# Research Note on Left-Digit Bias in Mortgage Rates*

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## 1 Mortgage Menus, Borrower Preference Heterogeneity, and Left-Digit Bias in Preferences

I begin by considering a two-dimensional space of mortgage rates, $r^m$, and initial balances, $B^o$. I begin with "standard" preferences in this space, ignoring the idea of the left-digit bias for the time being. These preferences are concave and increase in the direction of lower rates or larger balances. (I think this technically not quite right, but it makes a decent approximation.) The borrower faces a mortgage price schedule with rates increasing linearly in initial balances. Again, I ignore for the time being the step-wise nature of the GSE grids. The borrower's constrained optimization problem is to maximize utility subject to the mortgage price schedule, which they achieve in the standard way, by finding the point of tangency. I show an example of a borrower's mortgage choice, noting that borrower $i$ has positive utility at its choice of mortgage.

Next, I consider the heterogeneity of borrowers in the population. I index the various borrowers according to the coordinates in rate-balance space. I define zero utility as an indifference condition to taking no mortgage out at all. Each borrower type is represented by the coordinate corresponding to the point on their zero iso-utility curve with a tangency line parallel to the rate menu. There are two dimensions of heterogeneity that we can think of as populating the two dimensional space. On the one hand, borrowers may have different credit demand (due, for example, to different degrees of credit constraints). This will tend to make borrowers willing to take out larger loans at higher cost, varying as k stands in relation to j. On the other hand, the option value of borrowers' refinance

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option might vary due to the terms of an existing loan contract. This will make borrowers more or less motivated to take out a loan of the same size at a given rate. These borrowers will then vary as borrower i stands in relation to borrower j.

I consider the observed distribution of loans under the linear rate menu and standard preferences. Roughly, borrowers indexed above the mortgage menu follow the 135 degree line downward to the mortgage menu to choose their optimal mortgage. We observe the mass of these borrowers, contained in region A, in the empirical distribution.

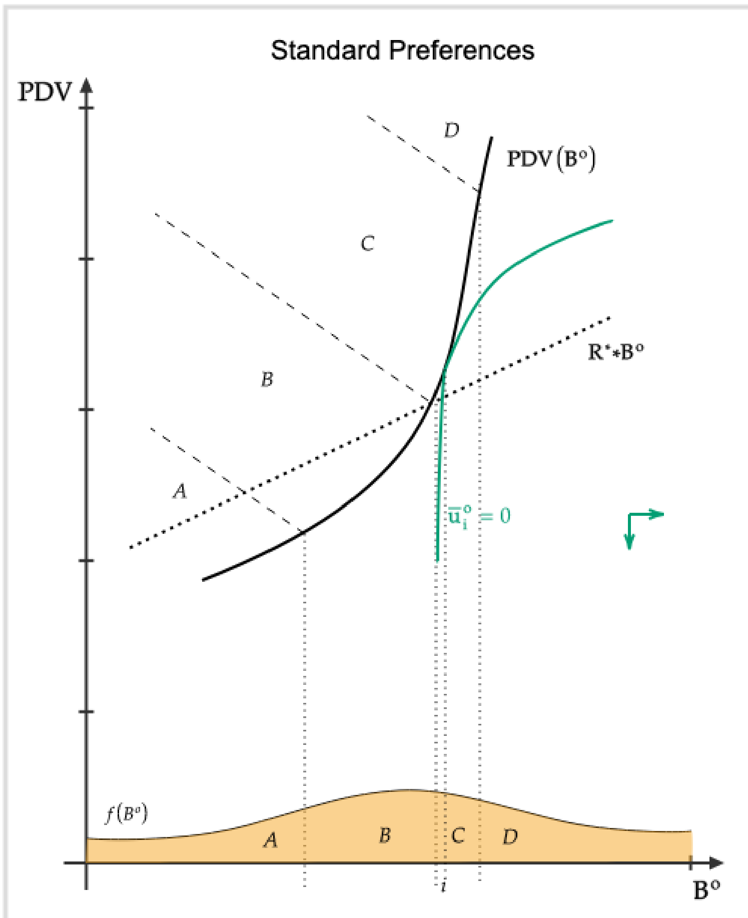


**Figure 1:** The benchmark distribution results from borrowers with standard preferences optimizating where to locate along a continuous and convex mortgage offer curve.

