New York University Graduate School of Arts and Sciences – New York, NY

Center for Experimental Humanities

Masters Project, May 2025

Categorical Approaches to Continental Philosophy: Formalizing Hegel, Marx, and Lacan

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A Master's Project in XE: Experimental Humanities & Social Engagement, in partial fulfillment of the requirements for the degree of Master of Arts at New York University

Abstract

In three chapters, this master's project aims to give a preliminary account of how innovations in mathematical structuralism (set theory, category theory, and type theory) may be used to formalize concepts in continental philosophical thought from Hegel's system of logic to Marx's political economy and Lacan's psychoanalytic theory of language. The project draws influence from the set-theoretical philosophy of Alain Badiou, while seeking to update the foundations in critical theory with more rigorous and higher-level mathematical theories. The chapter on Lacan begins with his own application of set theory in SXIX providing a critique of the limits of set theoretical foundations for modeling the psyche and constructing elementary category-theoretical diagrams of language, formulas of sexuation, and other key concepts in the Lacanian system. The chapter on Marx draws heavily from Gangle et al.'s work on modeling autopoietic systems with presheaves to argue that capitalism is a fundamentally autopoietic dynamical system. The chapter places particular emphasis on the role of primitive accumulation as an eigenbehavior of the recursive system. The chapter on Hegel looks closely at the Univalent Foundation's Project's *Homotopy Type Theory (HoTT) Book* as a new foundation for mathematics providing the tools to finally capture the fundamental concepts of universality, determinate negation, totality, and Spirit in Hegel's system of German Idealism. The mathematical analysis is supplemented by Robert Brandom's inferentialist reading of Hegel and insights from computation. All three chapters are guided by an effort to synthesize mathematical structuralism and philosophical structuralism into a single coherent intellectual project. With the help of Saunders Mac Lane's "one universe" hypothesis for category theory, and Voevodsky's univalence axiom (UA), the work revitalizes diverse notions of universality in critical theory.

Full project available at https://github.com/jhenkle/MA-Thesis-Work

Acknowledgments

My greatest appreciation to Leif Weatherby who introduced me to digital media theory when I was still dismissive of analytic philosophy, who walked me through the research and writing of this project, and whose generosity and wisdom was and continues to be an indispensable resource. Thanks as well to the attendees of the UC Berkeley German Studies Graduate Conference *Versuch*, LACK v, and the University of Minnesota Cultural Studies and Comparative Literature "Universality Renewed" Conference for their invaluable feedback. Special thanks to Christina Vagt, Yayha Madra, and Todd McGowan. Thank you to Alexander Galloway for taking me through Lacan's mathemes, and Hent de Vries for walking me through Hegel's *Phenomenology.* As always special thanks to my friend and constant interlocutor, Majorca Bateman-Coe.

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Absolute Mediation and Computational Universality: Hegel and Homotopy Type Theory

> The whole cycle constitutes life. It is neither what is first expressed, namely, the immediate continuity and unmixed character of its essence, nor is it the stably existing shape and what is "the discrete" existeing for itself, nor is it the pure process of all of this, nor again is it the simple gatehring together of these moments. Rather, it is the developing itself, then dissolving its development, and, in this movement, being the simple self-sustaining whole.
>
> G.W.F. Hegel, *The Phenomenology of Spirit*

> The role of space as an arena for quantitative "becoming" underlies the qualitative transformation of a spatial category into a homotopy category, on which extensive and intensive quantities reappear as homology and cohomology.
>
> F. William Lawvere, "Categories of Space and Quantity"

> The homotopy theory of the types in the universe should be fully and faithfully reflected by the equality on the universe.
>
> Vladimir Voevodsky, "Univalent Foundations"

Hegel Against Mathematics?

Hegel's dismissive evaluation of mathematics that "the *actual* is not something spatial as it is taken to be in mathematics; neither concrete sensuous intuition nor philosophy wastes any time with the kinds of non-actualities which are the things of mathematics,"[1] perhaps needs revision in light of the revolution of mathematical modernism beginning with Cantor and Frege in the 19th and 20th centuries. Hegel is correct to criticize both the rugged positivism of some tendencies in mathematics, and the shortcomings of what would become the axiomatic method, however in the time since Hegel's death mathematics has come

[1] Georg Wilhelm Friedrich Hegel. *The phenomenology of spirit.* Ed. and trans. by Terry P. Pinkard. First paperback edition. Cambridge Hegel translations. Cambridge New York Port Melbourne New Delhi Singapore: Cambridge University Press, 2018, p. 27

to understand its object as existing outside of empirical truths and privileges the concept of mathematical consistency instead. Consistency as a standard does not appear in conflict with Hegel's conception of science. That is to say, mathematics can make claims regarding the true contained by rules which separate it from the actual. Hegel's philosophical orientation remained anti-essentialist, thus it seems reasonable to assume he might revere contemporary mathematics for its daring move to abandon essences in actuality as such following the failure of the Hilbert program.

