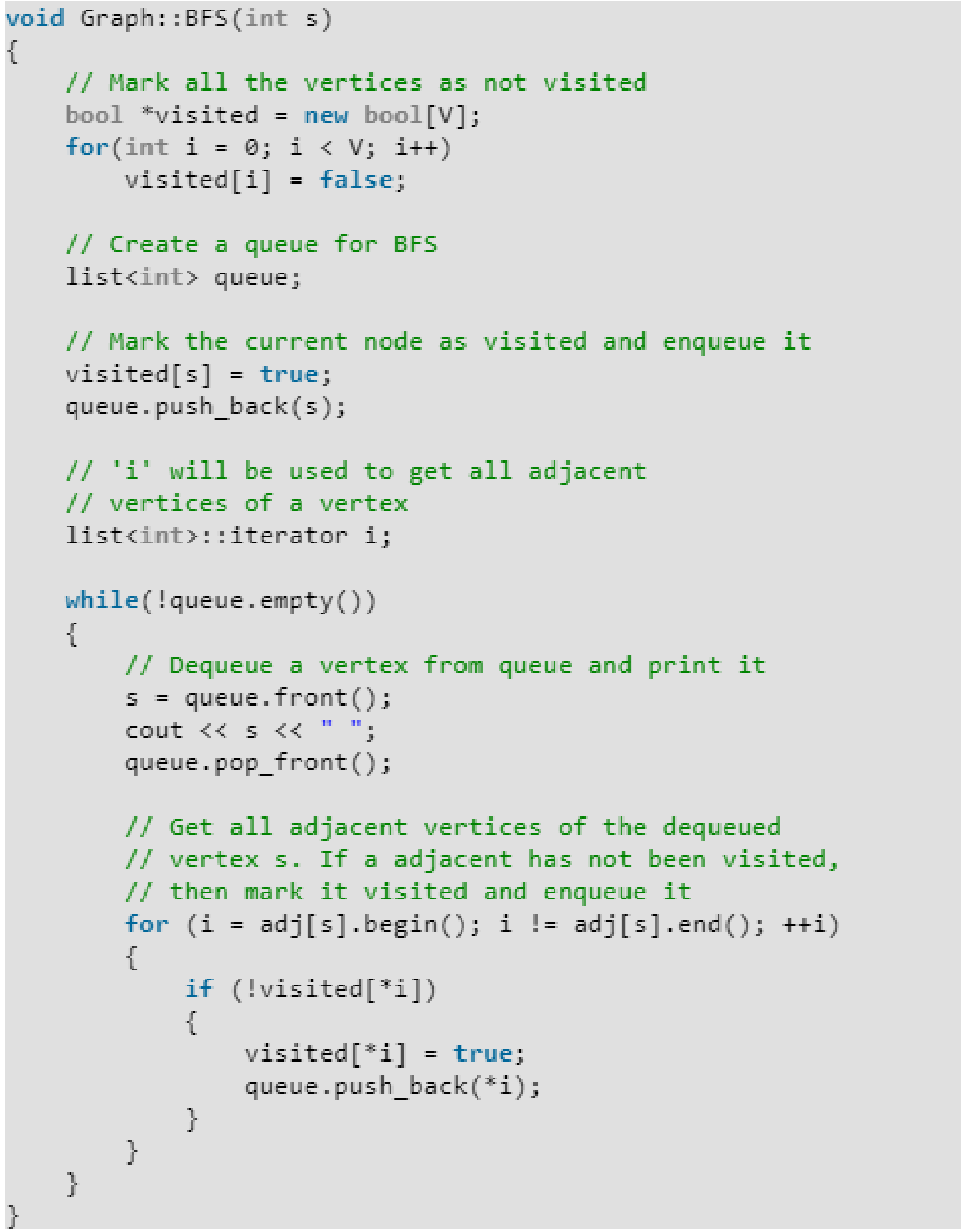
            }
        }
    }
}
```



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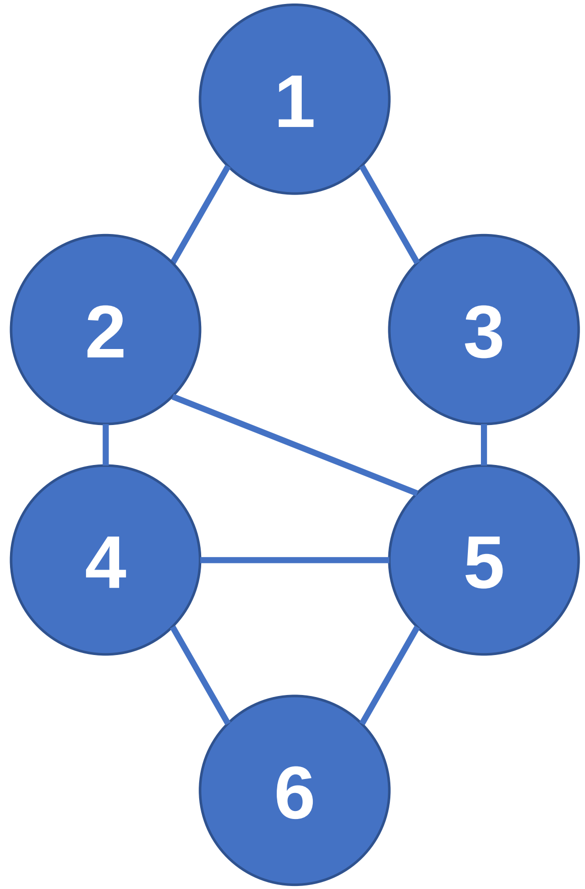
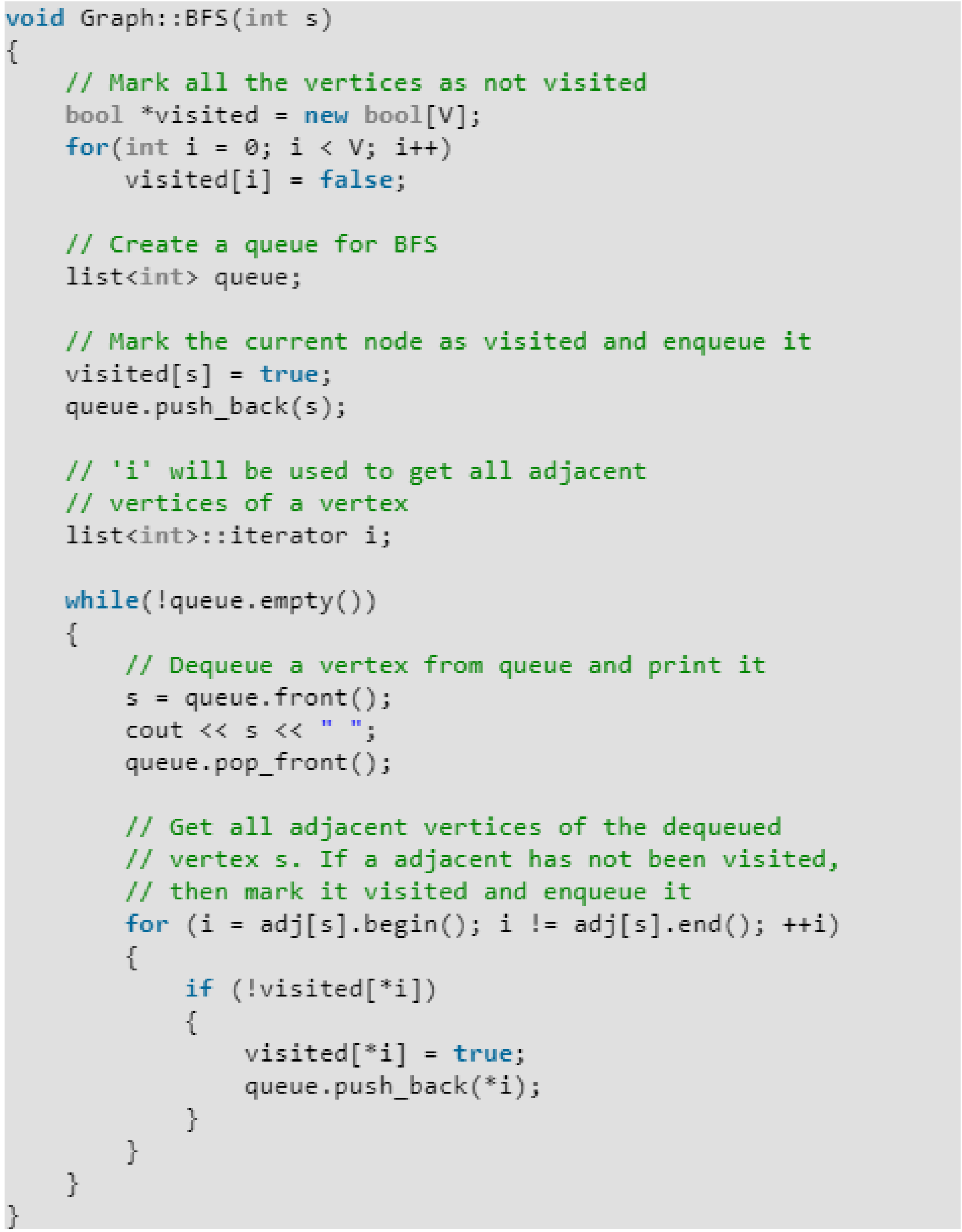


	1	2	3	4	5	6
Visited :	0	0	0	0	0	0

Queue :

Print :



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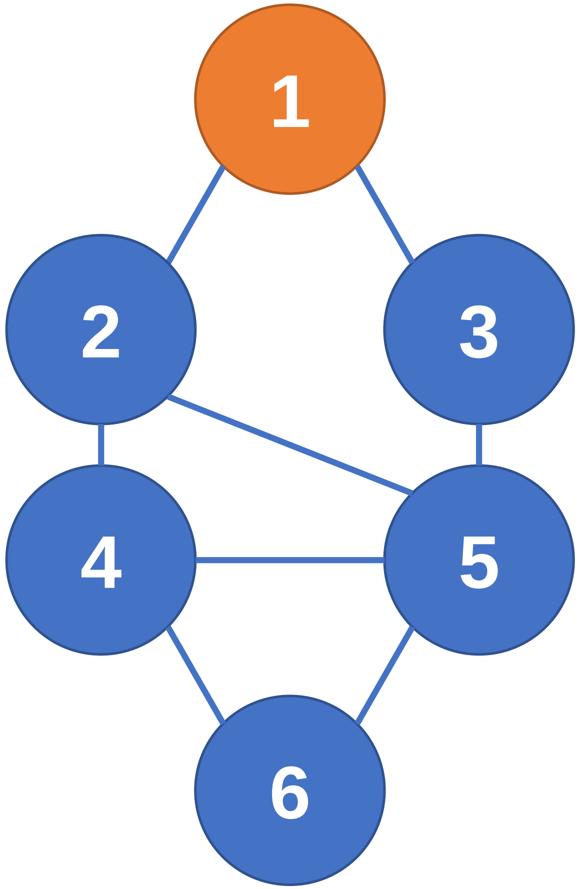
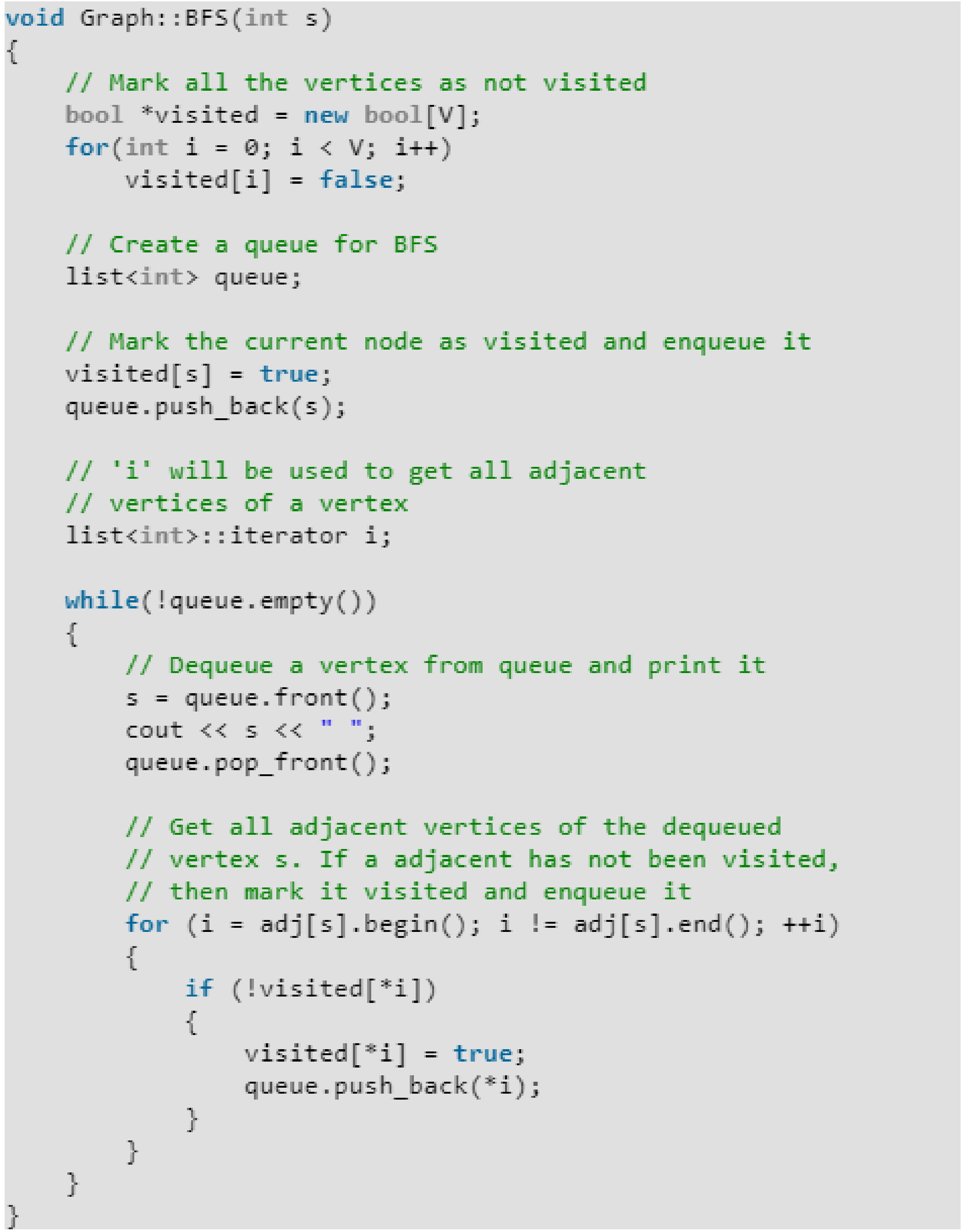


	1	2	3	4	5	6
Visited :	1	0	0	0	0	0

Queue : 1

Print : 1



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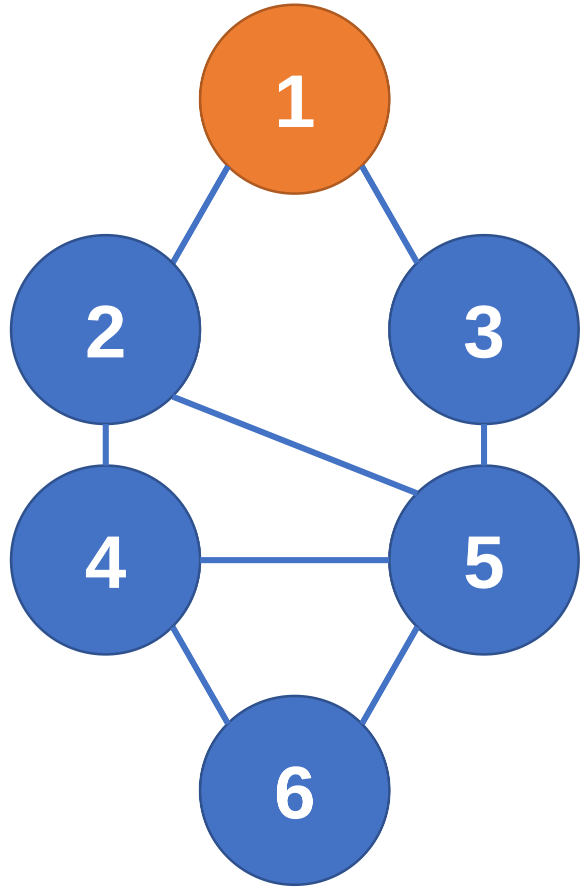
