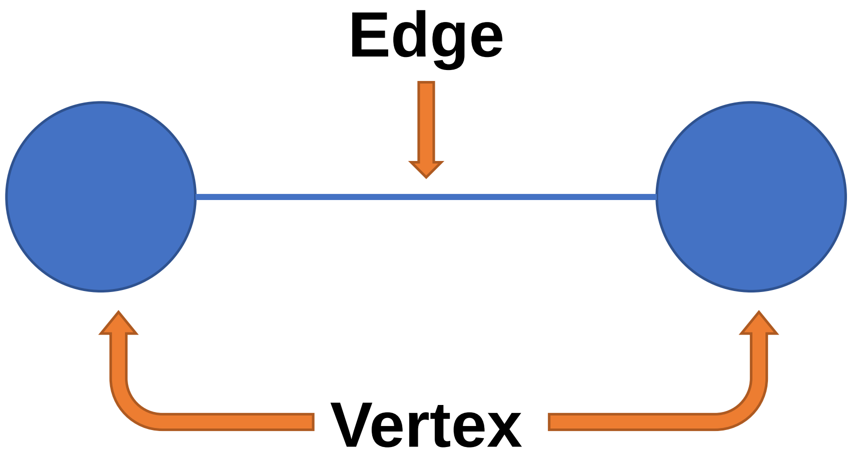
Graph Algorithm

SWE2016-44

Introduction

<u>Graph</u>: Represents pair-wise relationship between a set of objects

1. **Vertex (nodes)**
2. **Edges (arc)**



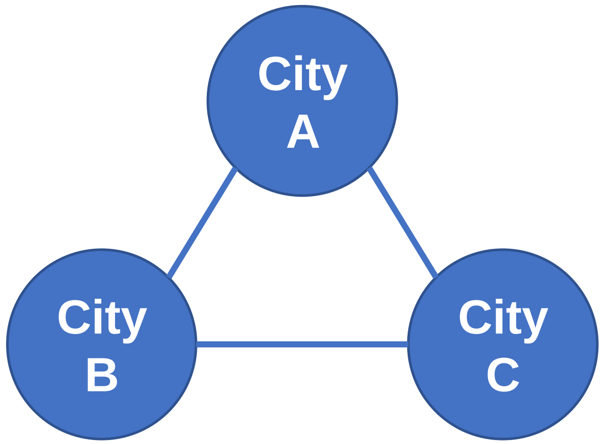
Introduction

<u>Directed Graph</u> (di-graph): have pair of ordered vertices (u,v)

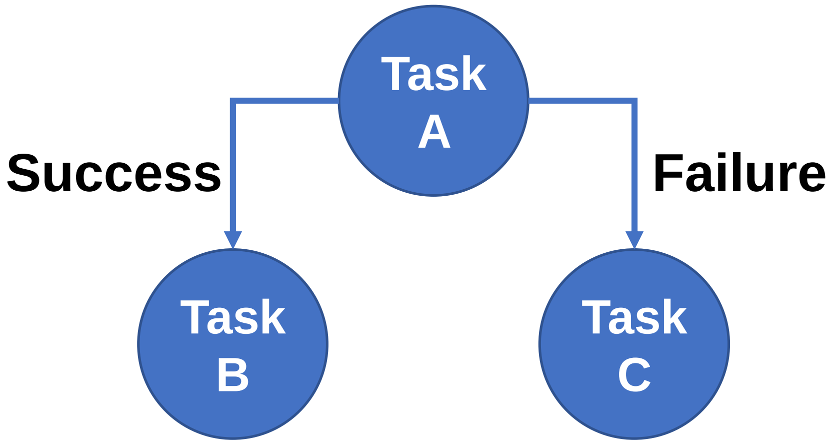
<u>Un-Directed Graph</u>: have pair of unordered vertices, (u,v) and (v,u) are same

Graph Applications

1. Social Networks – Facebook, LinkedIn, etc
2. City-Road Network



3. Precedence Constraints



Graph Representation

There are generally two ways to represent a graph data structure

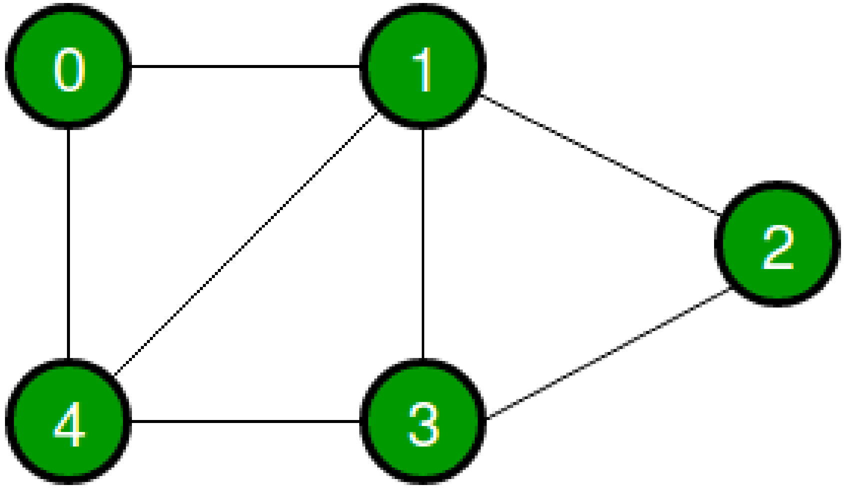
- **Adjacency Matrix**

- **Adjacency List**

Graph Representation

Adjacency Matrix: Represents graph with ‘V’ nodes into an VxV 0-1 matrix where A_{ij}=1 means that vertex ‘i’ and ‘j’ are connected.

Example



Graph

	0	1	2	3	4
0	0	1	0	0	1
1	1	0	1	1	1
2	0	1	0	1	0
3	0	1	1	0	1
4	1	1	0	1	0

Adjacency Matrix

Graph Representation

Adjacency Matrix

Pros:

- **Easy to implement.**
- **Removing an edge takes O(1) time.**
- **Queries like whether there is an edge from vertex ‘u’ to vertex ‘v’ are efficient and can be done O(1) ← Search**

Graph Representation

Adjacency Matrix

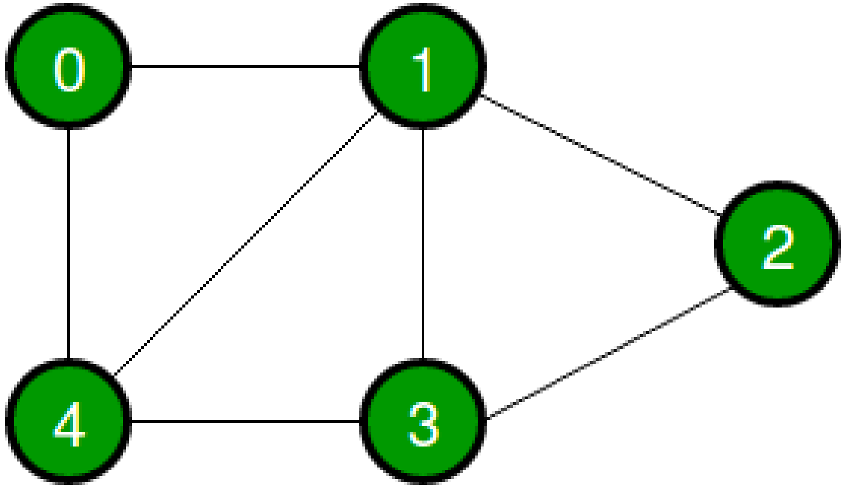
Cons:

- **Consumes more spaces $O(V^2)$.**
- **Adding a vertex is $O(V^2)$ ← if there is to add a new vertex, one has to increase the storage for a $|V|^2$ matrix to $(|V|+1)^2$**

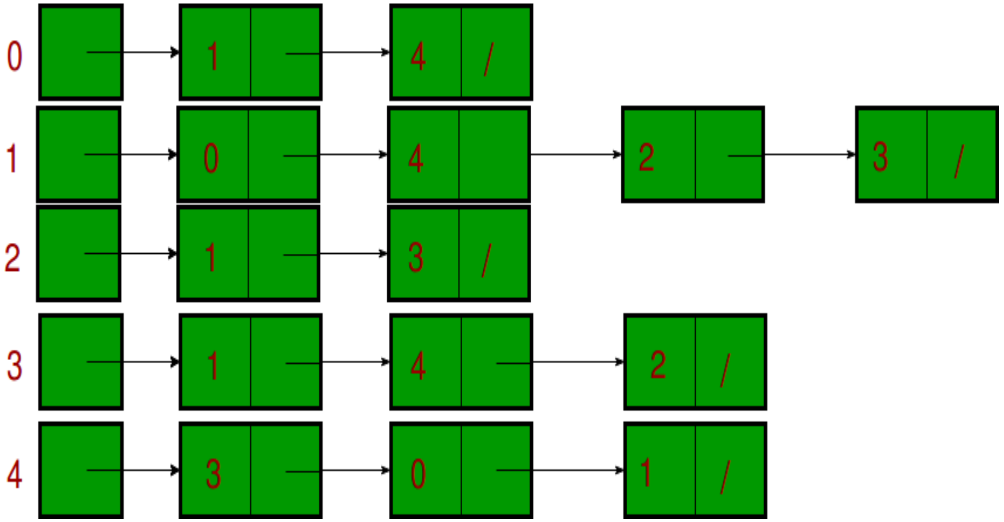
Graph Representation

Adjacency List: An array of linked lists is used. Size of the array is equal to number of vertices and each entry of array corresponds to a linked list of vertices adjacent to the index