Modern mathematics, namely by way of the invention of the axiomatic method, has transitioned away from the essentialist qualities of the geometric method of Spinoza and De Cartes towards a logical formalism, which without entirely evading Hegel's sharp criticism may be more compatible with Hegel's thought than his mathematical contemporaries.

Hegel specifically criticizes the way in which mathematics, in his time sought to conflate equality and knowledge. Stating, "it is also on account of that principle and element–and what is formal in mathematical convincingness consists in this–that knowing advances along the line of *equality*. Precisely because it does not move itself, what is lifeless does not make it all the way to the differences of essence, nor to essential opposition, or to inequality, nor to the transition of one opposition into its opposite, nor to qualitative, immanent self-movement."[2] Let it be our mission to show the ways in which recent developments in mathematics overcome the limitations Hegel identifies. That mathematics can formalize the rich versatility and dynamism of a system as sophisticated as spirit and Hegel's systematic logic.

Introducing HoTT

In the preface to *The Phenomenology of Spirit*, Hegel outlines his frustration with philosophical systems which rigidly conceptualize true and false as "itself fixed and set."[3] Writing, "it does not comprehend the diversity of philosophical systems as the progressive development of truth as much as it sees only contradiction in that diversity."[4] Stemming, in essence from the tradition of logical positivism, homotopy type theory (HoTT) maintains a clear and distinguishable boundary between true and false propositions. However, in its view of homotopy, HoTT privileges the level of a mathematical object over its content. Thus, the contractible types 0 and 1 are treated similarly, the propositions $\perp$ and $\top$ are treated similarly, and the sets $\mathbb{N}_0$ and $\mathbb{N}_1$ are treated similarly. The three examples provided are all reducible to claims to truth and falsehood, however up to homotopy they are regarded as members of their respective type. The homotopy type theoretic view that members of equal type are formally equal aligns closely with Hegel's view of spirit, in which he claims: "spirit has not

[2]Hegel, *The phenomenology of spirit*, p. 27
[3]Hegel, *The phenomenology of spirit*, p. 4
[4]Hegel, *The phenomenology of spirit*, p. 4

only gone beyond that to the opposite extreme of a reflection of itself into itself which is utterly devoid of substance; it has gone beyond that extreme too. Not only has its essential life been lost to it, it is conscious of this, and of the finitude that is its content."[5] HoTT's use of dependent types reduces even infinite sets $\mathbb{N}$ to a 0-Type stripping it of much of the informational content held in its set-theoretical definition while maintaining a knowledge of this information and its newly (de)limited status.

When operationalizing HoTT such as in identifying paths between homotopical spaces, we identify homotopic functors as mapping $H : X \times [0, 1] \to Y$ s.t. H describes a parameterized continuous deformation of a function f to g between the interval $t = 0$ and $t = 1$. This map can be thought of as a collection of functions, or in categorical terms morphisms between two homotopical spaces. This collection of paths $p \to Set$ is in essence the total motion from a proposition $\perp$ to a proposition $\top$. Thus, we can say, at its most elementary level, Hegel's position on the $\tau\varepsilon\lambda o\varsigma$ of truth or its diverse becoming is homotopic.