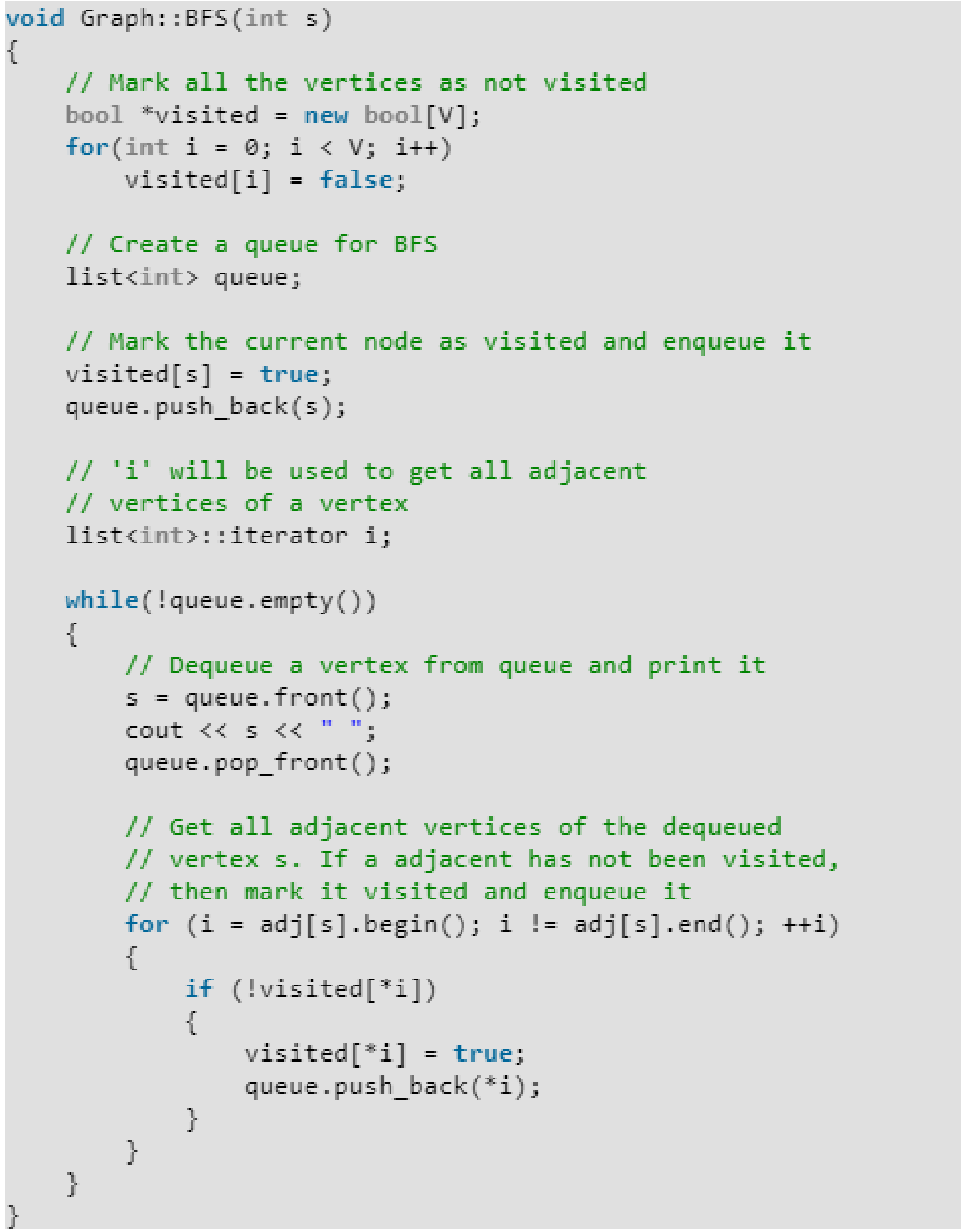


	1	2	3	4	5	6
Visited :	1	0	0	0	0	0

Queue :

Print : **1**



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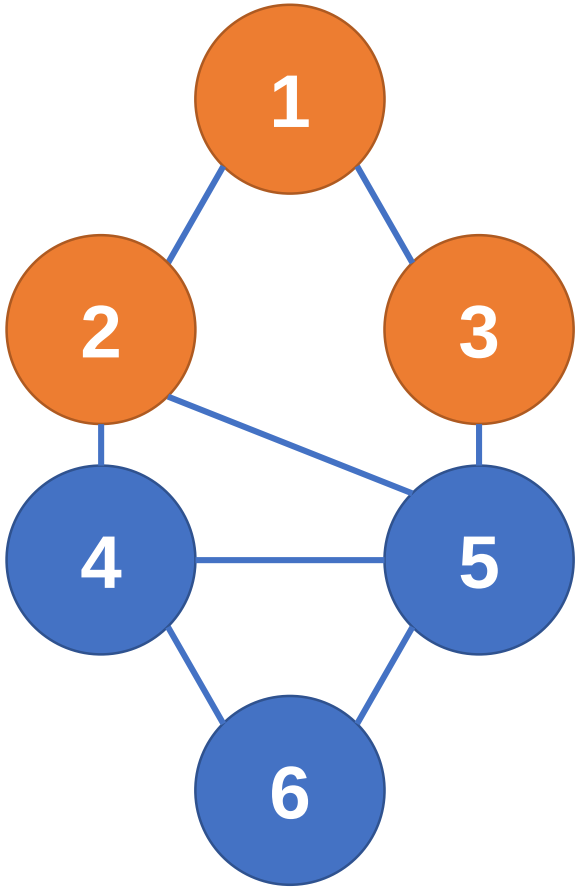
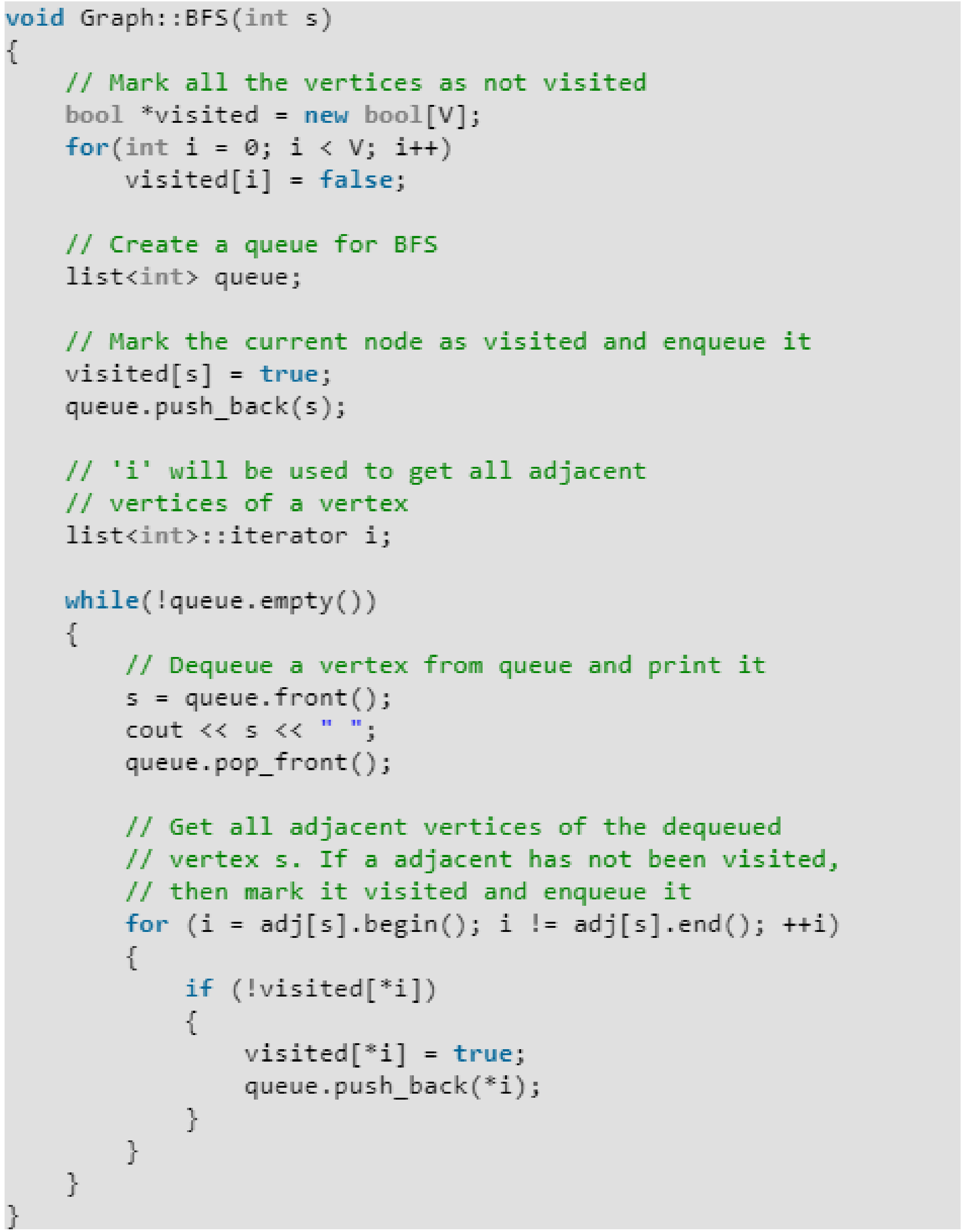


	1	2	3	4	5	6
Visited :	1	1	1	0	0	0

Queue : 2 3

Print : 1



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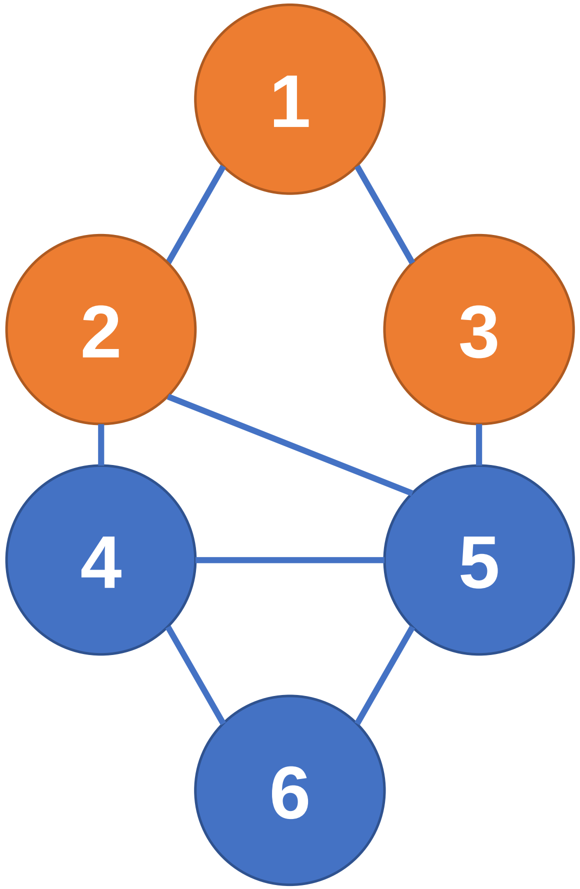
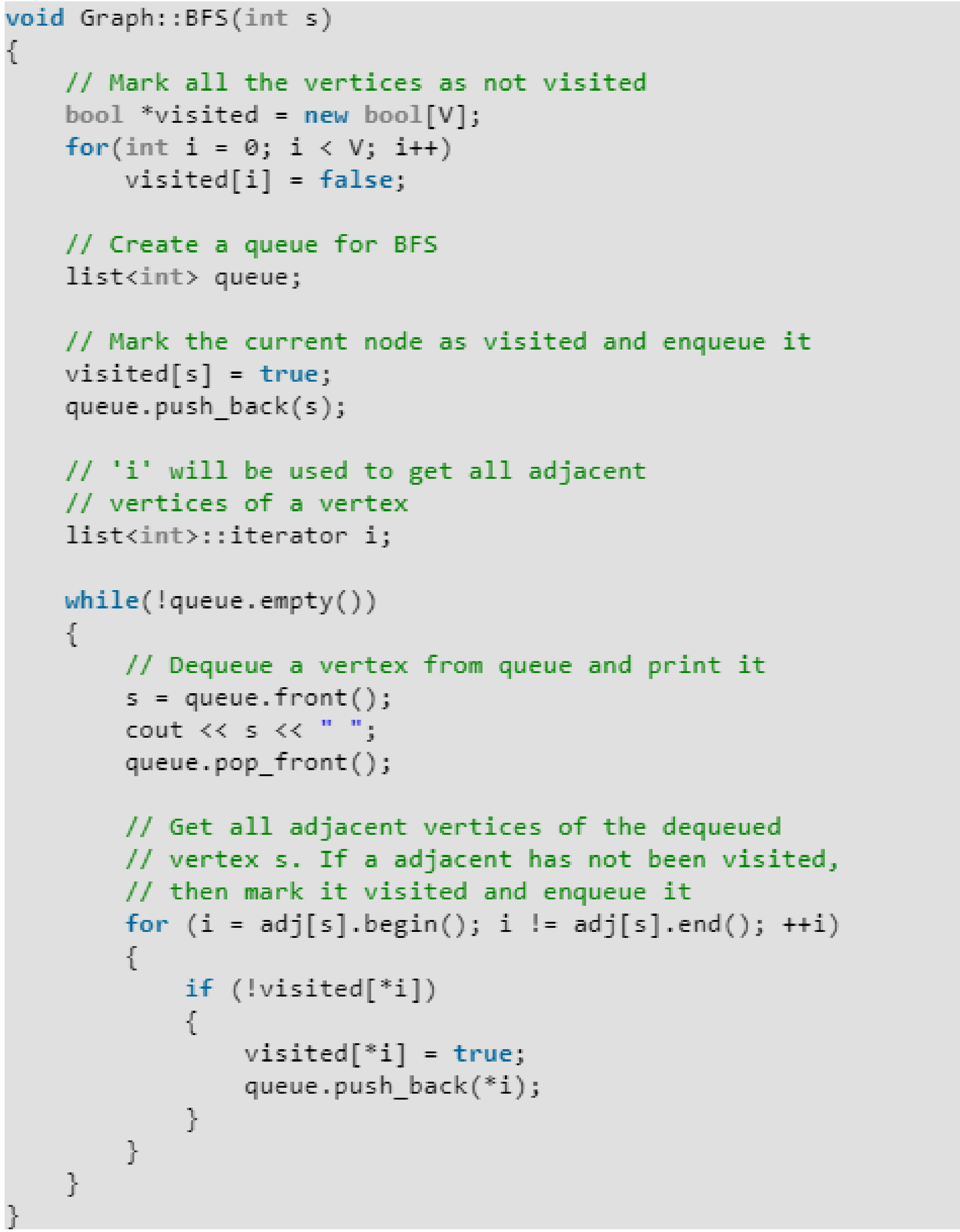


	1	2	3	4	5	6
Visited :	1	1	1	0	0	0

