SEARCHING

SYMBOL TABLES & BINARY SEARCH TREES

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Fall 2021

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THE SYMBOL TABLE CONCEPT

EXTENDING ARRAYS*

- In an array, an index (a number) corresponds to some value.
  - Terminology: call the index a ***key***.
- It would be useful to extend this idea: map any type of key to values.
- This is a ***symbol table***:
  - Insert: put(Key, Value) – associates a value with a key.
  - Search: get(Key) – returns the last value inserted with Key, null if missing.

- You may already be familiar with this idea from using dictionaries in Python.

*Don't throw away arrays after learning symbol tables. Arrays tend to match a computer's architecture and perform better when using integers and resizing is not needed.

USING A SYMBOL TABLE

//simple:
SymbolTable<Integer, String> titles = new SymbolTable();
title.put(222, “Data Structures & Algorithms”);

title.get(222); //returns “Data Structures & Algorithms”

The book gives a few examples of using symbol tables.

Really though, symbol tables are extensions of arrays – they have a million uses and a chart isn't needed.

We'll do one example in depth:

Compilers

application	purpose of search	key	value
dictionary	find definition	word	definition
book index	find relevant pages	term	list of page numbers
file share	find song to download	name of song	computer ID
account management	process transactions	account number	transaction details
web search	find relevant web pages	keyword	list of page names
compiler	find type and value	variable name	type and value

Typical symbol-table applications

Image from Sedgewick and Wayne.

SIMPLE SYMBOL TABLE API

public class ST<Key, Value>

	ST()	*create a symbol table*
void	put(Key key, Value val)	*put key-value pair into the table (remove* key *from table if value is* null*)*
Value	get(Key key)	*value paired with* key *(*null *if* key *is absent)*
void	delete(Key key)	*remove* key *(and its value) from table*
boolean	contains(Key key)	*is there a value paired with* key*?*
boolean	isEmpty()	*is the table empty?*
int	size()	*number of key-value pairs in the table*
Iterable<Key>	keys()	*all the keys in the table*

API for a generic basic symbol table

- Should a key correspond to more than one value?
- Should we allow values to be null?

- What order is the iterator returned from keys() in?

- Should Keys be mutable or immutable?
- Should Values be mutable or immutable?

Image from Sedgewick and Wayne.

APPLICATIONS IN COMPILERS

- As a standard part of most compilers, a symbol table of variables is built.
- The compiler indexes each variable in a method and uses it as a key to store the value assigned to it.
- This is useful since it gives us a place to work with the values/expressions within the variables.
- One of the more interesting uses is *Value Propagation*.

```
void compute_z() {
  int x = 2;
  int y = x + 5;
  int z = (y  + x) * 2;
  System.out.println(z);
}
```

Variable	Value
x : int	Constant: 2
y : int	Expr: x + 5
z : int	Expr: (y + x) * 2

VALUE PROPAGATION

```
String compute_z() {
   int x = 2;
   int y = x + 5;
   int z = (y  + x) * 2;
   System.out.println(z);
}
```

Imagine that we are filling in the symbol table. Let us follow these rules:

- If RHS is a constant, store the constant.
- If RHS is an expression, store by:
  - trying to replace any variables with the values they contain in the symbol table*
  - trying to compute operators with constant operands.

Variable	Value
x : int	Constant: 2
y : int	
z : int	

For y: x+5 → 2+5 → 7

Variable	Value
x : int	Constant: 2
y : int	Constant: 7
z : int	

For z: (y+x)*2 → (2+7)*2→18

Variable	Value
x : int	Constant: 2
y : int	Constant: 7
z : int	Constant: 18

Compile can remove x and y, and do z = 18.

*For the record, this can only happen if there is only one assignment of a variable before it is used in an expression.

ORDERED API

- Of course, we might want to know about values in terms of their order defined by keys.

- Ordered symbol tables give us this functionality.
- The methods with the red bar are new.

public class ST<Key extends Comparable<Key>, Value>

	ST()	*create an ordered symbol table*
void	put(Key key, Value val)	*put key-value pair into the table (remove* key *from table if value is* null*)*
Value	get(Key key)	*value paired with* key *(*null *if* key *is absent)*
void	delete(Key key)	*remove* key *(and its value) from table*
boolean	contains(Key key)	*is there a value paired with* key*?*
boolean	isEmpty()	*is the table empty?*
int	size()	*number of key-value pairs*
Key	min()	*smallest key*
Key	max()	*largest key*
Key	floor(Key key)	*largest key less than or equal to* key
Key	ceiling(Key key)	*smallest key greater than or equal to* key
int	rank(Key key)	*number of keys less than* key
Key	select(int k)	*key of rank* k
void	deleteMin()	*delete smallest key*
void	deleteMax()	*delete largest key*
int	size(Key lo, Key hi)	*number of keys in* [lo..hi]
Iterable<Key>	keys(Key lo, Key hi)	*keys in* [lo..hi]*, in sorted order*
Iterable<Key>	keys()	*all keys in the table, in sorted order*

API for a generic ordered symbol table

Image from Sedgewick and Wayne.

