Estimating Models with {logitr}

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Many FOSS options model estimation

R packages:

- {logitr}: Fastest, mixed logit, WTP space.
- {apollo}: Most flexible, great documentation.
- {mlogit}: The OG R package.
- {gmnl}: Generalized logit model (though slow).
- {mixl}: Good for big datasets (uses C for speed).

Python packages:

- {xlogit}: Basically Python version of {logitr}.

Stan: For the Bayesians.

Many FOSS options model estimation

R packages:

- {logitr}: Fastest, mixed logit, WTP space. ← I wrote this one, so I'm showcasing it!
- {apollo}: Most flexible, great documentation.
- {mlogit}: The OG R package.
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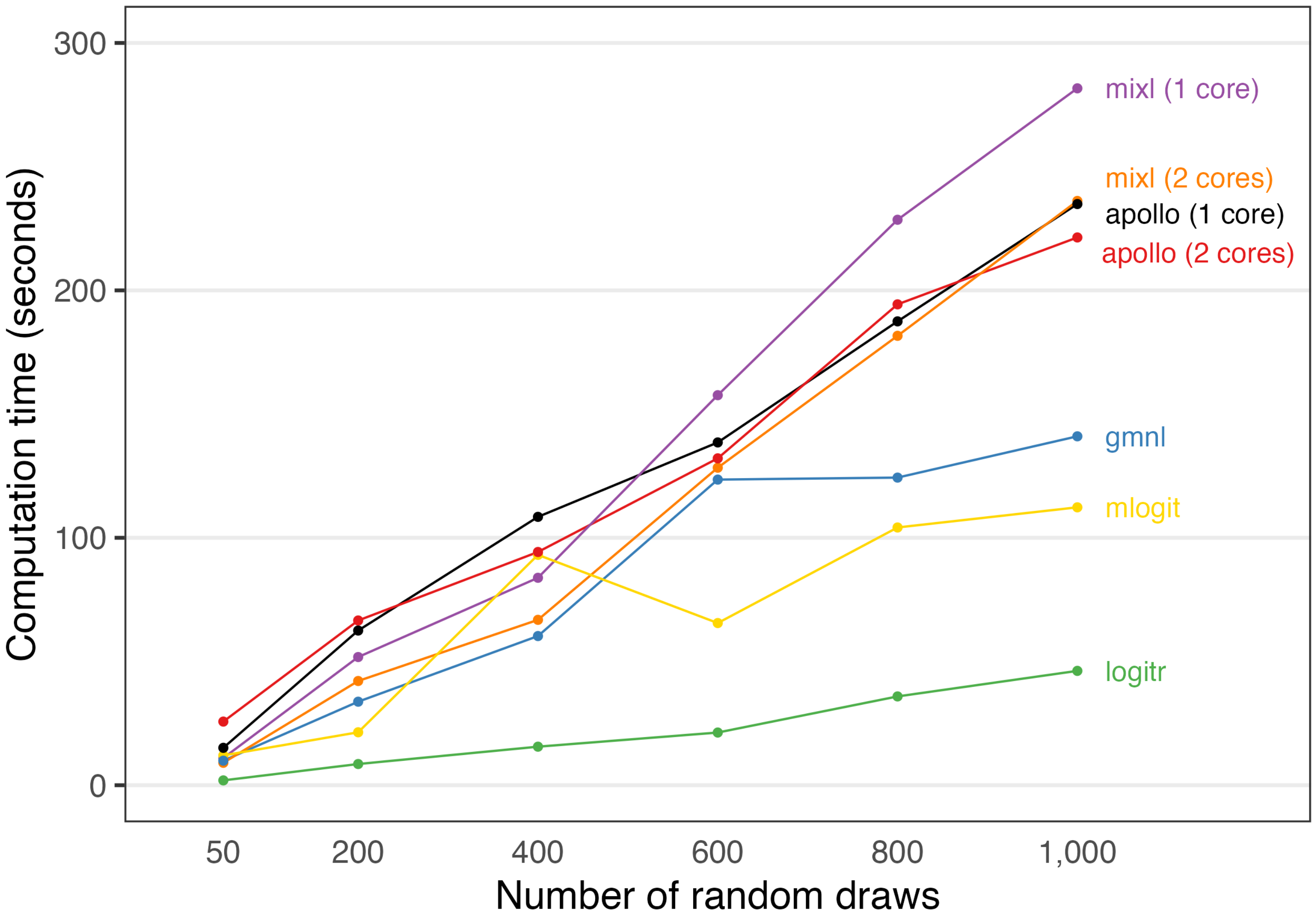
Stan: For the Bayesians.

