Chi-Squared Test

PSYC 2020-A01 / PSYC 6022-A01 | 2025-11-07 | Lab 12

Jessica Helmer

Outline

- Chi-Squared (χ^2) Test

Learning objectives:

R: χ^2 Tests

Chi-Squared (χ^2) Test

Testing inferences about proportions

- Involve categorical variables

Comparing observed frequencies (counts) to some expected (null hypothesis) frequencies

χ^2 **Test of Goodness of Fit**: expected frequencies are set by researcher (e.g., null hypothesis is equal frequencies across all groups)

χ^2 **Test of Independence**: expected frequencies are those implied by independence

Chi-Squared (χ^2) Test

Instead of a continuous variable, our outcome is a **count**

Continuous

- Height
- Petal length
- Test score

Counts

- Coin flip being heads
- Number of pets
- Goals scored

Only integers

χ^2 Test of Goodness of Fit

Do the observed frequencies of a categorical variable differ from what expected *a priori*?

Typically, this expectation is equal occurrence across groups (but doesn't have to be).

If equal occurrence, what would be our expected frequencies?

100 coin flips

Heads	Tails
50	50

Favorite primary color of 33 students

Red	Blue	Yellow
11	11	11

Can also hypothesize proportions and convert to frequencies once you have a sample size.

χ^2 Test of Goodness of Fit: Hypotheses

H_0: Observed data match the expected frequencies for the population

H_1: Observed data do not match the expected frequencies for the population

- Observed frequency is significantly different than expected

H_0: $\pi_j = \pi_{j_0}$ for all categories j (i.e., difference for all categories is 0), where

π_j = observed proportion

π_{j_0} = expected proportion

H_1: $\pi_j \neq \pi_{j_0}$ for all categories j

χ^2 Test of Goodness of Fit Generally

$$\chi^2 = \sum_{j=1}^{J} \frac{(O_j - N\pi_{j_0})^2}{N\pi_{j_0}} = \sum_{j=1}^{J} \frac{(O_j - E_j)^2}{E_j}$$

where

O_j = observed frequency

N = total sample size

E_j = expected frequency

j = individual category out of J total categories

The sum of the squared differences between the observed and expected frequencies, divided by the expected frequency, for each group.

χ^2 Test of Goodness of Fit Generally

$\mathrm{df} = J - 1$, where

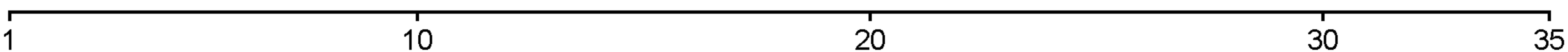
J = number of categories

Use to identify critical χ^2 value from χ^2 table, R, etc.

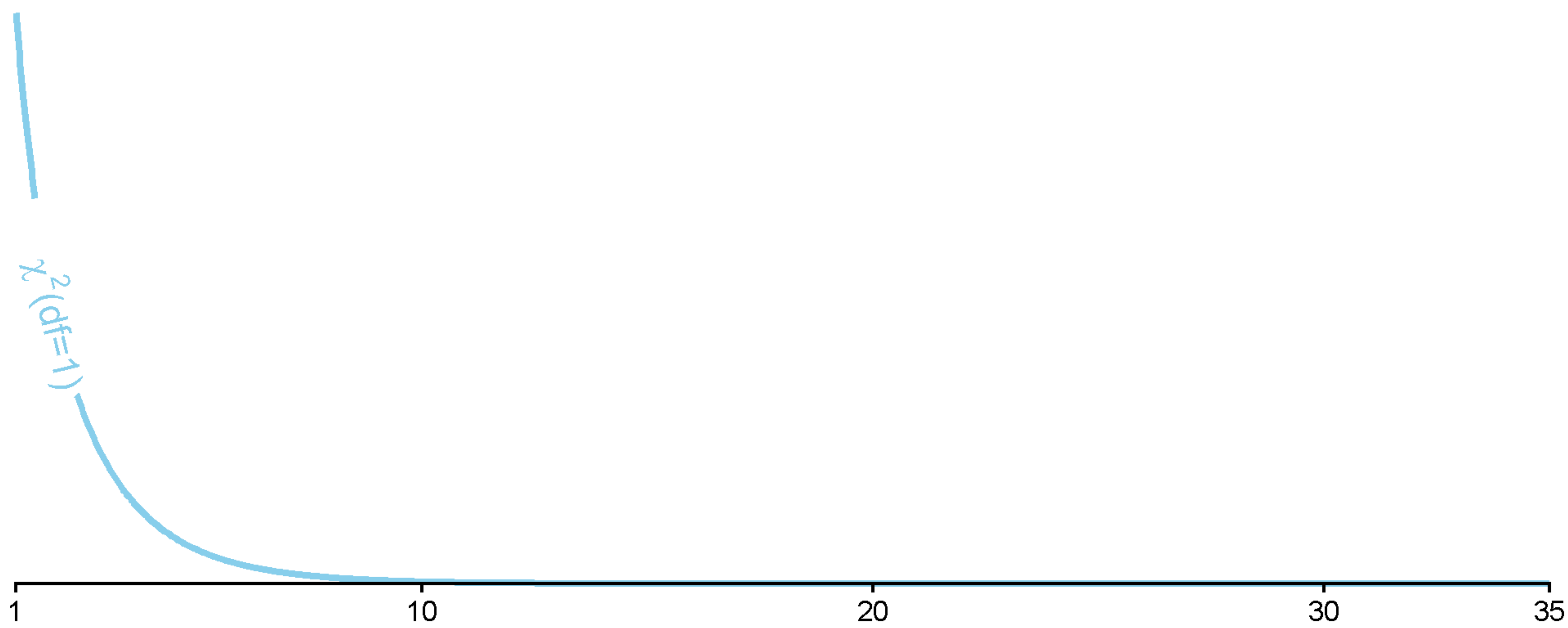
For χ^2, as df increases, so does the critical-χ^2 value (at same α)

Only ever a one-tailed test! For the upper end of the distribution.