ORDERED SYMBOL TABLES: METHODS

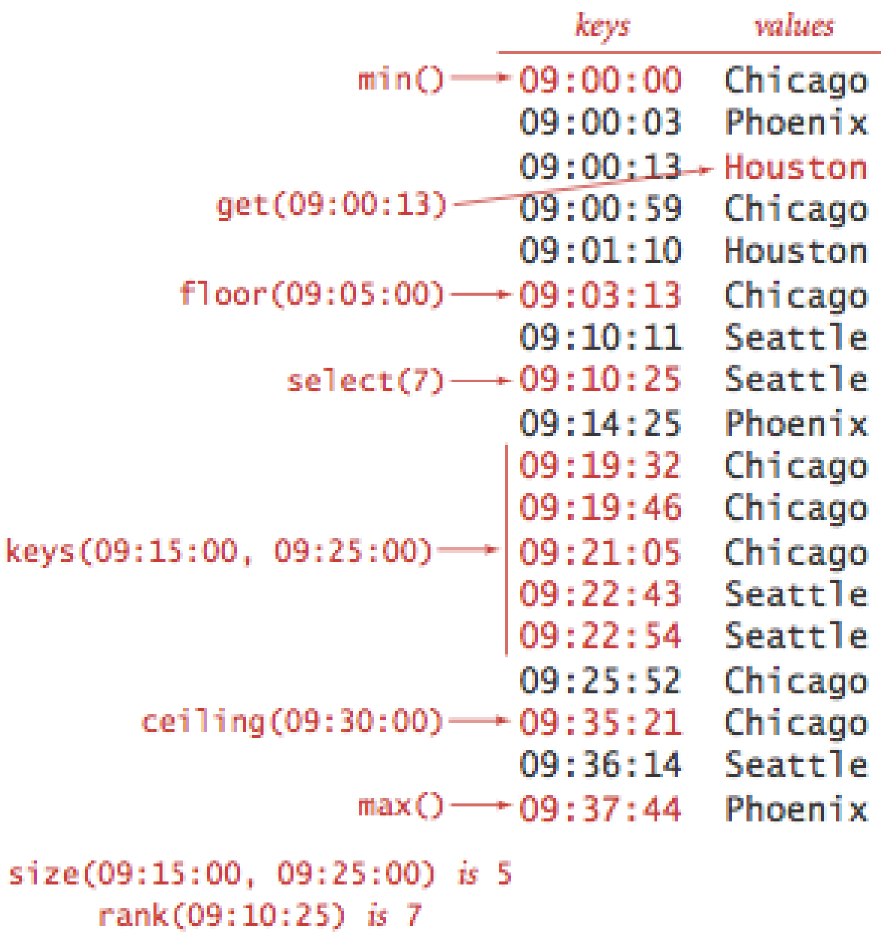


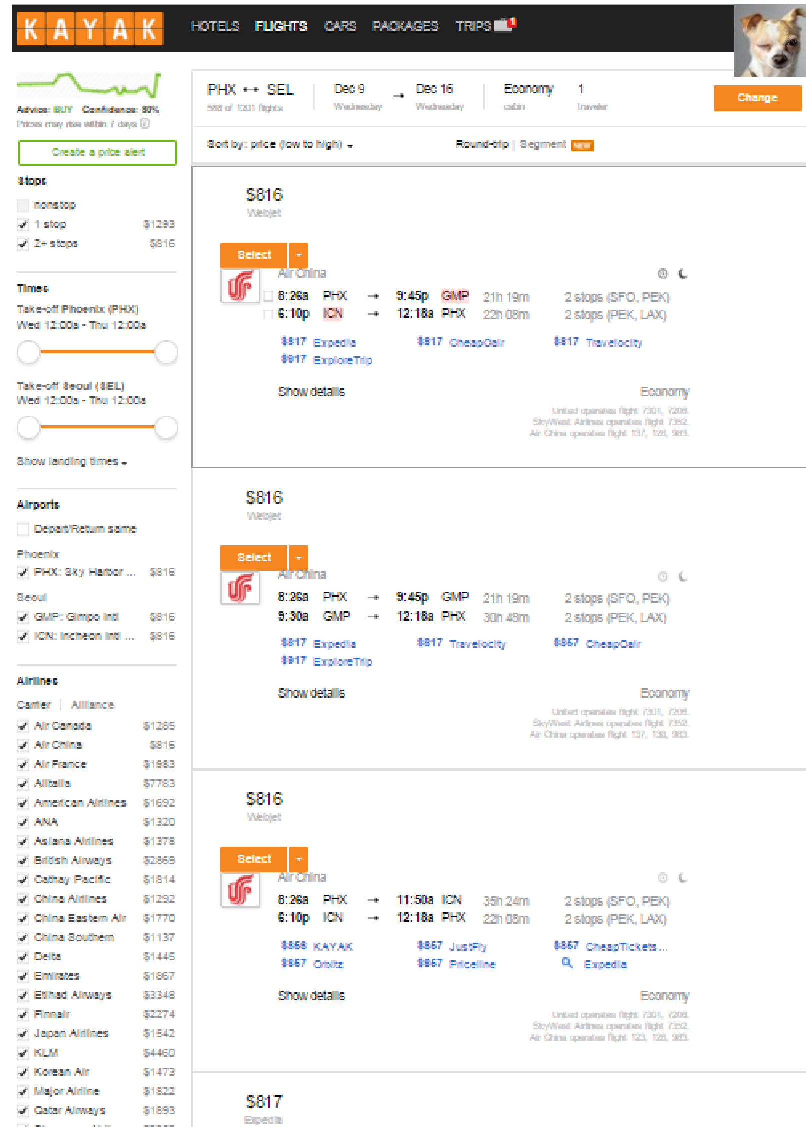
Image from Sedgewick and Wayne.

