Independent and Paired Sample *T*-Test

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Outline

- Assignment 10 Review
- Independent Sample *T*-Test
- Paired Sample *T*-Test

Learning objectives: **R:** *T*-Tests

Assignment 10 Review

[placeholder for Assignment 10 review]

T-Statistic Cutoffs

Say you want to find the cutoff *t* values: t(df = 3) and t(df = 37) for $\alpha = .05$, want both upper one-tailed and two-tailed

Which will have the larger magnitude cutoff values?

One-Tailed

All of our $\alpha = .05$ goes on the upper side

Need a cutoff for $.95$

```r
qt(p = .95, df = 3)
```

```
[1] 2.353363
```

```r
qt(p = .05, df = 37, lower.tail = F)
```

```
[1] 1.687094
```

$t_{crit}(3) = 2.35$

$t_{crit}(37) = 1.69$

Two-Tailed

$\alpha = 1 - .95 = 5\%$ on both sides

So $.05/2 = .025$ on each side

Need value for $.025$ and $.95 + .025$ ($[.025, .975]$)

```r
qt(p = c(.025, .975), df = 3)
```

```
[1] -3.182446  3.182446
```

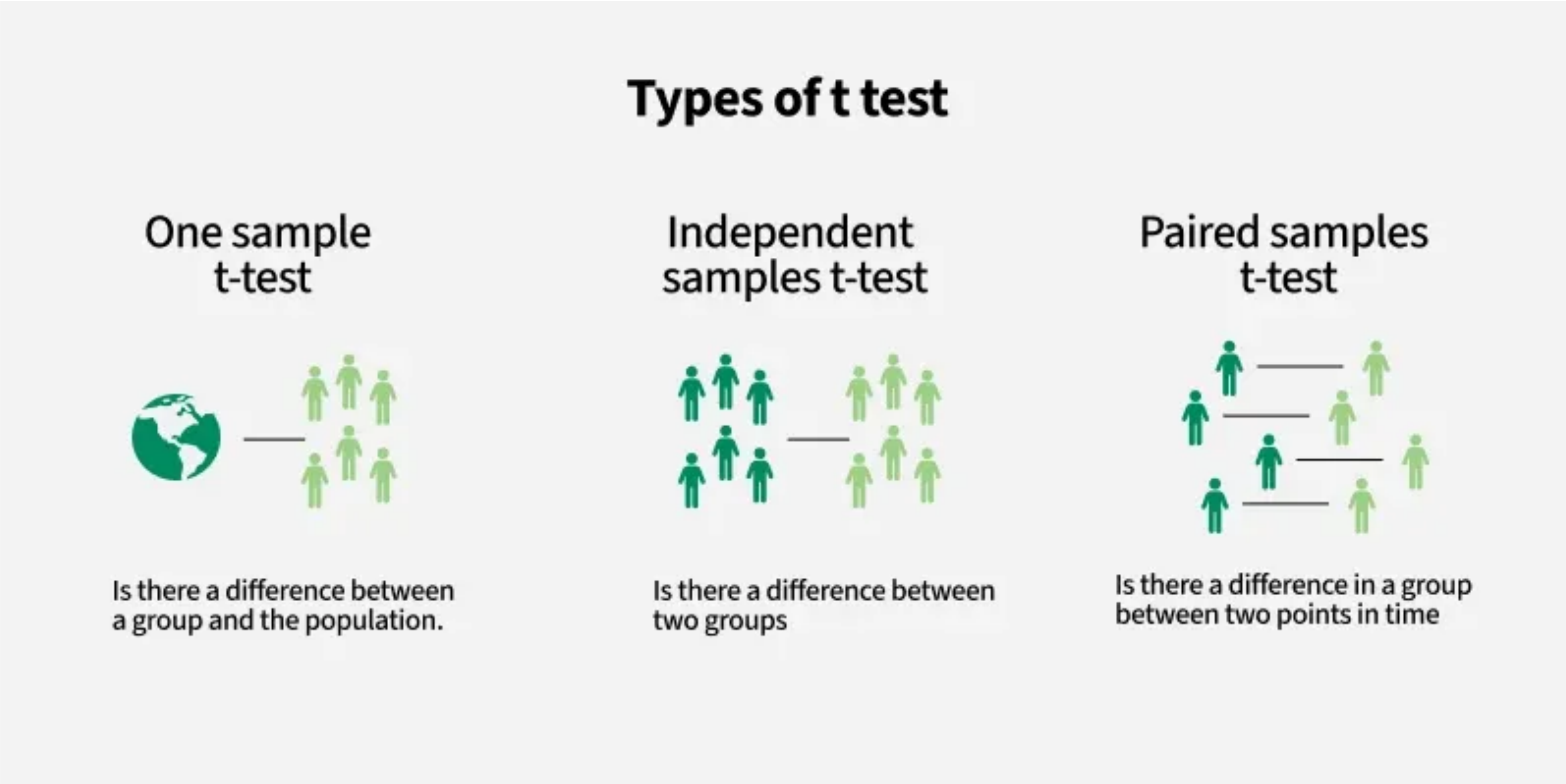
```r
qt(p = c(.025, .975), df = 37)
```

```
[1] -2.026192  2.026192
```

