Correlation and Regression

PSYC 2020-A01 / PSYC 6022-A01 | 2025-11-14 | Lab 13

Jessica Helmer

Outline

- Assignment 12 Review
- Correlation
- Regression

Learning objectives:

R: Correlation, regression

Assignment 12 Review

[placeholder for Assignment 12 review]

Correlation

Correlation

Measure of association: how strongly are two variables related?

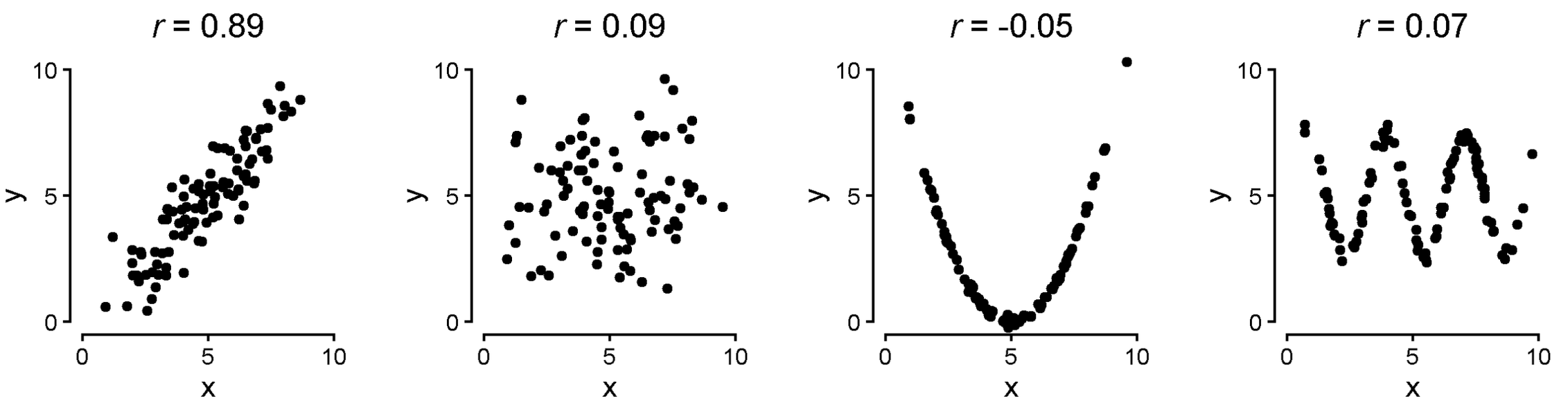
Indexes the linear relationship between two variables

We will cover correlation for two continuous variables

Correlation

Only measures **linear** relationships

Plot | Code



Correlation Generally

$$r_{xy} = \frac{\sum_{i=1}^{I}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y}$$

where

i = index of observation i out of I total observations

$\bar{x}$ = mean of x

$\bar{y}$ = mean of y

s_x = standard deviation of x

s_y = standard deviation of y

Correlation Generally

$$r_{xy} = \frac{\sum_{i=1}^{I}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y}$$

By subtracting the mean and dividing by the standard deviation, this formula is converting the observations to *z*-scores.

Correlation Example

$$r_{xy} = \frac{\sum_{i=1}^{I}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y}$$

Let’s find the correlation between iris’ sepal length and sepal width. For demonstration, let’s only use the first four rows.

```r
iris_small <- iris |>
  select(Sepal.Length, Sepal.Width) |>
  head(4)
iris_small
```

```
  Sepal.Length Sepal.Width
1          5.1         3.5
2          4.9         3.0
3          4.7         3.2
4          4.6         3.1
```

```r
x_bar <- mean(iris_small$Sepal.Length)
x_sd <- sd(iris_small$Sepal.Length)

y_bar <- mean(iris_small$Sepal.Width)
y_sd <- sd(iris_small$Sepal.Width)

iris_small <- iris_small |>
  mutate(sq_diff_x = Sepal.Length - x_bar,
         sq_diff_y = Sepal.Width - y_bar,
         product = sq_diff_x * sq_diff_y)
iris_small
```

```
  Sepal.Length Sepal.Width sq_diff_x sq_diff_y
product
1          5.1         3.5     0.275       0.3
0.0825
2          4.9         3.0     0.075      -0.2
-0.0150
3          4.7         3.2    -0.125       0.0
0.0000
4          4.6         3.1    -0.225      -0.1
0.0225
```

```r
sum(iris_small$product) / ((nrow(iris_small)
```

```
[1] 0.6263001
```