Just as Hegel states that:

> The bud disappears when the blossom breaks through, and one might say that the former is refuted by the latter. Likewise, through the fruit, the blossom itself may be declared to be a false existence of the plant, since the fruit emerges as the blossom's truth as it comes to replace the blossom itself. These forms are not only distinguished from each other, but, as incompatible with each other, they also supplant each other. However, at the same time their fluid nature makes them into moments of an organic unity in which they are not in conflict with each other, but rather, one is equally as necessary as the other, and it is this equal necessity which alone constitutes the whole.[6]

The motion and dynamism Hegel describes in the passage above is precisely what is modeled by homotopy. That is, a collection of continuous paths which deform a homotopical space (i.e. a sophisticated mathematical object) into another through an interval of time. The example above outlines an object bud (X), an object blossom (Y) and an object fruit (Z). We can model the homotopy $H : H_1 : X \times [0, 1] \to Y \to H_2 : Y \times [1, 2] \to Z$ s.t. $H : p \to Set : X \to Y \to Z$

For Hegel, as for HoTT the theoretical innovation comes in positing a rich dialectic of discrete and continuous logics. For HoTT we find this logic present in the concept of spaces (discrete) and homotopies (continuous). Hegel relates the dialectical relationship of the static and motion, writing "we see pure conciousness here posited in a twofold manner. At one time, it is posited as the restless *movement to and fro* which runs through all its moments, which have otherness in mind, an otherness which, in being grasped is sublated. At another time,

[5] Hegel, *The phenomenology of spirit*, p. 7
[6] Hegel, *The phenomenology of spirit*, p. 4

it is instead posited as the *motionless unity* which is certain of its truth."[7] A similar logic is at play in HoTT.

The appeal to homotopic paths as opposed to static homotopical spaces is a way to model Hegel's key argument: "nor is the *result* which is reached the *actual* whole itself; rather, the whole is the result together with the way the result comes to be."[8] More simply, the HoTT view allows us to overcome the model of elementary calculus where bud, flower, and fruit are points on a continuous curve, the set theoretical view:

Plant: {Bud, Flower, Fruit}

in favor of the type theoretical definition:

```
(Bud, Flower, Fruit) : P
P : Type plant
```

Hegel's best captures the necessity of going beyond set theoretical foundations to type theoretical foundations in a claim in the Self-Consciousness chapter of the *Phenomenology*: "The *being* of what was meant, along with the *singularity* and the *universality* opposed to that singularity in perception, as well as the *empty inner* of understanding, no longer are as the essence. Rather they are just moments of self-consciousness."[9] Where the transfinite induction of the natural numbers in set-theoretical foundations relies on an *empty inner*, the empty set, as the foundation of the system, HoTT seeks to capture homotopies as moments in a continuous deformation of space, or in Hegelian terms "moments of self-consciousness."

Similarly, in the conceptual difficulty of rendering the family "plant" as a set of discrete elements or a continuous curve, HoTT enables us to have it both ways, accounting for the discrete nature of members of the type "plant" and the continuous motion between them. The type-theoretic statement (Bud, Flower, Fruit) : Type plant does not require us to delineate all of the members of plant as is the case in set-theoretic definitions, but maintains the structure of the family. When this type-theoretic structure is joined by a homotopical description of bud, flower and fruit as spaces, we can formalize continuous mappings (paths) from bud $\rightsquigarrow$ flower $\rightsquigarrow$ fruit.

We can model this elementary case of homotopy paths between three objects of the same type in Coq thus:

```
(* Assume plant is a type *)
Variable plant : Type.

(* Assume P is a type *)
```

[7] Hegel, *The phenomenology of spirit*, p. 140
[8] Hegel, *The phenomenology of spirit*, p. 5
[9] Hegel, *The phenomenology of spirit*, p. 103

```
Variable P : Type.

(* Declare three objects of type P *)
Variables bud flower fruit : P.

(* Declare paths (equalities) between bud and flower, and flower
↪  and
fruit using Coq's equality *)
Variable p : bud = flower.
Variable q : flower = fruit.

(* We can model a path between bud and fruit using transitivity
↪  of equality *)
Definition r : bud = fruit := eq_trans p q.

(* Inductive definition of path, generalized over a type A *)
Inductive path {A : Type} (x : A) : A -> Type :=
| idpath : path x x.

(* Declare inductive paths between bud and flower, and flower and
↪  fruit *)
Variables p' : path bud flower.
Variables q' : path flower fruit.

(* Define path composition for the inductive path type *)
Definition compose_paths {A : Type} {x y z : A} (p : path x y) (q
↪  : path y z) : path x z :=
  match p with
  | idpath => q
  end.

(* Compose paths homotopically *)
Definition homotopical_path := compose_paths p' q'.

(* Declare two paths between bud and flower *)
Variables p1 p2 : path bud flower.

(* Define a higher path (path between paths) *)
Definition higher_path : path p1 p2 := idpath.

(* Define the composed path between bud and fruit *)
Definition composed_path : path bud fruit := compose_paths p' q'.
```