Queue : 2 3

Print : 1 2



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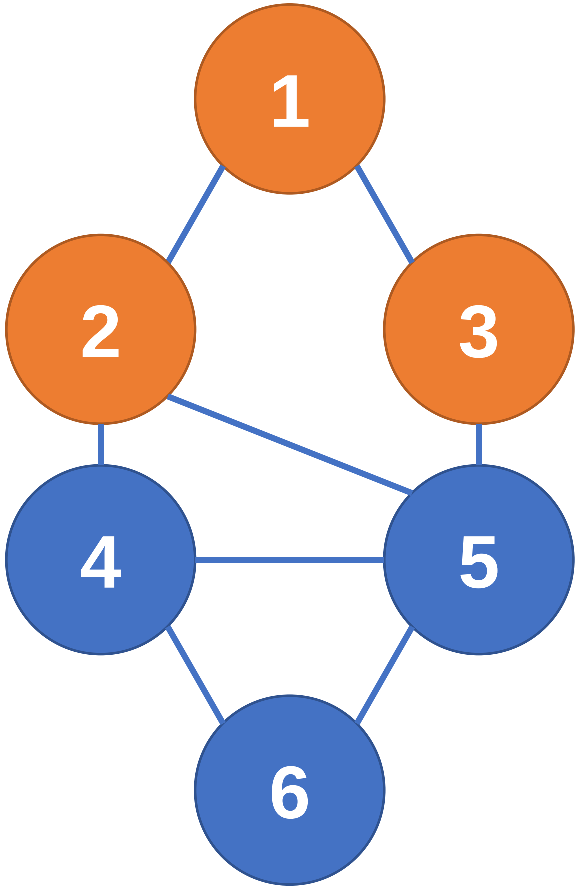
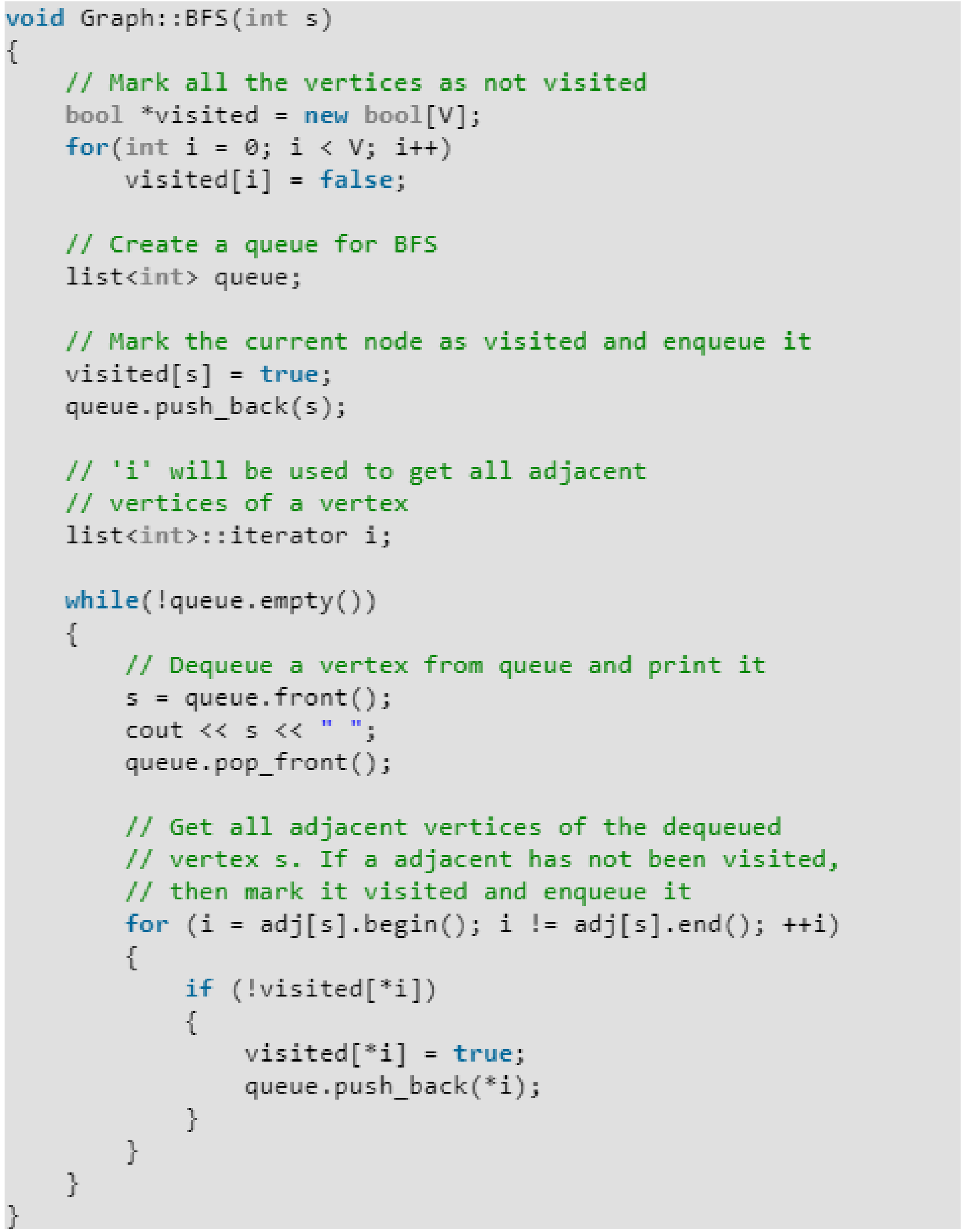


	1	2	3	4	5	6
Visited :	1	1	1	0	0	0

Queue : 3

Print : 1 2



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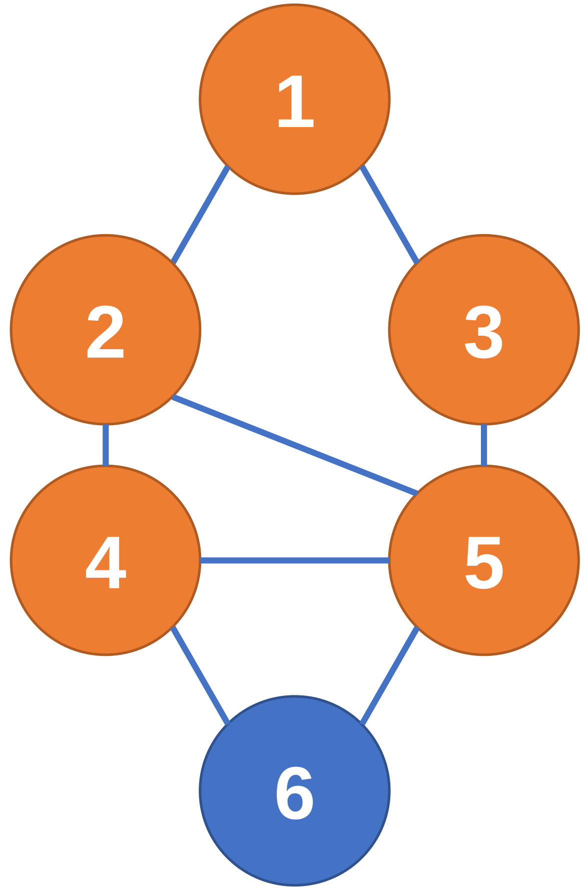
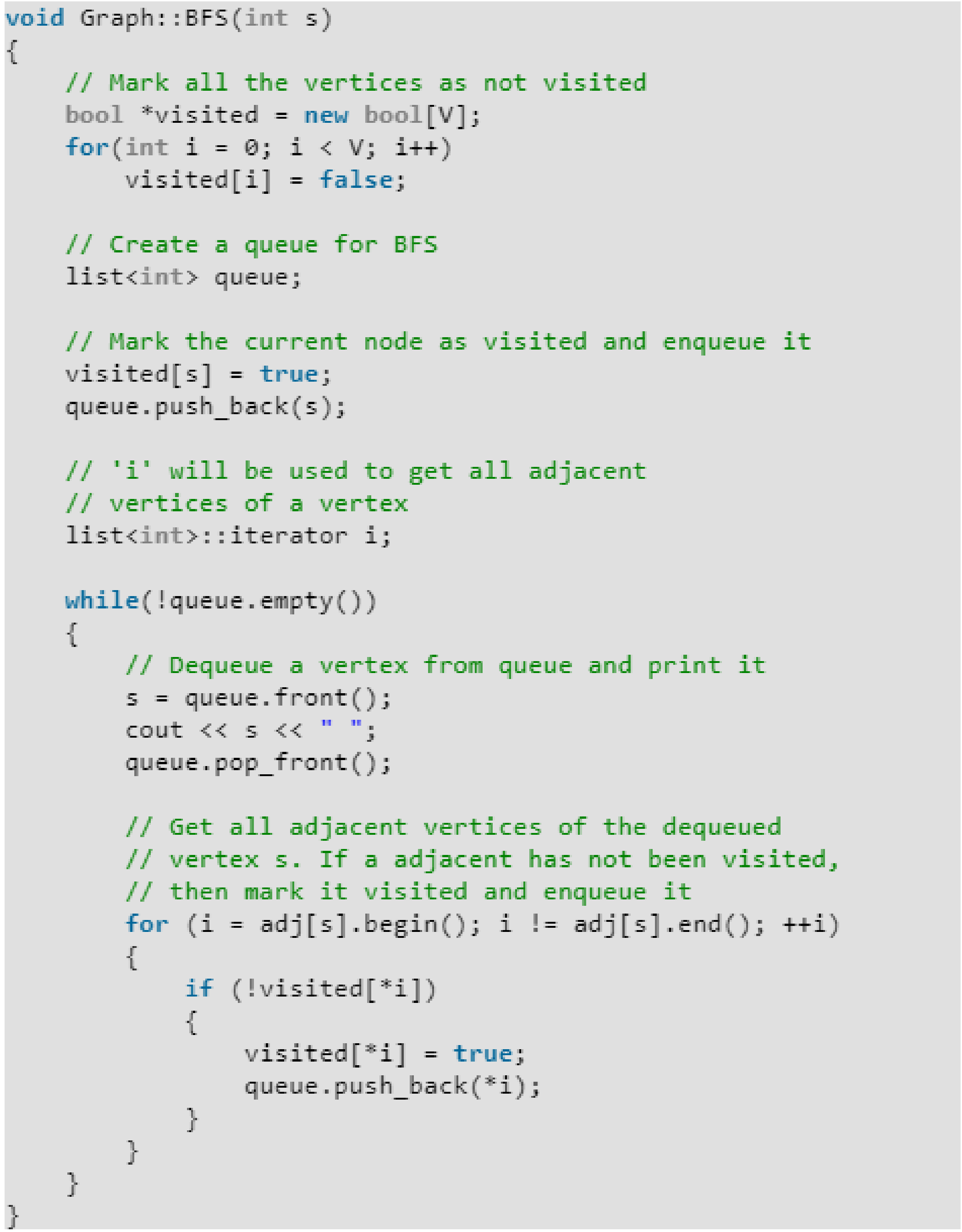


	1	2	3	4	5	6
Visited :	1	1	1	1	1	0

Queue : 3 4 5

Print : 1 2



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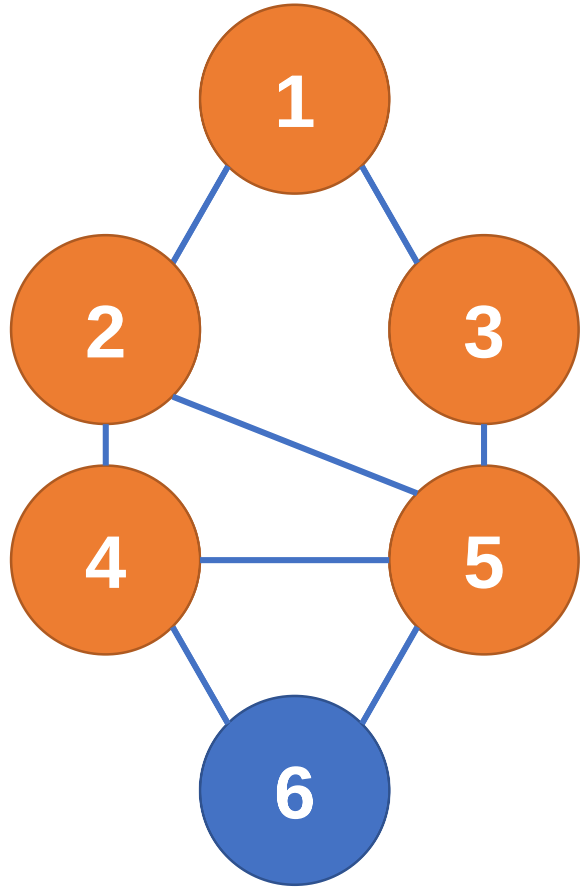