$t_{crit}(3) = [-3.18, 3.18]$

$t_{crit}(37) = [-2.03, 2.06]$

Types of *T*-Tests



Independent Sample *T*-Test

Might want to compare two samples of observations

- Post-graduation salaries of GT vs. UGA grads
- Treatment vs. control group
- Cats vs. dogs

We can represent each group with their mean on some outcome and compare the means

Consider whether the mean difference is significant but also have to account for different levels of variability within group

Independent Sample *T*-Test: Pooled SD

Need to pool the sample SDs while noting different sample sizes

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)} = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Independent Sample *T*-Test: Pooled SD

Note that when sample size is equal (i.e., $n_1 = n_2 = n$),

$$
\begin{aligned}
s_p^2\Big|_{n_1=n_2=n} &= \frac{(n-1)s_1^2+(n-1)s_2^2}{(n-1)+(n-1)} = \frac{(n-1)(s_1^2+s_2^2)}{(n-1)+(n-1)} \\
&= \frac{(n-1)(s_1^2+s_2^2)}{2(n-1)} = \frac{s_1^2+s_2^2}{2}
\end{aligned}
$$

Square root to get back to standard deviation

$$
s_p = \sqrt{s_p^2} = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{(n_1-1)+(n_2-1)}} = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}
$$

Independent Sample *T*-Test: Test statistic

$$t = \frac{\bar{x_1} - \bar{x_2}}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}} = \frac{\bar{x_1} - \bar{x_2}}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Compare against critical *t*-value with df = $n - 1$

Independent Sample *T*-Test: R Calculation

```r
summary(candy)
```

```
    candy               sales      
 Length:40          Min.   : 2.12  
 Class :character   1st Qu.: 5.48  
 Mode  :character   Median : 9.75  
                    Mean   :10.15  
                    3rd Qu.:15.32  
                    Max.   :17.83  
```

```r
ccorn <- candy |> filter(candy == "candy corn") |> pluck("sales")
choc <- candy |> filter(candy == "chocolate") |> pluck("sales")

ccorn
```

```
 [1] 5.51 4.09 5.57 6.85 4.83 8.60 4.53 2.12 7.60 3.24 4.82 5.56 6.17 4.53 6.15
[16] 3.75 6.61 5.39 6.44 4.82
```

```r
choc
```

```
 [1] 17.06 14.46 14.21 14.66 16.90 13.16 17.83 10.90 11.31 15.46 15.52 13.72
[13] 15.43 17.10 14.97 15.94 15.70 15.29 12.24 17.09
```

Independent Sample *T*-Test: R Calculation

```r
x_bar_ccorn <- mean(ccorn)
sd_ccorn <- sd(ccorn)
n_ccorn <- length(ccorn)

x_bar_choc <- mean(choc)
sd_choc <- sd(choc)
n_choc <- length(choc)

pooled_sd <- sqrt( ((n_ccorn - 1) * sd_ccorn^2 + (n_choc - 1) * sd_choc^2) /
                     ((n_ccorn - 1) + (n_choc - 1)) )
pooled_sd
```

```
[1] 1.732816
```

Independent Sample *T*-Test: R Calculation

```r
df <- (n_ccorn - 1) + (n_choc - 1)

t_crit <- qt(c(.025, .975), df)
t_crit
```

```
[1] -2.024394  2.024394
```

If our observed *t*-statistic exceeds the bounds of [-2.02, 2.02], we reject the null.

```r
t_stat <- (x_bar_ccorn - x_bar_choc) / (pooled_sd * sqrt(1 / n_ccorn + 1 / n_choc))
t_stat
```

```
[1] -17.49839
```

We reject the null! Can we tell which had a larger mean?