Though somewhat tedious in notation, encoding HoTT in a program like Coq, as pictured above explicitly details the richness of homotopical paths between objects in a step-by-step iterative process. What this sample code is meant to illustrate an elementary formal model, for the most elementary of Hegel's claims in *Phenomenology of Spirit*, the botanical example from the preface. Take the work here as a proof of concept, that a theory as complex as Hegel's dialectical motion can be formalized as a series of computational operations. Though not immediately apparent, just as Hegel's botanical example is used as an elementary model upon which a more detailed system is elaborated, the formal rules we defined and induced in Coq could be used as a basis for a much more plastic and dynamic proof verification model.

From Absolute to Generic Science

Formalizing (some of) Hegel's system of logic with HoTT is not meant to reduce concepts as dynamic as spirit to mere absolute equality. Hegel warns against this when he claims, "to examine any existence in the way in which it is in the *absolute* consists in nothing more than saying it is in fact being spoken of as, say, a 'something,' whereas in the absolute, in the $A = A$, there is no such 'something,' for in the absolute, everything is one."[10] This view, that is to say the view of HoTT is not to be confused with the formal view of the absolute "that is, to pass off its *absolute* as the night in which, as one says, all cows are black."[11] Homotopic equivalences are not positing a world of absolute monist substance. Rather it identifies equivalencies up to a limit we call "homotopy" which provides a unity of concepts without an absolute unity of substance. That is, even at night, all cows belong to the type "cow" but must not all be black.

In paths p we find a model of the subject-object relationship between X and Y. As Hegel notes, "the living substance is the being that is in truth *subject*, or, what amounts to the same thing, it is in truth actual only insofar as it is the movement of self-positing, or, that it is the mediation of itself and its becoming-other-to-itself."[12] Without formal notation this may be the clearest expression of what we mean by paths or the set of paths between two homotopical spaces. That is, a path, "as subject, it is pure, *simple negativity*."[13] Meaning it is through a continuous mapping that a homotopical space X is deformed into Y. This negative deterministic process is at once determined by difference and as Hegel says "this *self-restoring* sameness, the reflective turn into itself in its otherness."[14] The truth value of homotopic transformations of topoi is only in its continuity not in its unity or collapsability, or as Hegel states, "the true is not an *original unity* as such, or, not an *immediate* unity as such. It is the coming-to-be of itself."[15]

HoTT does not support reducing the content of substance to an absolute one-

[10] Hegel, *The phenomenology of spirit*, p. 11
[11] Hegel, *The phenomenology of spirit*, p. 12
[12] Hegel, *The phenomenology of spirit*, p. 12
[13] Hegel, *The phenomenology of spirit*, p. 12
[14] Hegel, *The phenomenology of spirit*, p. 12
[15] Hegel, *The phenomenology of spirit*, p. 12

ness, a total unity. Rather, the primary theoretical innovation of HoTT is to make generic the specific informational content of types identifying isomorphism up to homotopy. To *generify*, as computer scientists say, is not to abstract, but rather to perform an operation greatly generalizing information until its practical representation is mere form and structure and its specific content is reduced to knowledge and identity. Though related to the universal, to generify is not strictly to universalize. The generic introduces continuous paths of equivalency between discrete forms, in this sense it is a unity. Where Hegel criticizes mathematics for not dealing in "actualities" he also critiques a formal positivism which claims to know things-in-themselves. The generic offers an alternative, that may be closer to what Hegel intends, that is an acknowledgement of the impossibility of attaining absolute knowledge of the object, and a real knowledge of the movement of the object vis-á-vis its other.

A way to bridge the purely formal experiments in HoTT and the prosaic metaphysical claims in Hegel's *Phenomenology of Spirit* is Pittsburgh School philosopher Robert Brandom's analytical approach to Hegel. Brandom claims "at the very center of Hegel's thought (to begin with, his metaphysics and logic) is a radically new conception of the conceptual."[16] This is really the only aspect of Hegel's thought which my appeal to HoTT seeks to capture. Mathematical formalism, in the experimental and daring form undertaken by HoTT attempts to radically resignify concepts by increasingly generifying the foundations of mathematics to its univalence principle. Just as Brandom identifies "determinate negation" and "mediation" as the central analytical concepts from Hegel's *Phenomenology*, HoTT primary concern is the formalization of "determinate negation" and "mediation" between all mathematical objects.