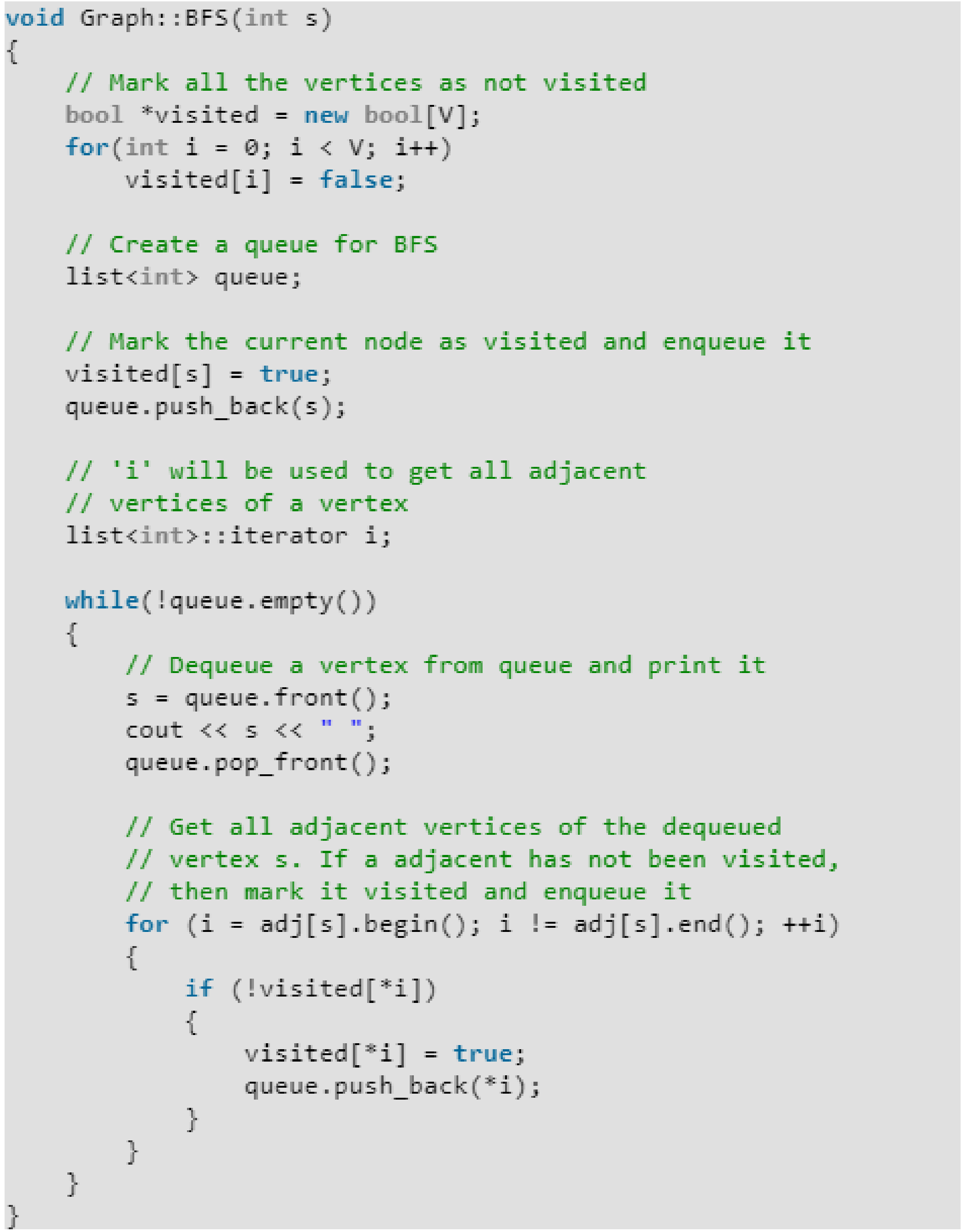


	1	2	3	4	5	6
Visited :	1	1	1	1	1	0

Queue : 4 5

Print : 1 2 3



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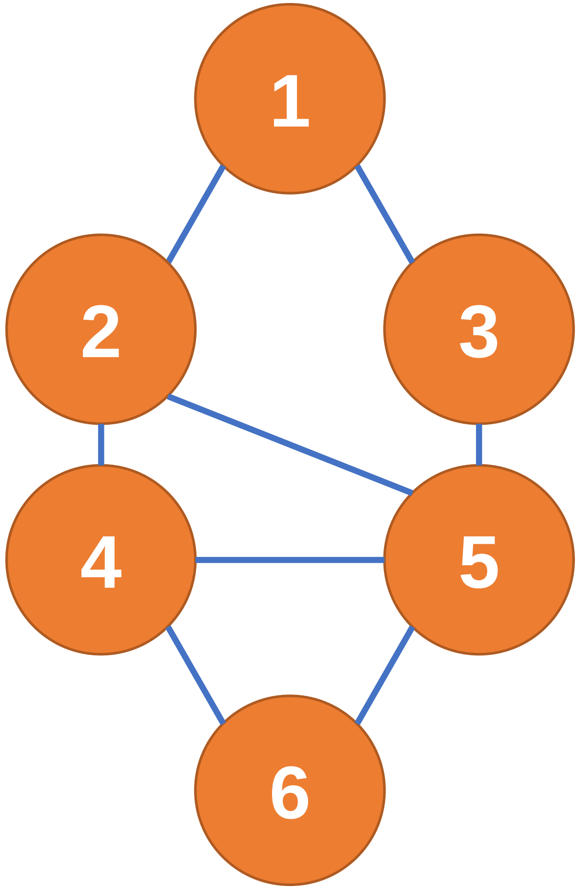
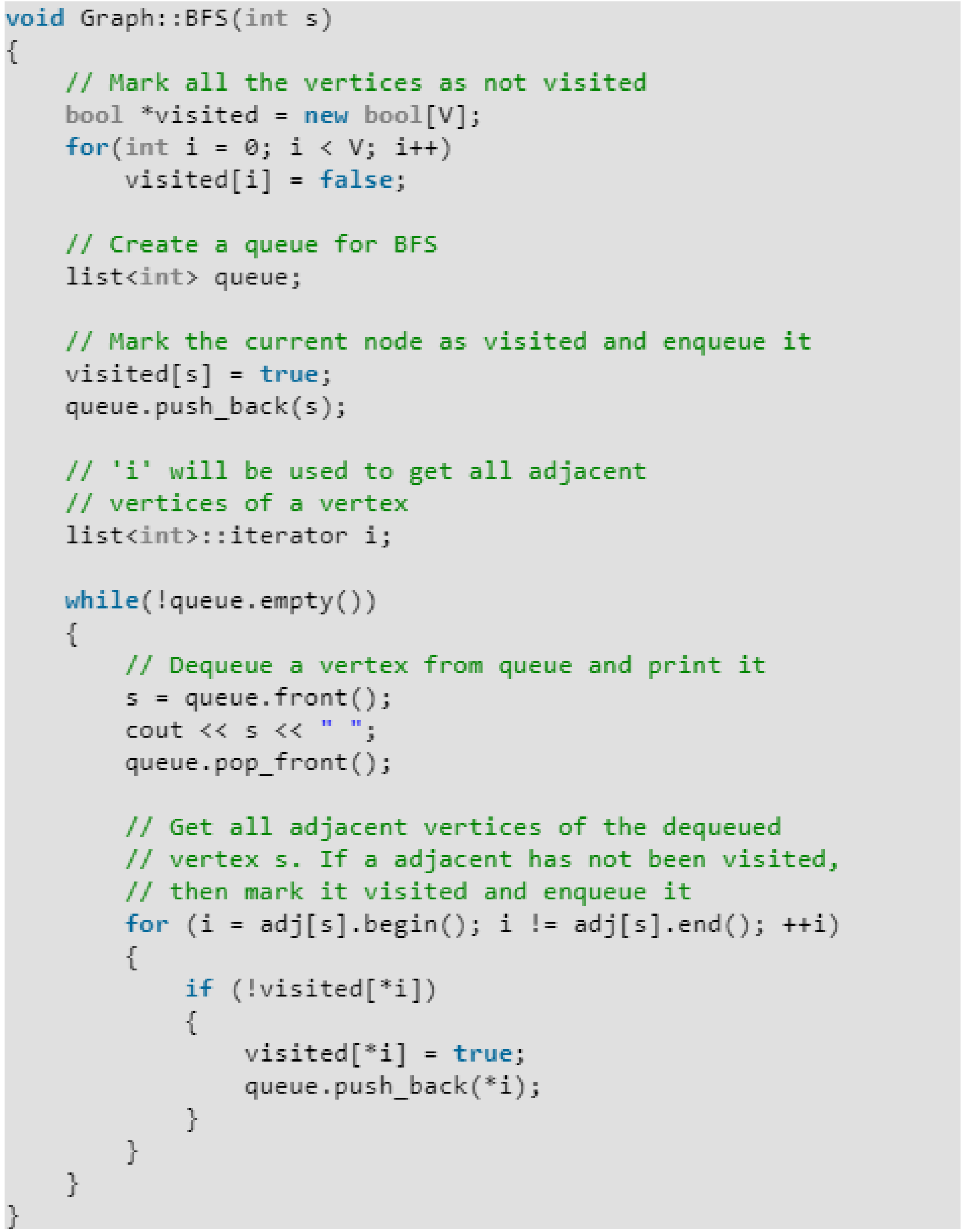


	1	2	3	4	5	6
Visited :	1	1	1	1	1	1

Queue : 5 6

Print : 1 2 3 4



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                queue.push_back(*i);
            }
        }
    }
}
```
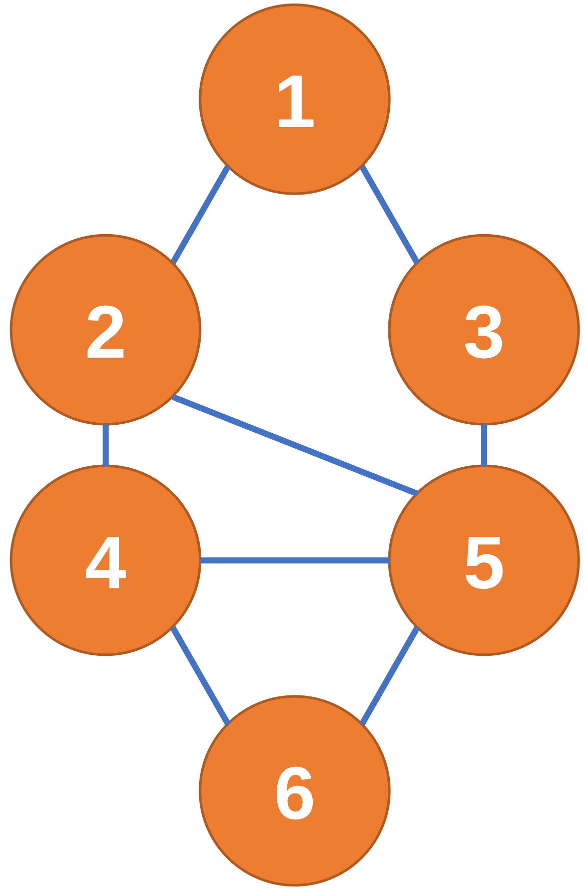
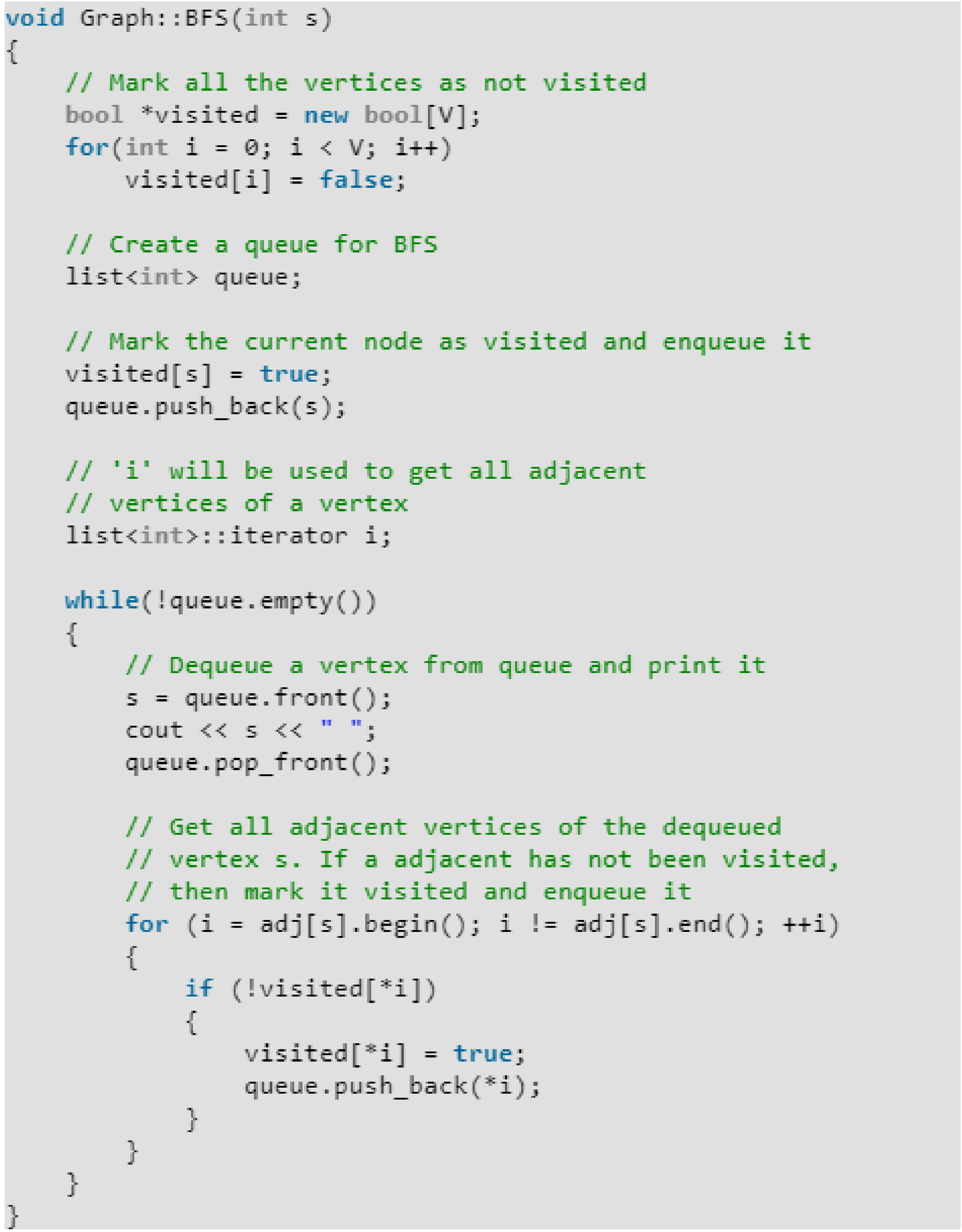