Correlation in R

$$r_{xy} = \frac{\sum_{i=1}^{I}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y}$$

Let's find the correlation between iris' sepal length and sepal width. For demonstration, let's only use the first four rows.

```
1 iris_small <- iris |>
2   select(Sepal.Length, Sepal.Width) |>
3   head(4)
4 iris_small
```

```
  Sepal.Length Sepal.Width
1          5.1         3.5
2          4.9         3.0
3          4.7         3.2
4          4.6         3.1
```

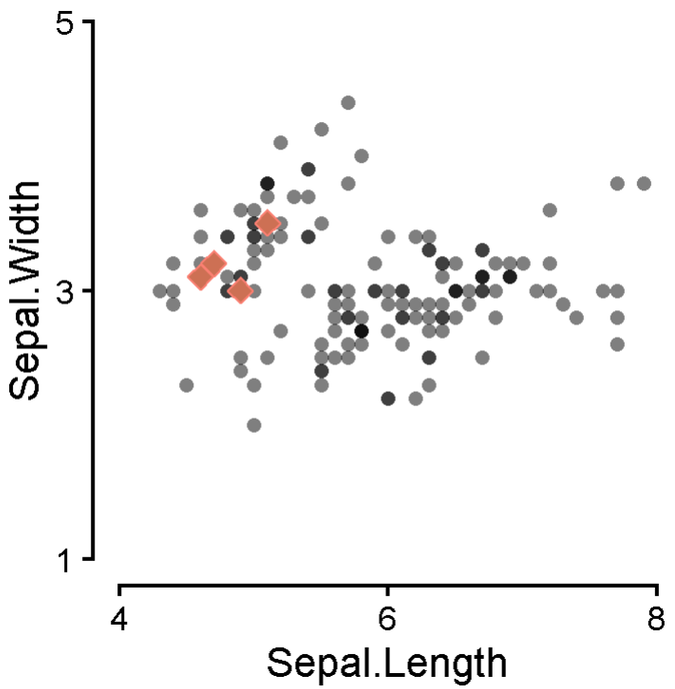
```
1 cor(iris_small$Sepal.Length, iris_small$Sepal
```

```
[1] 0.6263001
```

Correlation in R

As a note, those four points were not actually representative of the general trend!

Plot Code




```
1  cor(iris$Sepal.Length, iris$Sepal.Width)
```

```
[1] -0.1175698
```

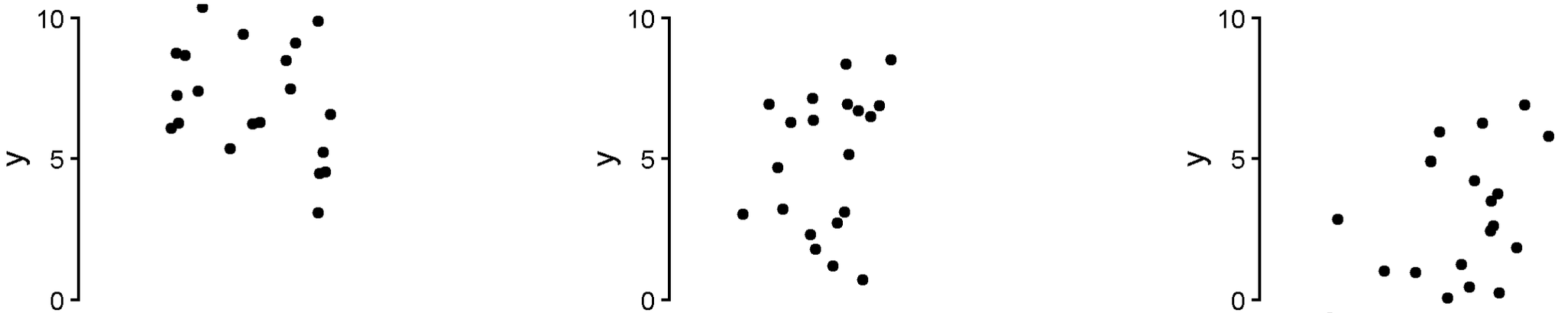
Regression

Regression

If you are working with just one variable, what is the best way you can represent the data?