In Brandom's reading, "by 'determinate negation,' Hegel means material incompatibility ... [that] it is impossible for one thing at one time to exhibit both properties."[17] HoTT is, at its core, a novel synthesis of homotopy and Martin-Löf type theory. In its appropriation of both mathematical traditions, HoTT models determinate negation in two ways, first by ordering mathematical objects into families of types of discrete level (its type-theoretic interpretation) and second, by constructing topoi as discrete forms which cannot be immanently unified (the homotopic interpretation). The synthesis of both views structures the HoTT project and naturalizes Hegel's conceptual innovation in the operations of the mathematics. Regarding the relationship between thought and objective states, Brandom writes:

> Hegel sees the deontic normative sense of "incompatible" and "consequence" that articulates the attitudes of knowing subjects and the alethic modal sense of those terms that articulates objective facts as deeply related. They are different *forms* that one identical conceptual *content* can take ... This hylomorphic structure of form and

[16] Robert Boyce Brandom. *A spirit of trust: a reading of Hegel's Phenomenology.* Cambridge, Massachusetts: The Belknap Press of Harvard University Press, 2019, p. 2
[17] Brandom, *A spirit of trust*, p. 2

> content underlies Hegel's expressive account of the relations between subjective thoughts and objective states of affairs in discursive practices of knowing and doing.[18]

What Brandom calls hylomrophism, is perhaps the primary occupation of HoTT and its prevailing innovation over previous foundations of mathematics such as set-theory and category theory. That is, through dependent typing, HoTT assigns all mathematical objects a type, which is a reference to its form extending from the knowledge of its content. Through homotopy, these forms are acted upon by a functor which is a set of paths which transform one form to another with only abstract expressive reference to their concrete content.

Dialectic of the Hierarchy of Equality Types

HoTT by way of the construction of equalities intimates a likeness to Hegel's dialectical relation between essence and appearance. For Hegel, essence is not an object of analysis or science devoid of the study of appearance. Type theorists have a similarly negative attitude towards lower level types that Hegel has to essences, referring to level -1 types as *mere* propositions. If an essence can be represented as an identity type of a level type (-2) it appears as a level (-1) type or proposition. Similarly, the identity type of a proposition (-1) is an appearance of level (0) type or a set. And so on up to groupoids and higher-level types. The point is that all higher types can be constructed from contractable types. This logic never treats types of any level as autonomous from the total hierarchy, even the contractable types have only one non-trivial feature, which is its role as a fixed-point in a category, sheaf, or presheaf.

In Martin-Löf type theory, types are constructable from natural numbers such that there is a type $\mathbb{N}$ such that $\mathbb{N}_0$ is the empty type $\varnothing$ which corresponds to the logical constant $\perp$ and $\mathbb{N}_1$ has one element $\{\varnothing\}$ and corresponds to the logical constant $\top$. From this base construction we can already see how sets and propositions can be constructed from contractible types. Through a type $C(x, y, z)$ where x and y are contractable types of type A and a proof $I(x, y)$ assigns a type; a function g assigns x of type A to an object of type $C(x, x, r(x))$; a function f which assigns the pair x, y and a proof z of type $I(x, y)$, we can see the preliminary form of a naive construction of the set $\mathbb{N}$ whereby through the extensionality of operators $(\Pi, \Sigma, \times, \exists, \text{and}, \forall)$ a complete bounded set theoretical universe $V \in \mathcal{U}$ can be constructed.

This elementary constructable universe sometimes when joined with the Zermelo-Frankel axioms is called CZF, is the basis of a hierarchy of types and a constructable universe which lends itself nicely to the kind of dialectical motion up to totality in Hegel's logical system.

Perhaps a more simple way of framing the operation of escalating types is the assignment of function types. If we can accept the premise that Hegel's dialectics

[18] Brandom, *A spirit of trust*, p. 3

is a model of dialectical motion, it would follow that function types ($\to$) are an important mathematical model which maps one object to another. In the HoTT hierarchy of types, if:

$a : \text{Type } A \wedge b : \text{Type } B$

$a \to b : \text{Type } (1 + \max A\ B)$

Here, we show that the motion between an object a in type A and an object b in type B generates a new function type of a level one higher than the highest level of a and b.

```
a: Type A
b: Type B
a -> b: Type (1+ max A B)
```