ORDERED SYMBOL TABLES: BEST PRICES

```
//simple:
SymbolTable prices = new SymbolTable();
prices.put(857, new Flight(AirChina, “931”));
prices.put(816, new Flight(United, “7301”));
prices.put(897, new Flight(Asiana, “5040”));
prices.put(980, new Flight(SkyWest, “7301”));
prices.put(817, new Flight(AirChina, “987”));

for(Integer i : prices.keys(0, 850))
            System.out.println(bst.get(i));

//will print out flights up to 850, in order.
```



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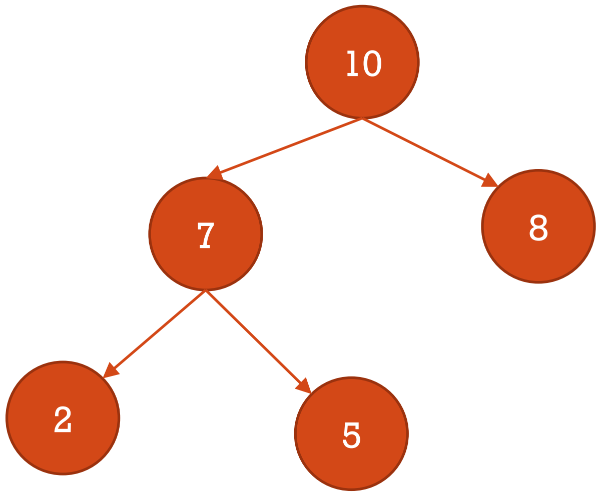
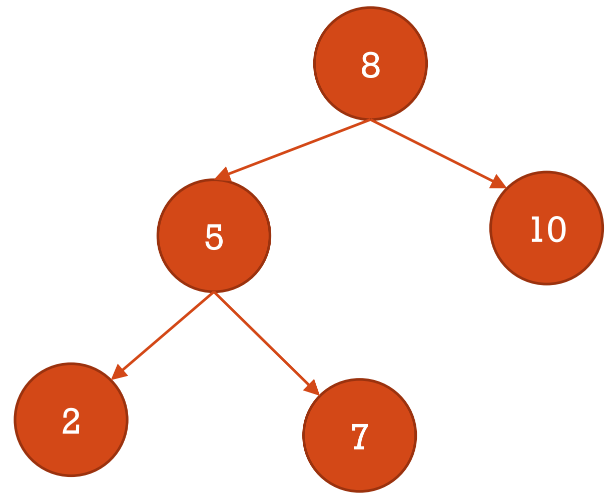
BINARY SEARCH TREES

HEAPS TO BINARY SEARCH TREES



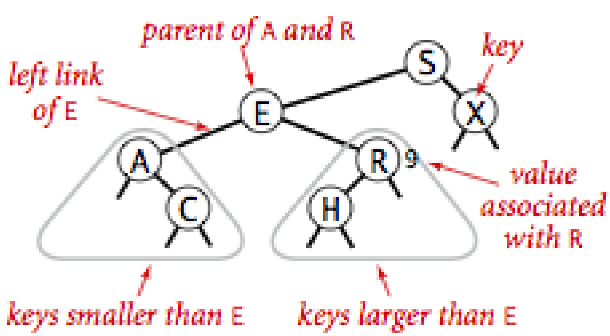


- In the last chapter, we discussed heaps as a way to structure data in a priority queue.
- Now we'll use BSTs which are a little more restrictive.
- Are heaps BSTs? Vice versa?

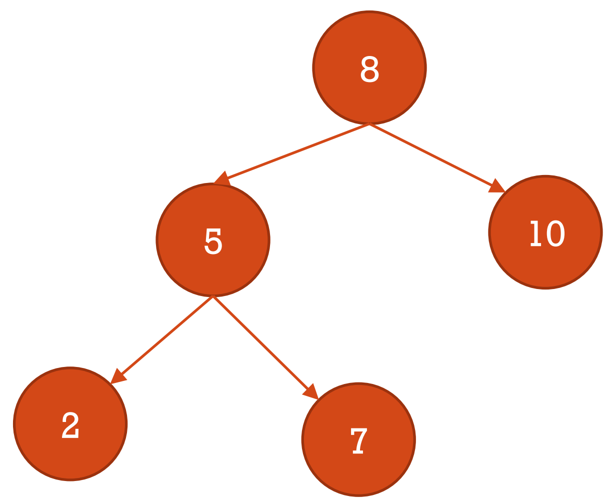
- Side note: one requirement of symbol tables is that each key maps to exactly one value. This useful in our implementations since we can assume the elements are distinct.