{logitr} is fast!



{logitr} supports two common forms of utility models

Preference Space

$$u_j = \boldsymbol{\beta}' \mathbf{x}_j + \alpha p_j + \varepsilon_j$$

WTP Space

$$u_j = \lambda \left(\boldsymbol{\omega}' \mathbf{x}_j - p_j \right) + \varepsilon_j$$

{logitr} has a similar UI with {cbcTools}

({cbcTools} uses {logitr} to simulate choices and assess power)

{cbcTools}

```r
power <- cbc_power(
    nbreaks = 10,
    n_q     = 6,
    data    = data,
    obsID   = "obsID",
    outcome = "choice",
    pars    = c("price", "type", "freshness'
)
```

{logitr}

```r
model <- logitr(
    data    = data,
    obsID   = "obsID",
    outcome = "choice",
    pars    = c("price", "type", "freshness'
)
```

Utility model refresher

Which would you choose?

$2.49

$2.99

$1.99

$3.99

Estimate marginal utilities

$$u_j = \boldsymbol{\beta}' \mathbf{x}_j + \alpha p_j + \varepsilon_j, \qquad \varepsilon_j \sim \text{Gumbel}\left(0, \frac{\pi^2}{6}\right)$$

```
#>                  Estimate Std. Error   z-value Pr(>|z|)
#> price          -0.3886257 0.02426923 -16.01311        0
#> brandhiland    -3.1167063 0.14496806 -21.49926        0
#> brandyoplait    1.4463603 0.08869767  16.30663        0
#> branddannon    0.6440868 0.05435965  11.84862        0
```