If we understand the typing as always escalating to one level higher through either the function operation or the operation $\in$ notated as : in type theory, we can see the operation of merely thinking a mathematical object in type theory involves thinking its identity and motion.

```
Type n: Type (n+1)
```

We can show the same proof of concept in Coq sample code like the following:

```
(* Declare two types 'a' and 'b', each in their respective
↪  universes 'A' and 'B' *)
Universe A B.

(* Declare 'a' of type Type A and 'b' of type Type B *)
Variable a : Type@{A}.
Variable b : Type@{B}.

(* The function type 'a -> b' lives in a universe that is one
↪  level higher than
the maximum of the universes A and B *)
Check (a -> b : Type@{1 + max(A, B)}).
```

Thus, we can show in HoTT that the movement of bud to flower to fruit, can be understood as a new higher-level type generated from the discrete identity of the three objects. Where the identity type of the total plant can be defined as identity plant : plant $\to$ plant.

```
(* First define the identity function of the type plant *)
Definition id {plant : Type} (x : plant) := x.
Definition identity_plant (x : plant) := x.

(* s.t. *)
identity_plant : plant -> plant
```

```
(* Then show that bud, flower, and fruit are all members of
↪  type plant *)
Inductive plant : Type :=
| bud : plant
| flower : plant
| fruit : plant.

(* Show the generic path p between the three elements *)
Definition p : bud = flower -> flower = fruit -> bud = fruit
↪  :=
fun Hbudflower Hflowerfruit => eq_trans Hbudflower
↪  Hflowerfruit.

(* Following the operation for assigning function types,
it follows that the mapping (bud to flower to fruit) should
↪  have the type plant -> plant *)

Definition plant_mapper (x : plant) : plant :=
match x with
| bud => flower
| flower => fruit
| fruit => suc(fruit)
end.
(* Here suc, is a function which returns the next (successor)
↪  element in the type plant *)

(* The identity map is of type plant -> plant *)
Check identity_plant.
(* identity_plant : plant -> plant *)

(* The mapping plant_mapper is also of type plant -> plant,
↪  demonstrating that functions
on the same type are at the same type level. *)
Check plant_mapper.
(* plant_mapper : plant -> plant *)
```

This hierarchy of types uses the botanical example from the introduction of *Phenomenology of Spirit* to show that both the operation of creating a function between two objects, and defining the identity of an object raises the level of the typed objects from the type plant to the type plant → plant. This lengthy proof highlights the rigour of formally constructing a model of Hegelian dialectics, while using the most elementary example.

$\neg(\neg)$

$$\begin{tikzcd}
\text{Geometric Method} \arrow[r] \arrow[d] & \text{Axiomatic Method} \arrow[d] \\
\text{Spinoza and De Cartes} \arrow[d, "\neg" description] & \text{Frege and Cantor} \arrow[d] \\
\text{Hegel} \arrow[r] & \text{Badiou}
\end{tikzcd}$$

Theories of homotopy are built on the formal relationality between mathematical objects called isomorphism. In Brandom's reading of modern philosophy, isomorphism is a foundational concept to semantic reason. Brandom claims that De Cartes "saw that what made algebraic understanding of geometric figures possible was a global *isomorphism* between the whole system of algebraic symbols and the whole system of geometric figures. That isomorphism defined a notion of form shared by the licit manipulations of strings of algebraic symbols and the constructions possible with geometric figures."[19] This representationalism, a system of concepts built on the correspondence of two sets of forms, is at the heart of Hegel's critique of Kant's epistemology. Notably, Hegel rejects immediacy—that there belongs a class of conceptual objects (appearances) which are immediately knowable. Summarizing Hegel's position, Brandom writes,

> when [*sic*] difference is construed as one of intelligibility in the strong sense—representings are intrinsically intelligible and representeds are not—then skepticism about genuine knowledge is a consequence. And he takes from Kant the idea that intelligibility is a matter of conceptual articulation: to be intelligible is to be in specifically conceptual shape. If this reading is correct, then Hegel's argument must show that to satisfy the Genuine Knowledge Condition, an epistemological theory must treat not only appearance (how things subjectively are, for consciousness), but also reality (how things objectively are, in themselves) as conceptually articulated.[20]

What separates homotopy and HoTT from general isomorphism, is its treatment of all matter of mathematical objects, from propositions, to sets, and higher inductive types (HITs) as functionally computable *up to homotopy.* In doing so, the operation of typing in HoTT elaborates its mathematical formalism in such a way that all of its structures are, as Brandom says, "conceptually articulated."