THE CONCEPT

- **Definition**: A *binary tree* is a tree where each node has at most two children.
- **Definition**: A binary search tree is a binary tree where each node's left child has a key less than the parent, and the right child has a key greater than the parent.



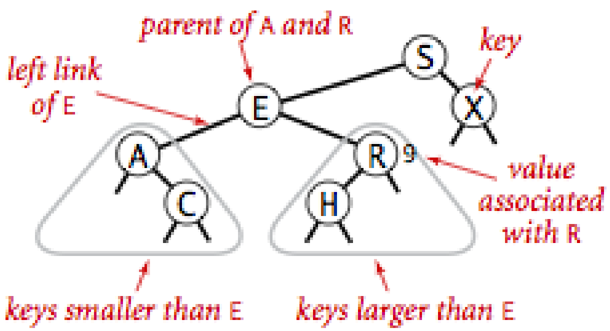
Anatomy of a binary search tree



THE CONCEPT

- **Recursive Definition**: a node is the root of a BST if:
  - A left child has a key less than than the parent, and the child is the root of a BST.
  - A right child has a key greater than the parent , and the child is the root of a BST.

- So… what?



Anatomy of a binary search tree

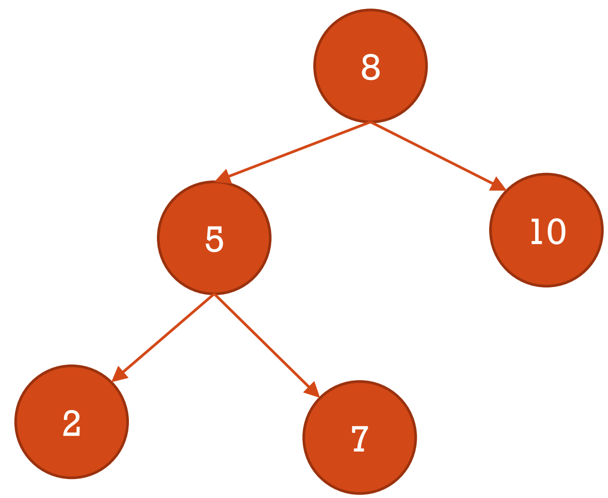
TRAVERSING TREES

- A common operation performed on trees is to visit all of their nodes. To explore a non-linear structure, we need rules to systematically explore the structure and be sure we don't miss any nodes.
  - How does this compare to exploring a linked list?

- There are three main algorithms for doing this:
  - **Preorder(root):**
    visit root, preorder(root.left), preorder(root.right)
  - **Inorder(root):**
    inorder(root.left), visit root, inorder(root.right)
  - **Postorder(root):**
    postorder(root.left), postorder(root.right), visit root

TRAVERSING TREES

- **Preorder(root):**
  visit root, preorder(root.left), preorder(root.right)



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IMPLEMENTATION

ORDERED SYMBOL TABLE

We're going to implement the whole thing – a ordered symbol table:

public class ST<Key extends Comparable<Key>, Value>

	ST()	*create an ordered symbol table*
void	put(Key key, Value val)	*put key-value pair into the table (remove* key *from table if value is null)*
Value	get(Key key)	*value paired with* key *(null if* key *is absent)*
void	delete(Key key)	*remove* key *(and its value) from table*
boolean	contains(Key key)	*is there a value paired with* key*?*
boolean	isEmpty()	*is the table empty?*
int	size()	*number of key-value pairs*
Key	min()	*smallest key*
Key	max()	*largest key*
Key	floor(Key key)	*largest key less than or equal to* key
Key	ceiling(Key key)	*smallest key greater than or equal to* key
int	rank(Key key)	*number of keys less than* key
Key	select(int k)	*key of rank* k
void	deleteMin()	*delete smallest key*
void	deleteMax()	*delete largest key*
int	size(Key lo, Key hi)	*number of keys in* [lo..hi]
Iterable<Key>	keys(Key lo, Key hi)	*keys in* [lo..hi]*, in sorted order*
Iterable<Key>	keys()	*all keys in the table, in sorted order*

API for a generic ordered symbol table

Image from Sedgewick and Wayne.