If you had to pick just one statistic, what might you choose?

Plot Code

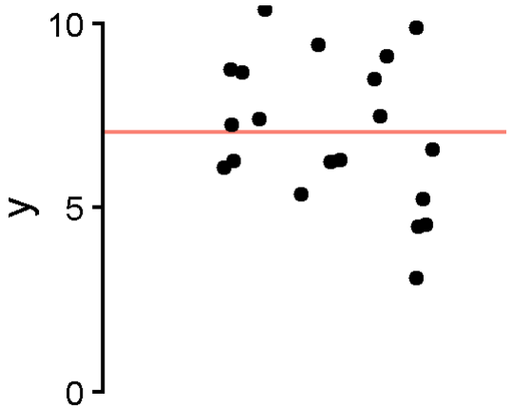

10
5
0
y
10
5
0
y
10
5
0
y

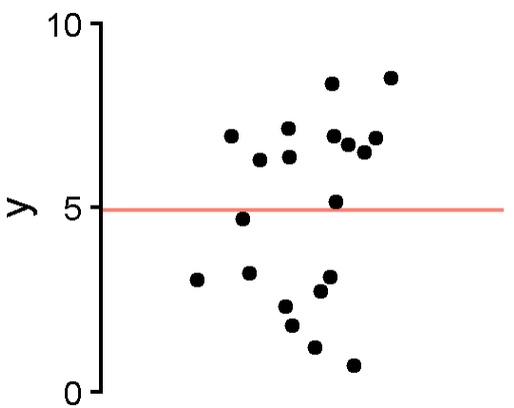
Regression

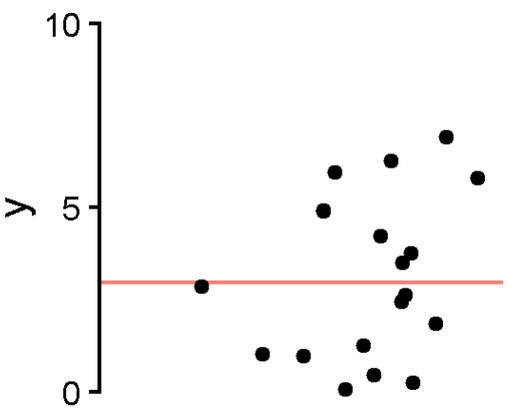
If we consider “best” to mean *minimizing the squared distances from the data*, the **mean** is the best fit.

The mean has the smallest squared errors from the observations.

