Dynamic Programming II

SWE2016-44

Longest Increasing Subsequence

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Subsequences: {10}, {10, 22}, {10, 9, 33}, {9, 21, 60}, {50, 60}, …

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Find the length of the longest subsequence of a given sequence such that all elements of the subsequence are sorted in increasing order.

For example,

Given sequence LIS = {10, 22, 9, 33, 21, 50, 41, 60}

Subsequences: {10}, {10, 22}, {10, 9, 33}, {9, 21, 60}, {50, 60}, …

Increasing Subsequences: {10}, {9, 33, 41}, {33, 41, 60}, {41}, …

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Subsequences: {10}, {10, 22}, {10, 9, 33}, {9, 21, 60}, {50, 60}, …

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Longest Increasing Subsequences: {10, 22, 33, 50, 60} or {10, 22, 33, 41, 60}

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For example,

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Subsequences: {10}, {10, 22}, {10, 9, 33}, {9, 21, 60}, {50, 60}, …

Increasing Subsequences: {10}, {9, 33, 41}, {33, 41, 60}, {41}, …

Longest Increasing Subsequences: {10, 22, 33, 50, 60} or {10, 22, 33, 41, 60}

So, Length of LIS = 5

Optimal Substructure Property

Let arr[0..n-1] be the input array; and L(i) be the length of the LIS ending at index i such that arr[i] is the last element of the LIS.

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L(i) can be recursively written as:

$L(i) = 1 + \max(L(j))$ where $0 < j < i$ and $arr[j] < arr[i]$; or
$L(i) = 1$, if no such j exists.

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To find the LIS for a given array, return max(L(i)) where 0<i<n.

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L(i) = 1, if no such j exists.

To find the LIS for a given array, return max(L(i)) where $0<i<n$.

→The LIS problem satisfies the optimal substructure property.

Overlapping Substructure Property

```
                lis(4)
          /        |
       lis(3)    lis(2)   lis(1)
      /            /
   lis(2) lis(1) lis(1)
   /
lis(1)
```

Longest Increasing Subsequence

Initialize LIS value

iterator								
arr[]	10	22	9	33	21	50	41	60
LIS	1	1	1	1	1	1	1	1

Longest Increasing Subsequence

For i=1:

iterator	j	i						
arr[]	10	22	9	33	21	50	41	60
LIS	1	1	1	1	1	1	1	1

Longest Increasing Subsequence

For i=1:

iterator	j	i						
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	1	1	1	1	1

Longest Increasing Subsequence

For i=2:

iterator	j		i					
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	1	1	1	1	1

Longest Increasing Subsequence

For i=2:

iterator		j	i					
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	1	1	1	1	1

Longest Increasing Subsequence

For i=3:

iterator	j			i				
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	1	1	1	1	1

Longest Increasing Subsequence

For i=3:

iterator	j			i				
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	2	1	1	1	1

Longest Increasing Subsequence

For i=3:

iterator		j		i				
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	2	1	1	1	1

Longest Increasing Subsequence

For i=3:

iterator		j		i				
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	1	1	1	1

Longest Increasing Subsequence

For i=3:

iterator			**j**	**i**				
arr[]	**10**	**22**	**9**	**33**	**21**	**50**	**41**	**60**
LIS	**1**	**2**	**1**	**3**	**1**	**1**	**1**	**1**

Longest Increasing Subsequence

For i=4:

iterator	j				i			
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	1	1	1	1

Longest Increasing Subsequence

For i=4:

iterator	j				i			
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	1	1	1

Longest Increasing Subsequence

For i=4:

iterator		j			i			
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	1	1	1

Longest Increasing Subsequence

For i=4:

iterator			j		i			
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	1	1	1

Longest Increasing Subsequence

For i=4:

iterator				j	i			
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	1	1	1

Longest Increasing Subsequence

For i=5:

iterator	j					i		
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	1	1	1

Longest Increasing Subsequence

For i=5:

iterator	j					i		
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	2	1	1

Longest Increasing Subsequence

For i=5:

iterator		j				i		
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	2	1	1

Longest Increasing Subsequence

For i=5:

iterator		j				i		
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	3	1	1

Longest Increasing Subsequence

For i=5:

iterator			j			i		
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	3	1	1

Longest Increasing Subsequence

For i=5:

iterator				j		i		
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	3	1	1

Longest Increasing Subsequence

For i=5:

iterator				j		i		
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	4	1	1

Longest Increasing Subsequence

For i=5:

iterator					j	i		
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	4	1	1

Longest Increasing Subsequence

For i=6:

iterator	j						i	
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	4	1	1

Longest Increasing Subsequence

For i=6:

iterator	j						i	
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	4	2	1

Longest Increasing Subsequence

For i=6:

iterator		j					i	
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	4	2	1

Longest Increasing Subsequence

For i=6:

iterator		**j**					**i**	
arr[]	**10**	**22**	**9**	**33**	**21**	**50**	**41**	**60**
LIS	**1**	**2**	**1**	**3**	**2**	**4**	**3**	**1**