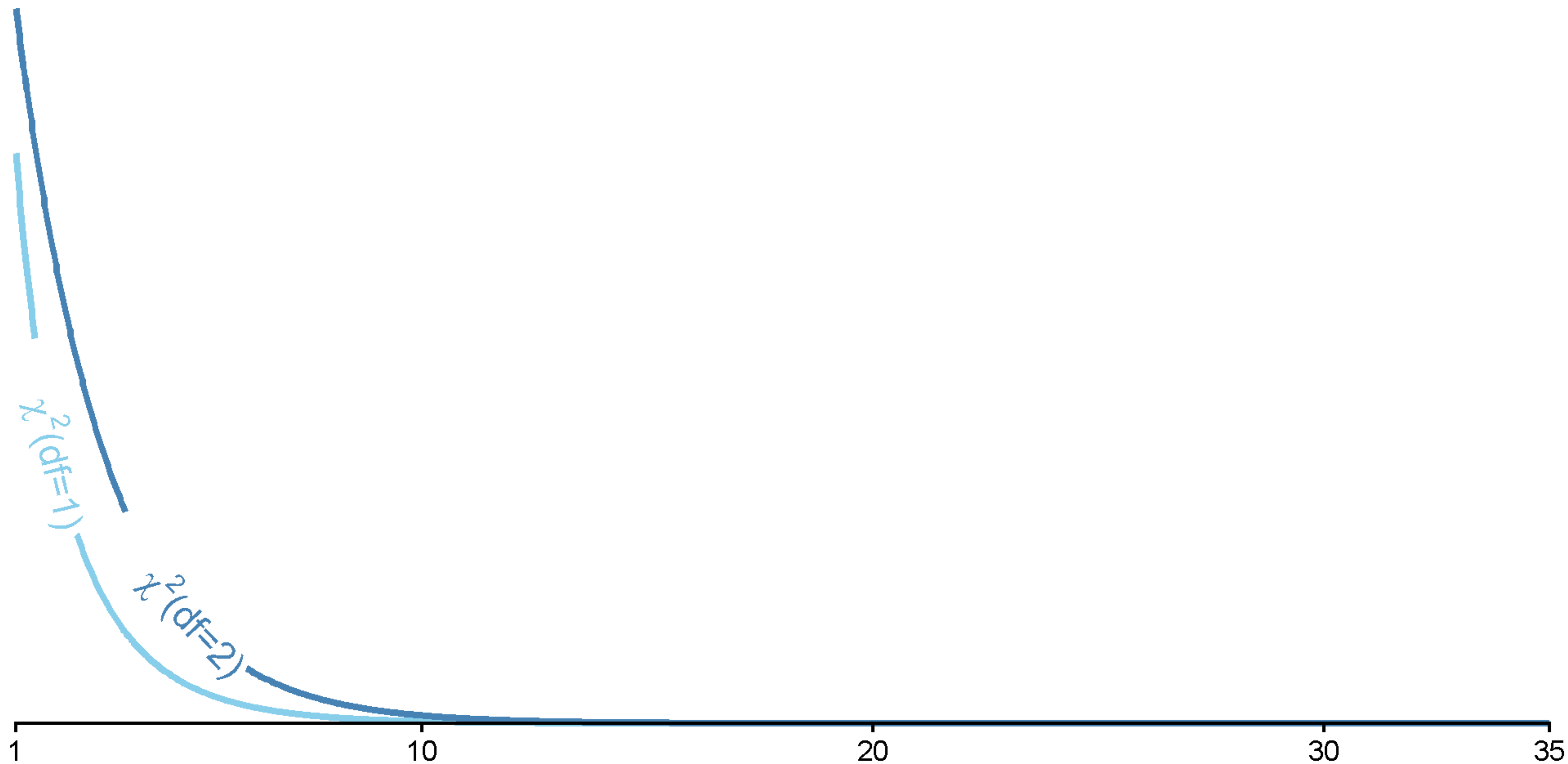
χ^2 Distribution



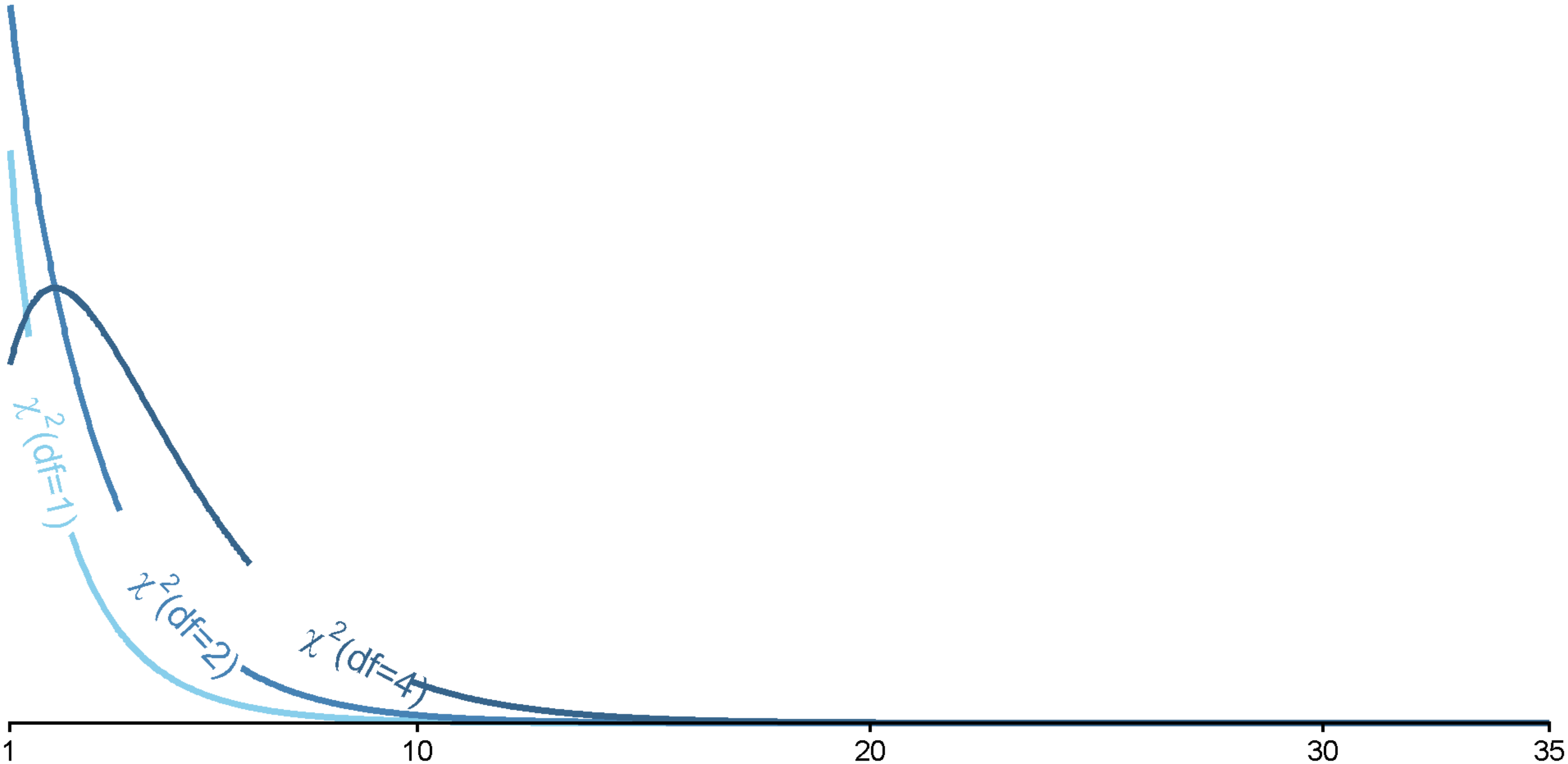
χ^2 Distribution



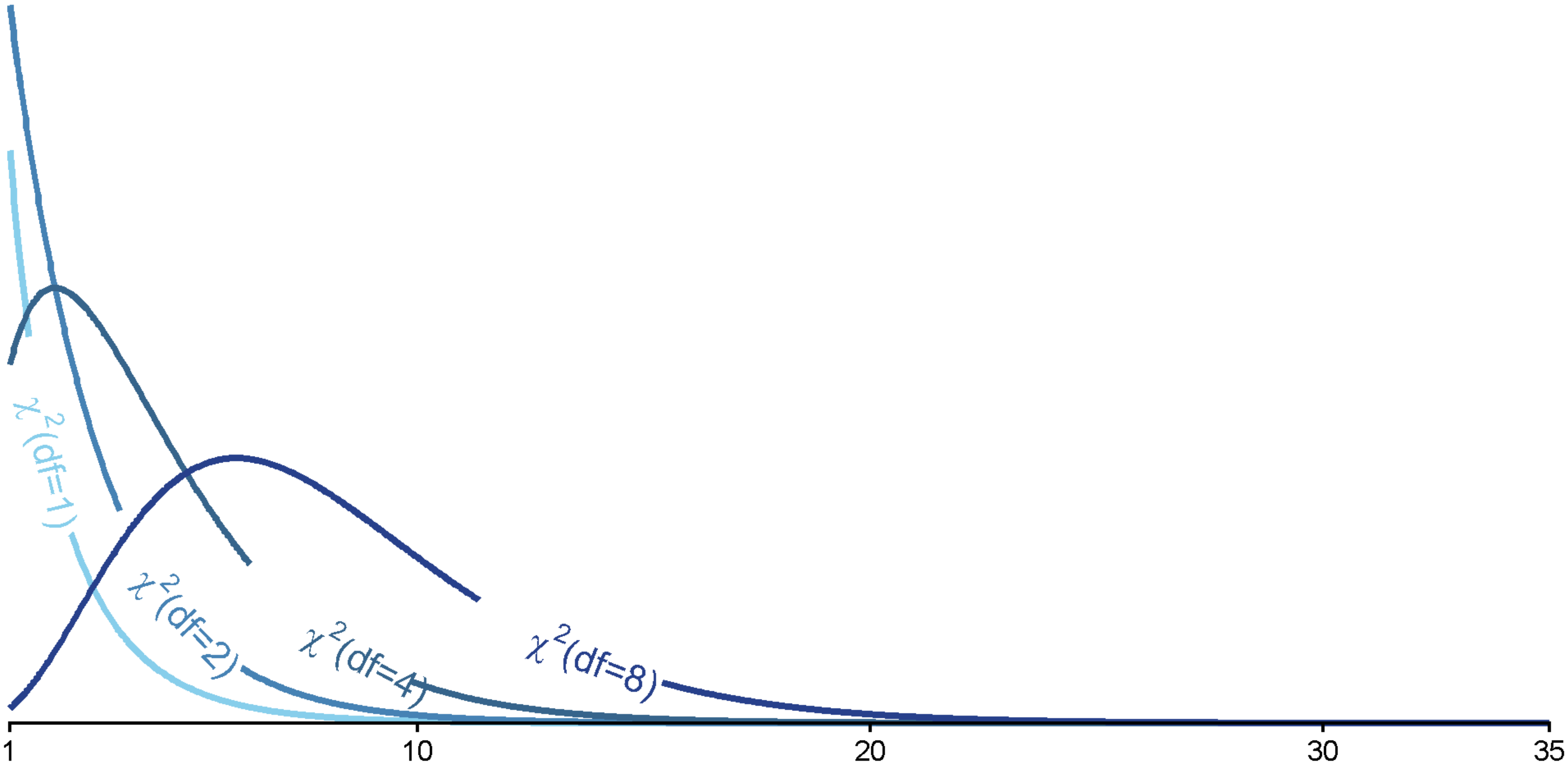
χ^2 Distribution



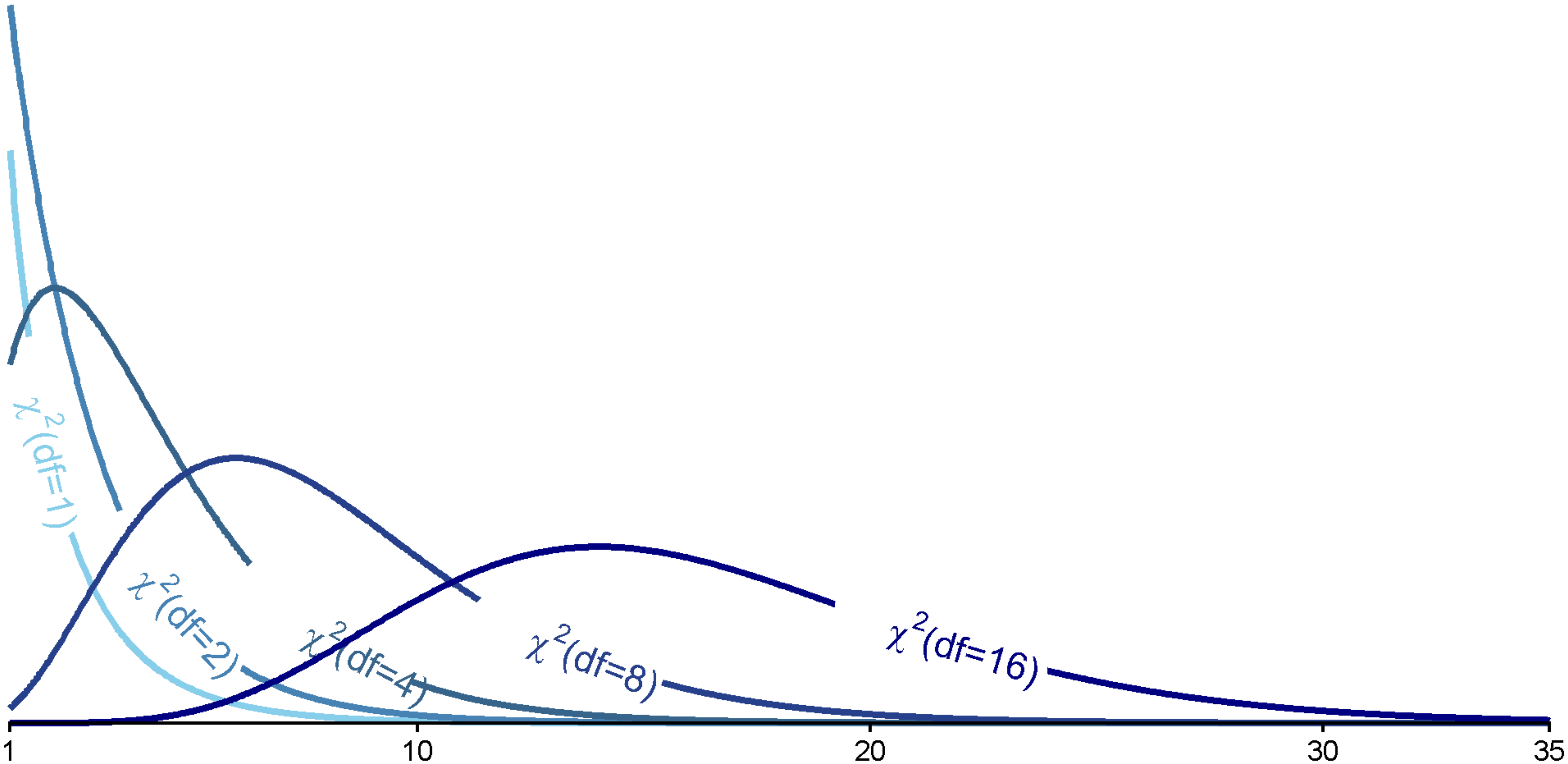
χ^2 Distribution



χ^2 Distribution



χ^2 Distribution



χ^2 Test of Goodness of Fit Cutoffs

Cutoff for test of heads vs. tails

$J = 2$

$\mathrm{df} = J - 1 = 1$

```r
qchisq(.95, 1)
```

```
[1] 3.841459
```

```r
qchisq(.05, 1, lower.tail = F)
```

```
[1] 3.841459
```

Cutoff for test of favorite color in the rainbow

$J = 7$ (ROYGBIV)

$\mathrm{df} = J - 1 = 6$

```r
qchisq(.95, 6)
```

```
[1] 12.59159
```

```r
qchisq(.05, 6, lower.tail = F)
```

```
[1] 12.59159
```

χ^2 Test of Goodness of Fit Example

You are an education psychologist interested in college students' choice of majors. You've grouped majors into five categories: STEM, Social Sciences, Humanities, Arts, and Business. You'd like to test whether there are differences in the population of students choosing those majors. You sample 100 students and ask what major they chose.