Plot | Code


10
y
5
0


10
y
5
0

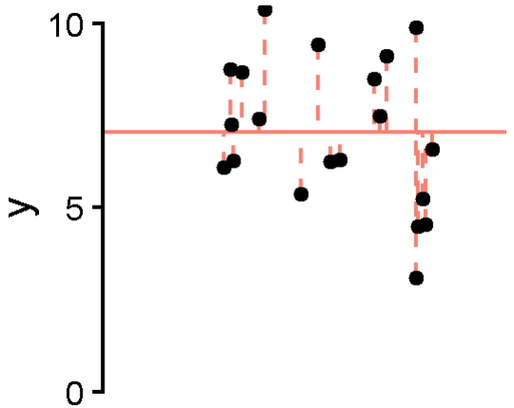

10
y
5
0

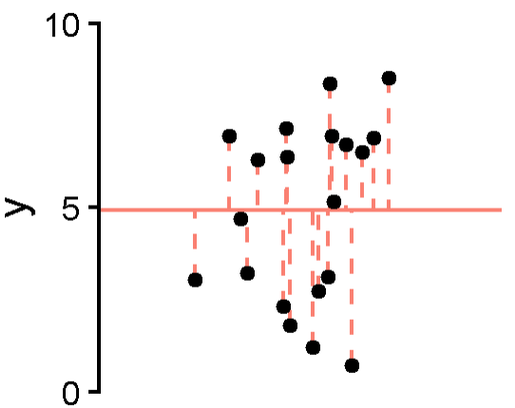
Regression

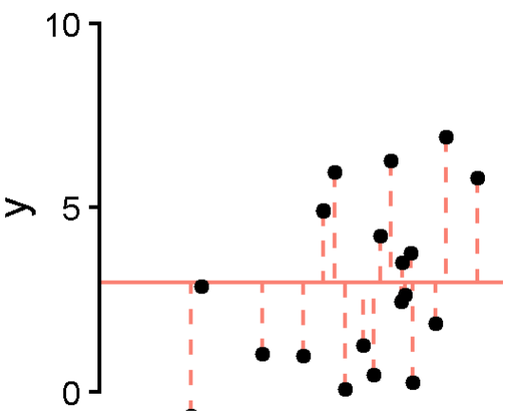
If we consider “best” to mean *minimizing the squared distances from the data*, the **mean** is the best fit.

The mean has the smallest squared errors from the observations.

Plot | Code


10
5
0
y


10
5
0
y

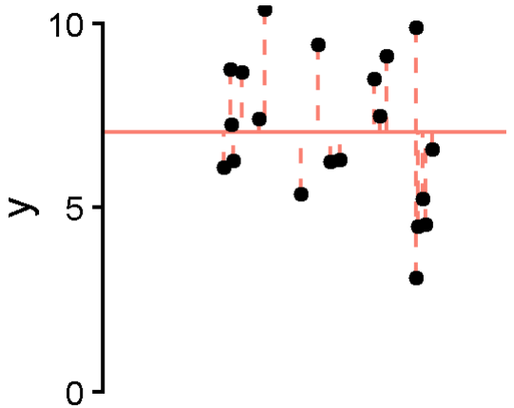

10
5
0
y

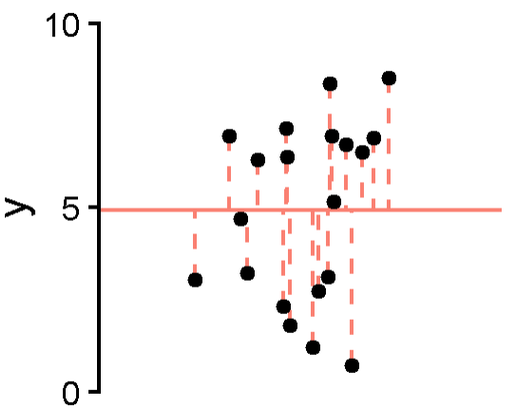
Regression

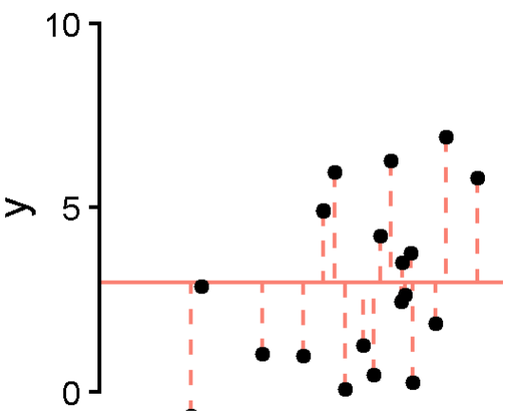
This is equivalent to the “mean intercept model” regression equation

$$y_i = b_0 + \epsilon_i$$

Plot Code


10
5
0
y


10
5
0
y


10
5
0
y

Intercept Model to Regression

When we have *one* variable of interest, the mean is the best single estimator.