Longest Increasing Subsequence

For i=6:

iterator			j				i	
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	4	3	1

Longest Increasing Subsequence

For i=6:

iterator				j			i	
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	4	3	1

Longest Increasing Subsequence

For i=6:

iterator				j			i	
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	4	4	1

Longest Increasing Subsequence

For i=6:

iterator					j		i	
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	4	4	1

Longest Increasing Subsequence

For i=6:

iterator						j	i	
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	4	4	1

Longest Increasing Subsequence

Final values:

iterator							j	i
arr[]	10	22	9	33	21	50	41	60
LIS	1	2	1	3	2	4	4	5

Recursive Implementation

```c
int _lis( int arr[], int n, int *max_ref)
{
    /* Base case */
    if (n == 1)
        return 1;

    // 'max_ending_here' is length of LIS ending with arr[n-1]
    int res, max_ending_here = 1;

    /* Recursively get all LIS ending with arr[0], arr[1] ...
       arr[n-2]. If   arr[i-1] is smaller than arr[n-1], and
       max ending with arr[n-1] needs to be updated, then
       update it */
    for (int i = 1; i < n; i++)
    {
        res = _lis(arr, i, max_ref);
        if (arr[i-1] < arr[n-1] && res + 1 > max_ending_here)
            max_ending_here = res + 1;
    }

    // Compare max_ending_here with the overall max. And
    // update the overall max if needed
    if (*max_ref < max_ending_here)
       *max_ref = max_ending_here;

    // Return length of LIS ending with arr[n-1]
    return max_ending_here;
}
```

```c
int lis(int arr[], int n)
{
    // The max variable holds the result
    int max = 1;

    // The function _lis() stores its result in max
    _lis( arr, n, &max );

    // returns max
    return max;
}
```

Recursive Implementation

```
int _lis( int arr[], int n, int *max_ref)
{
    /* Base case */
    if (n == 1)
        return 1;

    // 'max_ending_here' is length of LIS ending with arr[n-1]
    int res, max_ending_here = 1;

    /* Recursively get all LIS ending with arr[0], arr[1] ...
       arr[n-2]. If   arr[i-1] is smaller than arr[n-1], and
       max ending with arr[n-1] needs to be updated, then
       update it */
    for (int i = 1; i < n; i++)
    {
        res = _lis(arr, i, max_ref);
        if (arr[i-1] < arr[n-1] && res + 1 > max_ending_here)
            max_ending_here = res + 1;
    }

    // Compare max_ending_here with the overall max. And
    // update the overall max if needed
    if (*max_ref < max_ending_here)
       *max_ref = max_ending_here;

    // Return length of LIS ending with arr[n-1]
    return max_ending_here;
}
```

```
int lis(int arr[], int n)
{
    // The max variable holds the result
    int max = 1;

    // The function _lis() stores its result in max
    _lis( arr, n, &max );

    // returns max
    return max;
}
```

→ **Time Complextity: $O(2^n)$**

Dynamic Programming

```cpp
#include<bits/stdc++.h>
using namespace std;

/* lis() returns the length of the longest increasing
  subsequence in arr[] of size n */
int lis( int arr[], int n )
{
    int lis[n];

    lis[0] = 1;

    /* Compute optimized LIS values in bottom up manner */
    for (int i = 1; i < n; i++ )
    {
        lis[i] = 1;
        for (int j = 0; j < i; j++ )
            if ( arr[i] > arr[j] && lis[i] < lis[j] + 1)
                lis[i] = lis[j] + 1;
    }

    // Return maximum value in lis[]
    return *max_element(lis, lis+n);
}
```

Tabulation

Dynamic Programming

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        lis[i] = 1;
        for (int j = 0; j < i; j++ )
            if ( arr[i] > arr[j] && lis[i] < lis[j] + 1)
                lis[i] = lis[j] + 1;
    }