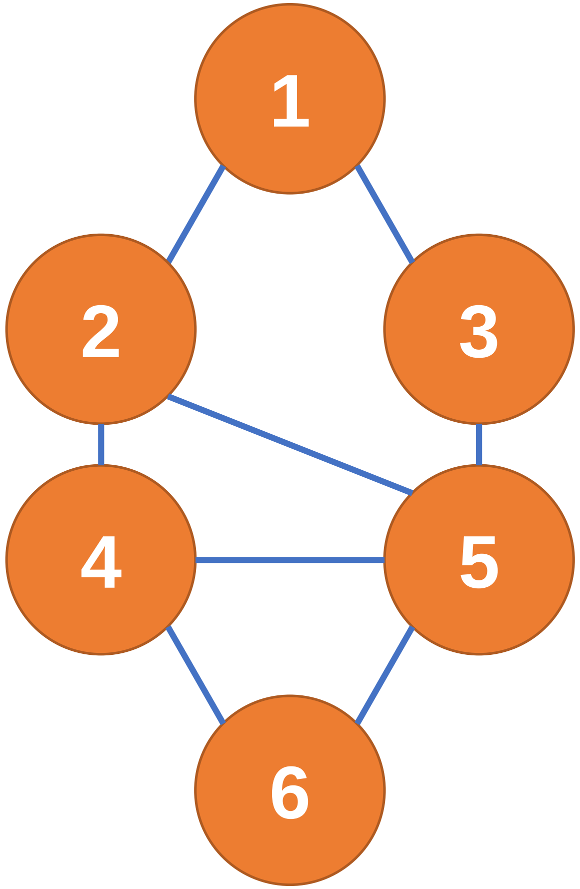
	1	2	3	4	5	6
Visited :	1	1	1	1	1	1

Queue : 6

Print : 1 2 3 4 5



```
void Graph::BFS(int s)
{
    // Mark all the vertices as not visited
    bool *visited = new bool[V];
    for(int i = 0; i < V; i++)
        visited[i] = false;

    // Create a queue for BFS
    list<int> queue;

    // Mark the current node as visited and enqueue it
    visited[s] = true;
    queue.push_back(s);

    // 'i' will be used to get all adjacent
    // vertices of a vertex
    list<int>::iterator i;

    while(!queue.empty())
    {
        // Dequeue a vertex from queue and print it
        s = queue.front();
        cout << s << " ";
        queue.pop_front();

        // Get all adjacent vertices of the dequeued
        // vertex s. If a adjacent has not been visited,
        // then mark it visited and enqueue it
        for (i = adj[s].begin(); i != adj[s].end(); ++i)
        {
            if (!visited[*i])
            {
                visited[*i] = true;
                queue.push_back(*i);
            }
        }
    }
}
```



	1	2	3	4	5	6
Visited :	1	1	1	1	1	1

Queue :

Print : 1 2 3 4 5 6

Complexity

Time Complexity: O(V+E)