- $y = b_0$ would be the line of best fit

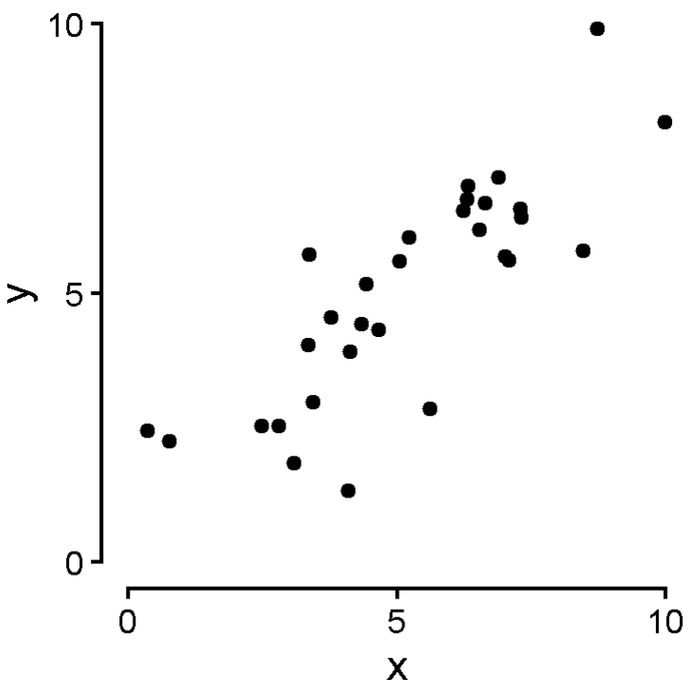
If we have *two* variables and we want to predict one from the other, we now may need a *slope* to find a line of best fit.

- This model will typically have both an intercept and a slope

Intercept Model to Regression

We have the same question: which line can represent points most effectively?

Plot | Code

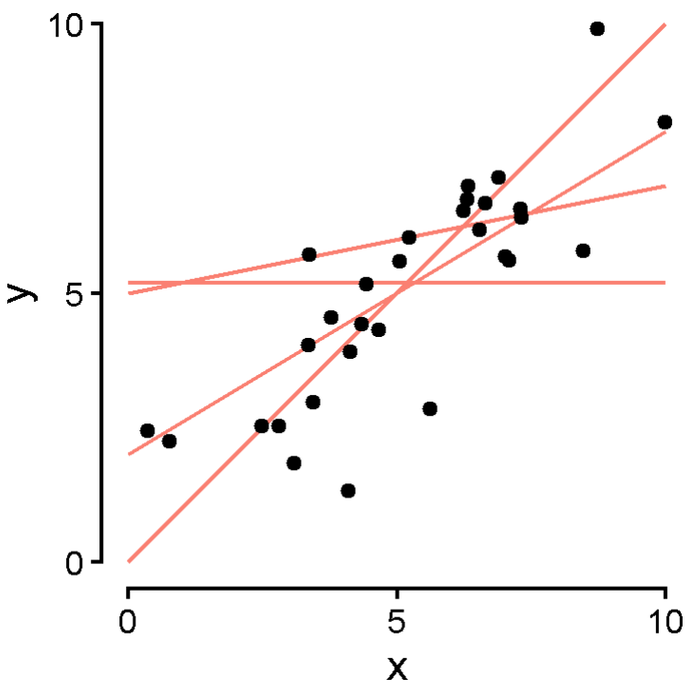


Lots of potential lines we could use

Intercept Model to Regression

We have the same question: which line can represent points most effectively?