Now, I consider how preferences might be adapted to account for left-digit bias. To do so, I depict the left-digit threshold, l*, in rate-balance space. I consider the preferences of a single borrower, i, in the space. I draw I's iso-utility curves, using dashed-lines below l* where preferences have been changed. There are two ways in which the borrowers preferences may be "distorted".

In the first instance, which I call a "sharp" left-digit bias, the borrower places a premium on being immediately below l* but otherwise doesn't care about their distance to l*. In the second instance, "smoothed" left digit bias, the borrower places a premium on being below l*, and this premium decays gradually as the borrower's mortgage rate gets further and further from the threshold.

The left-digit bias can be characterized by two parameters, the size of the disturbance, and the rate of decay. The size of the disturbance has a natural interpretation of being related to the borrower's willingness-to-pay for a low-left digit. I have to think more about how to describe this willingness-to-pay. It occurs to me, though, that the size of willingness-to-pay is of a lower order than the decrease in the mortgage balance size. On some level, we are asking how much worse off is a borrower with standard preferences if they are restricted from taking their optimal mortgage size. The answer is somewhat worse off, but because they are on their first-order condition, variations in "quantity" or mortgage size do not have first-order effects on welfare.

# 2 Left-Digit Bias and Bunching with a Continuous (convex) Mortgage Menu

Next, I explore how left-digit bias affects the empirical distribution of mortgage originations in the case of a linear mortgage menu. First, I consider "sharp" left-digit bias. I depict the mortgage menu and left-digit threshold. It is useful also to draw the iso-utility curve of borrower *i* who, under standard preferences, is indifferent to taking a mortgage or not (i.e. has a zero iso-utility curve tangent to the mortgage menu) and who, under left-digit bias, is shifted left-ward from the mortgage menu by precisely the size of the left-digit disturbance. It is also useful to construct the region X' and X" by rotating borrower *i*'s indifference curve 180 deg and following it to the mortgage menu.

Suppose we choose a borrower index that lies on the border of the region X' + X". By definition, this borrower's zero iso-utility curve passes through this point, and supposing that the shape of borrowers' preferences are all translational shifts, and the construction of X' + X", the borrower's zero iso-utility curve also passes through point q with standard preferences. This, in turn, means that their preferences augmented by left-digit bias will also attain point p. Borrowers indexed to the interior of X' + X" have corresponding points on the perimeter with positive utility who then strictly prefer mortgage p to no mortgage at all. Similarly, borrowers in regions in A' and B' can obtain higher utilities by choosing mortgage p because they benefit from the low left-digit.

This generates a pattern of bunching depicted below where regions A and B are undisturbed and there is excess mass at mortgage p. This excess mass is due to borrowers shifting their choice of mortgage size (A' + B'), as well as new borrowers originating mortgages who previously did not (X' + X"). There are a few other things worth noting here. The additional mass due to new borrowers is the result of the size of the disturbance term, the curvature of the standard iso-utility curve (which is often related to elasticities, though I'm not quite sure which in this instance), and the density of borrowers in the region of X' and X". Also, while I have used just a single "disturbance" term for all utility levels, it's certainly possible that, say, the importance of a rate is more important for borrowers just on the margin of originating a loan compared to those who are just adjusting their loan size. That is to say, regions X' and X" do not have to line up with A' and B' as they do in this example. Finally, note that some people are taking out larger mortgages with higher rates in this example in order to obtain the "low left-digit". This is just because of the way the left-digit bias disturbances are specified.