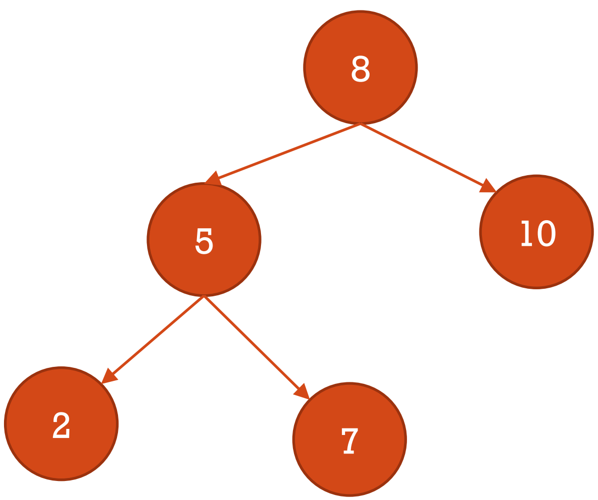
BST REPRESENTATION

- In the vernacular of the discussion we had during priority queues: we will represent our symbol table as a binary search tree and structure it as a binary tree.
- Each node will be a linked node with three references, a value, and a size.

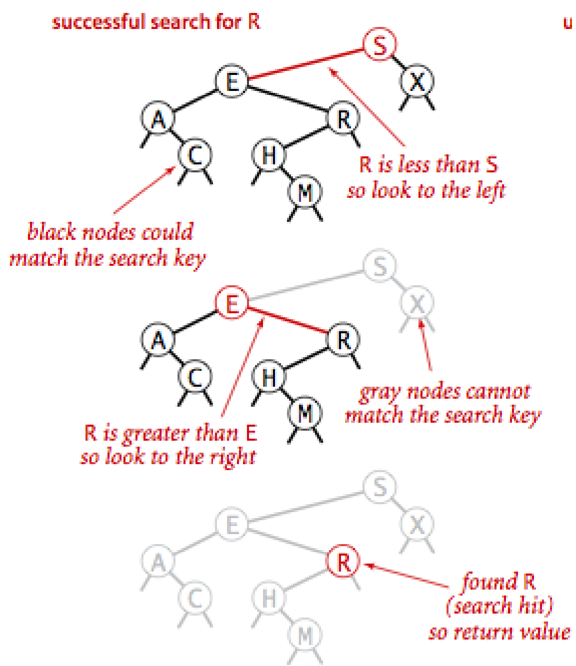
```
private class Node {
    private final Key key;
    private Value val;
    private Node left, right;
    private int N;

    public Node(Key key, Value val, int N) {
        this.key = key;
        this.val = val;
        this.N = N;
    }
}
```

GET() IN BSTS

- Per the API, we need to get "value paired with key".



GET() IMPLEMENTATION

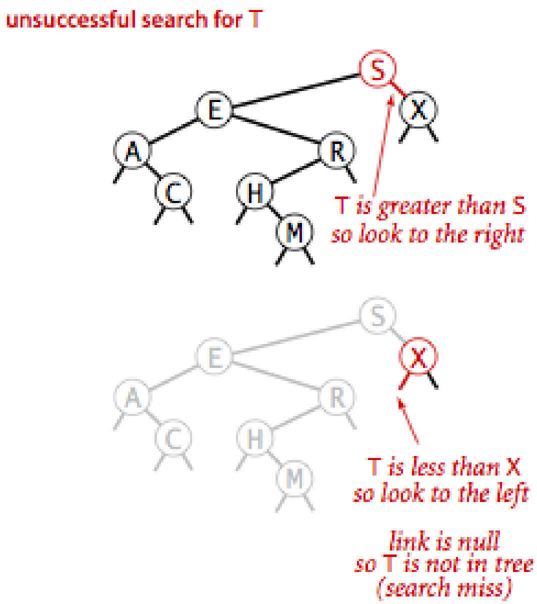


sucessful search

```
public Value get(Key key) {
        return get(root, key);
    }

    private Value get(Node x, Key key) {
        if (x == null)
            return null;
        int cmp = key.compareTo(x.key);
        if (cmp < 0)
            return get(x.left, key);
        else if (cmp > 0)
            return get(x.right, key);
        else return x.val;
    }
```


Image from Sedgewick and Wayne.

GET() IMPLEMENTATION



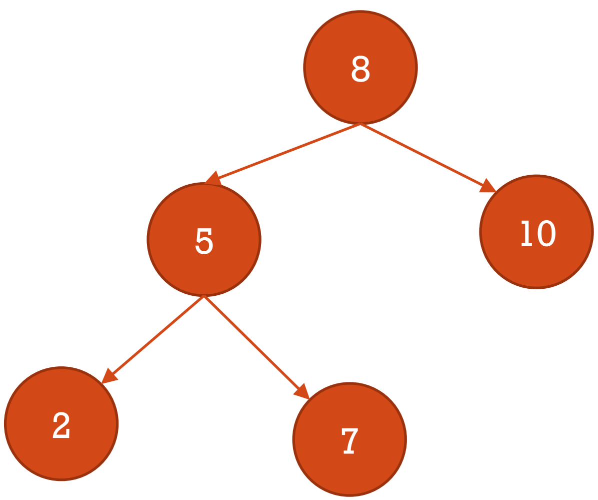
unsucessful search

```
public Value get(Key key) {
        return get(root, key);
    }

    private Value get(Node x, Key key) {
        if (x == null)
            return null;
        int cmp = key.compareTo(x.key);
        if (cmp < 0)
            return get(x.left, key);
        else if (cmp > 0)
            return get(x.right, key);
        else return x.val;
    }
```