Plot | Code



Lots of potential lines we could use

Intercept Model to Regression

Considering the relationship of y with another variable x

$$y_i = b_0 + b_1 \times x_i + \epsilon_i$$

where

i = index of observation i out of I total observations

y = outcome

x = predictor

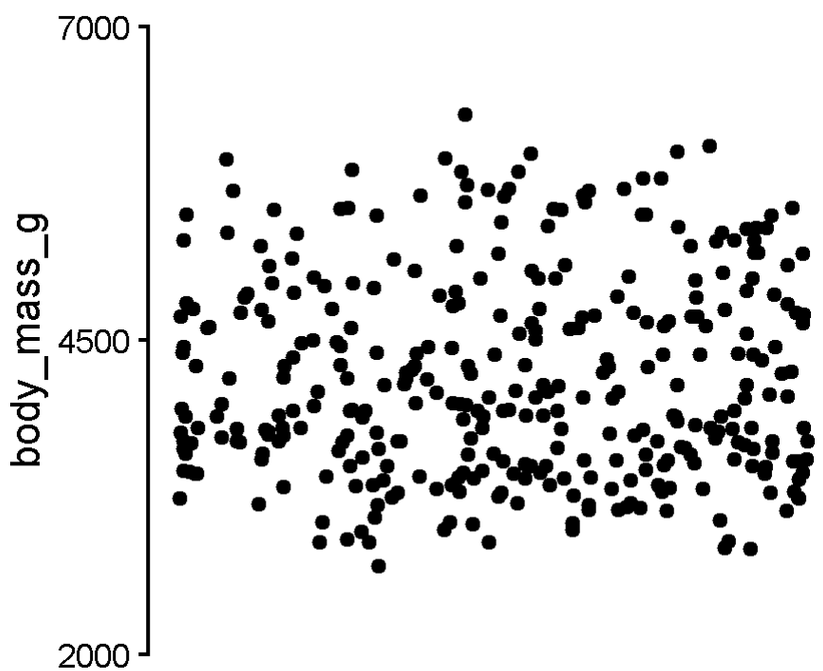
b_0 = intercept

b_1 = slope for x

Regression Example: `penguins`

Let's predict penguins' body mass!

Plot | Code



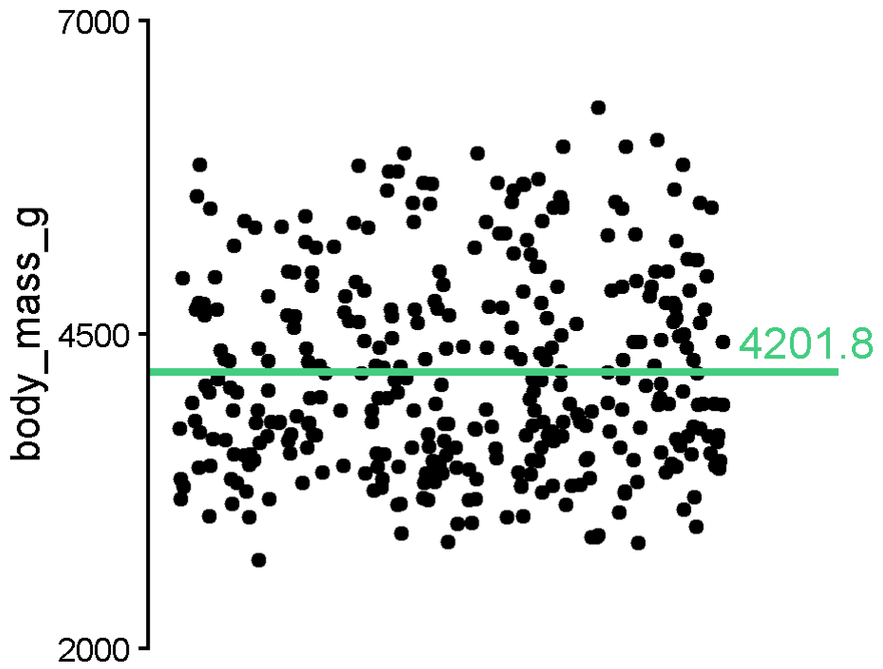
With just this one variable, what would be our best prediction for these data?

The mean!

Regression Example: `penguins`

Let's predict penguins' body mass!

Plot | Code



With just this one variable, what would be our best prediction for these data?

The mean!