First, what are your expected frequencies for each category?

	STEM	Social Sciences	Humanities	Arts	Business
Expected					
Observed					

χ^2 Test of Goodness of Fit Example

You are an education psychologist interested in college students' choice of majors. You've grouped majors into five categories: STEM, Social Sciences, Humanities, Arts, and Business. You'd like to test whether there are differences in the population of students choosing those majors. You sample 100 students and ask what major they chose.

First, what are your expected frequencies for each category?

	STEM	Social Sciences	Humanities	Arts	Business
Expected	*20*	*20*	*20*	*20*	*20*
Observed					

χ^2 Test of Goodness of Fit Example

You are an education psychologist interested in college students' choice of majors. You've grouped majors into five categories: STEM, Social Sciences, Humanities, Arts, and Business. You'd like to test whether there are differences in the population of students choosing those majors. You sample 100 students and ask what major they chose.

Next, what is your χ^2 cutoff value?

	STEM	**Social Sciences**	**Humanities**	**Arts**	**Business**
Expected	*20*	*20*	*20*	*20*	*20*
Observed					

χ^2 Test of Goodness of Fit Example

Cutoff in R

```
df <- 5 - 1
chisq_crit <- qchisq(.95, df)
chisq_crit
```

```
[1] 9.487729
```

χ^2 Test of Goodness of Fit Example

You are an education psychologist interested in college students' choice of majors. You've grouped majors into five categories: STEM, Social Sciences, Humanities, Arts, and Business. You'd like to test whether there are differences in the population of students choosing those majors. You sample 100 students and ask what major they chose.

Here are the observed values:

	STEM	Social Sciences	Humanities	Arts	Business
Expected	*20*	*20*	*20*	*20*	*20*
Observed	65	10	5	5	15

χ^2 Test of Goodness of Fit Example

$$\chi^2 = \sum_{j=0}^{J} \frac{(y_j - N\pi_j)^2}{N\pi_j} = \sum_{j=0}^{J} \frac{(y_j - m_j)^2}{m_j}$$

```
obs_chisq <- (65 - 20)^2 / 20 +
  (10 - 20)^2 / 20 +
  (5 - 20)^2 / 20 +
  (5 - 20)^2 / 20 +
  (15 - 20)^2 / 20
obs_chisq
```

```
[1] 130
```

```
obs_chisq > chisq_crit
```

```
[1] TRUE
```

We can reject the null in favor of the alternative that the number of students is not the same across majors.

χ^2 Test of Goodness of Fit R Function

```r
majordat <- data.frame(major = c("STEM", "Social.Sciences", "Humanities", "Arts", "Business"),
           observed = c(65, 10, 5, 5, 15))

chisq <- majordat |>
  select(observed) |>
  chisq.test()
chisq
```

```
    Chi-squared test for given probabilities

data:  select(majordat, observed)
X-squared = 130, df = 4, p-value < 2.2e-16
```

```r
chisq$expected
```

```
[1] 20 20 20 20 20
```

```r
chisq$statistic
```

```
X-squared 
      130 
```

```r
chisq$p.value
```

```
[1] 3.89406e-27
```

χ^2 Test of Goodness of Fit Example

Let's restart and say that you were a educational psychologist interested in how students in STEM schools choose their major. In this situation, you had expected more students to select a STEM major than other majors.

These are your new expected frequencies (60% STEM, evenly divided across the rest):

	STEM	Social Sciences	Humanities	Arts	Business
Expected	*60*	*10*	*10*	*10*	*10*
Observed	65	10	5	5	15

χ^2 Test of Goodness of Fit R Function

```
chisq.test(x = c(65, 10, 5, 5, 15),
           p = c(60, 10, 10, 10, 10),
           rescale.p = T)
```

```
    Chi-squared test for given probabilities

data:  c(65, 10, 5, 5, 15)
X-squared = 7.9167, df = 4, p-value = 0.09468
```

```
chisq.test(x = c(65, 10, 5, 5, 15),
           p = c(.60, .10, .10, .10, .10))
```

```
    Chi-squared test for given probabilities

data:  c(65, 10, 5, 5, 15)
X-squared = 7.9167, df = 4, p-value = 0.09468
```

- `x` = data of frequencies
- `p` = expected probabilities
- `rescale.p` = [`T`/`F`], will rescale `p` to probabilities if you prefer to input frequencies

χ^2 Test of Independence

Do the observed frequencies of a categorical variable differ from what expected via properties of independence?