Example



Graph



Adjacency List

Graph Representation

Adjacency List

Pros:

- **Saves space $O(|V|+|E|)$, worst case $O(V^2)$.**
- **Adding a vertex is easier.**

Graph Representation

Adjacency List

Cons:

- Queries like whether there is an edge from vertex u to vertex v are not efficient and can be done O(V) ← Search

Graph Implementation (C)

```c
struct AdjListNode
{
    int dest;
    struct AdjListNode* next;
};

// A structure to represent an adjacency list
struct AdjList
{
    struct AdjListNode *head;
};

// A structure to represent a graph. A graph
// is an array of adjacency lists.
// Size of array will be V (number of vertices
// in graph)
struct Graph
{
    int V;
    struct AdjList* array;
};

// A utility function to create a new adjacency list node
struct AdjListNode* newAdjListNode(int dest)
{
    struct AdjListNode* newNode =
     (struct AdjListNode*) malloc(sizeof(struct AdjListNode));
    newNode->dest = dest;
    newNode->next = NULL;
    return newNode;
}
```

```c
// A utility function that creates a graph of V vertices
struct Graph* createGraph(int V)
{
    struct Graph* graph =
        (struct Graph*) malloc(sizeof(struct Graph));
    graph->V = V;

    // Create an array of adjacency lists.  Size of
    // array will be V
    graph->array =
      (struct AdjList*) malloc(V * sizeof(struct AdjList));

    // Initialize each adjacency list as empty by
    // making head as NULL
    int i;
    for (i = 0; i < V; ++i)
        graph->array[i].head = NULL;

    return graph;
}

// Adds an edge to an undirected graph
void addEdge(struct Graph* graph, int src, int dest)
{
    // Add an edge from src to dest.  A new node is
    // added to the adjacency list of src.  The node
    // is added at the begining
    struct AdjListNode* newNode = newAdjListNode(dest);
    newNode->next = graph->array[src].head;
    graph->array[src].head = newNode;

    // Since graph is undirected, add an edge from
    // dest to src also
    newNode = newAdjListNode(src);
    newNode->next = graph->array[dest].head;
    graph->array[dest].head = newNode;
}
```

Graph Implementation (C++)

```
// A utility function to add an edge in an
// undirected graph.
void addEdge(vector<int> adj[], int u, int v)
{
    adj[u].push_back(v);
    adj[v].push_back(u);
}

// A utility function to print the adjacency list
// representation of graph
void printGraph(vector<int> adj[], int V)
{
    for (int v = 0; v < V; ++v)
    {
        cout << "\n Adjacency list of vertex "
             << v << "\n head ";
        for (auto x : adj[v])
           cout << "-> " << x;
        printf("\n");
    }
}

// Driver code
int main()
{
    int V = 5;
    vector<int> adj[V];
    addEdge(adj, 0, 1);
    addEdge(adj, 0, 4);
    addEdge(adj, 1, 2);
    addEdge(adj, 1, 3);
    addEdge(adj, 1, 4);
    addEdge(adj, 2, 3);
    addEdge(adj, 3, 4);
    printGraph(adj, V);
    return 0;
}
```

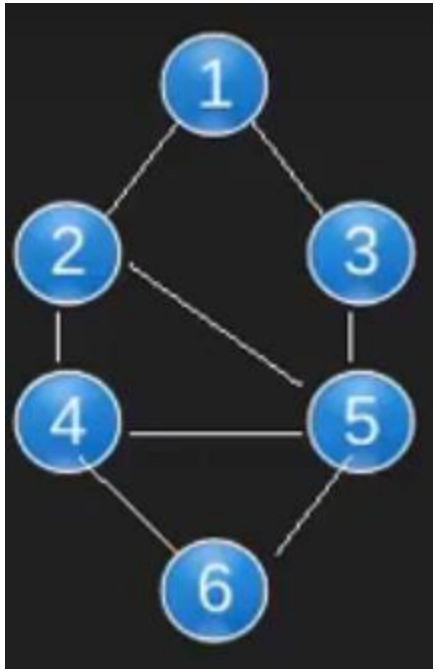
Breadth First Search (BFS)

Breadth First Search

Idea: Traverse nodes in layers

Problem: Since we have cycles, each node will be visited infinite times.

Solution: Use a Boolean visited array.



L1

L2

L3

L4

Implementation

```
void Graph::BFS(int s)
{
    // Mark all the vertices as not visited
    bool *visited = new bool[V];
    for(int i = 0; i < V; i++)
        visited[i] = false;

    // Create a queue for BFS
    list<int> queue;

    // Mark the current node as visited and enqueue it
    visited[s] = true;
    queue.push_back(s);

    // 'i' will be used to get all adjacent
    // vertices of a vertex
    list<int>::iterator i;

    while(!queue.empty())
    {
        // Dequeue a vertex from queue and print it
        s = queue.front();
        cout << s << " ";
        queue.pop_front();

        // Get all adjacent vertices of the dequeued
        // vertex s. If a adjacent has not been visited,
        // then mark it visited and enqueue it
        for (i = adj[s].begin(); i != adj[s].end(); ++i)
        {
            if (!visited[*i])
            {
                visited[*i] = true;
                queue.push_back(*i);
            }
        }
    }
}
```

```
void Graph::BFS(int s)
{
    // Mark all the vertices as not visited
    bool *visited = new bool[V];
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        // then mark it visited and enqueue it
        for (i = adj[s].begin(); i != adj[s].end(); ++i)
        {
            if (!visited[*i])
            {
                visited[*i] = true;
                queue.push_back(*i);
            }
        }
    }
}
```

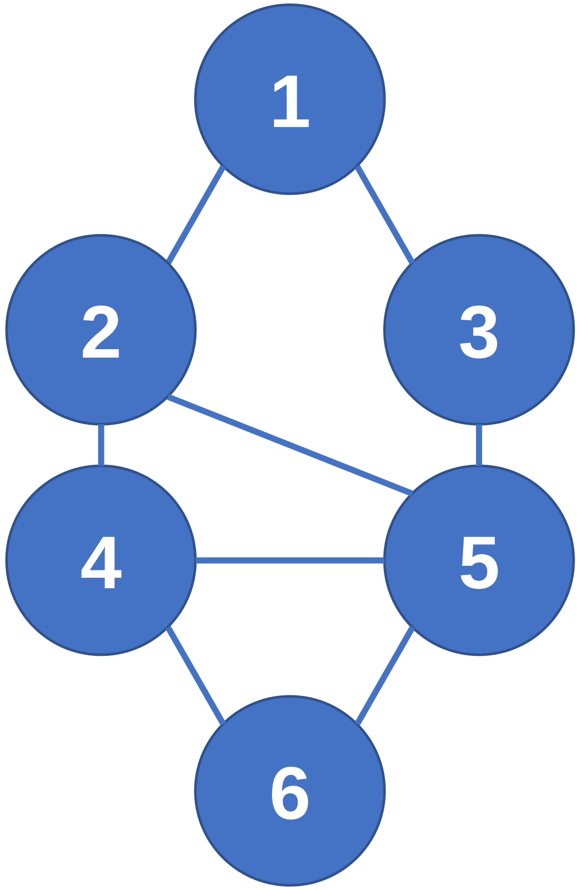

	1	2	3	4	5	6
Visited :	0	0	0	0	0	0

Queue :

Print :

```
void Graph::BFS(int s)
{
    // Mark all the vertices as not visited
    bool *visited = new bool[V];
    for(int i = 0; i < V; i++)
        visited[i] = false;

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        // then mark it visited and enqueue it
        for (i = adj[s].begin(); i != adj[s].end(); ++i)
        {
            if (!visited[*i])
            {
                visited[*i] = true;
                queue.push_back(*i);
            }
        }
    }
}
```

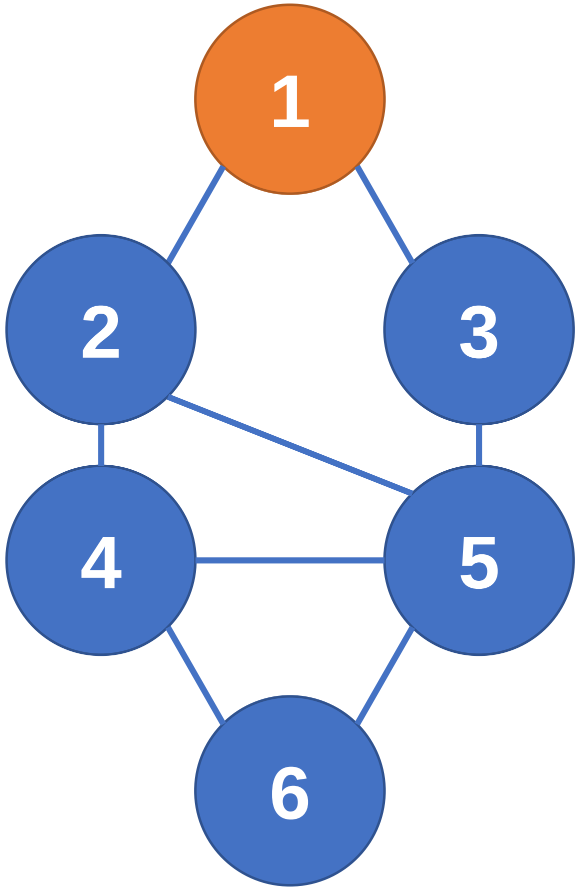

	1	2	3	4	5	6
Visited :	1	0	0	0	0	0

Queue : 1

Print : 1

```
void Graph::BFS(int s)
{
    // Mark all the vertices as not visited
    bool *visited = new bool[V];
    for(int i = 0; i < V; i++)
        visited[i] = false;

    // Create a queue for BFS
    list<int> queue;

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    queue.push_back(s);

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        queue.pop_front();

        // Get all adjacent vertices of the dequeued
        // vertex s. If a adjacent has not been visited,
        // then mark it visited and enqueue it
        for (i = adj[s].begin(); i != adj[s].end(); ++i)
        {
            if (!visited[*i])
            {
                visited[*i] = true;
                queue.push_back(*i);
            }
        }
    }
}
```