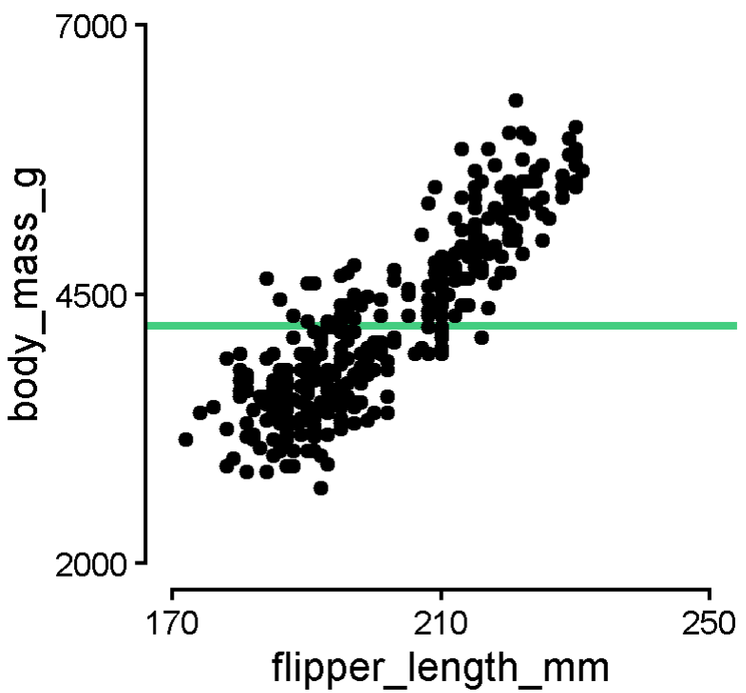
If we needed to predict any random penguin's body mass with only these data, our *best estimate* would be the mean, 4201.75.

Now, let's take into account another variable: a penguin's

Regression Example: `penguins`

Let's predict penguins' body mass!

Plot | Code



Now, let's take into account another variable: a penguin's flipper length

Just the mean no longer seems like our line of best fit

Regression Example in R

Let's predict penguins' body mass!

`lm(y ~ x, data = data)`

- `y` = outcome
- `x` = predictor, use `1` for just intercept
- `data` = dataframe that includes `x` and `y`

`summary(model)` to see results

Regression Example in R

Let's predict penguins' body mass!

```r
penguins_m0 <- lm(body_mass_g ~ 1, data = penguins)
summary(penguins_m0)
```

```
Call:
lm(formula = body_mass_g ~ 1, data = penguins)

Residuals:
    Min      1Q  Median      3Q     Max 
-1501.8  -651.8  -151.8   548.2  2098.2 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  4201.75      43.36   96.89   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Regression Example in R

Let’s predict penguins’ body mass!

```r
penguins_m1 <- lm(body_mass_g ~ flipper_length_mm, data = penguins)
summary(penguins_m1)
```

```
Call:
lm(formula = body_mass_g ~ flipper_length_mm, data = penguins)

Residuals:
     Min       1Q   Median       3Q      Max 
-1058.80  -259.27   -26.88   247.33  1288.69 

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)    
(Intercept)       -5780.831    305.815  -18.90   <2e-16 ***
flipper_length_mm    49.686      1.518   32.72   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Regression Example in R: Output anatomy

```r
summary(penguins_m1)
```

```
Call:
lm(formula = body_mass_g ~ flipper_length_mm,
data = penguins)

Residuals:
     Min       1Q   Median       3Q      Max
-1058.80  -259.27   -26.88   247.33  1288.69

Coefficients:
                   Estimate Std. Error t value
Pr(>|t|)
(Intercept)       -5780.831    305.815  -18.90
<2e-16 ***
```

```r
summary(penguins_m1)$coefficients
```

```
                    Estimate Std. Error   t
value      Pr(>|t|)
(Intercept)       -5780.83136 305.814504
-18.90306  5.587301e-55
flipper_length_mm    49.68557   1.518404
32.72223 4.370681e-107
```

```r
summary(penguins_m1)$r.squared
```

```
[1] 0.7589925
```

```r
summary(penguins_m1)$adj.r.squared
```

```
[1] 0.7582837
```

Regression Example in R: Line of best fit

Let's predict penguins' body mass!

