- **순열**
- **P(n,n) = n * (n−1) !**

1 2 3 4	2 1 3 4	3 1 2 4	4 1 2 3
1 2 4 3	2 1 4 3	3 1 4 2	4 1 3 2
1 3 2 4	2 3 1 4	3 2 1 4	4 2 1 3
1 3 4 2	2 3 4 1	3 2 4 1	4 2 3 1
1 4 2 3	2 4 1 3	3 4 1 2	4 3 1 2
1 4 3 2	2 4 3 1	3 4 2 1	4 3 2 1

1로 끝나는 수열	2로 끝나는 수열	3로 끝나는 수열	4로 끝나는 수열
2 3 4 **1**	1 3 4 **2**	1 2 4 **3**	1 2 3 **4**
2 4 3 **1**	1 4 3 **2**	1 4 2 **3**	1 3 2 **4**
3 2 4 **1**	3 1 4 **2**	2 1 4 **3**	2 1 3 **4**
3 4 2 **1**	3 4 1 **2**	2 4 1 **3**	2 3 1 **4**
4 2 3 **1**	4 1 3 **2**	4 1 2 **3**	3 1 2 **4**
4 3 2 **1**	4 3 1 **2**	4 2 1 **3**	3 2 1 **4**

- **순열 생성 재귀적 알고리즘1**

```
perm(n, r)
        if (r == 0) print_arr()
        else
                for (i : n – 1 ~ 0)
                        swap(a[i], a[n - 1])
                        t[r - 1] = a[n - 1]
                        perm(n - 1, r - 1)
                        swap(a[i], a[n - 1])
```

- **순열 생성 재귀적 알고리즘2**

```
perm(k)
        if (k == R) print_arr()
        else
                for (i : k ~ N - 1)
                        swap(k, i)
                        perm(k + 1)
                        swap(k, i)
```

- **순열 생성 재귀적 알고리즘2**

```
Visited[N-1]
perm(k)
        if (k == N) print_arr()
        else
                for (i : 0 ~ N - 1)
                        if (visited[i]) continue
                        t[k] = a[i]
                        visited[i] = true
                        perm(k + 1)
                        visited[i] = false
```

조합

- 서로 다른 n개의 원소 중 r개를 순서 없이 골라낸 것을 조합

$$nCr = \frac{n!}{(n-r)!\,r!}, (n \geq r)$$

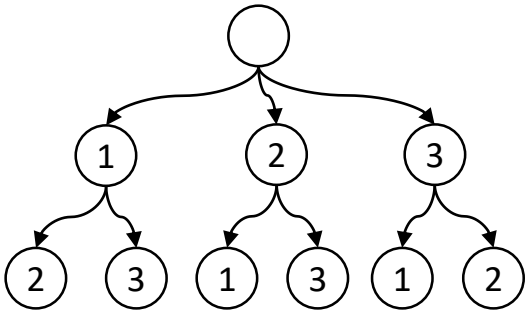
$$nCr = {}_{n-1}C_{r-1} + {}_{n-1}C_{r} \rightarrow$$ 재귀적 표현

$${}_nC_0 = 1$$

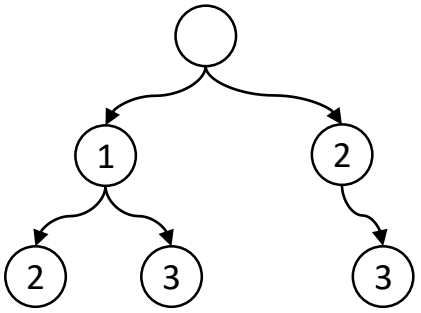
```
comb(n, r)
        IF r == 0  : print_array_t()
        ELIF n < r : RETURN
        ELSE
                tr[r - 1] ← an[n - 1]
                comb(n - 1, r - 1)
                comb(n - 1, r)
```