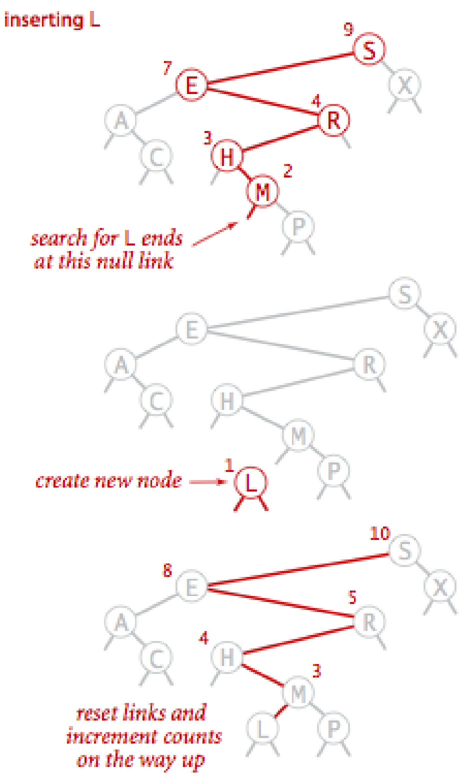

Image from Sedgewick and Wayne.

PUT() IN BSTS

- Per the API, we need to "put a key-value pair into the table".



PUT() IMPLEMENTATION

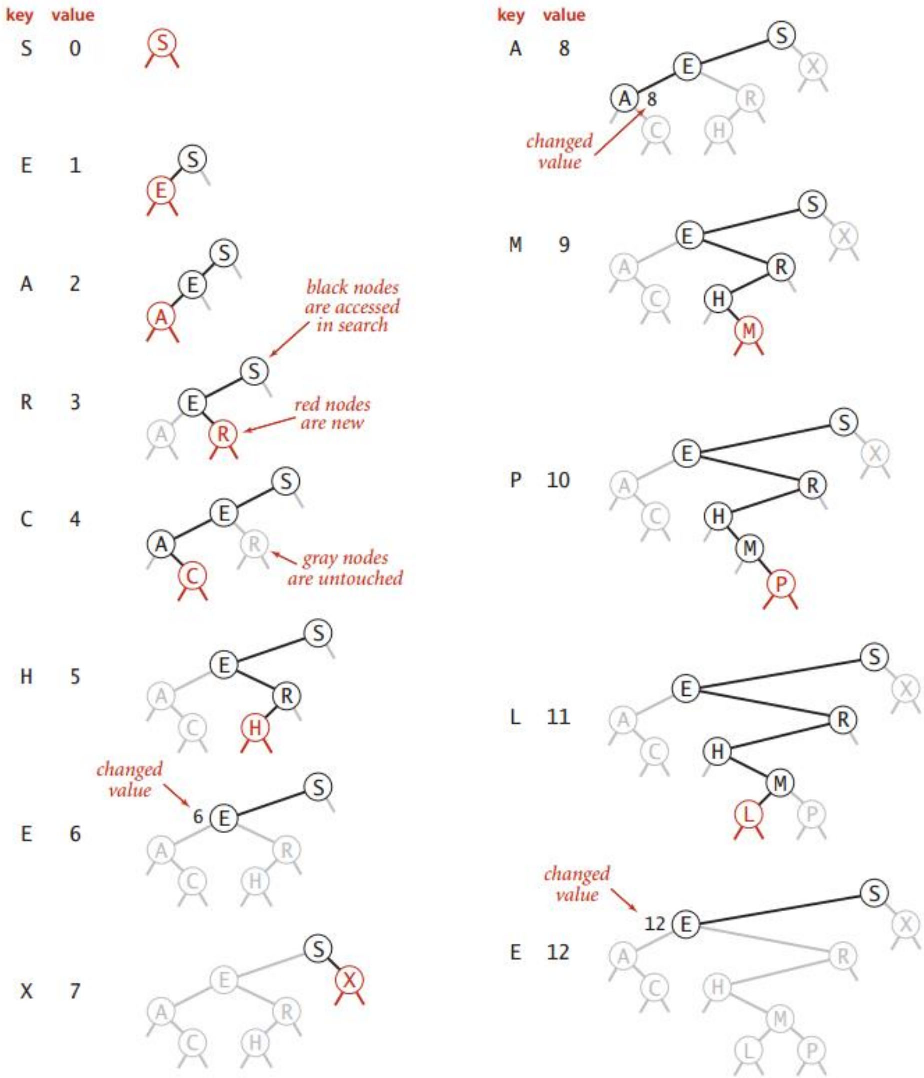


Insertion into a BST

```
public void put(Key key, Value val) {
    root = put(root, key, val);
}

private Node put(Node x, Key key, Value val) {
    if (x == null)
        return new Node(key, val, 1);

    int cmp = key.compareTo(x.key);
    if (cmp < 0)
        x.left = put(x.left, key, val);
    else if (cmp > 0)
        x.right = put(x.right, key, val);
    else
        x.val = val;
    x.N = size(x.left) + size(x.right) + 1;

    return x;
}
```


Image from Sedgewick and Wayne.