V: Vertices
E: Edges

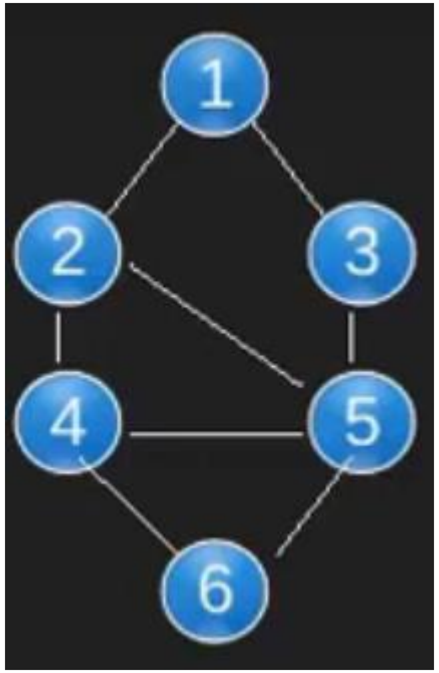
Depth First Search (DFS)

Depth First Search

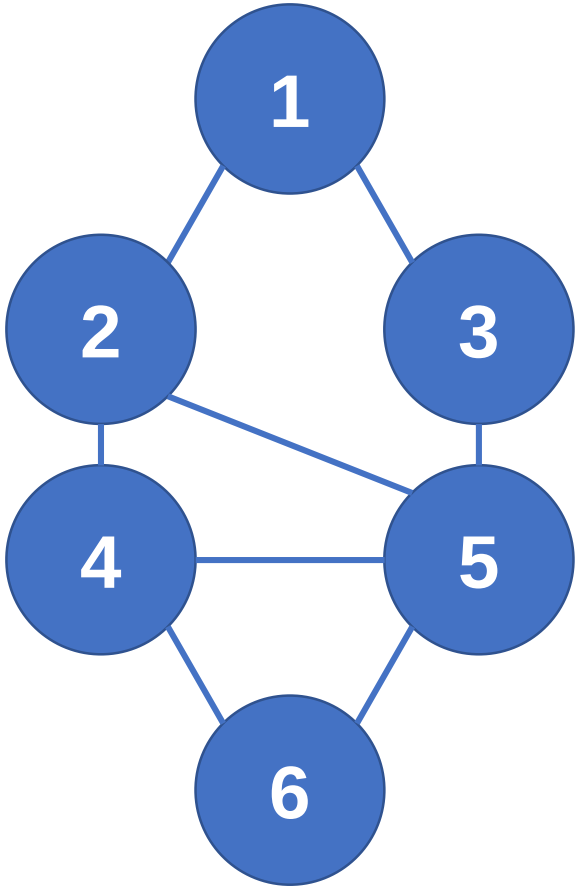
Idea: To go forward (in depth) while there is any such possibility, if not then, backtrack

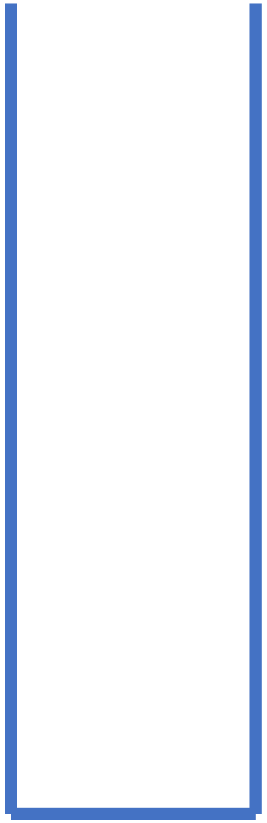
Problem: Since we have cycles, each node may be visited infinite times.

Solution: Use a Boolean visited array.



Depth First Search

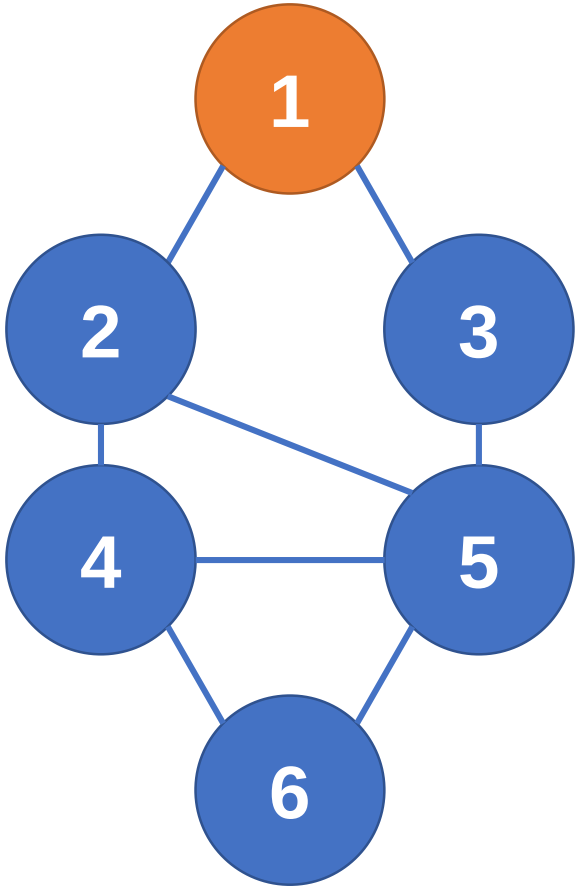


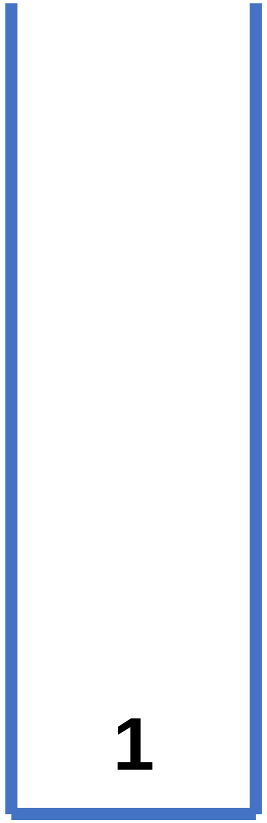


Stack

Output:

Depth First Search

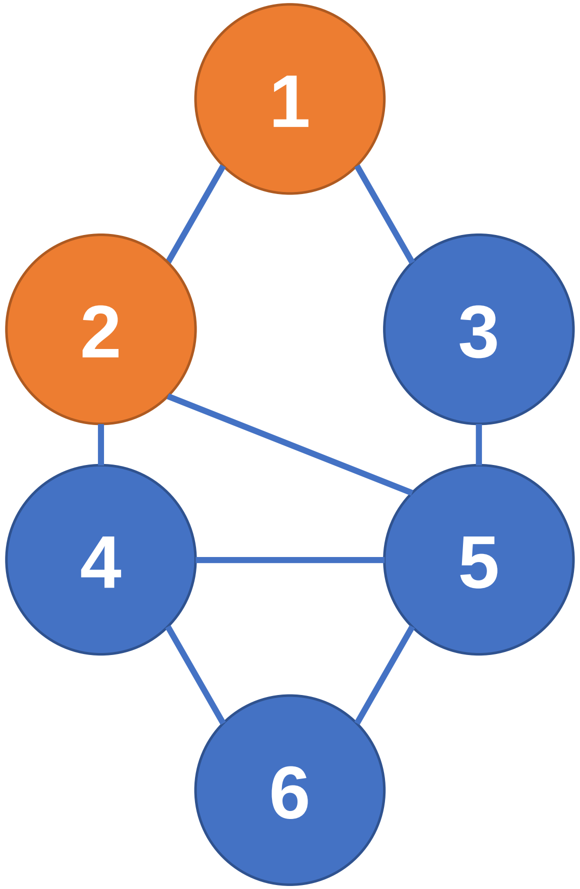


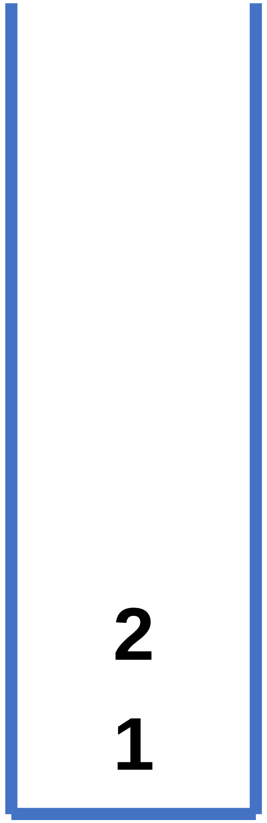

1

Stack

Output: 1

Depth First Search





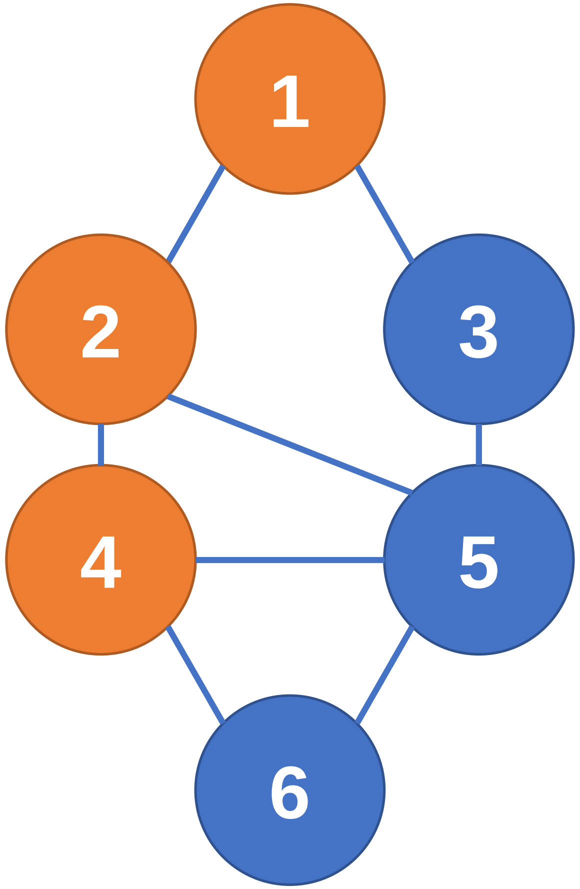
2

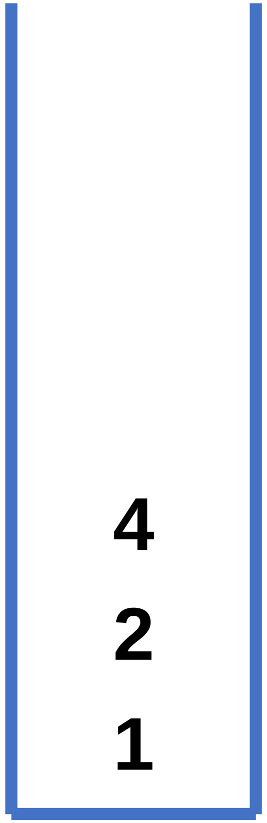
1

Stack

Output: 1 2

Depth First Search





4

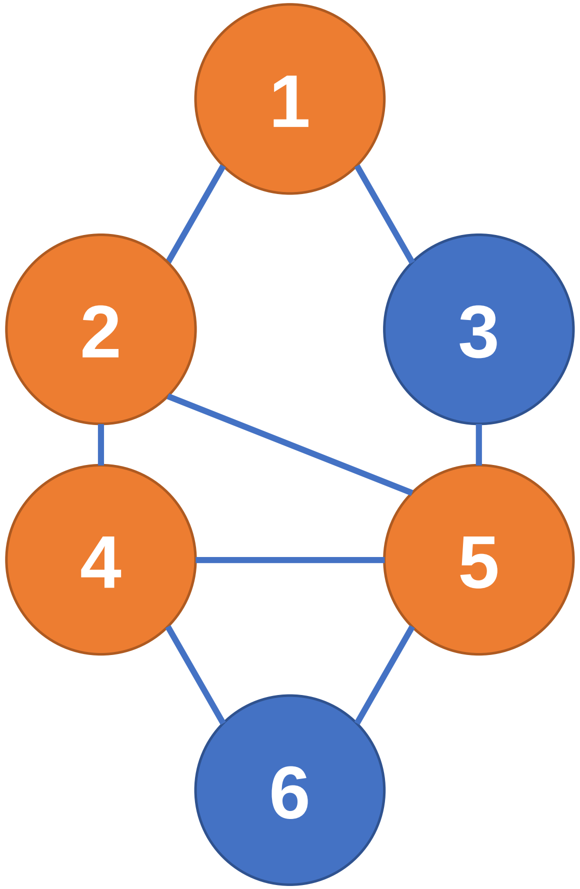
2

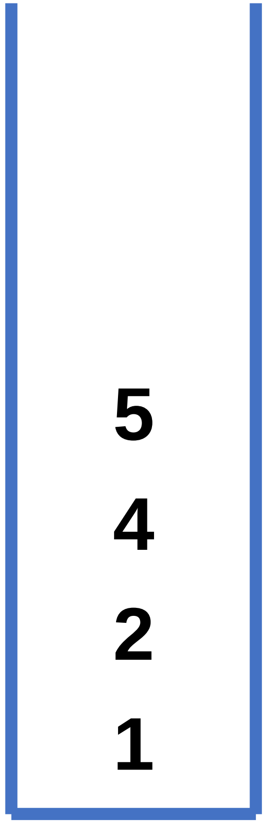
1

Stack

Output: 1 2 4

Depth First Search





5

4

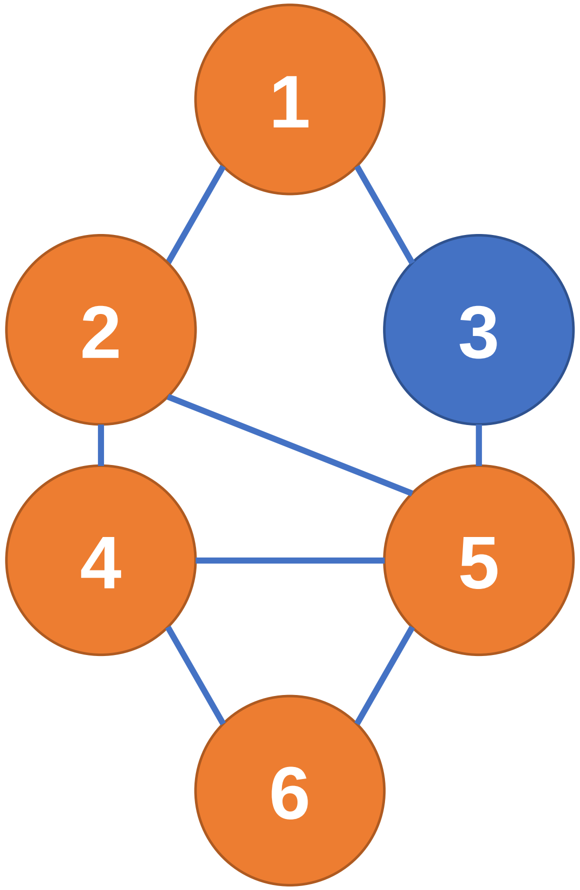
2

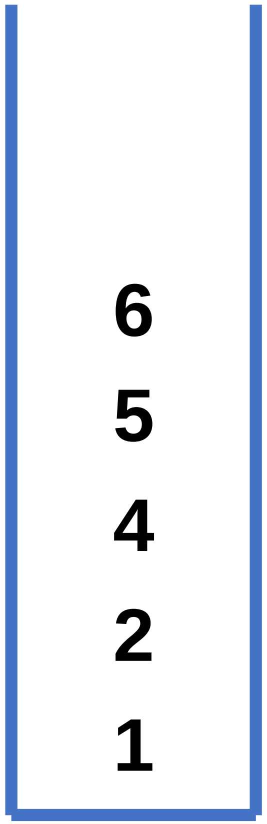
1

Stack

Output: 1 2 4 5

Depth First Search





6

5

4

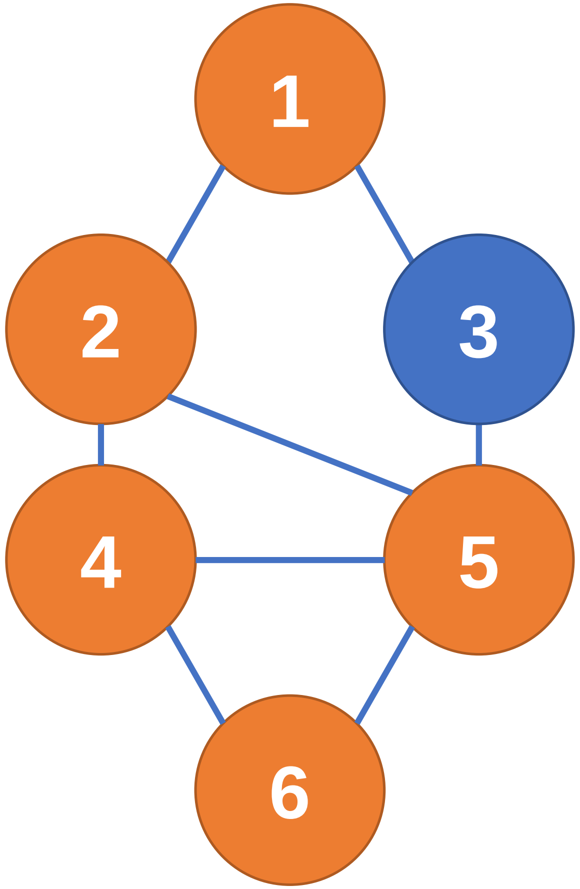
2

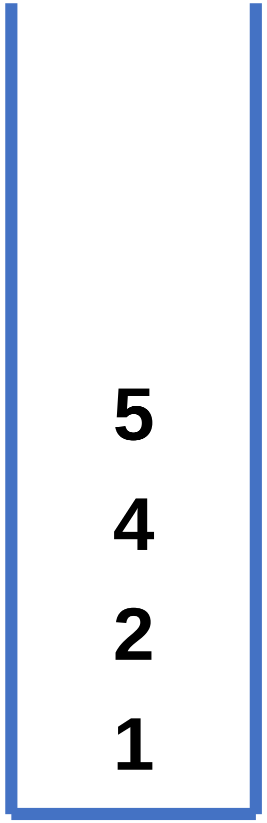
1

Stack

Output: 1 2 4 5 6

Depth First Search





5

4

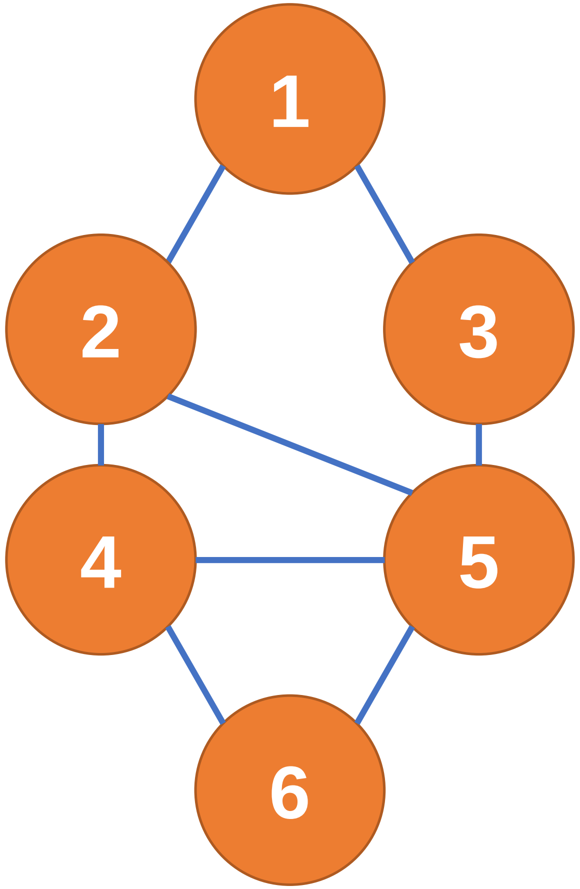
2

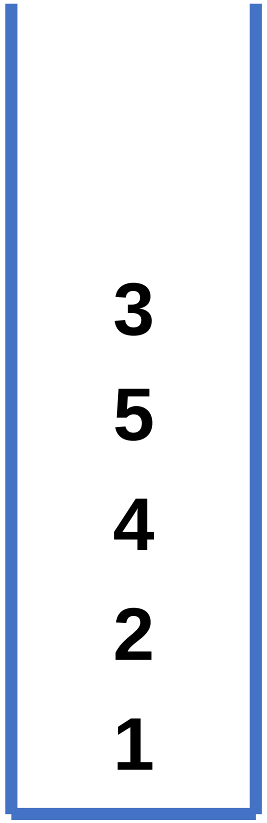
1

Stack

Output: 1 2 4 5 6

Depth First Search




3

5

4

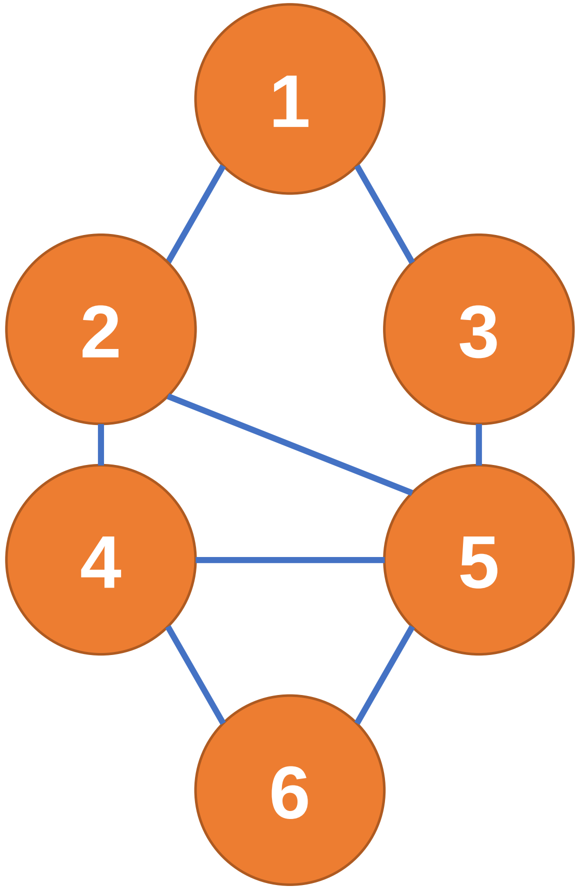
2

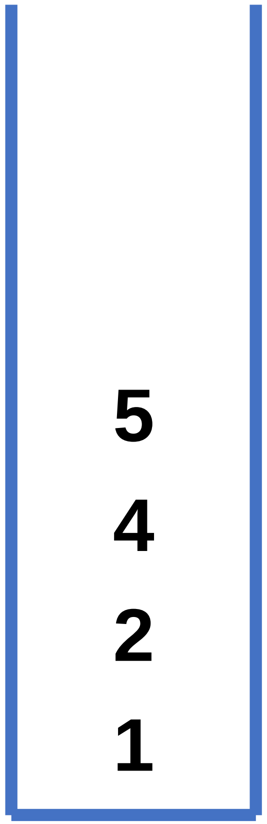
1

Stack

Output: 1 2 4 5 6 3

Depth First Search




5

4

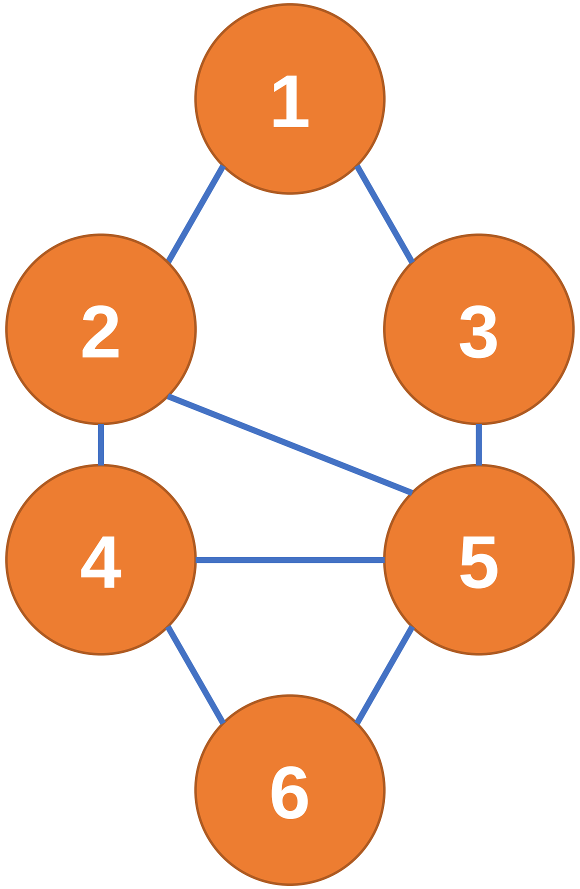
2

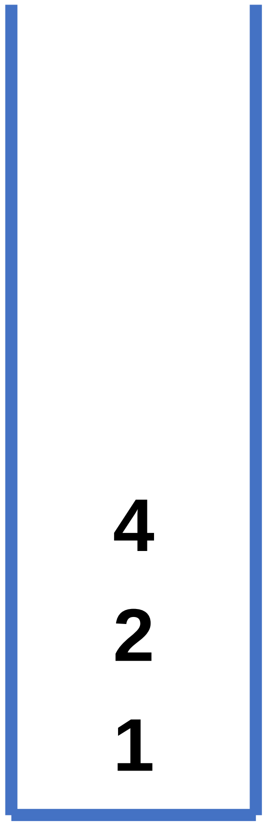
1

Stack

Output: 1 2 4 5 6 3

Depth First Search





4

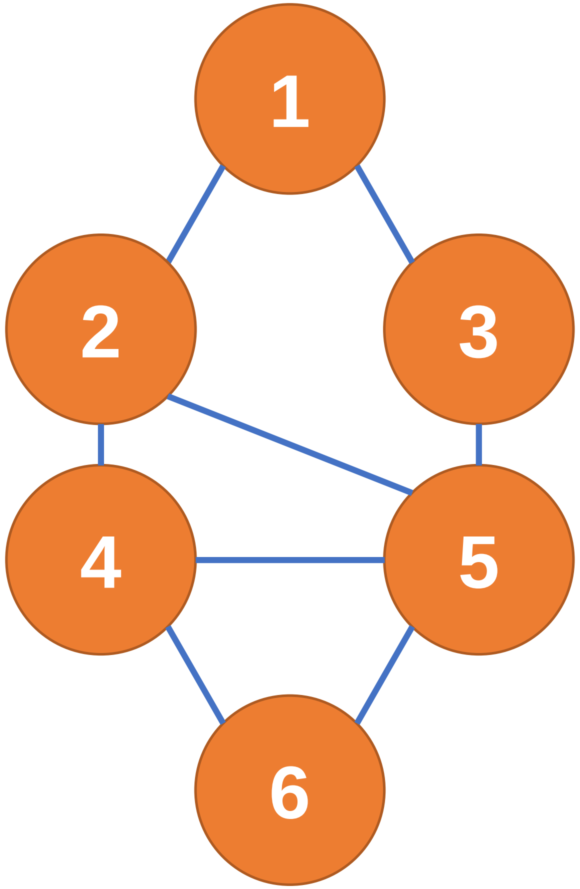
2

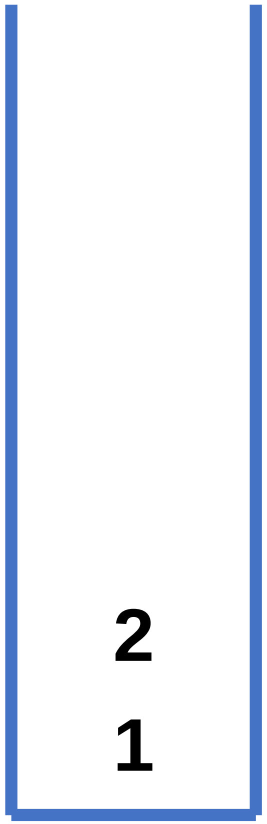
1

Stack

Output: 1 2 4 5 6 3