N = 3, R = 2

1 2
1 3
~~2 1~~
2 3
~~3 1~~
~~3 2~~



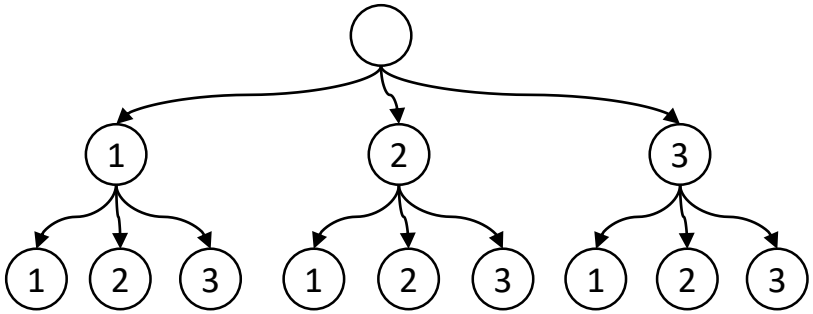
1 2
1 3
2 3



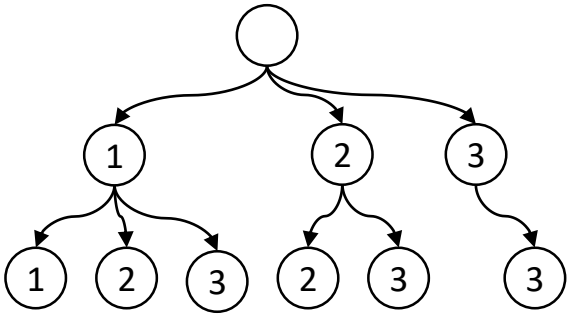
순열

조합

1 1
1 2
1 3
~~2 1~~
2 2
2 3
~~3 1~~
~~3 2~~
3 3



1 1
1 2
1 3
2 2
2 3
3 3

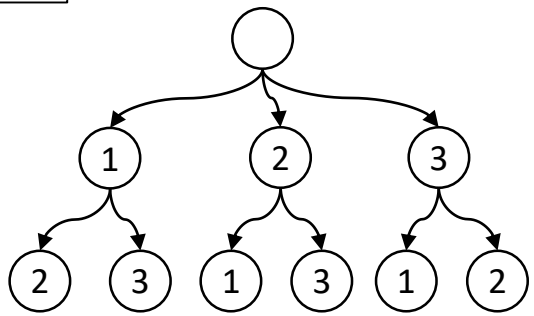


중복 순열

중복 조합

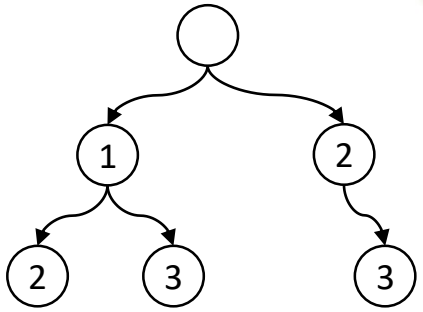
N = 3, R = 2

1 2
1 3
~~2 1~~
2 3
~~3 1~~
~~3 2~~



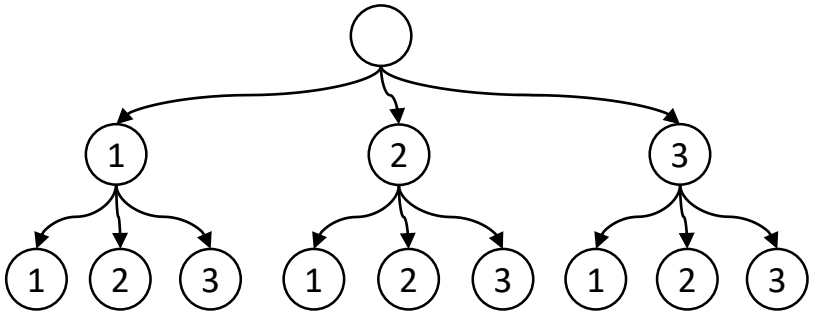

```
for x1 : 1 ~ 3
   for x2 : 1 ~ 3 단, x2 != x1
    print( x1, x2 )
```

1 2
1 3
2 3



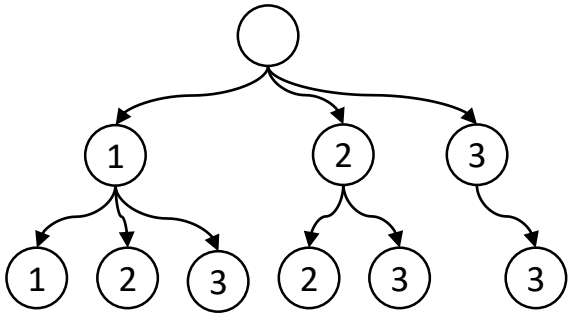

```
for x1 : 1 ~ 2
   for x2 : 2 ~ 3
    print( x1, x2 )
```

1 1
1 2
1 3
~~2 1~~
2 2
2 3
~~3 1~~
~~3 2~~
3 3




```
for x1 : 1 ~ 3
   for x2 : 1 ~ 3
    print( x1, x2 )
```

1 1
1 2
1 3
2 2
2 3
3 3



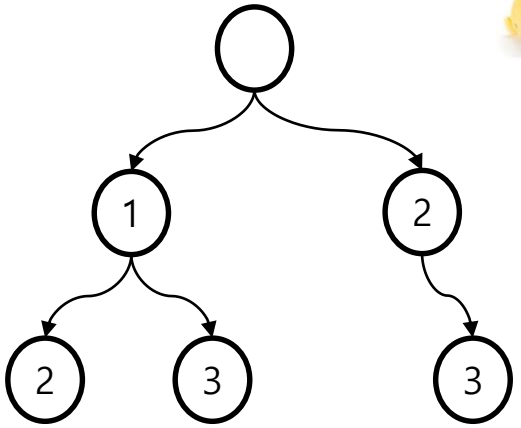

```
for x1 : 1 ~ 3
   for x2 : x1 ~ 3
    print( x1, x2 )
```