Convert marginal *utilities* to marginal *WTPs*

$$\hat{\omega} = \frac{\hat{\beta}}{-\hat{\alpha}}$$

```
#>                Estimate Std. Error  z-value  Pr(>|z|)
#> brandhiland  -8.01982    0.45951 -17.4529 < 2.2e-16 ***
#> brandyoplait  3.72173    0.15882  23.4330 < 2.2e-16 ***
#> branddannon   1.65734    0.16723   9.9105 < 2.2e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Alternative approach: **Estimate a WTP-Space Model**

Substitutions:

$$\boldsymbol{\omega} = \frac{\boldsymbol{\beta}}{-\alpha}$$

$$\lambda = -\alpha$$

"Preference Space"

$$u_j = \boldsymbol{\beta}'\mathbf{x}_j + \alpha p_j + \varepsilon_j$$

"WTP Space"

$$u_j = \lambda\left(\boldsymbol{\omega}'\mathbf{x}_j - p_j\right) + \varepsilon_j$$

What's the difference?

Preference Space

$$u_j = \boldsymbol{\beta}'\mathbf{x}_j + \alpha p_j + \varepsilon_j$$

$$\downarrow$$

$$\hat{\boldsymbol{\omega}} = \frac{\hat{\boldsymbol{\beta}}}{-\hat{\alpha}}$$

WTP Space

$$u_j = \lambda\left(\boldsymbol{\omega}'\mathbf{x}_j - p_j\right) + \varepsilon_j$$

Mixed logit:

Unreasonably large WTP variance across population

$$u_j = \boldsymbol{\beta}' \mathbf{x}_j + \alpha p_j + \varepsilon_j$$

$$\downarrow$$

$$\hat{\boldsymbol{\omega}} = \frac{\hat{\boldsymbol{\beta}}}{-\hat{\alpha}}$$

$$\hat{\boldsymbol{\beta}} \sim \mathcal{N}\left(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\Sigma}}\right)$$

$$\hat{\alpha} \sim \mathcal{N}\left(\hat{\mu}, \widehat{\sigma^2}\right)$$

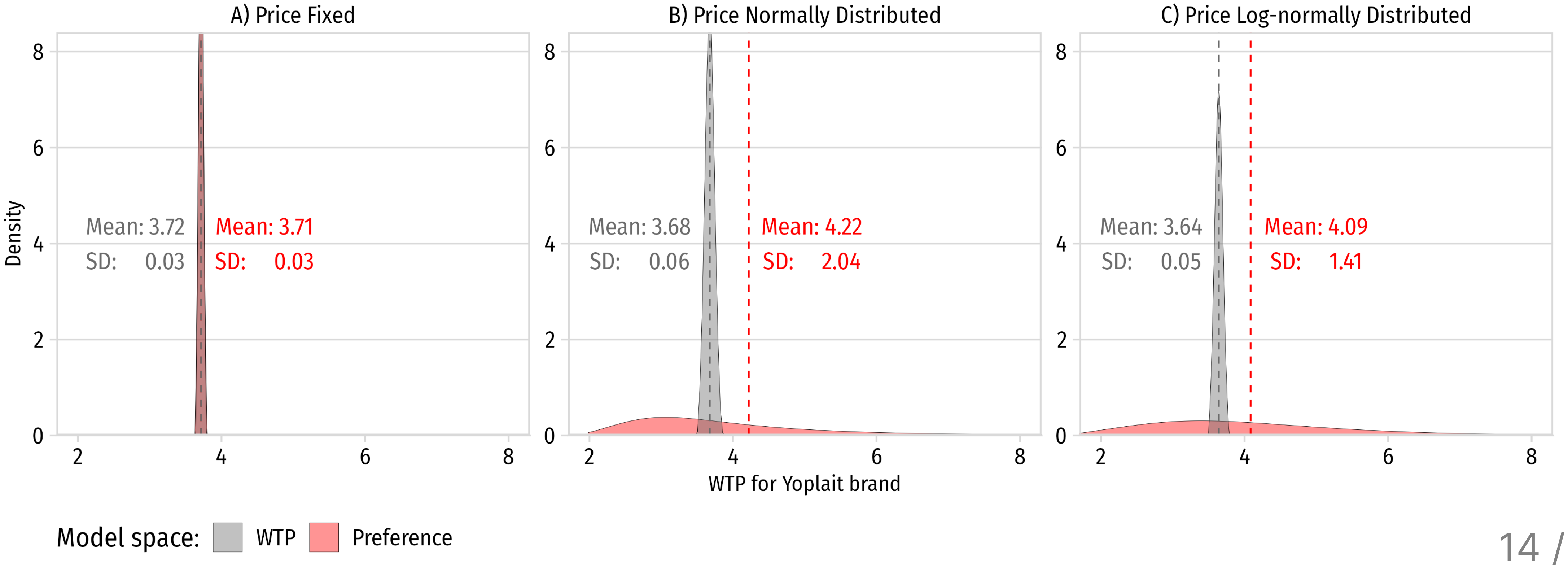
Preference space model produces unreasonably large variance in WTP

Preference Space

$$\hat{\beta} \sim \mathcal{N}\left(\hat{\mu}, \hat{\Sigma}\right)$$

WTP Space

$$\hat{\omega} \sim \mathcal{N}\left(\hat{\mu}, \hat{\Sigma}\right)$$



Practical Considerations

Practical Considerations

WTP space models produce immediately interpretable results

Unit: "Utility" (relative)

$$u_j = \boldsymbol{\beta}'\mathbf{x}_j + \alpha p_j + \varepsilon_j$$

