The `logitr` Package:

Obtaining Willingness to Pay Estimates from Preference Space and Willingness to Pay Space Utility Models

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Which would you choose?

$2.49

$2.99

$1.99

$3.99

Estimate marginal utilities

$$u_j = \boldsymbol{\beta}'\mathbf{x}_j + \alpha p_j + \varepsilon_j, \qquad \varepsilon_j \sim \text{Gumbel}\left(0, \frac{\pi^2}{6}\right)$$

```
#>                Estimate StdError    tStat pVal signif
#> price         -0.388627 0.024269 -16.0131    0    ***
#> brandhiland   -3.116712 0.144968 -21.4993    0    ***
#> brandyoplait   1.446362 0.088698  16.3066    0    ***
#> branddannon    0.644086 0.054360  11.8486    0    ***
```

Convert marginal *utilities* to marginal *WTPs*

$$\hat{\omega} = \frac{\hat{\beta}}{-\hat{\alpha}}$$

```
#>                Estimate StdError    tStat pVal signif
#> brandhiland   -8.019803 0.461772 -17.3675    0    ***
#> brandyoplait   3.721723 0.441000   8.4393    0    ***
#> branddannon    1.657337 0.184540   8.9809    0    ***
```

Alternative approach: **Estimate a WTP-Space Model**

Substitutions:

$$\boldsymbol{\omega} = \frac{\boldsymbol{\beta}}{-\alpha}$$

$$\lambda = -\alpha$$

"Preference Space"

$$u_j = \boldsymbol{\beta}'\mathbf{x}_j + \alpha p_j + \varepsilon_j$$

"WTP Space"

$$u_j = \lambda\left(\boldsymbol{\omega}'\mathbf{x}_j - p_j\right) + \varepsilon_j$$

What's the difference?

Preference Space

$$u_j = \boldsymbol{\beta}'\mathbf{x}_j + \alpha p_j + \varepsilon_j$$

$$\downarrow$$

$$\hat{\boldsymbol{\omega}} = \frac{\hat{\boldsymbol{\beta}}}{-\hat{\alpha}}$$

WTP Space

$$u_j = \lambda\left(\boldsymbol{\omega}'\mathbf{x}_j - p_j\right) + \varepsilon_j$$

Oops...WTP has undefined mean and variance

Preference Space

$$u_j = \boldsymbol{\beta}' \mathbf{x}_j + \alpha p_j + \varepsilon_j$$

$$\downarrow$$

$$\hat{\omega} = \frac{\hat{\beta}}{-\hat{\alpha}}$$

$\hat{\alpha}$ & $\hat{\beta}$ are assumed to have asymptotically normal distributions (Bockstael and Strand, 1987)

Implies error around $\hat{\omega}$ follows a Cauchy distribution

Mixed logit:

Unreasonably large WTP variance across population

$$u_j = \boldsymbol{\beta}' \mathbf{x}_j + \alpha p_j + \varepsilon_j$$

$$\downarrow$$

$$\hat{\boldsymbol{\omega}} = \frac{\hat{\boldsymbol{\beta}}}{-\hat{\alpha}}$$

$$\hat{\boldsymbol{\beta}} \sim \mathcal{N}\left(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\Sigma}}\right)$$