Depth First Search

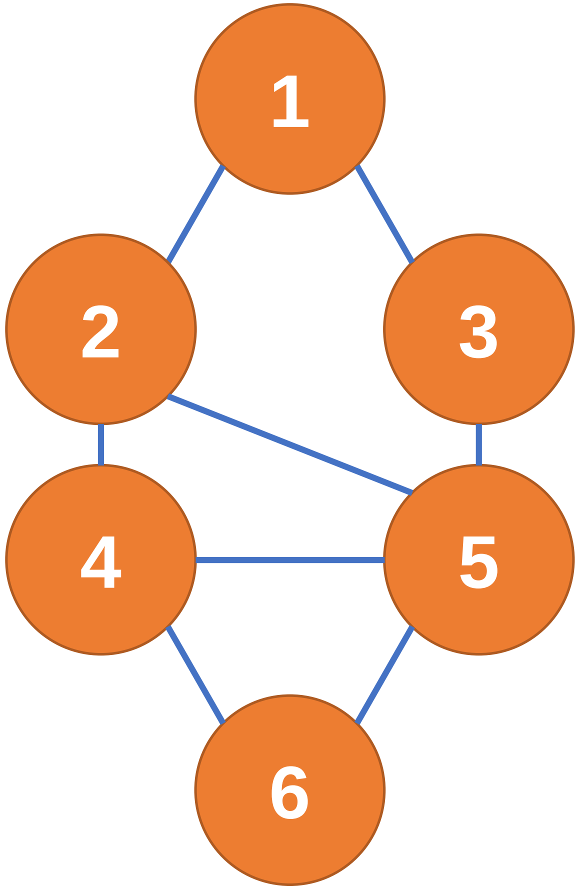


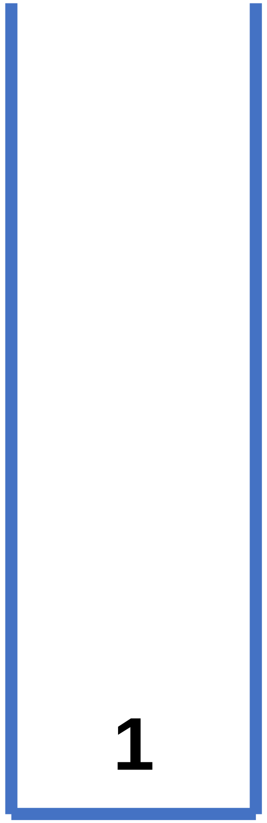


Stack

Output: 1 2 4 5 6 3

Depth First Search

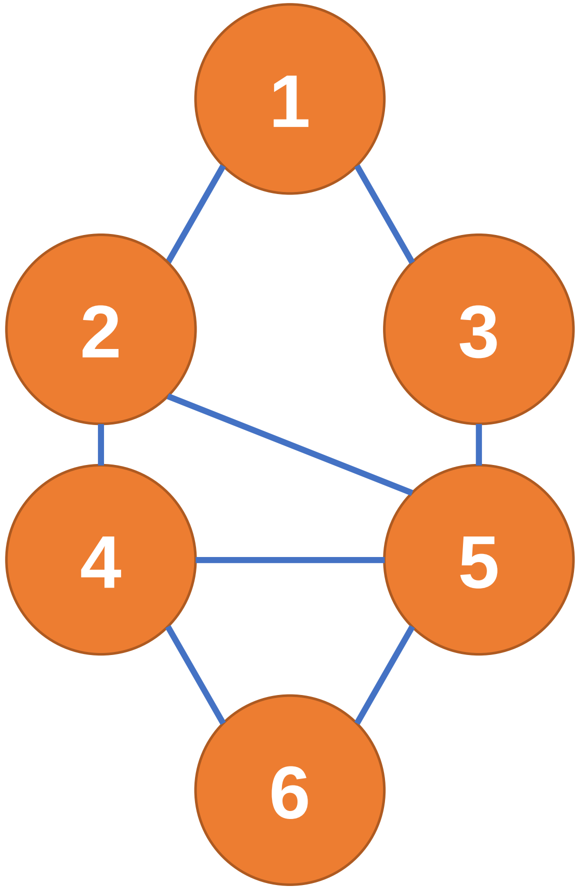


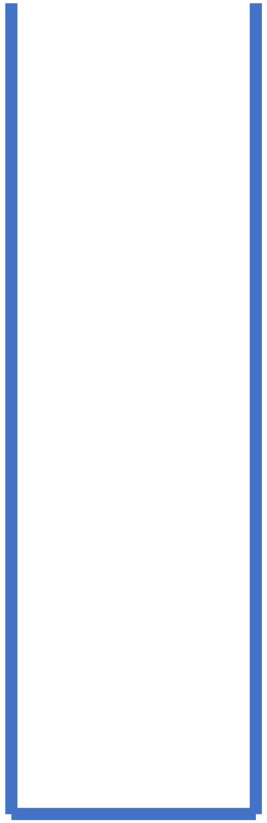


Stack

Output: 1 2 4 5 6 3

Depth First Search

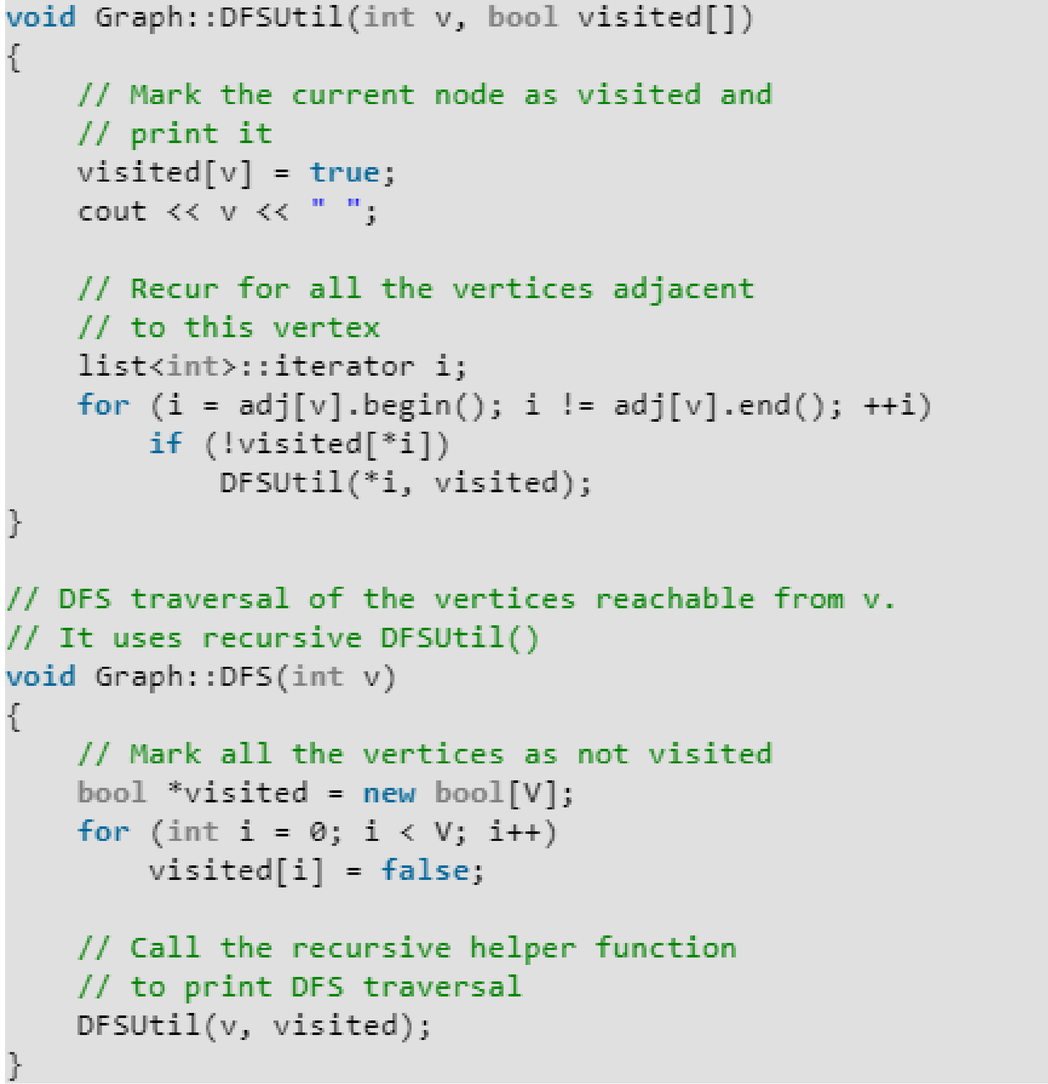




Stack

Output: 1 2 4 5 6 3

Implementation



```
void Graph::DFSUtil(int v, bool visited[])
{
    // Mark the current node as visited and
    // print it
    visited[v] = true;
    cout << v << " ";

    // Recur for all the vertices adjacent
    // to this vertex
    list<int>::iterator i;
    for (i = adj[v].begin(); i != adj[v].end(); ++i)
        if (!visited[*i])
            DFSUtil(*i, visited);
}

// DFS traversal of the vertices reachable from v.
// It uses recursive DFSUtil()
void Graph::DFS(int v)
{
    // Mark all the vertices as not visited
    bool *visited = new bool[V];
    for (int i = 0; i < V; i++)
        visited[i] = false;

    // Call the recursive helper function
    // to print DFS traversal
    DFSUtil(v, visited);
}
```

Complexity

Time Complexity: O(V+E)

V: Vertices
E: Edges

Reference

- Charles Leiserson and Piotr Indyk, “*Introduction to Algorithms*”, September 29, 2004

- https://www.geeksforgeeks.org