BST TRACE



One general concern: how will the resultant tree look?

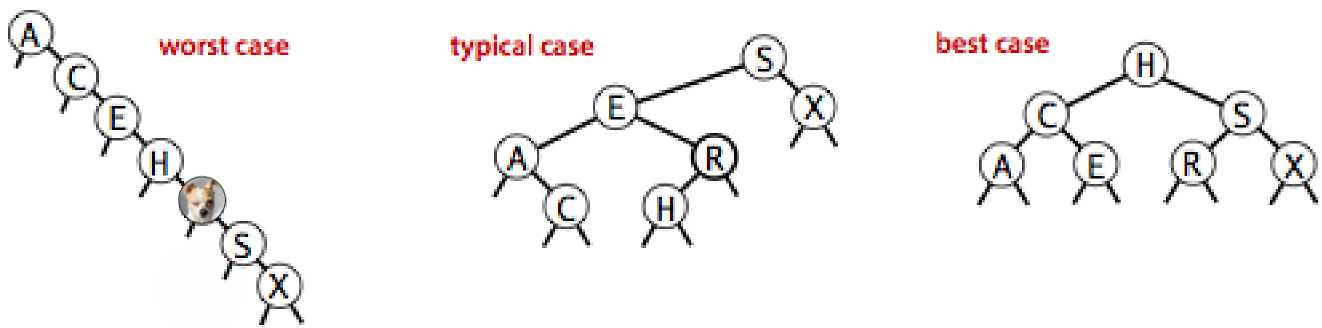
BST LAYOUT

There three cases for layout of a tree:

Worse case: a stilted tree.

Typical case: in between the other two cases.

Best case: a balanced tree.



We will focus on the typical (average) case for a successful search where we count comparisons.

- Where do unsuccessful searches and inserts fit in?
- Any guesses for the worse or best case Big-Oh right now?

The average case will be defined as a tree where the domain of keys is randomly inserted.

AVERAGE ANALYSIS FOR BST

Proof:

The sum of the depth of all nodes is the *internal path length*, call it C_N.

The average cost of a search hit is then:

- $1 + \frac{C_N}{N}$

As initial values, we have $C_0 = C_1 = 0$. We now want to write C_N:

- $N - 1$ (every node other than the root has to cross the root's edge)
- $((C_0 + C_{N-1}) + (C_1 + C_{N-2}) + \cdots (C_{N-1} + C_0))/N$ (take the average of possible subtrees)

The textbook rewrites as:

- $C_N = N - 1 + (C_0 + C_{N-1})/N + (C_1 + C_{N-2})/N + \cdots (C_{N-1} + C_0)/N$

a complete expression that looks like the quicksort recurrence.

We'll skip the algebra on this one:

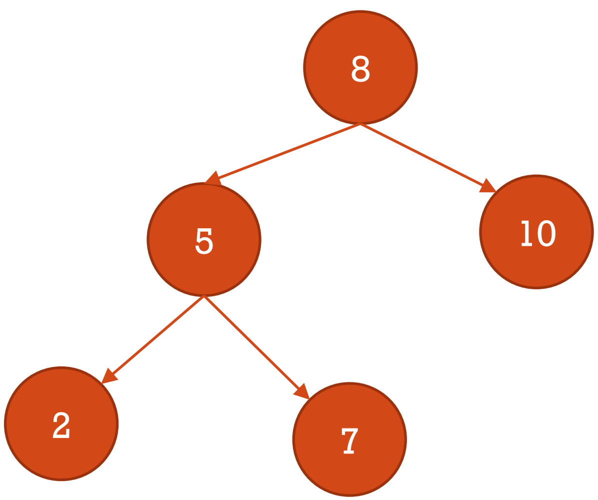
- $C_N \sim 2N ln N$

We can now evaluate our original expression, $1 + \frac{C_N}{N}$:

- $1 + \frac{2N ln}{N} \sim 2 ln N \approx 1.39 lg N$

MIN() IN BSTS

- Per the API, we need to find the "smallest key".



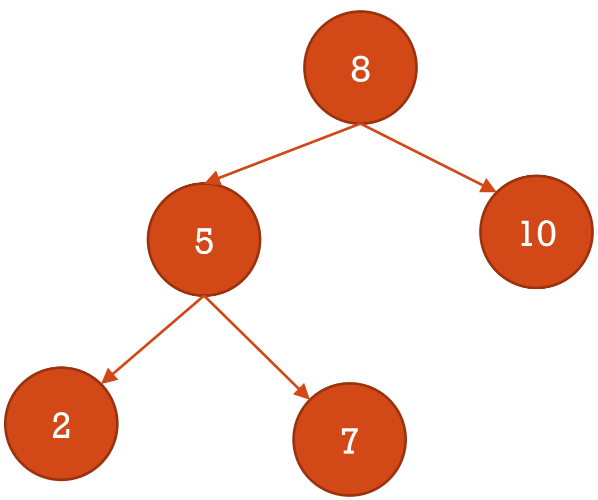
MIN() IMPLEMENTATION

- Finding the min is simple, just move left until you cannot do so.

```
public Key min() {
      return min(root).key;
    }

    private Node min(Node x) {
        if (x.left == null)
            return x;
        return min(x.left);
    }
```

DELETEMIN() IN BSTS

- Per the API, we need to “delete smallest key”.