$$\hat{\alpha} \sim \mathcal{N}\left(\hat{\mu}, \hat{\sigma^2}\right)$$

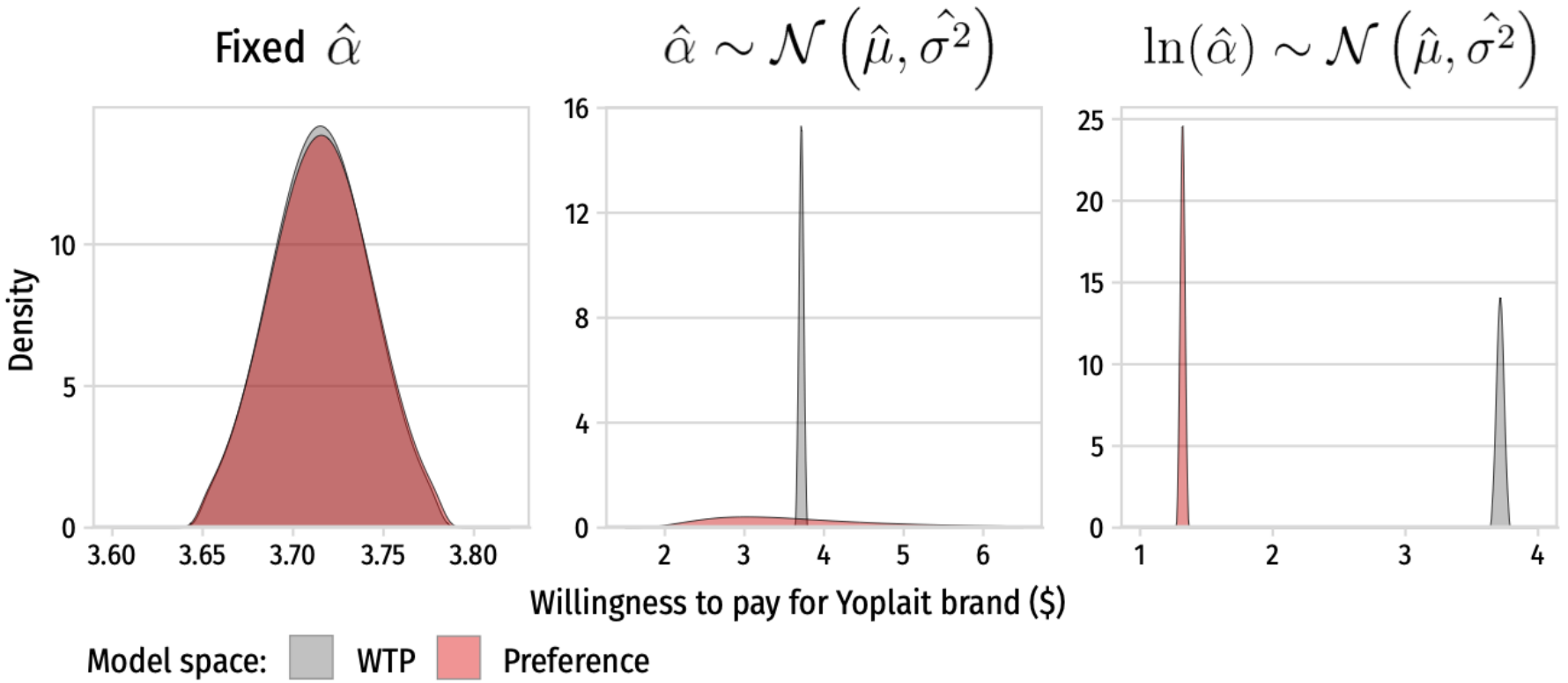
Preference space model produces unreasonably large variance in WTP

Preference Space

$$\hat{\beta} \sim \mathcal{N}\left(\hat{\mu}, \hat{\Sigma}\right)$$

WTP Space

$$\hat{\omega} \sim \mathcal{N}\left(\hat{\mu}, \hat{\Sigma}\right)$$



I'm not the first to identify these issues

WTP computed from preference space model has undefined moments

- **Carson and Czajkowski (2019)**. "A new baseline model for estimating willingness to pay from discrete choice models." *Journal of Environmental Economics and Management*, 95:57–61.

Preference space model produces unreasonably large variance in WTP

- **Train and Weeks (2005)**. "Discrete Choice Models in Preference and Willingness-to-Pay Space". In *Appl. Simul. Methods Environ. Resour. Econ.*, Chapter 1, pages 1–16.
- **Sonnier, Ainslie, & Otter (2007)**. "Heterogeneity distributions of willingness-to-pay in choice models." *Quant. Mark. Econ.*, 5(3):313–331.