- Now considering more than one type of category
- E.g., Number of pets by cat vs. dog people and undergrad vs. post-graduate

	Cat Person	**Dog Person**
Undergrad		
Post-Graduate		

Expected frequencies calculated from the marginal totals of the contingency table

- Expected values determined from data

χ^2 Test of Independence: Hypotheses

H_0: Categorical Variable 1 and Categorical Variable 2 are independent (i.e., have no association)

H_1: Categorical Variable 1 and Categorical Variable 2 are not independent (i.e., have some association)

- Do expected frequencies match those expected via independence?

Mathematically the same as before:

H_0: $\pi_j = \pi_{j_0}$ for all categories j; difference for all categories is 0, where

π_j = observed proportion

π_{j_0} = expected proportion

H_1: $p_j \neq \pi_{j_0}$ for all categories j

> Just changing the expected frequencies

χ^2 Test of Independence Generally

$$\chi^2 = \sum_{j_1=1}^{J_1}\sum_{j_2=1}^{J_2} \frac{(O_j - N\pi_{j_0})^2}{N\pi_{j_0}} = \sum_{j_1=1}^{J_1}\sum_{j_2=1}^{J_2} \frac{(O_j - E_j)^2}{E_j}$$

Basically identical formula to calculate the χ^2.

The difference is that our expected frequencies E_j are derived from the data. Just like before, we calculate the difference from each cell, but now we have two categories that make up the cell.

The sum of the squared differences between the observed and expected frequencies, divided by the expected frequency, for each cell.

χ^2 Test of Independence Cutoffs

What is the critical χ^2 value in a test of independence for a 3x2?

$$\text{df} = (J_1 - 1) \times (J_2 - 1) = J_1 \times J_2 - 2$$ or

$$\text{df} = (n_{rows} - 1) \times (n_{cols} - 1) = n_{rows} \times n_{cols} - 2$$

```r
df <- (3 - 1) * (2 - 1)
df
```

```
[1] 2
```

```r
qchisq(.95, df)
```

```
[1] 5.991465
```

χ^2 Test of Independence Example

You are another educational psychologist, and you're interested in examining your department's student body makeup. You wonder whether there is an association between students' level (graduate vs. undergraduate) and whether they are an in-state student or not. You sample 50 students from your department, and these are the frequencies you observe.

	In-State	Out-of-State
Undergrad	25	10
Graduate	5	10

First, what are your expected frequencies for each category?

χ^2 Test of Independence Example: Calculating Expected Frequencies

First, get the marginal totals for each column

	In-State	Out-of-State	*Total*
Undergrad	25	10	
Graduate	5	10	
Total			

χ^2 Test of Independence Example: Calculating Expected Frequencies

First, get the marginal totals for each column

	In-State	Out-of-State	*Total*
Undergrad	25	10	*35*
Graduate	5	10	*15*
Total	*30*	*20*	*50*

χ^2 Test of Independence Example: Calculating Expected Frequencies

	In-State	**Out-of-State**	***Total***
Undergrad	25	10	*35*
Graduate	5	10	*15*
Total	*30*	*20*	*50*

	In-State	***Out-of-State***	***Total***
Undergrad	*21*	*14*	*35*
Graduate	*9*	*6*	*15*
Total	*30*	*20*	*50*

Expected cell frequency = E_j = $\frac{\text{column total} \times \text{row total}}{N}$

- Undergrad & In-State = $\frac{35 \times 30}{50} = 21$
- Undergrad & Out-of-State = $\frac{35 \times 20}{50} = 14$
- Graduate & In-State = $\frac{15 \times 30}{50} = 9$
- Graduate & Out-of-State = $\frac{15 \times 20}{50} = 6$

χ^2 Test of Goodness of Fit Example

$$\chi^2 = \sum_{j=0}^{J} \frac{(y_j - N\pi_j)^2}{N\pi_j} = \sum_{j=0}^{J} \frac{(y_j - m_j)^2}{m_j}$$

```r
data.frame(type = c("Undergrad", "Graduate", "Total"),
           `In-State` = c(25, 5, 30),
           `Out-of-State` = c(10, 10, 20),
           Total = c(35, 15, 50))
```

```
       type In.State Out.of.State Total
1 Undergrad       25           10    35
2  Graduate        5           10    15
3     Total       30           20    50
```

```r
chisq_crit <- qchisq(.95, df = (2 - 1) * (2 - 1))
chisq_crit
```

```
[1] 3.841459
```

```r
obs_chisq <- (25 - 21)^2 / 21 +
  (10 - 14)^2 / 14 +
  (5 - 9)^2 / 9 +
  (10 - 6)^2 / 6
obs_chisq
```

```
[1] 6.349206
```

```r
obs_chisq > chisq_crit
```

```
[1] TRUE
```

χ^2 Test of Independence R Function

```
studdat <- data.frame(type = c("Undergrad", "Graduate"),
           `In-State` = c(25, 5),
           `Out-of-State` = c(10, 10))
studdat |>
  select(-type) |>
  chisq.test(correct = F)
```

- `correct` = [`T`/`F`], applies a "continuity correction" to the χ^2 value. Default is `T`. Change to `F`!

```
	Pearson's Chi-squared test

data:  select(studdat, -type)
X-squared = 6.3492, df = 1, p-value = 0.01174
```

We can reject the null in favor of the alternative that students' level (graduate vs. undergraduate) and whether they are an in-state student or not have some association.