```
#>                 Estimate Std. Error   z-value Pr(>|z|)
#> price         -0.3886257 0.02426923 -16.01311        0
#> brandhiland  -3.1167063 0.14496806 -21.49926        0
#> brandyoplait  1.4463603 0.08869767  16.30663        0
#> branddannon   0.6440868 0.05435965  11.84862        0
```

Units: $ (absolute)

$$u_j = \lambda\left(\boldsymbol{\omega}'\mathbf{x}_j - p_j\right) + \varepsilon_j$$

```
#>                Estimate Std. Error  z-value  Pr(>|z|)
#> scalePar       0.388626   0.024254  16.0230 < 2.2e-16 ***
#> brandhiland  -8.019815   0.459511 -17.4529 < 2.2e-16 ***
#> brandyoplait  3.721731   0.158824  23.4330 < 2.2e-16 ***
#> branddannon   1.657345   0.167231   9.9105 < 2.2e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Practical Considerations

WTPs can be directly compared across different models
(even estimates from different data sets)

$$u_j^* = \boldsymbol{\beta}^{*'} \mathbf{x}_j + \alpha^* p_j + \varepsilon_j^*, \qquad \varepsilon_j^* \sim \text{Gumbel}\left(0, \sigma^2 \frac{\pi^2}{6}\right)$$

Preference Space
Parameters proportional to σ

$$\left(\frac{u_j^*}{\sigma}\right) = \left(\frac{\boldsymbol{\beta}^*}{\sigma}\right)' \mathbf{x}_j + \left(\frac{\alpha^*}{\sigma}\right) p_j + \left(\frac{\varepsilon_j^*}{\sigma}\right)$$

$$u_j = \boldsymbol{\beta}' \mathbf{x}_j + \alpha p_j + \varepsilon_j$$

WTP Space
Parameters independent of σ

$$\left(\frac{u_j^*}{-\alpha^*}\right) = \left(\frac{\boldsymbol{\beta}^*}{-\alpha^*}\right)' \mathbf{x}_j + \left(\frac{\alpha^*}{-\alpha^*}\right) p_j + \left(\frac{\varepsilon_j^*}{-\alpha^*}\right)$$

$$u_j = \lambda \left(\boldsymbol{\omega}' \mathbf{x}_j - p_j\right) + \varepsilon_j$$

Practical Considerations

Neither space systematically predicts choice better

- **Train and Weeks (2005)** and **Sonnier et al. (2007)** found preference space model fit data better.
- **Das et al. (2009)** found nearly identical model fit on out-of-sample predictions with each model specification.

…but most software is built for

$$u_j = \boldsymbol{\beta}'\mathbf{x}_j + \alpha p_j + \varepsilon_j$$

not

$$u_j = \lambda\left(\boldsymbol{\omega}'\mathbf{x}_j - p_j\right) + \varepsilon_j$$

`logitr` to the rescue!

The logitr Package

Estimation of multinomial and mixed logit models in with "Preference" space or "Willingness-to-pay" (WTP) space utility parameterizations.

- Multinomial logit (MNL) models
- Mixed logit (MXL) models with normal and log-normal parameter distributions.
- Preference space and WTP space utility parameterizations.
- Weighted models to differentially weight individual observations.
- Uncorrelated or correlated heterogeneity covariances for mixed logit models.
- Functions for computing WTP from preference space models.
- Functions for predicting expected probabilities and outcomes for sets of alternatives based on an estimated model.
- A parallelized multistart optimization loop that uses different random starting points in each iteration to search for different local minima (useful for non-convex problems like MXL models or models with WTP space parameterizations).

Data format

Data must be arranged in a "long" format:

- Each row is an alternative from a choice observation.
- Choice observations do *not* have to be symmetric.

Required variables:

- `outcome`: A dummy variable for the chosen alternative (`1` or `0`).
- `obsID`: A sequence of repeated numbers identifying each unique choice observation, e.g. `1, 1, 2, 2, 3, 3`.
- `pars`: Any other variables to use as model covariates.