    // Return maximum value in lis[]
    return *max_element(lis, lis+n);
}
```

Tabulation

→ Time Complextity: $O(n^2)$

Longest Common Subsequence

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Given two sequences, find the length of longest subsequence present in both of them.

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Sequences = “abcdefg”, “abxdfg”

Common Subsequences = “a”, “b”, “d”, “f”, “g”, “ab”, “df”, “dfg”, “abd”, “abdfg”

Longest Common Subsequence

Given two sequences, find the length of longest subsequence present in both of them.

A subsequence is a sequence that appears in the same relative order, but not necessarily contiguous.

Sequences = “abcdefg”, “abxdfg”

Common Subsequences = “a”, “b”, “d”, “f”, “g”, “ab”, “df”, “dfg”, “abd”, “abdfg”

Longest Common Subsequences (LCS) = “abdfg”

Optimal Substructure Property

Let the input sequences be X[0..m-1] and Y[0..n-1]. And let L(X[0..m-1], Y[0..n-1]) be the length of LCS of the two sequences X and Y.

Optimal Substructure Property

Let the input sequences be X[0..m-1] and Y[0..n-1]. And let L(X[0..m-1], Y[0..n-1]) be the length of LCS of the two sequences X and Y.

If last characters of both sequences match (or X[m-1]==Y[n-1]) then L(X[0..m-1], Y[0..n-1]) = 1 + L(X[0..m-2], Y[0..n-2])

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If last characters of both sequences match (or X[m-1]==Y[n-1]) then L(X[0..m-1], Y[0..n-1]) = 1 + L(X[0..m-2], Y[0..n-2])

If last characters of both sequences don't match (or X[m-1]!=Y[n-1]) then L(X[0..m-1], Y[0..n-1]) = MAX (L(X[0..m-2], Y[0..n-1]), L(X[0..m-1], Y[0..n-2]))

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If last characters of both sequences don't match (or X[m-1]!=Y[n-1]) then L(X[0..m-1], Y[0..n-1]) = MAX (L(X[0..m-2], Y[0..n-1]), L(X[0..m-1], Y[0..n-2]))

→The LIS problem satisfies the optimal substructure property.

Overlapping Subproblems Property