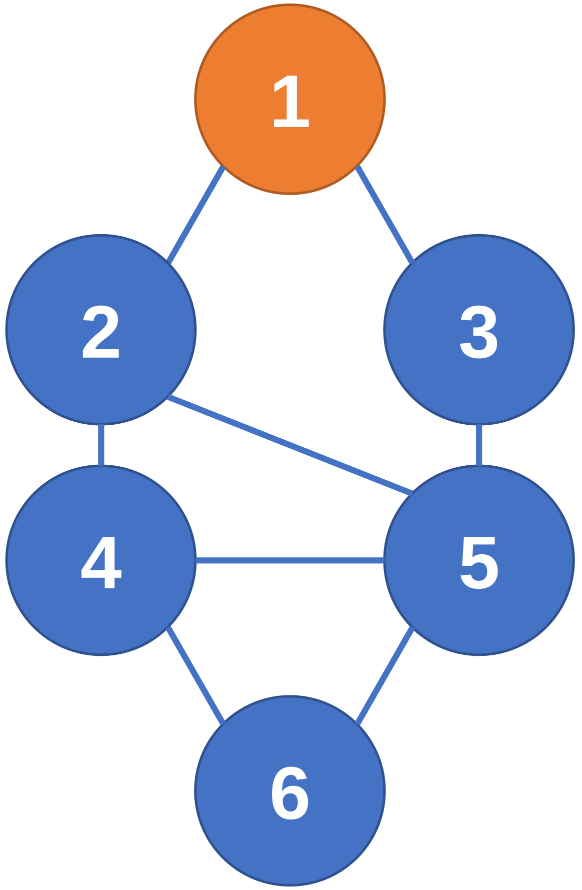

	1	2	3	4	5	6
Visited :	1	0	0	0	0	0

Queue :

Print : 1

```
void Graph::BFS(int s)
{
    // Mark all the vertices as not visited
    bool *visited = new bool[V];
    for(int i = 0; i < V; i++)
        visited[i] = false;

    // Create a queue for BFS
    list<int> queue;

    // Mark the current node as visited and enqueue it
    visited[s] = true;
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        queue.pop_front();

        // Get all adjacent vertices of the dequeued
        // vertex s. If a adjacent has not been visited,
        // then mark it visited and enqueue it
        for (i = adj[s].begin(); i != adj[s].end(); ++i)
        {
            if (!visited[*i])
            {
                visited[*i] = true;
                queue.push_back(*i);
            }
        }
    }
}
```

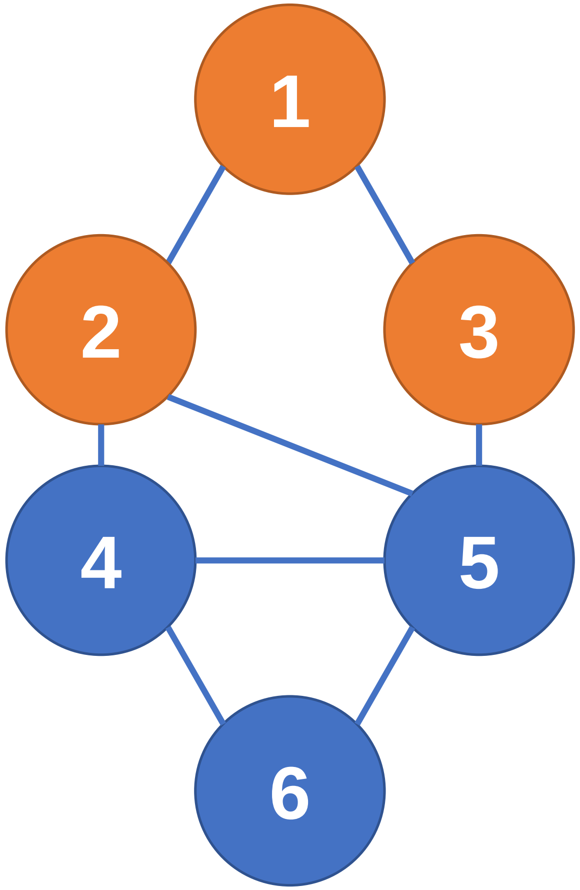

	1	2	3	4	5	6
Visited :	1	1	1	0	0	0

Queue : 2 3

Print : 1

```cpp
void Graph::BFS(int s)
{
    // Mark all the vertices as not visited
    bool *visited = new bool[V];
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        // vertex s. If a adjacent has not been visited,
        // then mark it visited and enqueue it
        for (i = adj[s].begin(); i != adj[s].end(); ++i)
        {
            if (!visited[*i])
            {
                visited[*i] = true;
                queue.push_back(*i);
            }
        }
    }
}
```

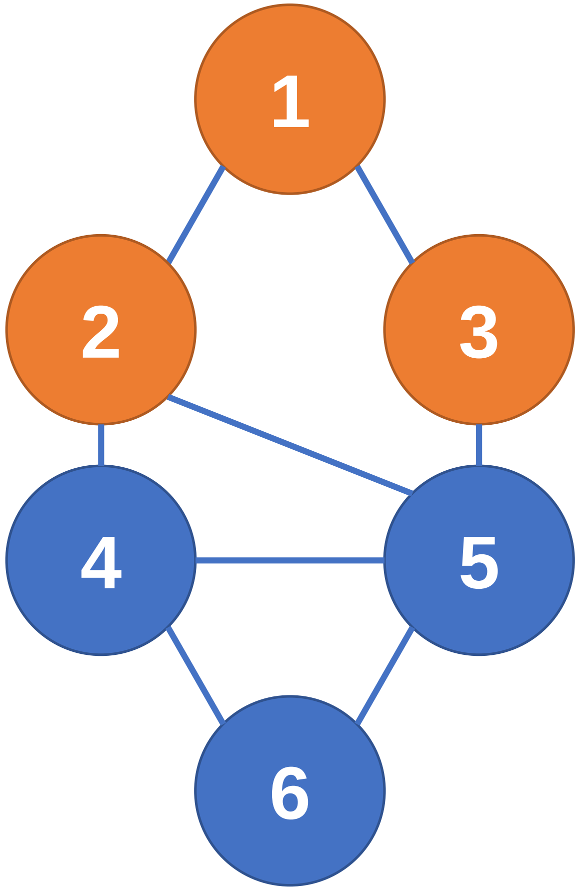

	1	2	3	4	5	6
Visited :	1	1	1	0	0	0

Queue : 2 3

Print : 1 2

```cpp
void Graph::BFS(int s)
{
    // Mark all the vertices as not visited
    bool *visited = new bool[V];
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        }
    }
}
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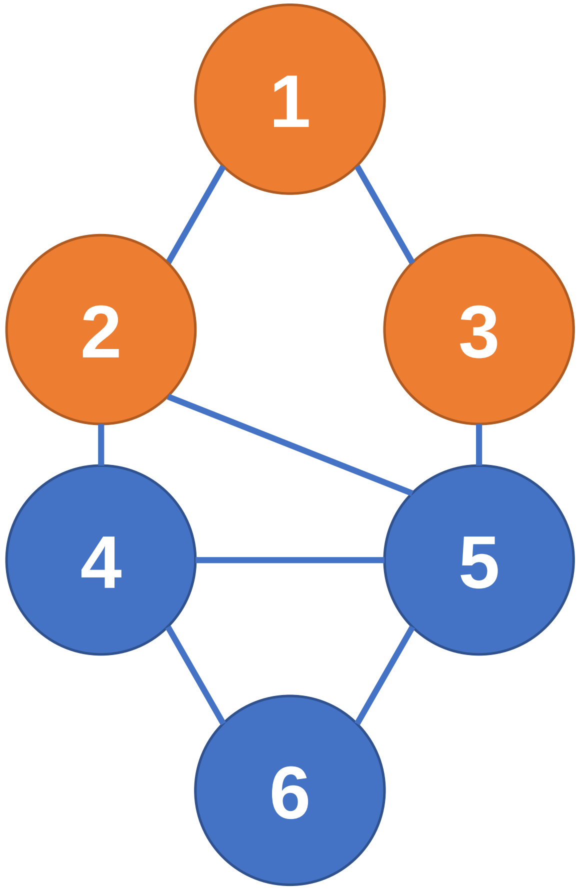

	1	2	3	4	5	6
Visited :	1	1	1	0	0	0

Queue : 3

Print : 1 2

```
void Graph::BFS(int s)
{
    // Mark all the vertices as not visited
    bool *visited = new bool[V];
    for(int i = 0; i < V; i++)
        visited[i] = false;

    // Create a queue for BFS
    list<int> queue;