Practical Considerations

Practical Considerations

WTP space models produce immediately interpretable results (with correct standard errors)

Unit: "Utility" (relative)

$$u_j = \boldsymbol{\beta}'\mathbf{x}_j + \alpha p_j + \varepsilon_j$$

```
#>                Estimate StdError    tStat pVal signif
#> price        -0.388627 0.024269 -16.0131    0    ***
#> brandhiland  -3.116712 0.144968 -21.4993    0    ***
#> brandyoplait  1.446362 0.088698  16.3066    0    ***
#> branddannon   0.644086 0.054360  11.8486    0    ***
```

Units: $ (absolute)

$$u_j = \lambda\left(\boldsymbol{\omega}'\mathbf{x}_j - p_j\right) + \varepsilon_j$$

```
#>                Estimate StdError    tStat pVal signif
#> lambda        0.388627 0.024380  15.9401    0    ***
#> brandhiland  -8.019803 0.461772 -17.3675    0    ***
#> brandyoplait  3.721723 0.441000   8.4393    0    ***
#> branddannon   1.657337 0.184540   8.9809    0    ***
```

Practical Considerations

WTPs can be directly compared across different models
(even estimates from different data sets)

$$u_j^* = \boldsymbol{\beta}^{*'}\mathbf{x}_j + \alpha^* p_j + \varepsilon_j^*, \qquad \varepsilon_j^* \sim \text{Gumbel}\left(0, \sigma^2 \frac{\pi^2}{6}\right)$$

Preference Space
Parameters proportional to σ

$$\left(\frac{u_j^*}{\sigma}\right) = \left(\frac{\beta^*}{\sigma}\right)' \mathbf{x}_j + \left(\frac{\alpha^*}{\sigma}\right) p_j + \left(\frac{\varepsilon_j^*}{\sigma}\right)$$

$$u_j = \boldsymbol{\beta}'\mathbf{x}_j + \alpha p_j + \varepsilon_j$$

WTP Space
Parameters independent of σ

$$\left(\frac{u_j^*}{-\alpha^*}\right) = \left(\frac{\beta^*}{-\alpha^*}\right)' \mathbf{x}_j + \left(\frac{\alpha^*}{-\alpha^*}\right) p_j + \left(\frac{\varepsilon_j^*}{-\alpha^*}\right)$$

$$u_j = \lambda\left(\boldsymbol{\omega}'\mathbf{x}_j - p_j\right) + \varepsilon_j$$

Practical Considerations

No theoretical basis for believing that marginal *utilities* versus marginal *WTPs* should follow standard distributions

$$\hat{\beta} \sim \mathcal{N}\left(\hat{\mu}, \hat{\Sigma}\right)$$

$$\hat{\omega} \sim \mathcal{N}\left(\hat{\mu}, \hat{\Sigma}\right)$$

Practical Considerations

Neither space systematically predicts choice better

- **Train and Weeks (2005)** and **Sonnier et al. (2007)** found preference space model fit data better.
- **Das et al. (2009)** found nearly identical model fit on out-of-sample predictions with each model specification.

...but most software is built for

$$u_j = \boldsymbol{\beta}' \mathbf{x}_j + \alpha p_j + \varepsilon_j$$

not

$$u_j = \lambda \left(\boldsymbol{\omega}' \mathbf{x}_j - p_j \right) + \varepsilon_j$$

`logitr` to the rescue!

The logitr Package

Estimation of multinomial and mixed logit models in with "Preference" space or "Willingness-to-pay" (WTP) space utility parameterizations.

- Homogeneous multinomial logit models.
- Heterogeneous mixed logit models (normal and log-normal parameter distributions).
- Preference & WTP space utility parameterizations.
- Optional multistart optimization loop.
- Computing and comparing WTP from preference space and WTP space models.
- Simulating expected shares.

Source code: https://github.com/jhelvy/logitr

Installation

Version 0.1.0 can be installed from the CRAN:

```
install.packages("logitr")
```

The development version can be installed from GitHub:

```
# install.packages("remotes")
remotes::install_github("jhelvy/logitr")
```

Data format

Data must be arranged in a "long" format:

- Each row is an alternative from a choice observation.
- Choice observations do *not* have to be symmetric.

Required variables:

- `choiceName`: A dummy variable for the chosen alternative (`1` or `0`).
- `obsIDName`: A sequence of repeated numbers identifying each unique choice observation, e.g. `1, 1, 2, 2, 3, 3`.
- `parNames`: Any other variables to use as model covariates.