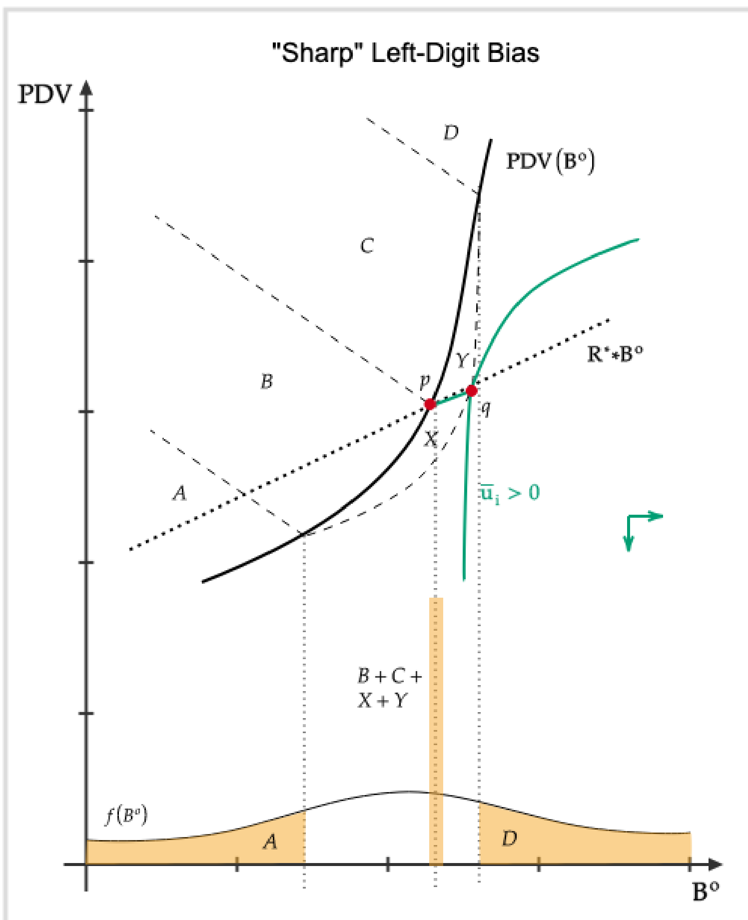


**Figure 2:** Bunching results in the observed distribution as a result of non-standard borrower preferences, which feature a "sharp" discontinuity due to preference for low left-digit mortgage rates. Additional mass is priced in. There is upward and downward bunching.

Next, I consider the empirical distribution of mortgage originations in the presence of "smooth" left-digit bias and a linear mortgage menu. Again, in addition to the menu and the left digit threshold, l*, I depict the preferences of borrower *i*, whose zero iso-utility curves are tangent to the mortgage menu and whose point of tangency to the mortgage menu shifted rightward by exactly the size of the left-digit disturbance lies on the left-digit threshold. Note that the indifference curve below the l* is at first tangent to the mortgage menu by construction. It is also useful to construct the region X" again by rotating borrower *i*'s standard preferences 180d around the threshold.

By a similar argument to the previous example, all borrowers in region X" will elect to hold mortgage p even though they did not hold a mortgage before. Similarly, borrowers in region B' will reduce their mortgage balances and bunch at point p. Somewhat loosely, I believe there is an argument that no borrowers in region A will locate at the mass point. Roughly, we can think about constructing region X' as before and noting that borrowers in region X' and A' can achieve a higher utility than under standard preferences by first maximizing utility under standard preferences along the perimeter of X'. They are guaranteed access to this utility at mortgage p because of the disturbance term. Note, though, that while borrower *i* had an iso-utility curve tangent to the mortgage menu at p, borrowers in region A' and X' will have tangency points here. As a result, they can further improve on this strategy by scaling down their mortgage size.

This generates several distortions in the distribution of observed mortgages. There is still a mass point corresponding to mortgage p. This is still due to both changes in the extensive (B') and intensive margin (X"). There is now only a single region of missing mass, and this is due to borrowers in B' scaling down their mortgage. We no longer have borrowers in region bunching upward. It is no longer the case, however, that the rest of the distribution is un-disturbed. In the region below the bunching, there is now additional mass due to borrowers in X'. And borrowers in region A' may adjust their borrowing demand relative to the "standard" preferences. It's worth noting that it's probably the case that loan size/rate adjusts upward for some borrowers relative to standard preferences, even if there is not stark upward bunching, because the size of the benefit increases as the rate on the loan increases toward l*.