Data format

```r
head(yogurt, 10)
```

```
#>    choice obsID alt price   brand
#> 1       0     1   1   8.1  dannon
#> 2       0     1   2   6.1  hiland
#> 3       1     1   3   7.9  weight
#> 4       0     1   4  10.8 yoplait
#> 5       1     2   1   9.8  dannon
#> 6       0     2   2   6.4  hiland
#> 7       0     2   3   7.5  weight
#> 8       0     2   4  10.8 yoplait
#> 9       1     3   1   9.8  dannon
#> 10      0     3   2   6.1  hiland
```

- `outcome = "choice"`
- `obsID = "obsID"`
- `pars = c("price", "brand")`

Multinomial logit in **Preference Space**

```r
mnl_pref <- logitr(
  data    = yogurt,
  outcome = "choice",
  obsID   = "obsID",
  pars    = c("price", "brand")
)

summary(mnl_pref)
```

$$u_j = \boldsymbol{\beta}'\mathbf{x}_j + \alpha p_j + \varepsilon_j$$

```
#> =================================================
#>
#> Model estimated on: Fri Jun 09 10:12:27 2023
#>
#> Using logitr version: 1.1.0
#>
#> Call:
#> logitr(data = yogurt, outcome = "choice", obsID = "obsID", pars = c("p
#>     "brand"))
#>
#> Frequencies of alternatives:
#>        1        2        3        4
#> 0.402156 0.029436 0.229270 0.339138
#>
#> Exit Status: 3, Optimization stopped because ftol_rel or ftol_abs was
#>
#> Model Type:    Multinomial Logit
#> Model Space:          Preference
#> Model Run:                1 of 1
#> Iterations:                   20
#> Elapsed Time:        0h:0m:0.01s
#> Algorithm:        NLOPT_LD_LBFGS
#> Weights Used?:             FALSE
#> Robust?                    FALSE
#>
#> Model Coefficients:
#>               Estimate Std. Error z-value  Pr(>|z|)
#> price        -0.388626   0.024269 -16.013 < 2.2e-16 ***
#> brandhiland  -3.116706   0.144968 -21.499 < 2.2e-16 ***
#> brandyoplait  1.446360   0.088698  16.307 < 2.2e-16 ***
#> branddannon   0.644087   0.054360  11.849 < 2.2e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Multinomial logit in **WTP Space**

```r
library(logitr)

mnl_wtp <- logitr(
  data     = yogurt,
  outcome  = "choice",
  obsID    = "obsID",
  pars     = "brand",
  scalePar = "price"
)

summary(mnl_wtp)
```

$$u_j = \lambda\left(\boldsymbol{\omega}'\mathbf{x}_j - p_j\right) + \varepsilon_j$$

```
#> =================================================
#>
#> Model estimated on: Fri Jun 09 10:12:41 2023
#>
#> Using logitr version: 1.1.0
#>
#> Call:
#> logitr(data = yogurt, outcome = "choice", obsID = "obsID", pars = "bra
#>     scalePar = "price")
#>
#> Frequencies of alternatives:
#>        1        2        3        4
#> 0.402156 0.029436 0.229270 0.339138
#>
#> Exit Status: 3, Optimization stopped because ftol_rel or ftol_abs was
#>
#> Model Type:     Multinomial Logit
#> Model Space:   Willingness-to-Pay
#> Model Run:                 1 of 1
#> Iterations:                    40
#> Elapsed Time:         0h:0m:0.02s
#> Algorithm:          NLOPT_LD_LBFGS
#> Weights Used?:               FALSE
#> Robust?                      FALSE
#>
#> Model Coefficients:
#>               Estimate Std. Error z-value  Pr(>|z|)
#> scalePar      0.388633   0.024269  16.013 < 2.2e-16 ***
#> brandhiland  -8.019717   0.455549 -17.605 < 2.2e-16 ***
#> brandyoplait  3.721711   0.157655  23.607 < 2.2e-16 ***
#> branddannon   1.657290   0.165712  10.001 < 2.2e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#>
```

Caution

Log-likelihood function for WTP space models is **non-convex** 😔

Use a Multistart