Working With Count Data

```r
dat |> head(20)
```

```
   student           major           level
1        1            Arts     STEM School
2        2 Social Sciences     STEM School
3        3 Social Sciences Non-STEM School
4        4 Social Sciences Non-STEM School
5        5            STEM     STEM School
6        6            Arts     STEM School
7        7 Social Sciences Non-STEM School
8        8            Arts Non-STEM School
9        9            Arts     STEM School
10      10 Social Sciences Non-STEM School
11      11        Business     STEM School
12      12 Social Sciences Non-STEM School
13      13      Humanities Non-STEM School
```

Working With Count Data: `table()`

Some ways to generate frequencies from categorical variables

```r
table(dat$major)
```

```

           Arts        Business      Humanities Social Sciences            STEM 
             19              20              17              26              18 
```

```r
table(dat$major) |>
  data.frame()
```

```
             Var1 Freq
1            Arts   19
2        Business   20
3      Humanities   17
4 Social Sciences   26
5            STEM   18
```

Working With Count Data: `table()`

```r
table(dat$major, dat$level)
```

```
                  Non-STEM School STEM School
  Arts                          8          11
  Business                      7          13
  Humanities                    8           9
  Social Sciences              12          14
  STEM                         11           7
```

```r
table(dat$major, dat$level) |>
  data.frame()
```

```
              Var1            Var2 Freq
1             Arts Non-STEM School    8
2         Business Non-STEM School    7
3       Humanities Non-STEM School    8
4  Social Sciences Non-STEM School   12
5             STEM Non-STEM School   11
6             Arts     STEM School   11
7         Business     STEM School   13
8       Humanities     STEM School    9
9  Social Sciences     STEM School   14
10            STEM     STEM School    7
```

Working With Count Data: `summarize()`

Some ways to generate frequencies from categorical variables

```
dat |>
  summarize(.by = c(major, level),
            n = n())
```

```
            major          level  n
1            Arts     STEM School 11
2 Social Sciences     STEM School 14
3 Social Sciences Non-STEM School 12
4            STEM     STEM School  7
5            Arts Non-STEM School  8
6        Business     STEM School 13
7      Humanities Non-STEM School  8
8        Business Non-STEM School  7
9            STEM Non-STEM School 11
10     Humanities     STEM School  9
```

```
dat |>
  summarize(.by = c(major, level),
            n = n()) |>
  arrange(major, level)
```

```
             major          level  n
1             Arts Non-STEM School  8
2             Arts     STEM School 11
3         Business Non-STEM School  7
4         Business     STEM School 13
5       Humanities Non-STEM School  8
6       Humanities     STEM School  9
7             STEM Non-STEM School 11
8             STEM     STEM School  7
9  Social Sciences Non-STEM School 12
10 Social Sciences     STEM School 14
```

Working With Count Data: `count()`

`count(x, grouping_var1, grouping_var2, etc.)`

- `x` = dataframe
- `grouping_vars` = variable names to group by

Basically shorthand for:

```
df |>
  summarise(.by = c(a, b),
            n = n())
```

Also seems to `arrange` by default

Working With Count Data: `count()`

`count(x, grouping_var1, grouping_var2, etc.)`

- `x` = dataframe
- `grouping_vars` = variable names to group by

```r
dat |>
  summarize(.by = c(major, level),
            n = n())
```

```
             major           level  n
1             Arts     STEM School 11
2  Social Sciences     STEM School 14
3  Social Sciences Non-STEM School 12
4             STEM     STEM School  7
5             Arts Non-STEM School  8
6         Business     STEM School 13
7       Humanities Non-STEM School  8
8         Business Non-STEM School  7
9             STEM Non-STEM School 11
10      Humanities     STEM School  9
```

```r
dat |>
  count(major, level)
```

```
             major           level  n
1             Arts Non-STEM School  8
2             Arts     STEM School 11
3         Business Non-STEM School  7
4         Business     STEM School 13
5       Humanities Non-STEM School  8
6       Humanities     STEM School  9
7             STEM Non-STEM School 11
8             STEM     STEM School  7
9  Social Sciences Non-STEM School 12
10 Social Sciences     STEM School 14
```

Working With Count Data: `summarize()`

If going to put into the `chisq.test()` function, want to pivot and get rid of additional columns.

Many ways to do this, but one example:

```
chisqdat <- dat |>
  count(major, level) |>
  pivot_wider(names_from = level,
              values_from = n) |>
  select(-major)
chisqdat
```

```
# A tibble: 5 × 2
  `Non-STEM School` `STEM School`
              <int>         <int>
1                 8            11
2                 7            13
3                 8             9
4                11             7
5                12            14
```

```
chisq.test(chisqdat, correct = F)
```