Independent Sample *T*-Test: R Function

```r
t.test(sales ~ candy, candy, var.equal = T)
```

```

	Two Sample t-test

data:  sales by candy
t = -17.498, df = 38, p-value < 2.2e-16
alternative hypothesis: true difference in means between group candy corn and group chocolate is not equal to 0
95 percent confidence interval:
 -10.697796  -8.479204
sample estimates:
mean in group candy corn  mean in group chocolate 
                  5.3590                  14.9475 
```

Paired Sample *T*-Test

Comparing the same subjects under two conditions (typically two timepoints)

- Stress levels before vs. after a test
- Reaction times before vs. after caffiene consumption
- Anxiety levels before vs. after a mindfulness intervention

We can still use the mean to represent the observations in each condition and make comparisons

This time, focus on difference scores (individual differences in the observations between the two conditions)

Paired Sample *T*-Test

Need difference score for each observation $d_i = x_2 - x_1$

Then, get mean difference $\bar{d} = \frac{d_1 + d_2 + \ldots + d_n}{n}$ and standard deviation of those difference scores $s_d = \sqrt{\frac{(d_1 - \bar{d})^2 + (d_2 - \bar{d})^2 + \ldots + (d_n - \bar{d})^2}{n-1}}$

$$t = \frac{\bar{d} - \mu_{d_0}}{\frac{s_d}{\sqrt{n}}} = (\text{usually}) \frac{\bar{d} - 0}{\frac{s_d}{\sqrt{n}}} = \frac{\bar{d}}{\frac{s_d}{\sqrt{n}}}$$

Compare against critical *t*-value with df = $n - 1$

Paired Sample *T*-Test: R Calculation

```r
mean_diff <- mean(pumpkin$diff)
sd_diff <- sd(pumpkin$diff)
df <- nrow(pumpkin) - 1

qt(c(.025, .975), df)
```

```
[1] -2.093024  2.093024
```

```r
t_stat <- (mean_diff - 0) / (sd_diff / sqrt(nrow(pumpkin)))
t_stat
```

```
[1] 11.59213
```

Paired Sample *T*-Test: R Function

```r
t.test(pumpkin$Oct1, pumpkin$Oct31, paired = T, var.equal = T)
```

```
	Paired t-test

data:  pumpkin$Oct1 and pumpkin$Oct31
t = 11.592, df = 19, p-value = 4.636e-10
alternative hypothesis: true mean difference is not equal to 0
95 percent confidence interval:
 2.55216 3.67684
sample estimates:
mean difference 
         3.1145 
```

Assumptions

[placeholder for assumtions: align with lecture slides when available]

Cohen's *D* Revisited

Now that we have two groups, R functions for Cohen's *D* will be more helpful to us. The `cohens_d()` function from the `effectsize` package allows for many formats of inputs.

```
(x_bar_ccorn - x_bar_choc) / pooled_sd
```

```
[1] -5.533478
```

```
effectsize::cohens_d(sales ~ candy, data = ca
```

```
Cohen's d |         95% CI
--------------------------
-5.53     | [-6.91, -4.14]

- Estimated using pooled SD.
```

```
mean_diff / sd_diff
```

```
[1] 2.592079
```

```
effectsize::cohens_d(pumpkin$Oct1, pumpkin$Oc
```

```
Cohen's d |       95% CI
------------------------
2.59      | [1.66, 3.51]
```

T-Test: Interpretation and write-up

An [independent samples / repeated measures] *t*-test analysis was conducted between [group 1] and [group 2]. We [fail to reject / reject] H0 [in favor of H1], t([df]) = [_____], p [< 0.05 / p-value]. There [is / is not] a significant difference between the mean [DV] of [group 1] and [group 2] in our sample.

Happy Halloween!