    // Mark the current node as visited and enqueue it
    visited[s] = true;
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    // 'i' will be used to get all adjacent
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                queue.push_back(*i);
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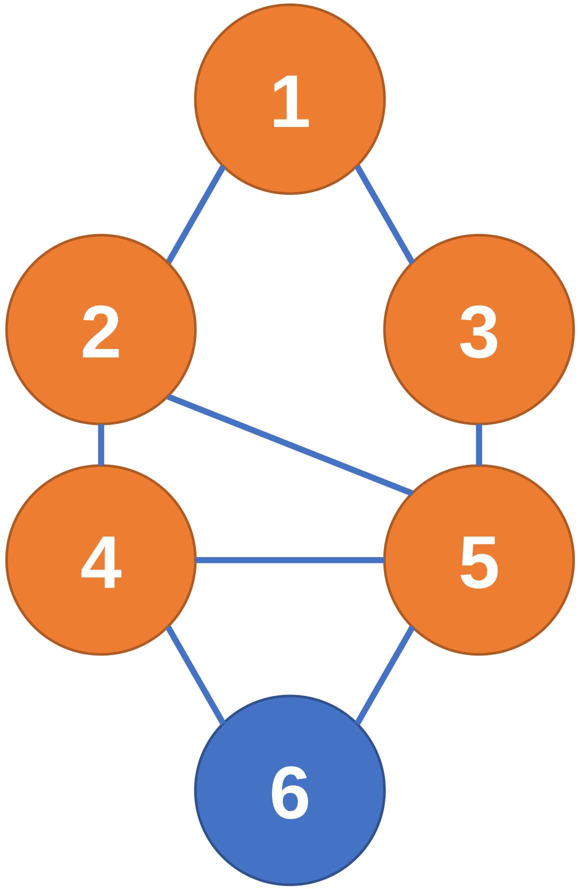
        }
    }
}
```



	1	2	3	4	5	6
Visited :	1	1	1	1	1	0

Queue : 3 4 5

Print : 1 2

```
void Graph::BFS(int s)
{
    // Mark all the vertices as not visited
    bool *visited = new bool[V];
    for(int i = 0; i < V; i++)
        visited[i] = false;

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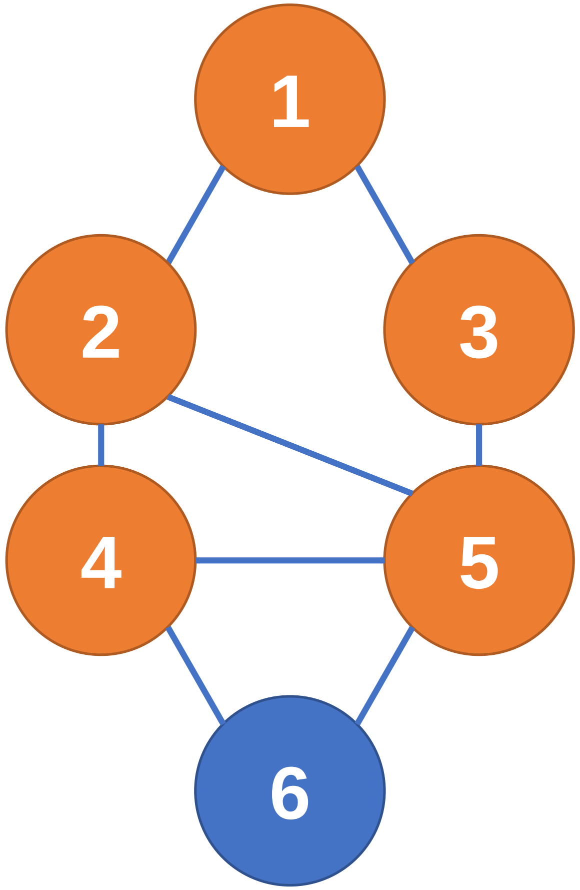

	1	2	3	4	5	6
Visited :	1	1	1	1	1	0

Queue : 4 5

Print : 1 2 3

```
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{
    // Mark all the vertices as not visited
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        }
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}
```

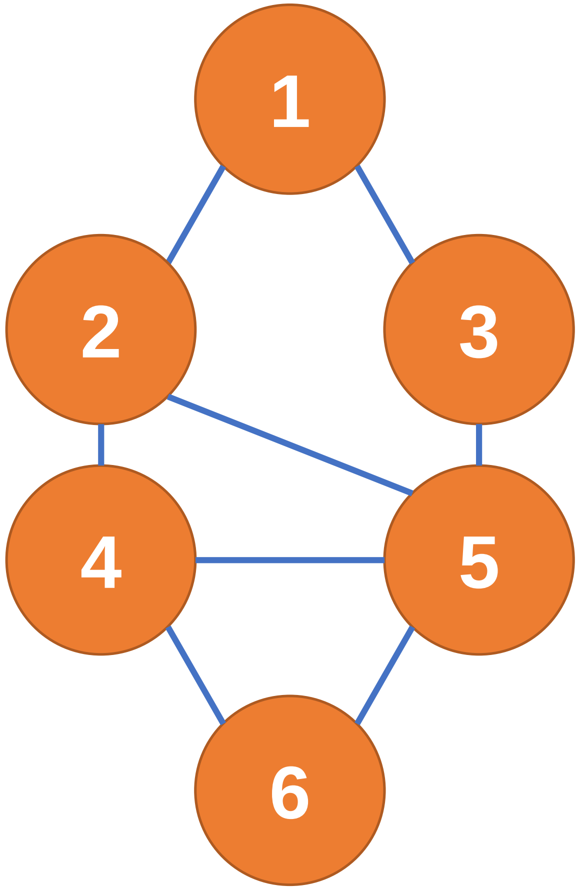

	1	2	3	4	5	6
Visited :	1	1	1	1	1	1

Queue : 5 6

Print : 1 2 3 4

```
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{
    // Mark all the vertices as not visited
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    }
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```

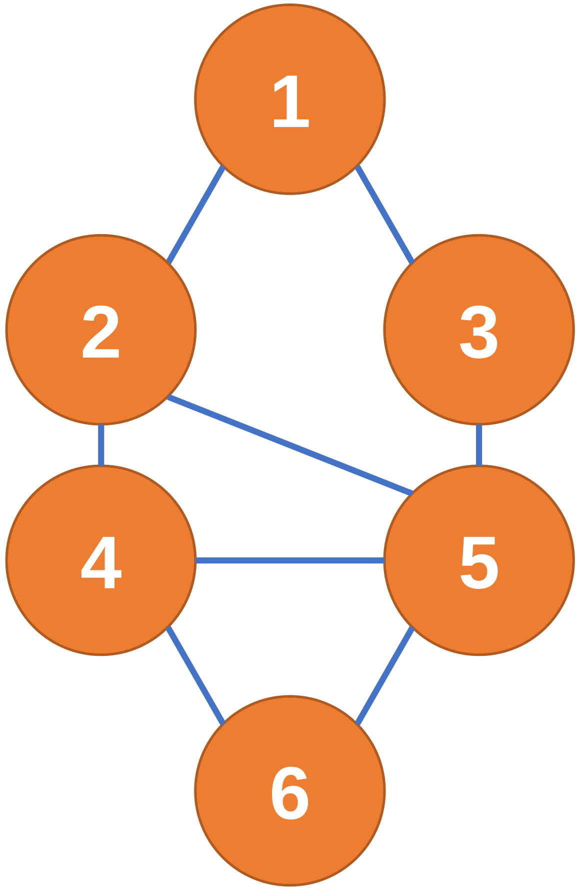

	1	2	3	4	5	6
Visited :	1	1	1	1	1	1

Queue : 6

Print : 1 2 3 4 5

```cpp
void Graph::BFS(int s)
{
    // Mark all the vertices as not visited
    bool *visited = new bool[V];
    for(int i = 0; i < V; i++)
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```
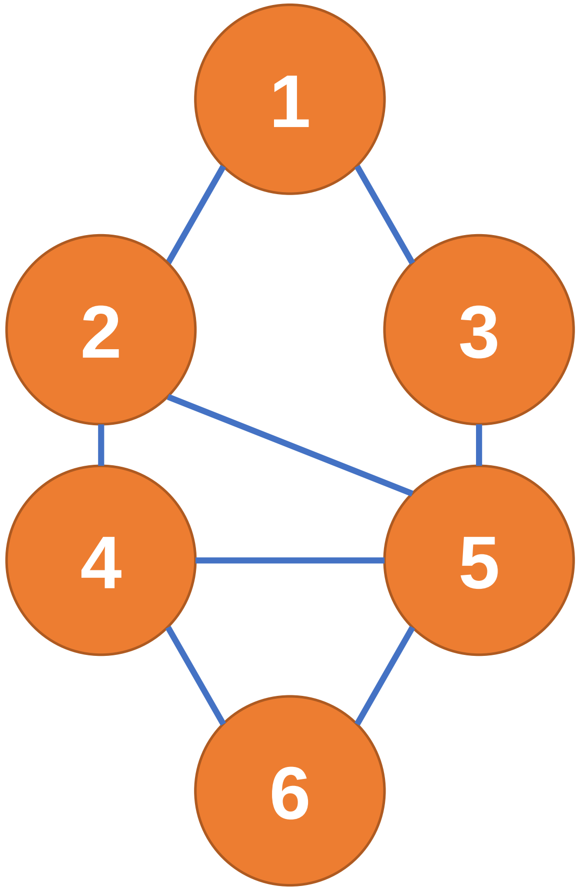



