

Independent and Paired Sample *T*-Test

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Outline

- Assignment 10 Review
- Independent Sample T -Test
- Paired Sample T -Test

Learning objectives:

R: T -Tests

Assignment 10 Review

Great work!

T-Statistic Cutoffs

Say you want to find the cutoff t values: $t(df = 3)$ and $t(df = 37)$ for $\alpha = .05$, both upper one-tailed and two-tailed.

Which will have the larger magnitude cutoff values?

One-Tailed

All of our $\alpha = .05$ goes on the upper side

Need a cutoff for .95

```
1 qt(p = .95, df = 3)
```

```
[1] 2.353363
```

```
1 qt(p = .05, df = 37, lower.tail = F)
```

```
[1] 1.687094
```

$$t_{crit}(df = 3) = 2.35$$

$$t_{crit}(df = 37) = 1.69$$

Two-Tailed

$\alpha = 1 - .95 = 5\%$ on both sides

So $.05/2 = .025$ on each side

Need value for .025 and $.95 + .025$ ([.025, .975])

```
1 qt(p = c(.025, .975), df = 3)
```

```
[1] -3.182446  3.182446
```

```
1 qt(p = c(.025, .975), df = 37)
```

```
[1] -2.026192  2.026192
```

$$t_{crit}(df = 3) = [-3.18, 3.18]$$

$$t_{crit}(df = 37) = [-2.03, 2.03]$$

T-Statistic Cutoffs: Table

t Table

cum. prob	<i>t</i> _{.50}	<i>t</i> _{.75}	<i>t</i> _{.80}	<i>t</i> _{.85}	<i>t</i> _{.90}	<i>t</i> _{.95}	<i>t</i> _{.975}	<i>t</i> _{.99}	<i>t</i> _{.995}	<i>t</i> _{.999}	<i>t</i> _{.9995}
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
Z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

Types of T-Tests

Types of t test

One sample t-test



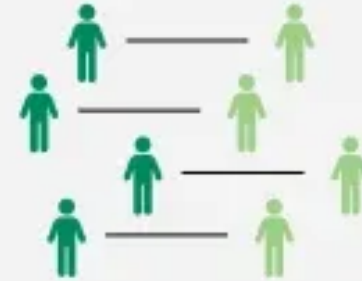
Is there a difference between a group and the population.

Independent samples t-test



Is there a difference between two groups

Paired samples t-test



Is there a difference in a group between two points in time

Independent Sample *T*-Test

Might want to compare two samples of observations

- Post-graduation salaries of GT vs. UGA grads
- Treatment vs. control group
- Cats vs. dogs

We can represent each group with their mean on some outcome and compare the means

Consider whether the mean difference is significant but also have to account for different levels of variability within group

Independent Sample *T*-Test: Pooled SD

Need to pool the sample SDs while noting different sample sizes

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)} = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Independent Sample *T*-Test: Pooled SD

Note that when sample size is equal (i.e., $n_1 = n_2 = n$),

$$\begin{aligned} s_p^2 \Big|_{n_1=n_2=n} &= \frac{(n-1)s_1^2 + (n-1)s_2^2}{(n-1) + (n-1)} = \frac{(n-1)(s_1^2 + s_2^2)}{(n-1) + (n-1)} \\ &= \frac{(n-1)(s_1^2 + s_2^2)}{2(n-1)} = \frac{s_1^2 + s_2^2}{2} \end{aligned}$$

Square root to get back to standard deviation

$$s_p = \sqrt{s_p^2} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

Independent Sample *T*-Test: Test statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}} = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Compare against critical *t*-value with $df = (n_1 - 1) + (n_2 - 2)$

Independent Sample *T*-Test: R Calculation

```
1 summary(candy)
```

candy	sales
Length:40	Min. : 2.12
Class :character	1st Qu.: 5.48
Mode :character	Median : 9.75
	Mean :10.15
	3rd Qu.:15.32
	Max. :17.83

```
1 ccorn <- candy |> filter(candy == "candy corn") |> pluck("sales")
2 choc <- candy |> filter(candy == "chocolate") |> pluck("sales")
```

```
1 ccorn
```

```
[1] 5.51 4.09 5.57 6.85 4.83 8.60 4.53 2.12 7.60 3.24 4.82 5.56 6.17 4.53 6.15
[16] 3.75 6.61 5.39 6.44 4.82
```

```
1 choc
```

```
[1] 17.06 14.46 14.21 14.66 16.90 13.16 17.83 10.90 11.31 15.46 15.52 13.72
[13] 15.43 17.10 14.97 15.94 15.70 15.29 12.24 17.09
```


Independent Sample *T*-Test: R Calculation

```
1 x_bar_ccorn <- mean(ccorn)
2 sd_ccorn <- sd(ccorn)
3 n_ccorn <- length(ccorn)
4
5 x_bar_choc <- mean(choc)
6 sd_choc <- sd(choc)
7 n_choc <- length(choc)
8
9 pooled_sd <- sqrt( ((n_ccorn - 1) * sd_ccorn^2 + (n_choc - 1) * sd_choc^2) /
10                  ((n_ccorn - 1) + (n_choc - 1)) )
11 pooled_sd
```

```
[1] 1.732816
```