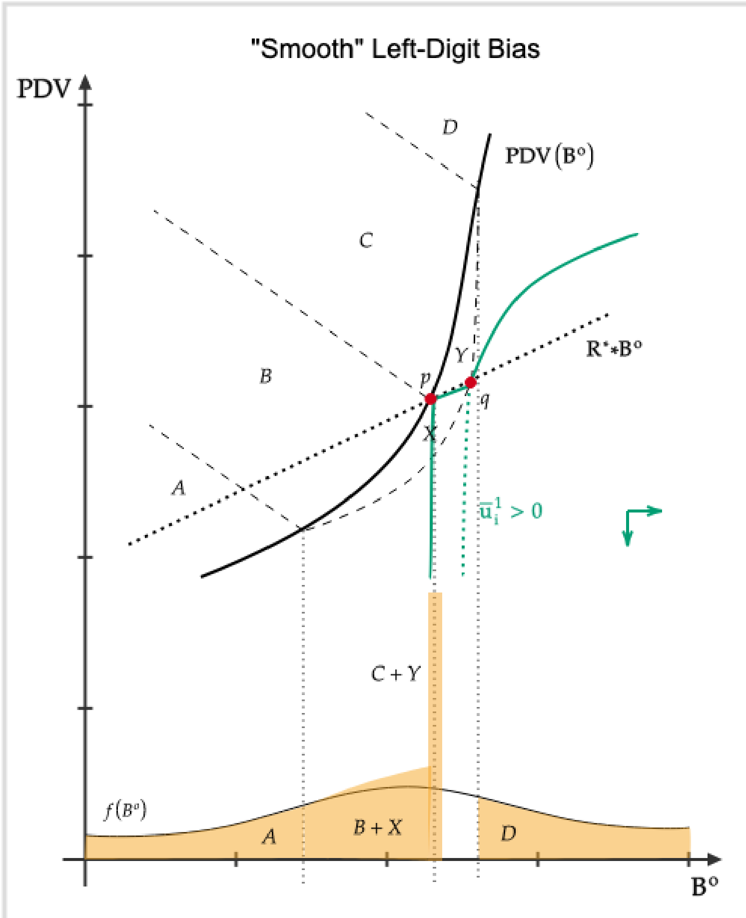


**Figure 3:** Bunching results in a different observed distribution due to nonstsandard borrower preferences, which feature a discontinuity due to a preference for low left-digit mortgage rates. This resolution of the discontinuity is "smooth" in this instance. Some additional mass is priced in. There is downward bunching and though there is some upward disturbance of the distribution below the mass point, this does not displace mass onto the mass point.

# 3 Welfare Analysis of Left-Digit Bias

Consider borrower $i$ whose preferences have the following properties:

1. The "standard preferences" of borrower $i$ have a zero iso-utility curve tangent to the mortgage offer curve.

2. Denote the "shifted" mortgage offer curve as the mortgage offer curve shifted by the size of the left-digit bias disturbance term. The "standard preferences" of borrower $i$ have a point of tangency to the "shifted" mortgage offer curve precisely along the threshold rate curve.

Although we identify borrower $i$ by the properties of their "standard preferences", we now consider their behavior under preferences exhibiting left-digit bias. We map the borrower's changing mortgage choice behavior into a willingness-to-pay (WTP) for a mortgage with a low left-digit.

The distortion introduced by the left-digit bias means that the borrower will pay an additional down-payment amount of $\Delta D$. Note, though, this means the borrower has lower payment obligations at a later date. Considering a simple setting, this is an amount of $\Delta D * R^*$.

To make a comparison with the additional costs, the future payment obligations should be valued at the present according to the borrower's discount factor. The present-valued savings are thus $\Delta D \frac{R^*}{R^s}$. And the net payment is $\Delta D * (1 - \frac{R^*}{R^s}) \in (0, \Delta D)$.

Additionally, this is a willingness-to-pay out of constrained resources, evident because the borrower is paying an elevated rate for marginal mortgage borrowing. To capture what the borrower is willing-to-pay out of unconstrained resources, an object potentially of interest to the social-planner, we multiply by the wedge between the borrower's subjective discount rate and the market rate $\frac{R^s}{R^f}$. Overall, we get $\Delta D(1 - \frac{R^*}{R^s})\frac{R^s}{R^f}$.

To make this estimate, we need the size of the adjustment or distortion induced by left-digit bias, $\Delta D$, the threshold rate, $R^*$, the borrower's subjective discount factor, $R^s$, and the market discount factor $R^f$. The threshold rate is known and an appropriate market discount factor can be chosen. The borrower's subjective discount factor, $R^s$, can be chosen from the slope of the mortgage menu where the borrower chooses to locate. What remains is the estimate the size of the adjustment induced, $\Delta D$.

The size of the bunching should offer some insight, but there are a few challenges in estimating $\Delta D$. First of all, $\Delta D$ is slightly larger than the horizontal length of the missing mass due to the curvature of the preferences. In principle, with information about the curvature of the preferences and mortgage menu, it may be possible to make an estimate of the difference between $\Delta D$ and the length of the missing mass.

In practice, it is difficult to observe missing mass and so the bunching mass and non-bunching density are very often used as a way to back-out the horizontal length of the missing mass. In this case, however, the bunching mass is due not just to the missing mass, but also additional mass - non-borrowers who became borrowers only because of the left-digit bias. Finally, distortions due to left-digit bias may affect the mass to the left of the bunching, making it difficult to get an accurate measurement of the counter-factual density of borrowers.