```
                             lcs("AXYT", "AYZX")
                           /
          lcs("AXY", "AYZX")              lcs("AXYT", "AYZ")
          /                                /
lcs("AX", "AYZX") lcs("AXY", "AYZ")   lcs("AXY", "AYZ") lcs("AXYT", "AY")
```

Longest Common Subsequence

Example: Consider the input strings L_1 with length m and L_2 with length n such that L_1="AGGTAB" and L_2="GXTXAYB"

Longest Common Subsequence

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1. **There can be two cases. The last characters match or the last characters do not match.**

Longest Common Subsequence

Example: Consider the input strings L_1 with length m and L_2 with length n such that L_1=“AGGTAB” and L_2=“GXTXAYB”

1. **There can be two cases. The last characters match or the last characters do not match.**
2. **If the last characters match: Increment the length of LCS by 1 and process L_1[m-1] and L_2[n-1].**

Longest Common Subsequence

Example: Consider the input strings L_1 with length m and L_2 with length n such that L_1="AGGTAB" and L_2="GXTXAYB"

1. **There can be two cases. The last characters match or the last characters do not match.**
2. **If the last characters match: Increment the length of LCS by 1 and process $L_1[m-1]$ and $L_2[n-1]$.**
3. **If the last characters do not match: Find max($L_1[m-1]$ $L_2[n]$, $L_1[m]$ $L_2[n-1]$).**

Dynamic Programming

Approach:

- **If the last characters match:**

  LCS[i][j] = LCS[i-1][j-1] + 1

- **If the last characters do not match:**

  LCS[i][j] = max(LCS[i-1][j], LCS[i][j-1])

Dynamic Programming

LCS	ø	A	G	G	T	A	B
ø	0	0	0	0	0	0	0
G	0						
X	0						
T	0						
X	0						
A	0						
Y	0						
B	0						

Dynamic Programming

LCS	ø	A	G	G	T	A	B
ø	0	0	0	0	0	0	0
G	0	0					
X	0						
T	0						
X	0						
A	0						
Y	0						
B	0						

Dynamic Programming

LCS	ø	A	G	G	T	A	B
ø	0	0	0	0	0	0	0
G	0	0	1				
X	0						
T	0						
X	0						
A	0						
Y	0						
B	0						

Dynamic Programming

LCS	ø	A	G	G	T	A	B
ø	0	0	0	0	0	0	0
G	0	0	1	1			
X	0						
T	0						
X	0						
A	0						
Y	0						
B	0						

Dynamic Programming

LCS	ø	A	G	G	T	A	B
ø	0	0	0	0	0	0	0
G	0	0	1	1	1		
X	0						
T	0						
X	0						
A	0						
Y	0						
B	0						

Dynamic Programming

LCS	ø	A	G	G	T	A	B
ø	0	0	0	0	0	0	0
G	0	0	1	1	1	1	
X	0						
T	0						
X	0						
A	0						
Y	0						
B	0						

Dynamic Programming

LCS	ø	A	G	G	T	A	B
ø	0	0	0	0	0	0	0
G	0	0	1	1	1	1	1
X	0						
T	0						
X	0						
A	0						
Y	0						
B	0						

Dynamic Programming

LCS	ø	A	G	G	T	A	B
ø	0	0	0	0	0	0	0
G	0	0	1	1	1	1	1
X	0	0					
T	0						
X	0						
A	0						
Y	0						
B	0						

Dynamic Programming

LCS	ø	A	G	G	T	A	B
ø	0	0	0	0	0	0	0
G	0	0	1	1	1	1	1
X	0	0	1				
T	0						
X	0						
A	0						
Y	0						
B	0						

Dynamic Programming

LCS	ø	A	G	G	T	A	B
ø	0	0	0	0	0	0	0
G	0	0	1	1	1	1	1
X	0	0	1	1	1	1	1
T	0						
X	0						
A	0						
Y	0						
B	0						

Dynamic Programming

LCS	ø	A	G	G	T	A	B
ø	0	0	0	0	0	0	0
G	0	0	1	1	1	1	1
X	0	0	1	1	1	1	1
T	0	0					
X	0						
A	0						
Y	0						
B	0						

Dynamic Programming

LCS	ø	A	G	G	T	A	B
ø	0	0	0	0	0	0	0
G	0	0	1	1	1	1	1
X	0	0	1	1	1	1	1
T	0	0	1				
X	0						
A	0						
Y	0						
B	0						

Dynamic Programming

LCS	ø	A	G	G	T	A	B
ø	0	0	0	0	0	0	0
G	0	0	1	1	1	1	1
X	0	0	1	1	1	1	1
T	0	0	1	1			
X	0						
A	0						
Y	0						
B	0						

Dynamic Programming

LCS	ø	A	G	G	T	A	B
ø	0	0	0	0	0	0	0
G	0	0	1	1	1	1	1
X	0	0	1	1	1	1	1
T	0	0	1	1	2		
X	0						
A	0						
Y	0						
B	0						

Dynamic Programming

LCS	ø	A	G	G	T	A	B
ø	0	0	0	0	0	0	0
G	0	0	1	1	1	1	1
X	0	0	1	1	1	1	1
T	0	0	1	1	2	2	2
X	0						
A	0						
Y	0						
B	0						

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LCS	ø	A	G	G	T	A	B
ø	0	0	0	0	0	0	0
G	0	0	1	1	1	1	1
X	0	0	1	1	1	1	1
T	0	0	1	1	2	2	2
X	0	0	1	1	2	2	2
A	0						
Y	0						
B	0						

Dynamic Programming

LCS	ø	A	G	G	T	A	B
ø	0	0	0	0	0	0	0
G	0	0	1	1	1	1	1
X	0	0	1	1	1	1	1
T	0	0	1	1	2	2	2
X	0	0	1	1	2	2	2
A	0	1	1	1	2	3	3
Y	0						
B	0						

Dynamic Programming

LCS	ø	A	G	G	T	A	B
ø	0	0	0	0	0	0	0
G	0	0	1	1	1	1	1
X	0	0	1	1	1	1	1
T	0	0	1	1	2	2	2
X	0	0	1	1	2	2	2
A	0	1	1	1	2	3	3
Y	0	1	1	1	2	3	3
B	0						

Dynamic Programming

LCS	ø	A	G	G	T	A	B
ø	0	0	0	0	0	0	0
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Y	0	1	1	1	2	3	3
B	0	1	1	1	2	3	4

Dynamic Programming

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GTAB → Length of LCS = 4

Recursive Implementation

```c
/* Returns length of LCS for X[0..m-1], Y[0..n-1] */
int lcs( char *X, char *Y, int m, int n )
{
    if (m == 0 || n == 0)
        return 0;
    if (X[m-1] == Y[n-1])
        return 1 + lcs(X, Y, m-1, n-1);
    else
        return max(lcs(X, Y, m, n-1), lcs(X, Y, m-1, n));
}