Independent Sample *T*-Test: R Calculation

```
1 df <- (n_ccorn - 1) + (n_choc - 1)
2
3 t_crit <- qt(c(.025, .975), df)
4 t_crit
```

```
[1] -2.024394  2.024394
```

If our observed *t*-statistic exceeds the bounds of [-2.02, 2.02], we reject the null.

```
1 t_stat <- (x_bar_ccorn - x_bar_choc) / (pooled_sd * sqrt(1 / n_ccorn + 1 / n_choc))
2 t_stat
```

```
[1] -17.49839
```

We reject the null! Can we tell which had a larger mean?

Independent Sample *T*-Test: R Function

```
t.test(outcome ~ group_var, data, var.equal = T)
```

```
1 t.test(sales ~ candy, candy, var.equal = T)
```

Two Sample t-test

data: sales by candy

t = -17.498, df = 38, p-value < 2.2e-16

alternative hypothesis: true difference in means between group candy corn and group chocolate is not equal to 0

95 percent confidence interval:

-10.697796 -8.479204

sample estimates:

mean in group candy corn	mean in group chocolate
5.3590	14.9475

Paired Sample *T*-Test

Comparing the same subjects under two conditions (typically two timepoints)

- Stress levels before vs. after a test
- Reaction times before vs. after caffeine consumption
- Anxiety levels before vs. after a mindfulness intervention

We can still use the mean to represent the observations in each condition and make comparisons

This time, focus on difference scores (individual differences in the observations between the two conditions)

Paired Sample T -Test

Need difference score for each observation $d_i = x_2 - x_1$

Then, get mean difference $\bar{d} = \frac{d_1 + d_2 + \dots + d_n}{n}$ and standard deviation of those difference scores $s_d = \sqrt{\frac{(d_1 - \bar{d})^2 + (d_2 - \bar{d})^2 + \dots + (d_n - \bar{d})^2}{n-1}}$

$$t = \frac{\bar{d} - \mu_{d_0}}{\frac{s_d}{\sqrt{n}}} = (\text{usually}) \frac{\bar{d} - 0}{\frac{s_d}{\sqrt{n}}} = \frac{\bar{d}}{\frac{s_d}{\sqrt{n}}}$$

Compare against critical t -value with $df = n - 1$

Paired Sample *T*-Test: R Calculation

```
1 pumpkin <- pumpkin |>
2   mutate(diff = Oct1 - Oct31)
3
4 mean_diff <- mean(pumpkin$diff)
5 sd_diff <- sd(pumpkin$diff)
6 df <- nrow(pumpkin) - 1
7
8 qt(c(.025, .975), df)
```

```
[1] -2.093024  2.093024
```

```
1 t_stat <- (mean_diff - 0) / (sd_diff / sqrt(nrow(pumpkin)))
2 t_stat
```

```
[1] 11.59213
```

We can reject the null in favor of the alternative that there is a difference in pumpkin weights between October 1st and October 31st in the population!

Paired Sample *T*-Test: R Function

```
t.test(y1_vector, y2_vector, paired = T, var.equal = T)
```

```
1 t.test(pumpkin$Oct1, pumpkin$Oct31, paired = T, var.equal = T)
```

Paired t-test

data: pumpkin\$Oct1 and pumpkin\$Oct31

t = 11.592, df = 19, p-value = 4.636e-10

alternative hypothesis: true mean difference is not equal to 0

95 percent confidence interval:

2.55216 3.67684

sample estimates:

mean difference

3.1145

Cohen's *D* Revisited

Now that we have two groups, R functions for Cohen's *D* will be more helpful to us. The `cohens_d()` function from the `effectsize` package allows for many formats of inputs.

```
(x_bar_ccorn - x_bar_choc) / pooled_sd
```

```
[1] -5.533478
```

```
effectsize::cohens_d(sales ~ candy,  
                      data = candy)
```

Cohen's d		95% CI
-5.53		[-6.91, -4.14]

- Estimated using pooled SD.

```
mean_diff / sd_diff
```

```
[1] 2.592079
```

```
effectsize::cohens_d(pumpkin$Oct1, pumpkin$Oct31,  
                      paired = T)
```

Cohen's d		95% CI
2.59		[1.66, 3.51]

T-Test: Interpretation and write-up

An [independent samples / repeated measures] *t*-test analysis was conducted between [group 1] and [group 2]. We [**fail to reject** / **reject**] H_0 [**in favor of H_1**], $t([df]) = [_____]$, p [**< 0.05** / **p-value**]. We [**did not**] observe[**d**] a significant difference between the mean [DV] of [group 1] and [group 2] in our sample.

Happy Halloween!