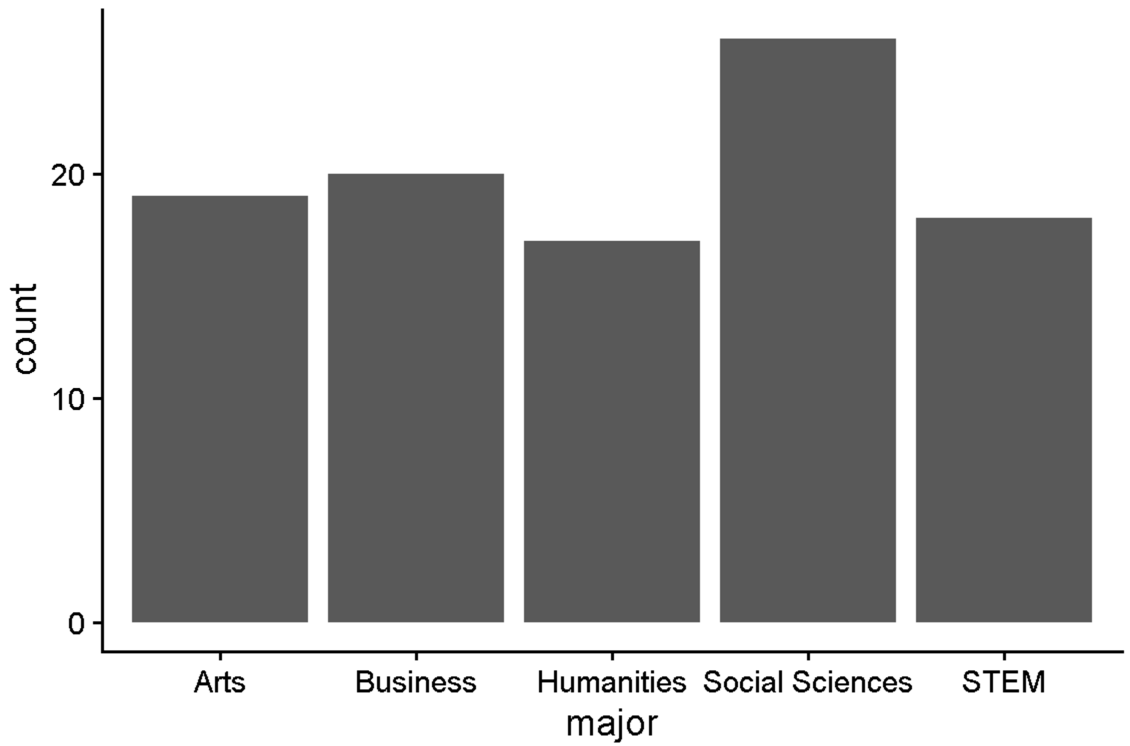
```

	Pearson's Chi-squared test

data:  chisqdat
X-squared = 2.7529, df = 4, p-value = 0.6
```

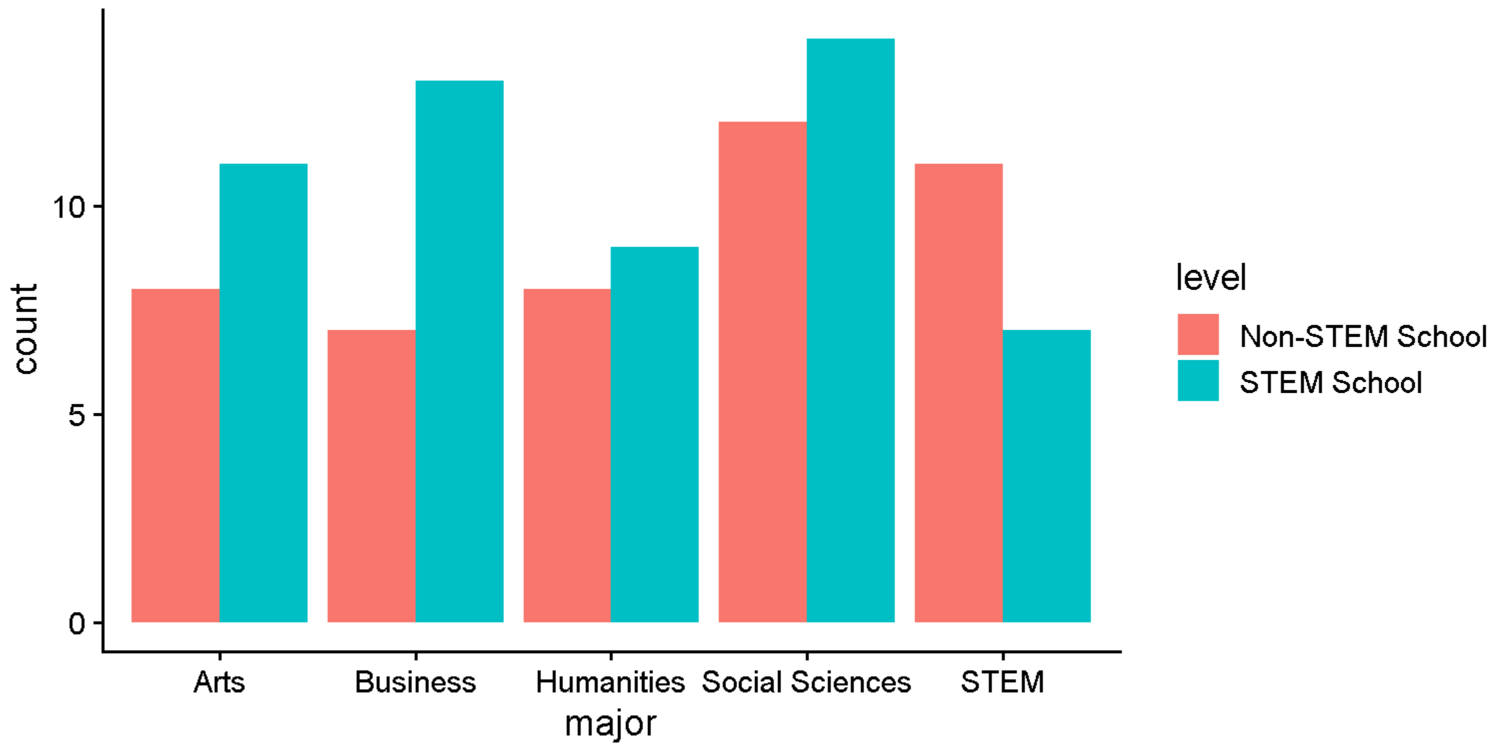
Working With Count Data: `geom_bar()`

Plot | Code



Working With Count Data: `geom_bar()`

Plot | Code



Assignment 12