/* Utility function to get max of 2 integers */
int max(int a, int b)
{
    return (a > b)? a : b;
}
```

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}

/* Utility function to get max of 2 integers */
int max(int a, int b)
{
    return (a > b)? a : b;
}
```

→ **Time Complextity: $O(2^n)$**

Dynamic Programming

```c
/* Returns length of LCS for X[0..m-1], Y[0..n-1] */
int lcs( char *X, char *Y, int m, int n )
{
    int L[m + 1][n + 1];
    int i, j;

    /* Following steps build L[m+1][n+1] in
       bottom up fashion. Note that L[i][j]
       contains length of LCS of X[0..i-1]
       and Y[0..j-1] */
    for (i = 0; i <= m; i++)
    {
        for (j = 0; j <= n; j++)
        {
        if (i == 0 || j == 0)
            L[i][j] = 0;

        else if (X[i - 1] == Y[j - 1])
            L[i][j] = L[i - 1][j - 1] + 1;

        else
            L[i][j] = max(L[i - 1][j], L[i][j - 1]);
        }
    }

    /* L[m][n] contains length of LCS
    for X[0..n-1] and Y[0..m-1] */
    return L[m][n];
}
```

Tabulation

Dynamic Programming

```c
/* Returns length of LCS for X[0..m-1], Y[0..n-1] */
int lcs( char *X, char *Y, int m, int n )
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    int L[m + 1][n + 1];
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    for (i = 0; i <= m; i++)
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        {
        if (i == 0 || j == 0)
            L[i][j] = 0;

        else if (X[i - 1] == Y[j - 1])
            L[i][j] = L[i - 1][j - 1] + 1;

        else
            L[i][j] = max(L[i - 1][j], L[i][j - 1]);
        }
    }

    /* L[m][n] contains length of LCS
    for X[0..n-1] and Y[0..m-1] */
    return L[m][n];
}
```
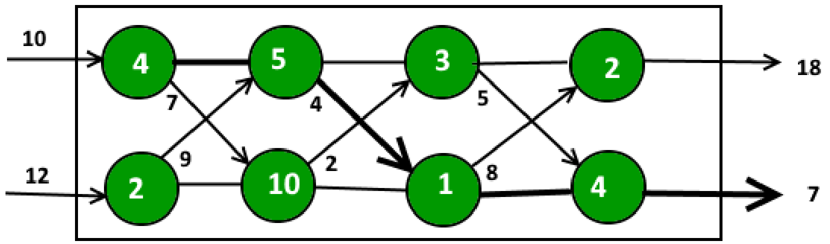

Tabulation

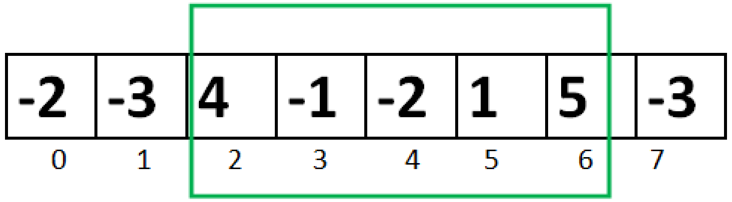
→ Time Complextity: O(mn)

Other Dynamic Programming Questions

- **Assembly Line Scheduling**



- **Largest Sum Contiguous Subarray**



4 + (-1) + (-2) + 1 + 5 = 7

Maximum Contiguous Array Sum is 7

Other Dynamic Programming Questions

- **0-1 Knapsack Problem**

0-1 Knapsack Problem

value[] = {60, 100, 120};
weight[] = {10, 20, 30};
W = 50;

Solution: 220

Weight = 10; Value = 60;
Weight = 20; Value = 100;
Weight = 30; Value = 120;
Weight = (20+10); Value = (100+60);
Weight = (30+10); Value = (120+60);
Weight = (30+20); Value = (120+100);
Weight = (30+20+10) > 50

- **Building Bridges**

```
8     1     4     3     5     2     6     7
<---- Cities on the other bank of river---->
--------------------------------------------
  <--------------- River--------------->
--------------------------------------------
1     2     3     4     5     6     7     8
<------- Cities on one bank of river------->
```

Reference

- Charles Leiserson and Piotr Indyk, “*Introduction to Algorithms*”, September 29, 2004

- https://www.geeksforgeeks.org