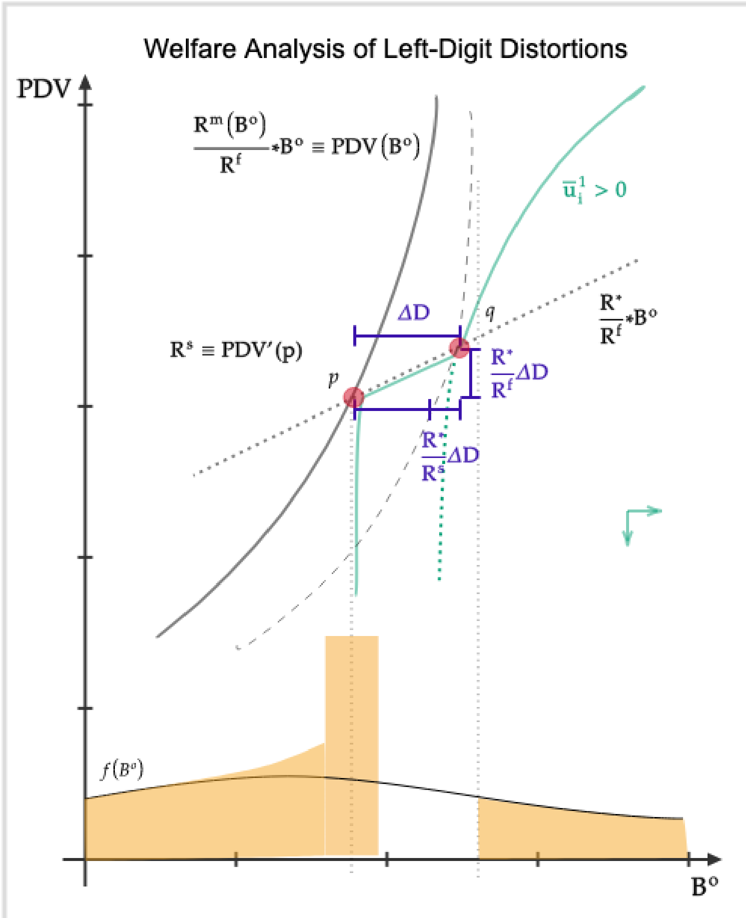


**Figure 4:** An analysis of the welfare implications of distortions due to left-digit bias is provided together with the resulting pattern of bunching. For the marginal borrower, the left-digit bias induces a willingness to make a larger down-payment in order to obtain a lower mortgage rate. The cost of this additional down-payment must be weighed against the reduction in payment obligations for having reduced the size of the mortgage. The relevant discount factors can be imputed from market data and the missing mass reveals the size of the down-payment distortion.

# 4 Left-Digit Bias and Bunching with a Notched Mortgage Menu

Next, I begin to consider the implications of a step-wise mortgage menu. I have (faintly) sketched a linear mortgage menu at the outer limit of the step-wise menu. I mark the set of all borrowers that can attain an offered mortgage with (weakly) positive utility. Nearly all the borrowers who could obtain loans under linear utility can still obtain loans under this step-wise menu. Some borrowers, particularly mid-step, are priced out. And, importantly, borrowers now bunch exclusively at the steps. (It's worth noting that this does not quite correspond with the data, where there is bunching but also borrowers mid-step. I think this may begin to demonstrate some of the shortcomings of writing standard-looking preferences in rate-balance space rather than, say, obligation_pdv-balance space. Still, I think it's possible to get some basic intuitions.)

Finally, I consider the problem with a step-wise mortgage menu and preferences with a "smoothed" left-digit bias. For simplicity, in the example, I choose l* to be just above one of the thresholds. The main theme, that distortions happen on both the "extensive" and "intensive" margins, persists. In some ways, because of the strict bunching at mass points, it is a bit easier to pin down where the additional bunching mass shows up.