	1	2	3	4	5	6
Visited :	1	1	1	1	1	1

Queue :

Print : 1 2 3 4 5 6

Complexity

Time Complexity: O(V+E)

V: Vertices
E: Edges

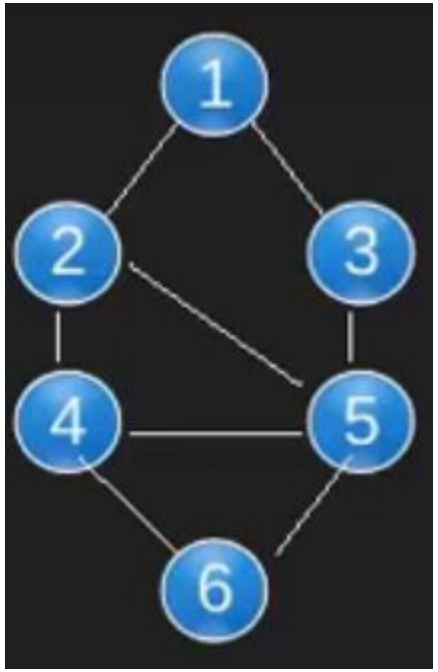
Depth First Search (DFS)

Depth First Search

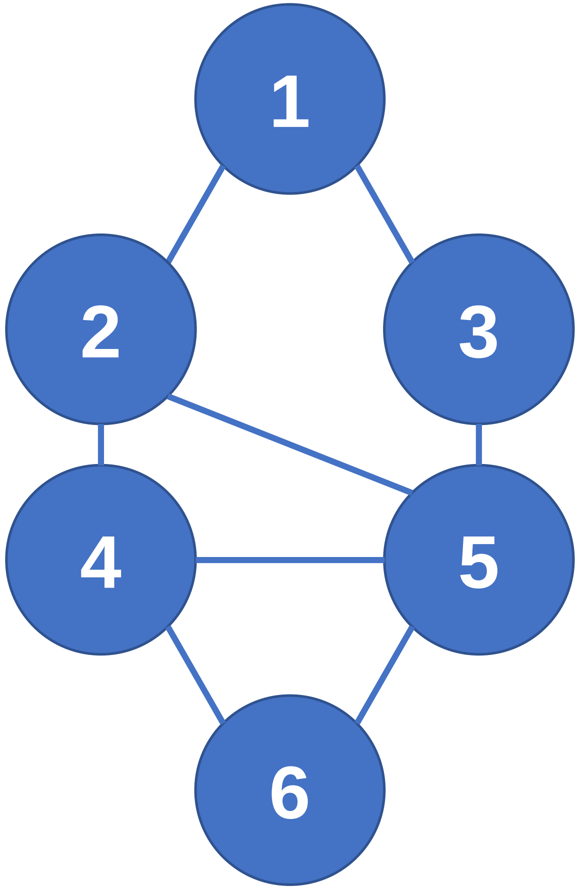
Idea: To go forward (in depth) while there is any such possibility, if not then, backtrack

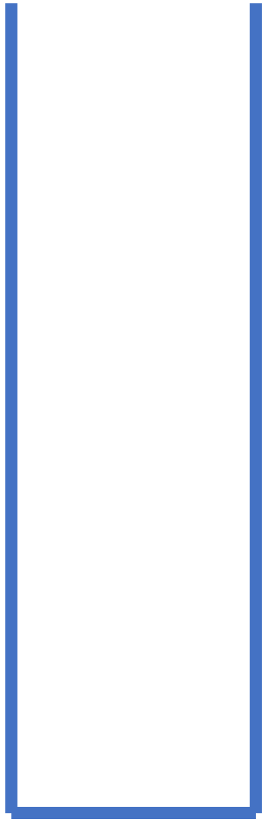
Problem: Since we have cycles, each node may be visited infinite times.

Solution: Use a Boolean visited array.



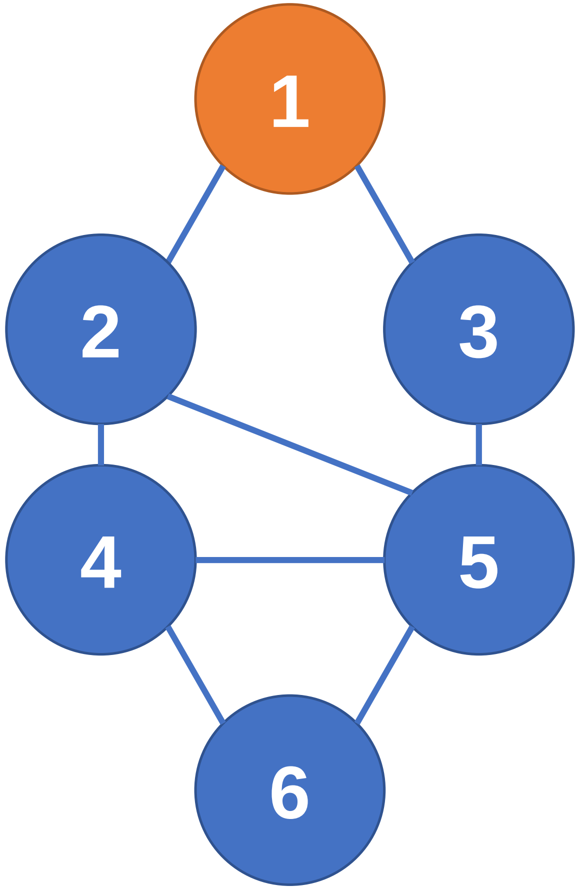
Depth First Search

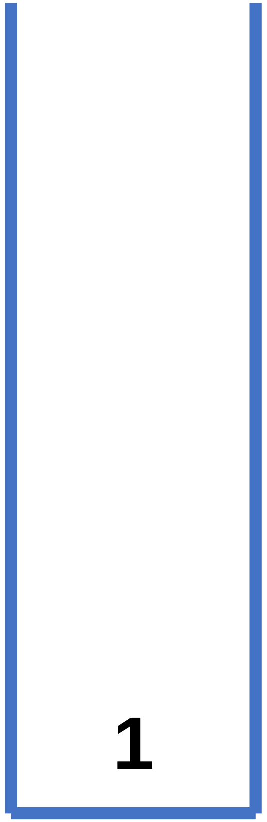




Output:

Depth First Search

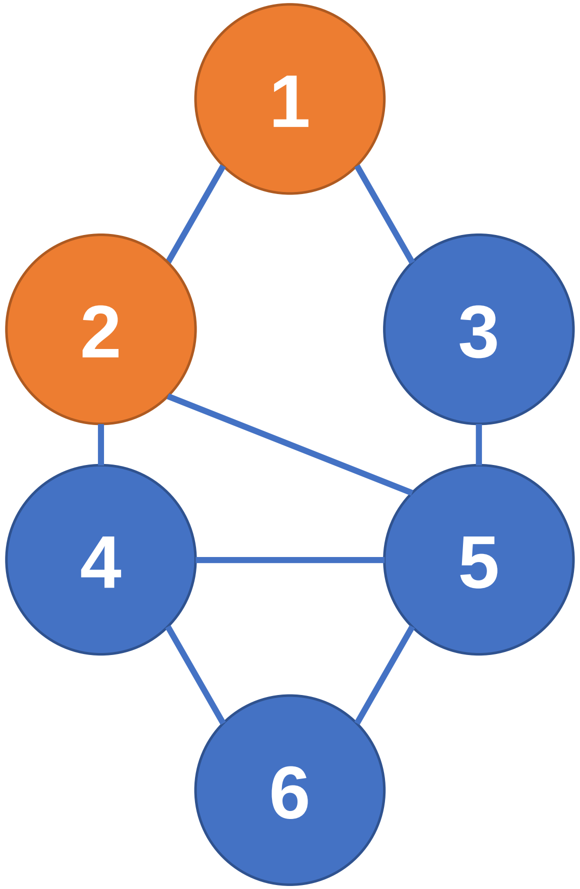


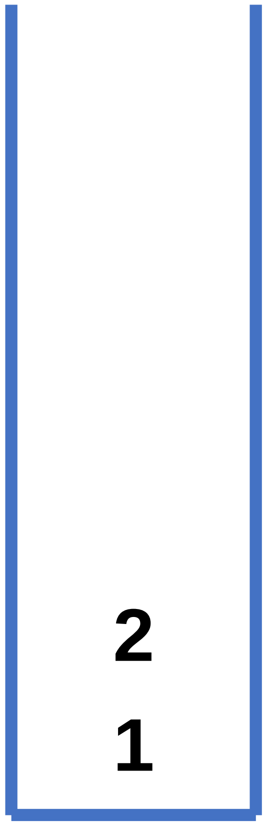


Stack

Output: 1

Depth First Search

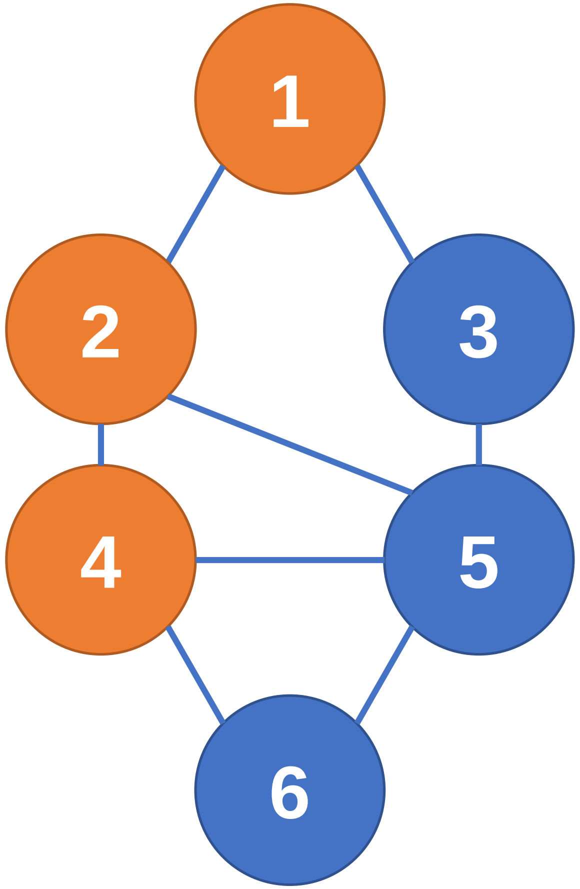


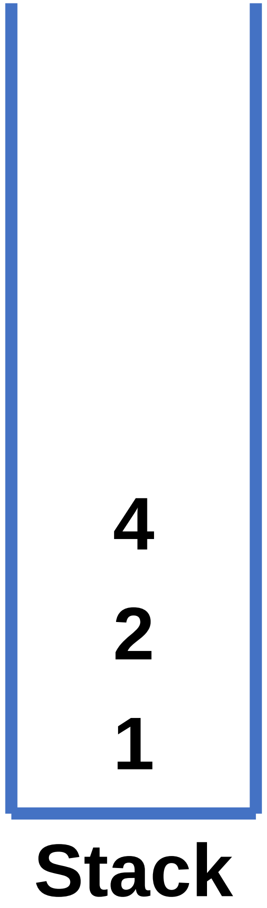


Stack

Output: 1 2

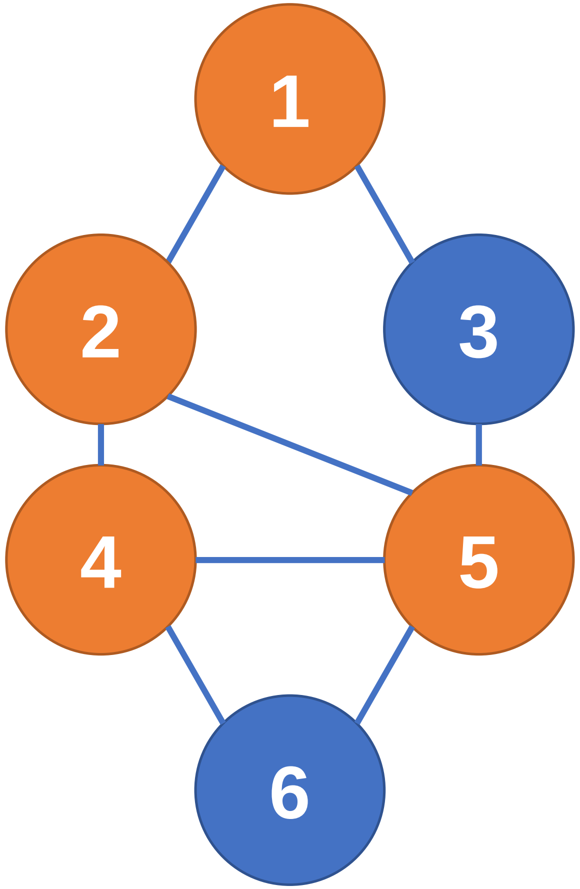
Depth First Search

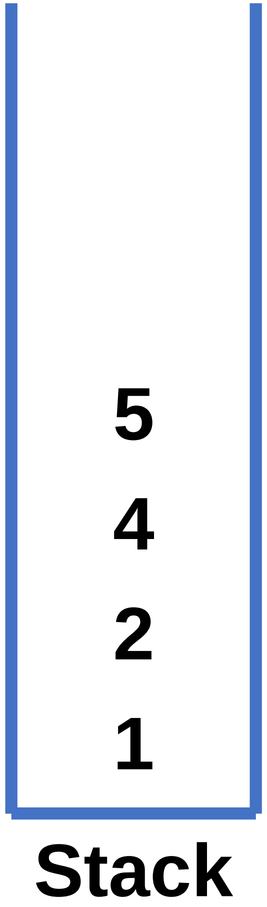




Output: 1 2 4

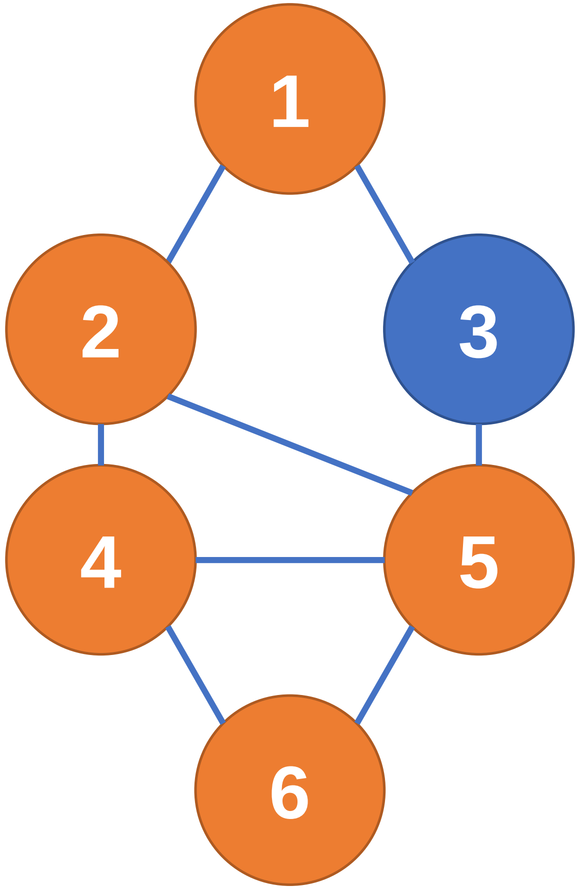
Depth First Search

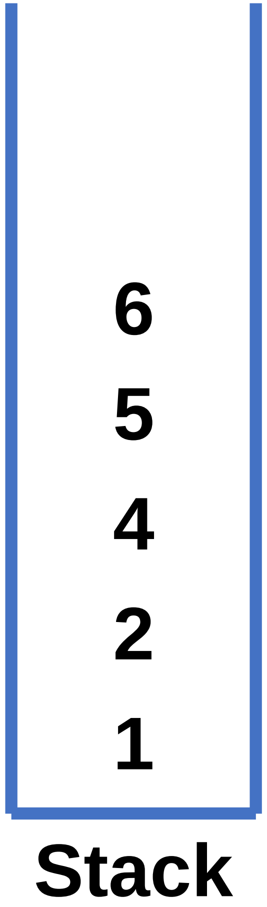




Output: 1 2 4 5

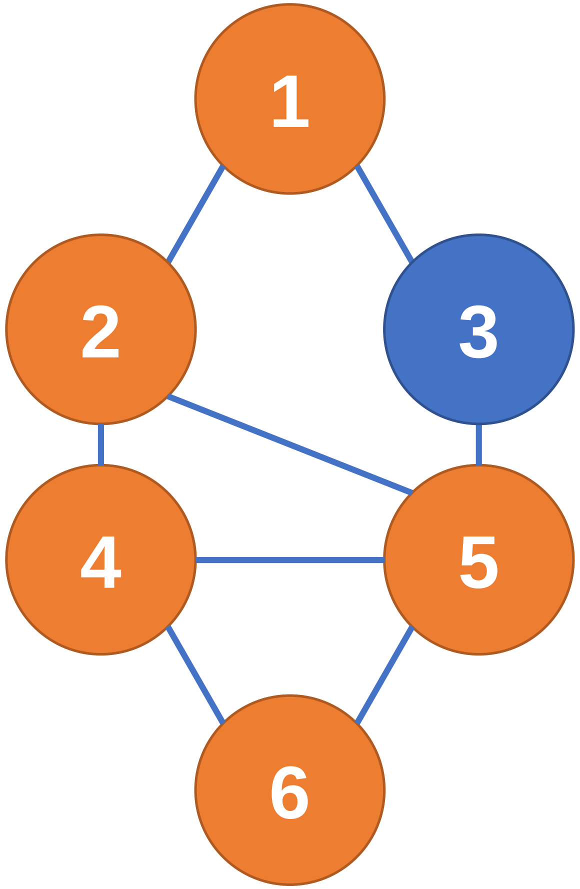
Depth First Search





Output: 1 2 4 5 6

Depth First Search

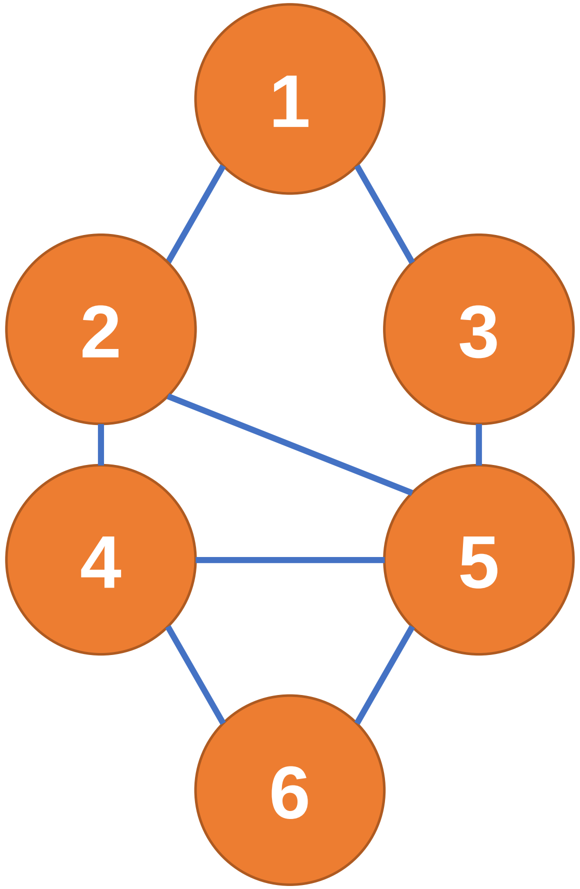


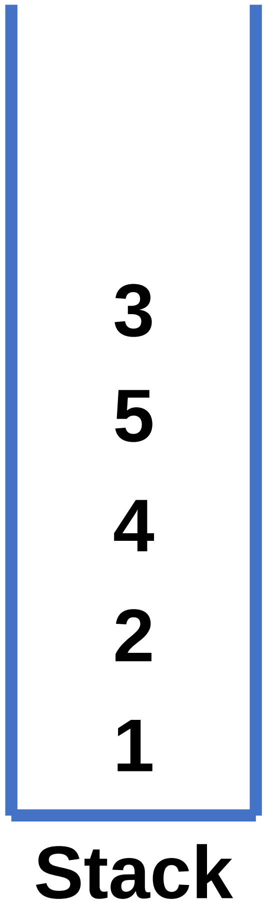
5
4
2
1

Stack

Output: 1 2 4 5 6