From this point of departure, Hegel distinguishes between "determinate negation" and "formal negation," in which the former describes the material incom-

[19]Brandom, *A spirit of trust*, p. 39
[20]Brandom, *A spirit of trust*, pp. 44–5

patibility between concepts (discrete) and the latter describes logical inconsistency. HoTT treats mathematical objects as discrete, in the sense of determinate negation, isomorphic up to homotopy, meaning to the boundary of their logical inconsistency. The formal treatment of objects in HoTT acknowledges the content of information while understanding this content to be trivial up to the point at which they are formally negated, to use Hegel's terminology. The semantic relationship between concepts in HoTT are called paths. When the only path between two mathematical objects (concepts) is an identity path, we say the two objects are identical. In all other cases, the collection of paths between two objects details what Hegel would call, their "mediation." In HoTT we call two objects, whose paths detail a continuous deformation of one into the other (mediation) "homotopy equivalent." The former situation is considered a trivial case, where the latter situation—the situation of determinate negation—is the primary object of HoTT's analysis. By the univalence axiom, we say in HoTT that "identity is equivalent to equivalence,"[21] such that "isomorphic things can be identified."[22]

The conceptual revolution in HoTT does not do away with Hegel's primary concerns with representational modes of semantics. Rather, it formalizes material incompatibility (determinate negation) and, to a lesser extent logical inconsistency (formal/abstract negation), in the same system of foundations of mathematics. HoTT privileges determinate negation to formal negation, as did Hegel. In the language of HoTT "homotopy equivalent" becomes an important shorthand for the more detailed concept "isomorphic up to homotopy." By this we mean, in the topological sense that there is an identity-preserving map between spaces A and B and between a and b in space A. Brandom alludes to this exact relationship between Hegel and mathematical homotopy in a footnote, which this essay is meant to be an expansion on.

In the main text of SoT, Brandom writes:

> Isomorphism between deontic normative conceptual relations of incompatibility and consequence among commitments and alethic modal relations of incompatibility and consequence among states of affairs determines how one takes things objectively to be. Practically acquiring and altering one's commitments in accordance with a certain set of deontic norms of incompatibility and consequence is taking the objective alethic modal relations articulating the conceptual content of states of affairs to be the isomorphic ones.[23]

Continuing the line of thought in the footnote which states:

> Really, "homomorphic," because in general subjects need not be

[21]The Univalent Foundations Program. *Homotopy Type Theory: Univalent Foundations of Mathematics*. Institute for Advanced Study: `https://homotopytypetheory.org/book`, 2013, p. 4

[22]Univalent Foundations Program, *Homotopy Type Theory: Univalent Foundations of Mathematics*, p. 5

[23]Brandom, *A spirit of trust*, p. 60

> aware of (apperceive, conceptually represent) all the alethic modal relations of incompatibility and consequence that objectively obtain. But I mean "homomorphic" in the technical mathematical sense of a structure-preserving mapping from one relational structure (whose elements are subjective commitments labeled by declarative sentences, and whose relations are deontic normative relations of incompatibility and consequence) to another (whose elements are objective states of affairs—in virtue of the homomorphism, labelable by the same declarative sentences, and whose relations are alethic modal relations of incompatibility and consequence). The structure preserved is those relations. To say that the homomorphism h is "structure-preserving" in this sense means that if aRb in the commitment-structure, where R is normative incompatibility (or consequence) in that structure, then $h(a)\ R'h(b)$, where R' is alethic incompatibility (or consequence) in the objective conceptual structure.[24] [25]

Brandom's claims here outline the very beginnings of what could be a rich mathematical systematization of Hegel's thought, rooted in the most elementary claim that Hegel's epistemological theory as briefly detailed in the introduction to *Phenomenology of Spirit* closely parallels the mathematical concept of homotopy. It follows that taking homotopy as its partial theoretical foundation, HoTT offers tools which have the possibility to extend and further develop an analytical mode for formalizing Hegel's thought.