Data format

```
head(yogurt, 10)
```

```
#>    choice obsID alt price   brand
#> 1       0     1   1   8.1  dannon
#> 2       0     1   2   6.1  hiland
#> 3       1     1   3   7.9  weight
#> 4       0     1   4  10.8 yoplait
#> 5       1     2   1   9.8  dannon
#> 6       0     2   2   6.4  hiland
#> 7       0     2   3   7.5  weight
#> 8       0     2   4  10.8 yoplait
#> 9       1     3   1   9.8  dannon
#> 10      0     3   2   6.1  hiland
```

- `choiceName = "choice"`
- `obsIDName = "obsID"`
- `parNames = c("price", "brand")`

Multinomial logit in **Preference Space**

```
library(logitr)

mnl_pref <- logitr(
  data       = yogurt,
  choiceName = "choice",
  obsIDName  = "obsID",
  parNames   = c("price", "brand"))
)

summary(mnl_pref)
```

$$u_j = \boldsymbol{\beta}'\mathbf{x}_j + \alpha p_j + \varepsilon_j$$

```
#> ================================================
#> MODEL SUMMARY:
#>
#> Model Space:    Preference
#> Model Run:          1 of 1
#> Iterations:             20
#> Elapsed Time:  0h:0m:0.12s
#> Exit Status:             3
#> Weights Used?:       FALSE
#>
#> Model Coefficients:
#>               Estimate StdError    tStat pVal signif
#> price        -0.388627 0.024269 -16.0131    0    ***
#> brandhiland  -3.116712 0.144968 -21.4993    0    ***
#> brandyoplait  1.446362 0.088698  16.3066    0    ***
#> branddannon   0.644086 0.054360  11.8486    0    ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#>
#> Model Fit Values:
#>
#> Log.Likelihood.         -2665.1101915
#> Null.Log.Likelihood.    -3343.7419990
#> AIC.                     5338.2204000
#> BIC.                     5361.3732000
#> McFadden.R2.                0.2029558
#> Adj..McFadden.R2            0.2017595
#> Number.of.Observations.  2412.0000000
```

Multinomial logit in **WTP Space**

```r
library(logitr)

mnl_wtp <- logitr(
  data       = yogurt,
  choiceName = "choice",
  obsIDName  = "obsID",
  parNames   = "brand",
  priceName  = "price",
  modelSpace = "wtp")
)

summary(mnl_wtp)
```

$$u_j = \lambda \left(\boldsymbol{\omega}' \mathbf{x}_j - p_j \right) + \varepsilon_j$$

```
#> ================================================
#> MODEL SUMMARY:
#>
#> Model Space:   Willingness-to-Pay
#> Model Run:                 6 of 10
#> Iterations:                   36
#> Elapsed Time:         0h:0m:0.23s
#> Exit Status:                   3
#> Weights Used?:             FALSE
#>
#> Model Coefficients:
#>               Estimate StdError    tStat pVal signif
#> lambda        0.388631 0.024269  16.0132    0    ***
#> brandhiland  -8.019752 0.455555 -17.6044    0    ***
#> brandyoplait  3.721705 0.157656  23.6065    0    ***
#> branddannon   1.657322 0.165715  10.0010    0    ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#>
#> Model Fit Values:
#>
#> Log.Likelihood.         -2665.1101915
#> Null.Log.Likelihood.    -3343.7419990
#> AIC.                     5338.2204000
#> BIC.                     5361.3732000
#> McFadden.R2.                0.2029558
#> Adj..McFadden.R2            0.2017595
#> Number.of.Observations.  2412.0000000
```

Caution

Log-likelihood function for WTP space models is **non-convex** 😔

Use a Multistart

```
library(logitr)

mnl_wtp <- logitr(
  data       = yogurt,
  choiceName = "choice",
  obsIDName  = "obsID",
  parNames   = "brand",
  priceName  = "price",
  modelSpace = "wtp",
  options = list(numMultiStarts = 10)
)

summary(mnl_wtp)
```