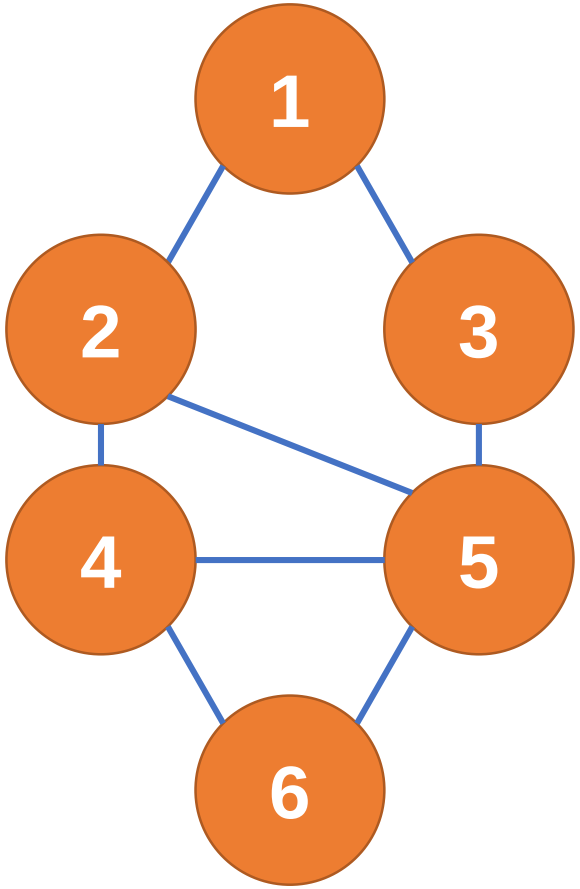
Depth First Search

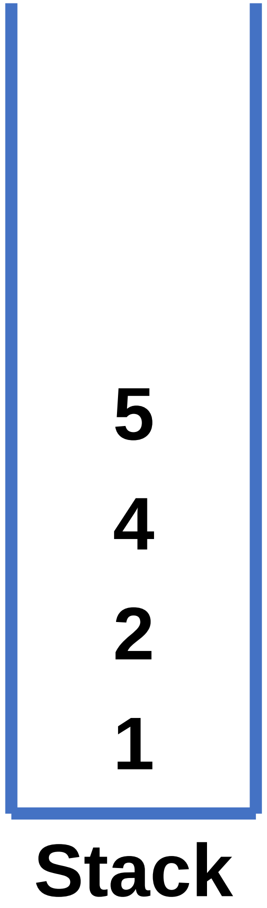




Output: 1 2 4 5 6 3

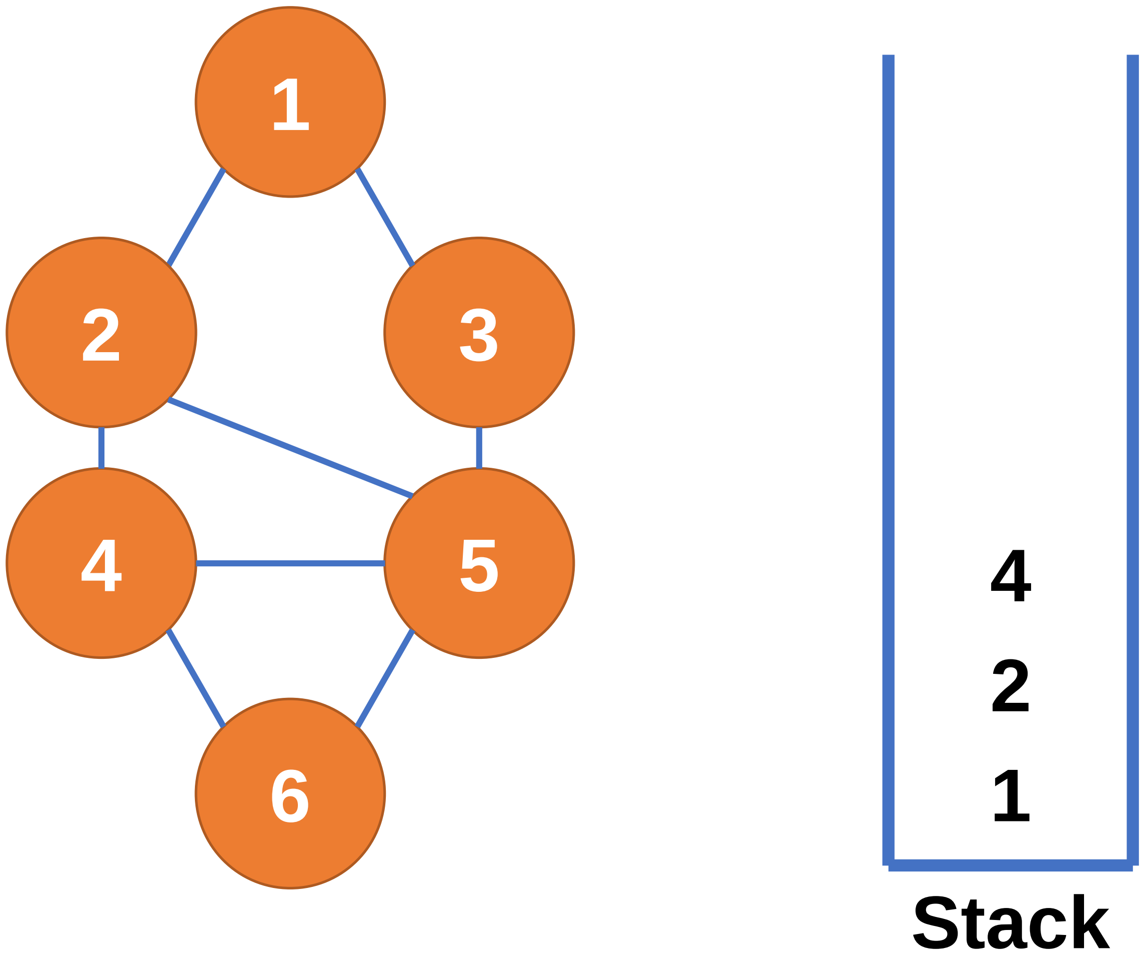
Depth First Search





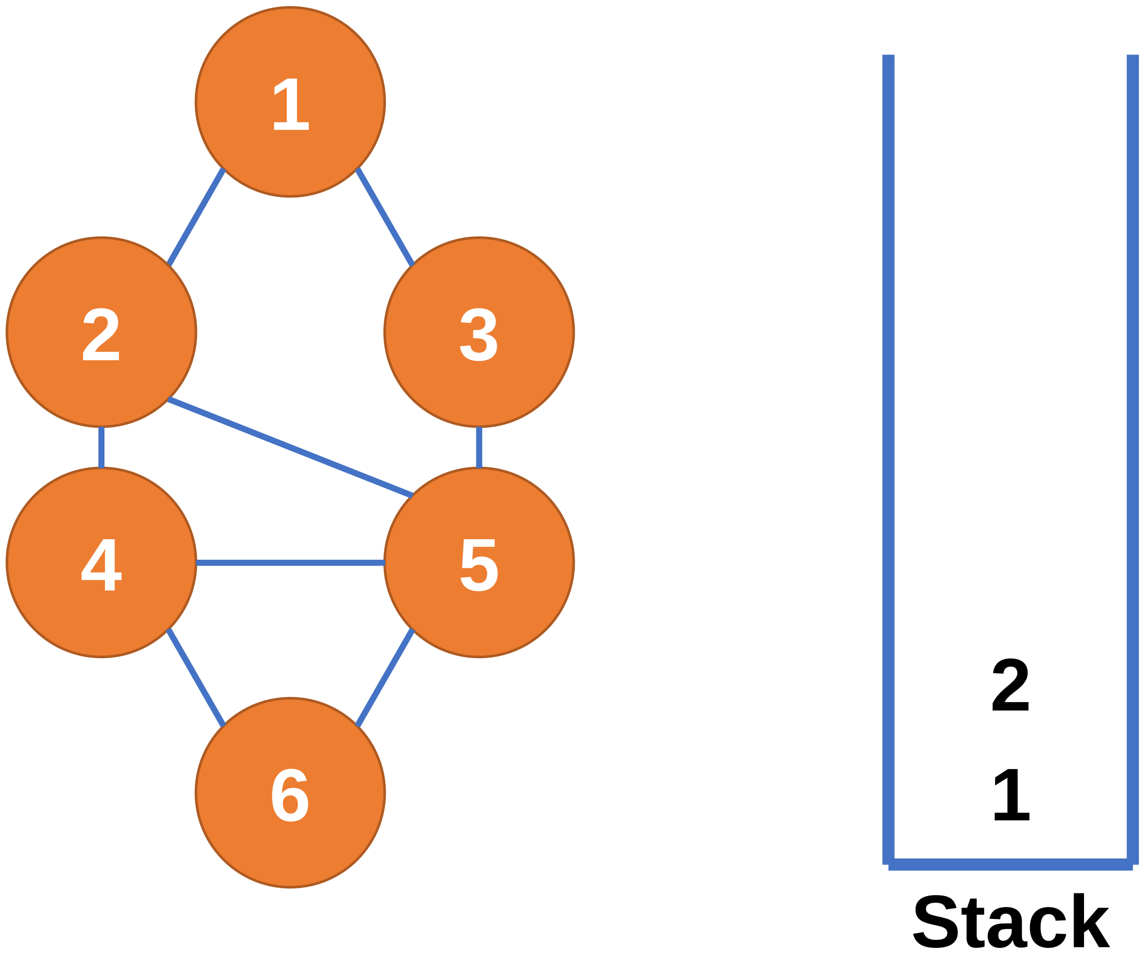
Output: 1 2 4 5 6 3

Depth First Search



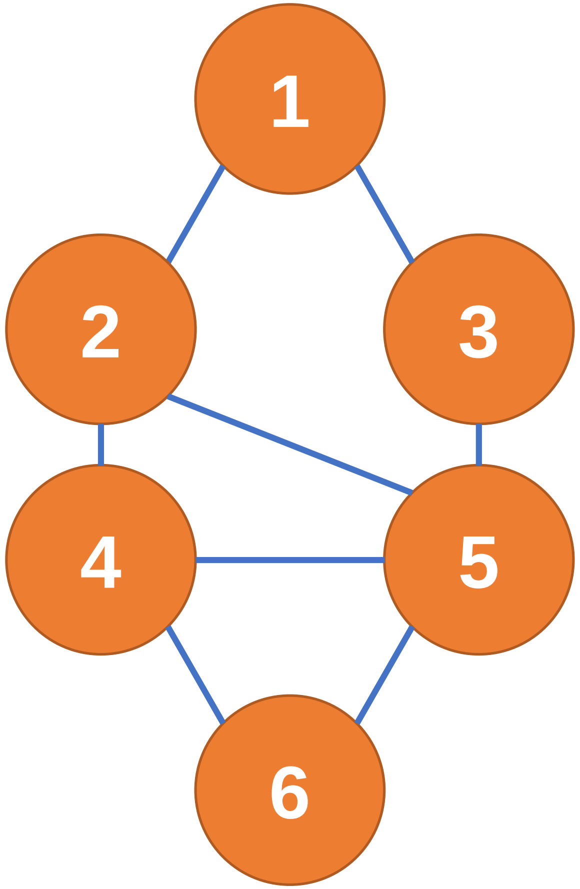
Output: 1 2 4 5 6 3

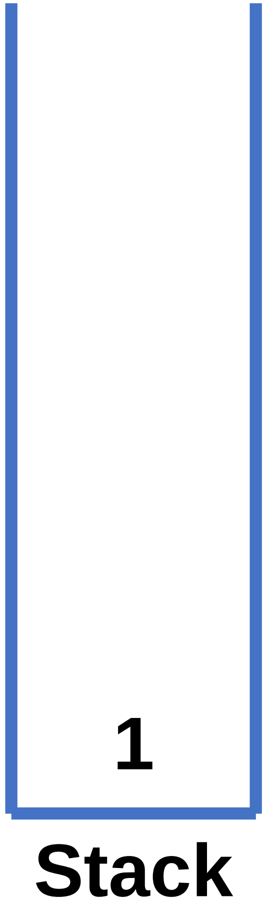
Depth First Search



Output: 1 2 4 5 6 3

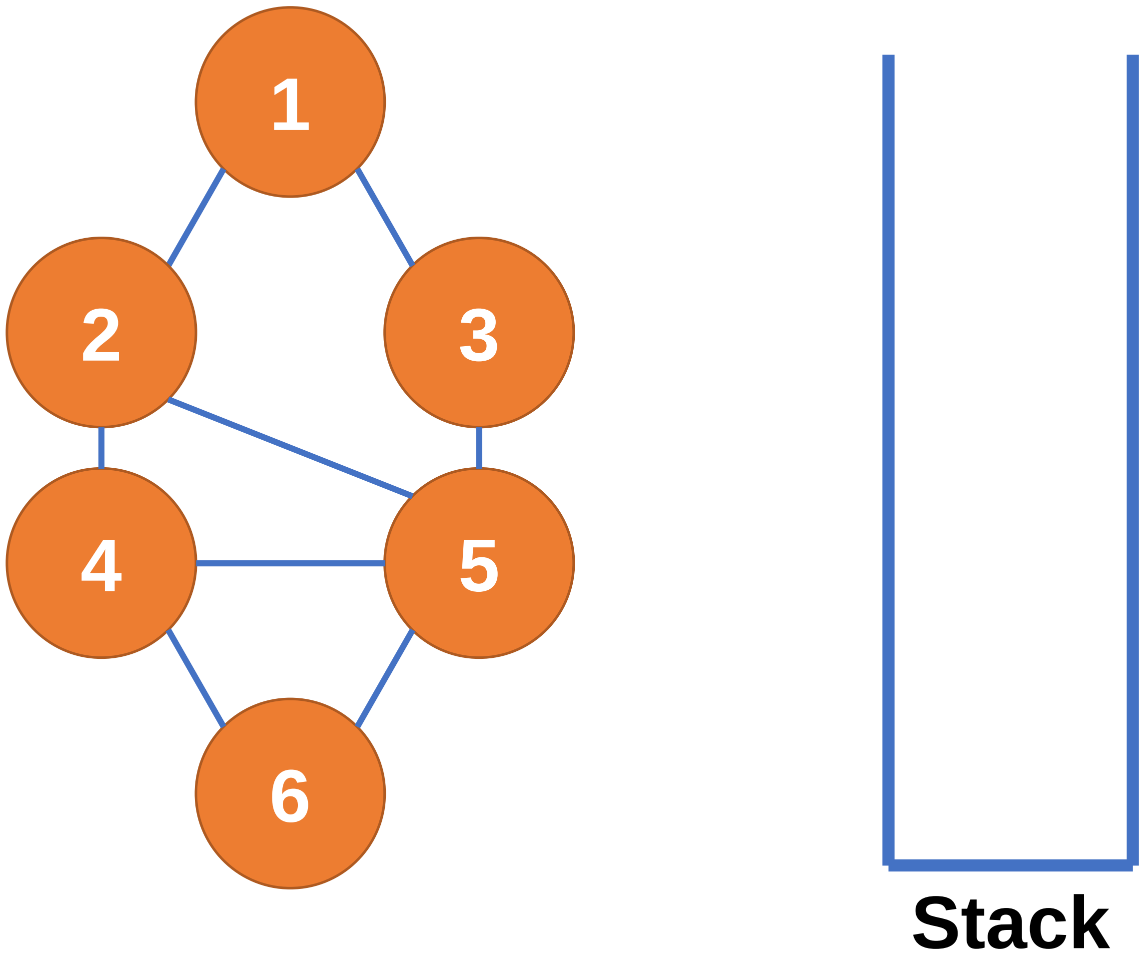
Depth First Search





Output: 1 2 4 5 6 3

Depth First Search



Output: 1 2 4 5 6 3

Implementation

```
void Graph::DFSUtil(int v, bool visited[])
{
    // Mark the current node as visited and
    // print it
    visited[v] = true;
    cout << v << " ";

    // Recur for all the vertices adjacent
    // to this vertex
    list<int>::iterator i;
    for (i = adj[v].begin(); i != adj[v].end(); ++i)
        if (!visited[*i])
            DFSUtil(*i, visited);
}

// DFS traversal of the vertices reachable from v.
// It uses recursive DFSUtil()
void Graph::DFS(int v)
{
    // Mark all the vertices as not visited
    bool *visited = new bool[V];
    for (int i = 0; i < V; i++)
        visited[i] = false;

    // Call the recursive helper function
    // to print DFS traversal
    DFSUtil(v, visited);
}
```

Complexity

Time Complexity: O(V+E)

V: Vertices
E: Edges

Reference

- Charles Leiserson and Piotr Indyk, “*Introduction to Algorithms*”, September 29, 2004

- https://www.geeksforgeeks.org