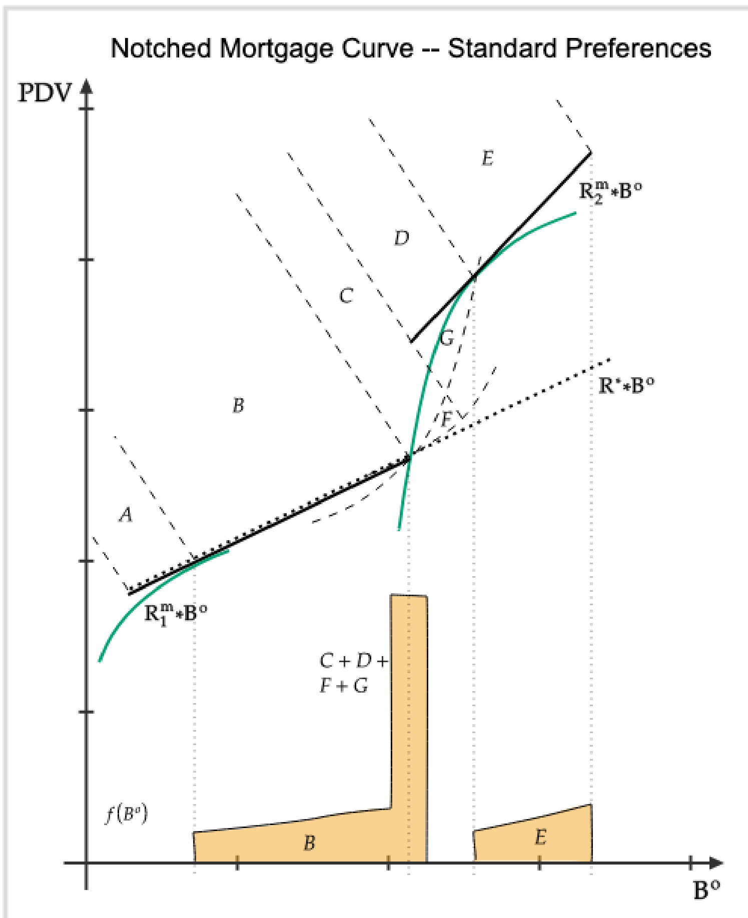


**Figure 5:** This benchmark distribution results from borrowers optimizing their choice of mortgage along a notched mortgage menu. The mortgage menu features increasing rates at given loan-to-value thresholds. The benchmark distribution of borrower mass features bunching even in the case of standard preferences because of the notches in the menu.

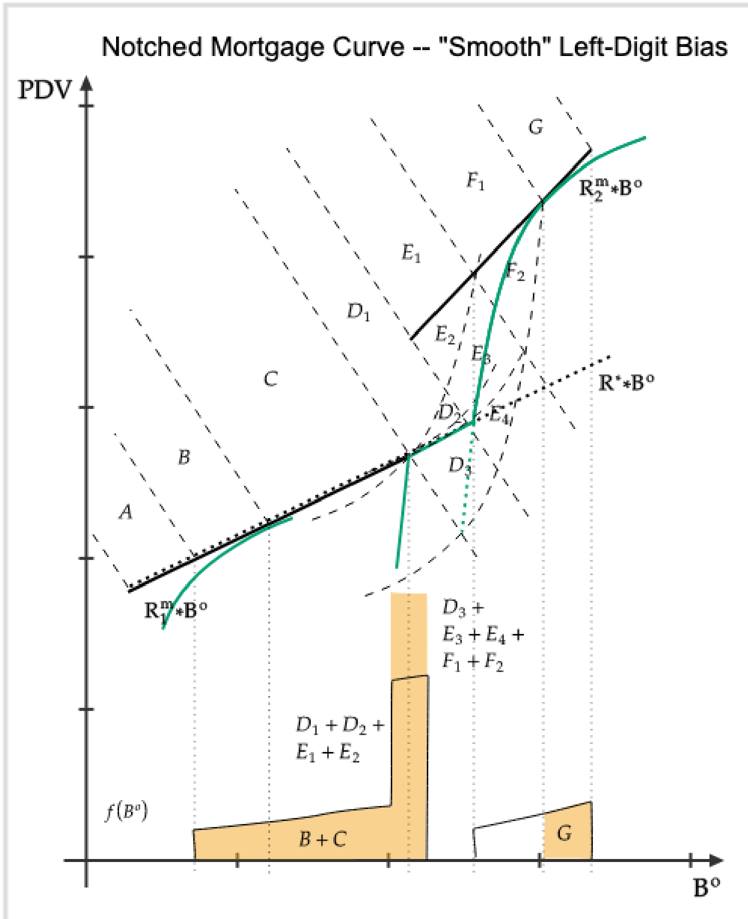


**Figure 6:** Further bunching results in the observed distribution when non-standard preferences featuring left-digit bias are introduced. In this case, additional mass is priced in and borrowers substitute from above to below the threshold mortgage rate.