$$u_j = \lambda\left(\boldsymbol{\omega}'\mathbf{x}_j - p_j\right) + \varepsilon_j$$

```
#> ================================================
#> SUMMARY OF ALL MULTISTART RUNS:
#>
#>    run    logLik iterations status
#> 1    1 -2833.932         80      4
#> 2    2 -2665.110         34      3
#> 3    3 -2665.110         34      3
#> 4    4 -2665.110         32      3
#> 5    5 -2834.285         76      3
#> 6    6 -2665.110         36      3
#> 7    7 -2665.110         30      3
#> 8    8 -2834.989         69      3
#> 9    9 -2834.567         70      3
#> 10  10 -2834.183         75      4
#> ---
#> Use statusCodes() to view the meaning of the status codes
#>
#> Below is the summary of run 6 of 10 multistart runs
#> (the run with the largest log-likelihood value)
#> ================================================
#> MODEL SUMMARY:
#>
#> Model Space:   Willingness-to-Pay
#> Model Run:                 6 of 10
#> Iterations:                   36
#> Elapsed Time:         0h:0m:0.23s
#> Exit Status:                   3
#> Weights Used?:             FALSE
#>
#> Model Coefficients:
#>              Estimate StdError    tStat pVal signif
#> lambda       0.388631 0.024269  16.0132    0    ***
```

Mixed logit in **Preference Space**

```r
library(logitr)

mxl_pref <- logitr(
  data       = yogurt,
  choiceName = "choice",
  obsIDName  = "obsID",
  parNames   = c("price", "brand"),
  randPars   = c(brand = "n"),
  options = list(numMultiStarts = 10)
)

summary(mxl_pref)
```

$$u_j = \boldsymbol{\beta}'\mathbf{x}_j + \alpha p_j + \varepsilon_j$$

$$\hat{\boldsymbol{\beta}} \sim \mathcal{N}\left(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\Sigma}}\right)$$

```
#> ================================================
#> MODEL SUMMARY:
#>
#> Model Space:   Preference
#> Model Run:        5 of 10
#> Iterations:            43
#> Elapsed Time:   0h:0m:17s
#> Exit Status:            3
#> Weights Used?:      FALSE
#>
#> Model Coefficients:
#>                    Estimate StdError    tStat   pVal signif
#> price             -0.389810 0.044362  -8.7870 0.0000    ***
#> brandhiland_mu    -3.124798 0.243803 -12.8169 0.0000    ***
#> brandyoplait_mu    1.449109 0.140574  10.3085 0.0000    ***
#> branddannon_mu     0.655452 0.138290   4.7397 0.0000    ***
#> brandhiland_sigma  0.056181 1.177854   0.0477 0.9620
#> brandyoplait_sigma 0.011127 0.388879   0.0286 0.9772
#> branddannon_sigma  0.172808 2.458086   0.0703 0.9440
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#>
#> Model Fit Values:
#>
#> Log.Likelihood.         -2665.1077011
#> Null.Log.Likelihood.    -3343.7419990
#> AIC.                     5344.2154000
#> BIC.                     5384.7329000
#> McFadden.R2.                0.2029565
#> Adj..McFadden.R2            0.2008631
#> Number.of.Observations.  2412.0000000
#>
```

Mixed logit in **WTP Space**

```
library(logitr)

mxl_wtp <- logitr(
  data       = yogurt,
  choiceName = "choice",
  obsIDName  = "obsID",
  parNames   = "brand",
  priceName  = "price",
  randPars   = c(brand = "n"),
  modelSpace = "wtp",
  options = list(numMultiStarts = 10)
)

summary(mxl_wtp)
```

$$u_j = \lambda\left(\boldsymbol{\omega}'\mathbf{x}_j - p_j\right) + \varepsilon_j$$

$$\hat{\boldsymbol{\omega}} \sim \mathcal{N}\left(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\Sigma}}\right)$$

```
#> ================================================
#> MODEL SUMMARY:
#>
#> Model Space:   Willingness-to-Pay
#> Model Run:                6 of 10
#> Iterations:                    51
#> Elapsed Time:           0h:0m:27s
#> Exit Status:                    3
#> Weights Used?:              FALSE
#>
#> Model Coefficients:
#>                    Estimate StdError    tStat   pVal signif
#> lambda             0.391114 0.037629  10.3938 0.0000    ***
#> brandhiland_mu    -8.001475 0.579751 -13.8016 0.0000    ***
#> brandyoplait_mu    3.715322 0.181690  20.4487 0.0000    ***
#> branddannon_mu     1.682783 0.169619   9.9210 0.0000    ***
#> brandhiland_sigma  0.182098 2.623297   0.0694 0.9447
#> brandyoplait_sigma 0.034227 0.975524   0.0351 0.9720
#> branddannon_sigma  0.598862 3.586766   0.1670 0.8674
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#>
#> Model Fit Values:
#>
#> Log.Likelihood.         -2665.1066726
#> Null.Log.Likelihood.    -3343.7419990
#> AIC.                     5344.2133000
#> BIC.                     5384.7308000
#> McFadden.R2.                0.2029568
#> Adj..McFadden.R2            0.2008634
#> Number.of.Observations.  2412.0000000
#>
```

