$$u_j^* = \boldsymbol{\beta}^{*\prime}\mathbf{x}_j + \alpha^* p_j + \varepsilon_j^*, \qquad \varepsilon_j^* \sim \text{Gumbel}\left(0, \sigma^2 \tfrac{\pi^2}{6}\right) \tag{1}$$

$$\left(\frac{u_j^*}{\sigma}\right) = \left(\frac{\boldsymbol{\beta}^*}{\sigma}\right)' \mathbf{x}_j + \left(\frac{\alpha^*}{\sigma}\right) p_j + \left(\frac{\varepsilon_j^*}{\sigma}\right), \qquad \left(\frac{\varepsilon_j^*}{\sigma}\right) \sim \text{Gumbel}\left(0, \tfrac{\pi^2}{6}\right) \tag{2}$$

$$u_j = \boldsymbol{\beta}'\mathbf{x}_j + \alpha p_j + \varepsilon_j \qquad \varepsilon_j \sim \text{Gumbel}\left(0, \tfrac{\pi^2}{6}\right) \tag{3}$$

$$\left(\frac{u_j^*}{\alpha^*}\right) = \left(\frac{\boldsymbol{\beta}^*}{\alpha^*}\right)' \mathbf{x}_j - p_j + \left(\frac{\varepsilon_j^*}{\alpha^*}\right), \qquad \left(\frac{\varepsilon_j^*}{\alpha^*}\right) \sim \text{Gumbel}\left(0, \tfrac{\sigma^2}{(\alpha^*)^2} \tfrac{\pi^2}{6}\right) \tag{4}$$

$$u_j = \lambda\left(\boldsymbol{\omega}'\mathbf{x}_j - p_j\right) + \varepsilon_j \qquad \varepsilon_j \sim \text{Gumbel}\left(0, \tfrac{\pi^2}{6}\right) \tag{5}$$

$$P_{jc} = \frac{\exp(v_j)}{\sum\limits_{k \in \mathcal{J}_c} \exp(v_k)}, \qquad \forall c \in \{1, 2, 3, \ldots C\}, \quad j \in \mathcal{J}_c \tag{6}$$

$$\frac{P_{jc}}{P_{kc}} = \frac{\exp(v_j)}{\exp(v_k)}, \qquad \forall c \in \{1, 2, 3, \ldots C\}, \quad j \in \mathcal{J}_c \tag{7}$$

$$P_{jc} = \int \left(\frac{\exp(v_j)}{\sum\limits_{k \in \mathcal{J}_c} \exp(v_k)} \right) f(\boldsymbol{\theta}) d\boldsymbol{\theta}, \qquad \forall c \in \{1, 2, 3, \ldots C\}, \quad j \in \mathcal{J}_c \tag{8}$$

$$L = \sum_{c=1}^{C} \sum_{i \in \mathcal{N}_c} \sum_{j \in \mathcal{J}_c} y_{ijc} \ln P_{jc} \tag{9}$$

$$u_j = v_j + \varepsilon_j \qquad \varepsilon_j \sim \text{Gumbel}\left(0, \tfrac{\pi^2}{6}\right) \tag{10}$$

$$v_j = \boldsymbol{\beta}'\mathbf{x}_j + \alpha p_j \tag{11}$$