- **조합 생성 재귀적 알고리즘1**

$$C(n,r) = C(n-1,r-1) + C(n-1,r)$$

```
comb(n, r)
        if( r == 0) print_arr()
        else if (n < r) return
        else
                t[r-1] = a[n-1]
                comb(n-1, r-1)
                comb(n-1, r)
```

- **조합 생성 재귀적 알고리즘2**
  - 초기값 : k = 0, s = 0, N, R




```
comb(k, s) // 깊이, 시작숫자
        if (k == R) print_arr()
        else
                for (int i : s ~ N - R + k )
                        t[k] = a[i]
                        comb(k + 1, i + 1)
```

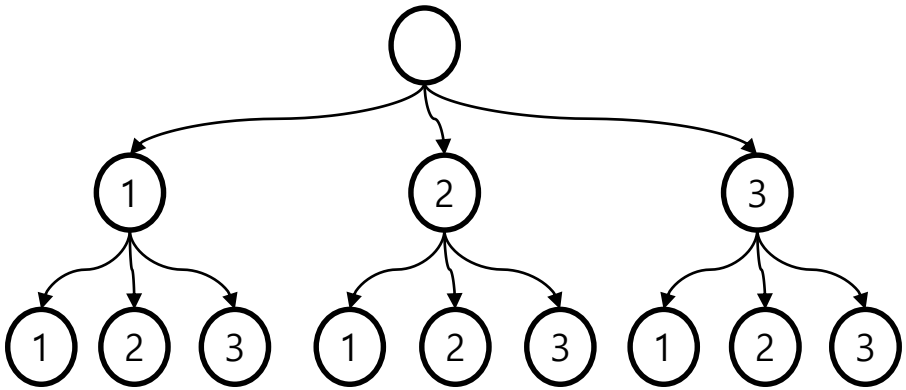
- **중복 순열 생성 재귀적 알고리즘1**

```
perm(n, r)
      if (r == 0) print()
      else
            for (i : n – 1 ~ 0)
                  swap(a[i], a[n - 1])
                  t[r - 1] = a[n - 1]
                  perm(n - 1, r - 1)
                  swap(a[i], a[n - 1])
```

```
PI(n, r)
      if (r == 0) print()
      else
            for (i : n – 1 ~ 0)
                  swap(a[i], a[n - 1])
                  t[r - 1] = a[n - 1]
                  PI(n, r - 1)
                  swap(a[i], a[n - 1])
```

- **중복 순열 생성 재귀적 알고리즘2** n^r

```
PI(k) // 깊이
        if (k == R) print_arr()
        else
                for (i : 0 ~ N - 1)
                        t[k] = a[i]
                        pi_r(k + 1)
```



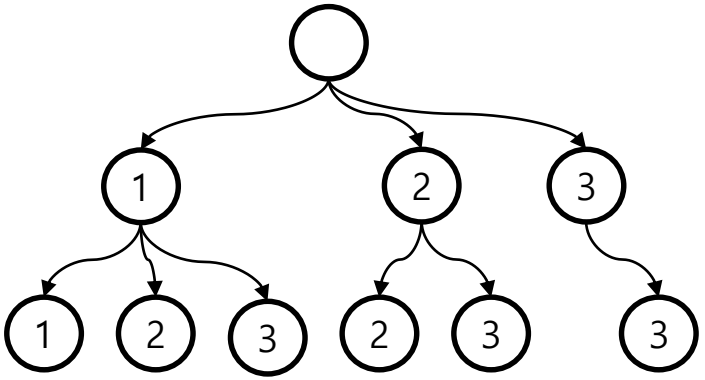
- **중복조합 생성 재귀적 알고리즘1**

```
comb(n, r)
        if( r == 0) print_arr()
        else if (n < r) return
        else
                t[r-1] = a[n-1]
                comb(n-1, r-1)
                comb(n-1, r)
```

```
H(n, r)
        if( r == 0) print_arr()
        else if (n == 0) return
        else
                t[r-1] = a[n-1]
                H(n, r-1)
                H(n-1, r)
```

$$H(n,r) = H(n,n-1) + H(n-1,r)$$

- **중복조합 생성 재귀적 알고리즘2**
  - 초기값 : K = 0, s = 1, N, R




```
H(k, s) // 깊이, 시작숫자
        if (k == R) print_arr()
        else
                for (int i : s ~ N )
                        t[k] = a[i]
                        H(k + 1, i)
```