Convenient helper functions

`wtp()`: Compute WTP from Preference Space model

$$\hat{\omega} = \frac{\hat{\beta}}{-\hat{\alpha}}$$

```
wtp(mnl_pref, priceName = "price")
```

```
#>                Estimate StdError    tStat pVal signif
#> lambda         0.388627 0.024230  16.0393    0    ***
#> brandhiland   -8.019803 0.460116 -17.4300    0    ***
#> brandyoplait   3.721723 0.438991   8.4779    0    ***
#> branddannon    1.657337 0.183939   9.0103    0    ***
```

`wtpCompare()`: Compare WTP from Preference & WTP space models

```
wtpCompare(mnl_pref, mnl_wtp, priceName = "price")
```

```
#>                      pref           wtp difference
#> lambda           0.388627     0.3886309  3.940e-06
#> brandhiland     -8.019803    -8.0197522  5.083e-05
#> brandyoplait     3.721723     3.7217045 -1.848e-05
#> branddannon      1.657337     1.6573215 -1.547e-05
#> logLik       -2665.110192 -2665.1101915  2.000e-08
```

`simulateShares()`: Expected shares for a set of alternatives

Define a set of alternatives

```
alts <- subset(
  yogurt, obsID == 42,
  select = c('price', 'brand'))

alts
```

```
#>   price   brand
#> 1   6.3  dannon
#> 2   6.1  hiland
#> 3   7.9  weight
#> 4  11.5 yoplait
```

Compute expected shares

```
simulateShares(
  model = mnl_pref,
  alts = alts,
  alpha = 0.025
)
```

```
#>         share_mean  share_low share_high
#> Alt: 1 0.62391502 0.56615070 0.67466090
#> Alt: 2 0.01568871 0.01210113 0.02011584
#> Alt: 3 0.17593678 0.16127684 0.18999846
#> Alt: 4 0.18445949 0.13460611 0.24531509
```

Thanks!

`logitr` documentation: https://jhelvy.github.io/logitr/

Slides: https://jhelvy.github.io/2021-sawtooth-conf

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Extra slides

A note about the error scale

$$u_j^* = \boldsymbol{\beta}^{*\prime}\mathbf{x}_j + \alpha^* p_j + \varepsilon_j^*, \qquad \varepsilon_j^* \sim \text{Gumbel}\left(0, \sigma^2 \frac{\pi^2}{6}\right)$$

Preference Space

$$\left(\frac{u_j^*}{\sigma}\right) = \left(\frac{\boldsymbol{\beta}^*}{\sigma}\right)' \mathbf{x}_j + \left(\frac{\alpha^*}{\sigma}\right) p_j + \left(\frac{\varepsilon_j^*}{\sigma}\right), \qquad \left(\frac{\varepsilon_j^*}{\sigma}\right) \sim \text{Gumbel}\left(0, \frac{\pi^2}{6}\right)$$

$$u_j = \boldsymbol{\beta}'\mathbf{x}_j + \alpha p_j + \varepsilon_j, \qquad \varepsilon_j \sim \text{Gumbel}\left(0, \frac{\pi^2}{6}\right)$$

WTP Space

$$\left(\frac{u_j^*}{-\alpha^*}\right) = \left(\frac{\boldsymbol{\beta}^*}{-\alpha^*}\right)' \mathbf{x}_j + \left(\frac{\alpha^*}{-\alpha^*}\right) p_j + \left(\frac{\varepsilon_j^*}{-\alpha^*}\right), \qquad \left(\frac{\varepsilon_j^*}{-\alpha^*}\right) \sim \text{Gumbel}\left(0, \frac{\sigma^2}{(-\alpha^*)^2} \frac{\pi^2}{6}\right)$$

$$u_j = \lambda\left(\boldsymbol{\omega}'\mathbf{x}_j - p_j\right) + \varepsilon_j, \qquad \varepsilon_j \sim \text{Gumbel}\left(0, \frac{\pi^2}{6}\right)$$