We can extract predicted values to create our line of best fit with `predict()`

`predict(model, newdata)`

- `newdata` = dataframe containing theoretical values of the predictor(s)

- Should have the same column name(s)

```
1 predict(penguins_m1, newdata = data.frame(flipper_length_mm = seq(170, 240, 1)))
```

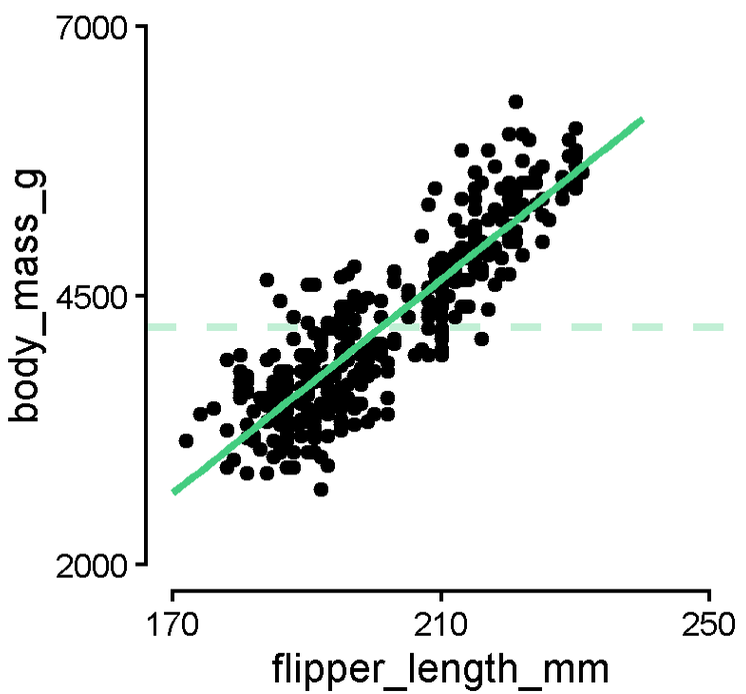
```
       1        2        3        4        5        6        7        8 
2665.715 2715.400 2765.086 2814.772 2864.457 2914.143 2963.828 3013.514 
       9       10       11       12       13       14       15       16 
3063.199 3112.885 3162.571 3212.256 3261.942 3311.627 3361.313 3410.998 
      17       18       19       20       21       22       23       24 
3460.684 3510.370 3560.055 3609.741 3659.426 3709.112 3758.797 3808.483 
      25       26       27       28       29       30       31       32 
```

3858.169	3907.854	3957.540	4007.225	4056.911	4106.596	4156.282	4205.967
33	34	35	36	37	38	39	40
4255.653	4305.339	4355.024	4404.710	4454.395	4504.081	4553.766	4603.452
41	42	43	44	45	46	47	48
4653.138	4702.823	4752.509	4802.194	4851.880	4901.565	4951.251	5000.937
49	50	51	52	53	54	55	56

Regression Example in R: Line of best fit

```r
predicted_data <- data.frame(flipper_length_mm = seq(170, 240, 1)) |>
  mutate(predicted_body_mass = predict(penguins_m1, newdata = data.frame(flipper_length_mm)))
```

Plot | Code



Regression Example in R: Adding a predictor

We can add predictors by adding them to the right hand side of the formula

```r
penguins_m2 <- lm(body_mass_g ~ flipper_length_mm + bill_length_mm, data = penguins)
summary(penguins_m2)
```

```
Call:
lm(formula = body_mass_g ~ flipper_length_mm + bill_length_mm,
    data = penguins)

Residuals:
    Min      1Q  Median      3Q     Max
-1090.5  -285.7   -32.1   244.2  1287.5

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)       -5736.897    307.959 -18.629   <2e-16 ***
flipper_length_mm    48.145      2.011  23.939   <2e-16 ***
```

Regression Example: Writeup

Controlling for bill length, the effect of flipper length on penguins body mass was significant ($p < .001$). For every mm increase in flipper length, we expect to see a 46.15g increase in body mass. Controlling for flipper length, the effect of bill length on penguins' body mass was not significant ($p = .244$). The expected body mass when flipper length and bill length are both zero is -5736.897.

> **Note**
>
> Why is the intercept so crazy? Because flipper length and bill length would never actually be zero!

Assignment 13