```r
mnl_wtp <- logitr(
  data      = yogurt,
  outcome   = "choice",
  obsID     = "obsID",
  pars      = "brand",
  scalePar = "price",
  numMultiStarts = 10
)

summary(mnl_wtp)
```

$$u_j = \lambda\left(\boldsymbol{\omega}'\mathbf{x}_j - p_j\right) + \varepsilon_j$$

```
#> =================================================
#>
#> Model estimated on: Fri Jun 09 10:13:43 2023
#>
#> Using logitr version: 1.1.0
#>
#> Call:
#> logitr(data = yogurt, outcome = "choice", obsID = "obsID", pars = "bra
#>     scalePar = "price", numMultiStarts = 10)
#>
#> Frequencies of alternatives:
#>        1        2        3        4
#> 0.402156 0.029436 0.229270 0.339138
#>
#> Summary Of Multistart Runs:
#>    Log Likelihood Iterations Exit Status
#> 1        -2665.11         40           3
#> 2        -2665.11         39           3
#> 3        -2665.11         43           3
#> 4        -2665.11         47           3
#> 5        -2665.11         54           3
#> 6        -2665.11         42           3
#> 7        -2665.11         39           3
#> 8        -2665.11         44           3
#> 9        -2665.11         39           3
#> 10       -2665.11         38           3
#>
#> Use statusCodes() to view the meaning of each status code
#>
#> Exit Status: 3, Optimization stopped because ftol_rel or ftol_abs was
#>
#> Model Type:     Multinomial Logit
#> Model Space:   Willingness-to-Pay
#> Model Run:                10 of 10
```

Mixed logit in **Preference Space**

```r
mxl_pref <- logitr(
  data      = yogurt,
  outcome   = "choice",
  obsID     = "obsID",
  pars      = c("price", "brand"),
  randPars = c(brand = "n"),
  numMultiStarts = 10
)
```

$$u_j = \boldsymbol{\beta}'\mathbf{x}_j + \alpha p_j + \varepsilon_j$$

$$\hat{\boldsymbol{\beta}} \sim \mathcal{N}\left(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\Sigma}}\right)$$

Mixed logit in **WTP Space**

```r
mxl_wtp <- logitr(
  data      = yogurt,
  outcome   = "choice",
  obsID     = "obsID",
  pars      = "brand",
  scalePar = "price",
  randPars = c(brand = "n"),
  randScale = "ln",
  numMultiStarts = 10
)
```

$$u_j = \lambda\left(\boldsymbol{\omega}'\mathbf{x}_j - p_j\right) + \varepsilon_j$$

$$\hat{\boldsymbol{\omega}} \sim \mathcal{N}\left(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\Sigma}}\right)$$

Convenient helper functions

`predict()`: Expected shares for a set of alternatives

Define a set of alternatives

```r
data <- subset(
  yogurt, obsID == 42,
  select = c('price', 'brand', 'obsID'))

data
```

```
#>   price   brand obsID
#> 1   6.3  dannon    42
#> 2   6.1  hiland    42
#> 3   7.9  weight    42
#> 4  11.5 yoplait    42
```

Predict probabilities

```r
predict(
  mnl_pref,
  newdata = data,
  obsID = "obsID",
  returnData = TRUE
)
```

```
#>   obsID predicted_prob price   brand
#> 1    42     0.62391435   6.3  dannon
#> 2    42     0.01568877   6.1  hiland
#> 3    42     0.17593683   7.9  weight
#> 4    42     0.18446005  11.5 yoplait
```

Your turn

- Be sure to have downloaded and unzipped the practice code.
- Open the `2023-qux-conf-conjoint.Rproj` file to open RStudio.
- In RStudio, open the `estimating-models.R` file.
- Experiment with estimating different models (use either one of the example datasets included in the package, or simulate your own data!)

{logitr} documentation:
https://jhelvy.github.io/logitr/

Back to workshop website:
https://jhelvy.github.io/2023-qux-conf-conjoint/

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