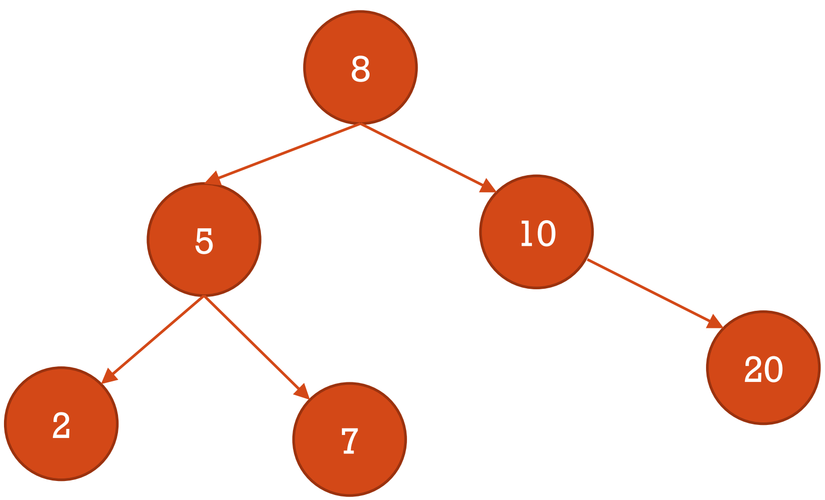


DELETEMIN() IMPLEMENTATION

```
public void deleteMin() {
        root = deleteMin(root);
    }

    private Node deleteMin(Node x) {
        if (x.left == null) return x.right;
        x.left = deleteMin(x.left);
        x.N = size(x.left) + size(x.right) + 1;
        return x;
    }
```

DELETE() IN BSTS

- Per the API, we need to "remove key (and value) from table".
- This will be our most complicated method – there are actually three different cases to handle (explicitly or implicitly).

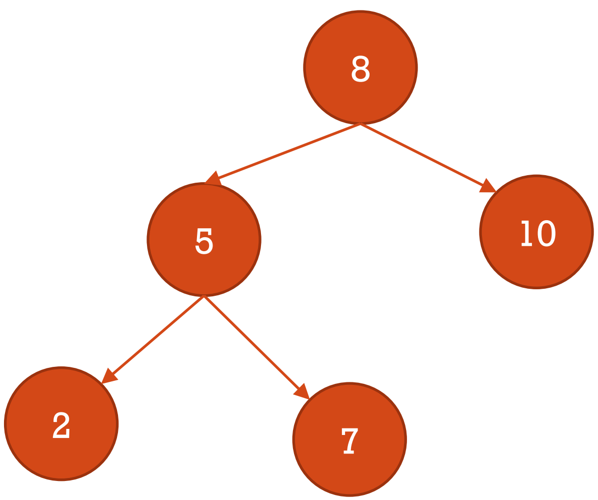


DELETE() IMPLEMENTATION

```
public void delete(Key key) {
        root = delete(root, key);
    }

    private Node delete(Node x, Key key) {
        if (x == null) return null;
        int cmp = key.compareTo(x.key);
        if (cmp < 0) x.left = delete(x.left, key);
        else if (cmp > 0) x.right = delete(x.right, key);
        else
        {
            if (x.right == null) return x.left;
            if (x.left == null) return x.right;
            Node t = x;
            x = min(t.right);
            x.right = deleteMin(t.right);
            x.left = t.left;
        }
        x.N = size(x.left) + size(x.right) + 1;
        return x;
    }
```

KEYS() IN BSTS

- Per the API, we need to find all the "keys in [lo..hi]".



KEYS() IMPLEMENTATION

```java
public Iterable<Key> keys(Key lo, Key hi) {
    Queue<Key> queue = new LinkedList<>();
    keys(root, queue, lo, hi);
    return queue;
}

private void keys(Node x, Queue<Key> queue, Key lo, Key hi) {
    if (x == null) return;
    int cmplo = lo.compareTo(x.key);
    int cmphi = hi.compareTo(x.key);
    if (cmplo < 0) keys(x.left, queue, lo, hi);
    if (cmplo <= 0 && cmphi >= 0) queue.add(x.key);
    if (cmphi > 0) keys(x.right, queue, lo, hi);
}
```

PERFORMANCE SUMMARY

Algorithm (data structure)	avg: search hit	avg: insert	Efficiently support ordered operations?
sequential search (unordered linked list)	N/2	N	No
binary search (ordered array)	lg N	N	Yes
binary search trees	1.39lg N	1.39lg N	Yes