What makes paths distinct from the categorical notion of homomorphisms, is its pragmatist application of mappings. That is to say, a path is not strictly a morphism in the categorical sense—a mapping where every element of the domain maps to a corresponding element in a codomain. Rather, paths deal in the pre-Cartesian notion of spaces rather than sets. We can consider a homotopy, the collection of all paths between two spaces, not as mapping discrete elements in set A to discrete elements in set B, but as a complete equivalence map of the continuous deformation of space A into space B where all mathematical objects can be redereed as homotopical spaces. This model more closely aligns with Brandom's view of Hegel's pragmatism: the contrast by which "there is a kind of sense-dependence of modal vocabulary on what is expressed by normative vocabulary, not a kind of reference-dependence."[26] Paths capture sense-dependence by no longer structuring mappings as a one-to-one correspondence between a reference and a referent.

The isomorphism of things and ideas, Brandom attributes to Spinoza. This Spinozist notion *omnis determinatio est negatio* can be understood in terms of

[24]Brandom, *A spirit of trust*, p. 773

[25]Here, homomorphism refers to equivalence in category theory where mappings between objects are called "morphisms." In HoTT, these mappings between homotopy equivalent spaces are called paths.

[26]Brandom, *A spirit of trust*, p. 82

categorical homomorphism. However, Hegel's thought adopts Spinoza's position with a Kantian supplement, namely the marrying of semantics to pragmatics. As Hegel's theory of consciousness seeks to synthesize these two ideas it seems fitting to apply HoTT's notion of homotopy equivalence where paths can express not only the determinateness of concepts and their isomorphic relation, but also the movement (consciousness) by which things appear as ideas. If you recall, in the beginning of this essay, I gestured to Hegel's critique of the propositional truth value of equality in mathematics. As Brandom claims, "instead of thinking of truth as an achievable state or status, Hegel wants us to think of it as characteristic of a process: the process of *experience*, in which appearances 'arise and pass away.'"[27] Hence, HoTT's richer notion of homotopy equivalence provides a Hegelian antidote to the rigidness of propositional equality, whereby spaces (appearances) deform continuously into another shape whilst maintaining structural equivalence.

For instance, we can look at the rather trivial homotopic mapping by which the propositions true and false are mapped: the Boolean. $f : \{0,1\}^k \to \{0,1\}$ We can also reformulate the Boolean as $f : \{\bot, \top\}^k \to \{\bot, \top\}$ In HoTT we notate the Boolean as

$$2 : \mathcal{U}$$

$$\text{s.t.}$$

$$\prod_{x:2} (x = 0_2) + (x = 1_2) \tag{1}$$

we read this as "the Boolean in HoTT notation is formulated as 2 in the universe $\mathcal{U}$ such that, if x is in Boolean, x is either 0_2 or 1_2." Using induction and lambda notation we can write the element of Boolean as:

$$\mathrm{ind}_2(\lambda x.(x = 0_2) + (x = 1_2), \mathrm{inl}(\mathrm{refl}0_2), \mathrm{inr}(\mathrm{refl}1_2)) \tag{2}$$

The Boolean enables us to see a simple homotopy equivalence which maps all possible combinations of propositional truth statements, while also acknowledging that in HoTT this is only one trivial example of the many homotopy equivalences.[28] Let it be an illustration of the renewed dynamism of truth in HoTT relative to the restrictive and limited concept of truth by propositional equivalence Hegel critiques in the preface to the *Phenomenology*.

Categories of Consciousness

One primary way to import categorical logic into the Hegelian system is to model Hegel's concepts as categories. Taking as one example, we can sketch a category K for Knowing with initial and terminal objects C_0 and C_1 denoting stages of consciousness, and morphisms b_0 and b_1 as such:

[27] Brandom, *A spirit of trust*, p. 100

[28] Univalent Foundations Program, *Homotopy Type Theory: Univalent Foundations of Mathematics*, p. 35

$$K : C_0 \overset{b_0}{\underset{b_1}{\rightrightarrows}} C_1 \tag{3}$$

$$\begin{tikzcd}
 & & & (b_0, b_1) \arrow[dl] \arrow[dr] & & & \\
 & & (b_0, K) \arrow[dl] \arrow[dr] & & (K, b_1) \arrow[dl] \arrow[dr] & & \\
 & C_0 \arrow[dr, "b_0"] & & K^2 \arrow[dl, "K^{\overline{\partial}_0}"'] \arrow[dr, "K^{\overline{\partial}_1}"] & & C_1 \arrow[dl, "b_1"'] & \\
 & & K & & K & &
\end{tikzcd}$$

Figure 1: Knowing

Following Hegel's claim, "consciousness *distinguishes* something from itself while at the same time it *relates* itself to it. Or, as it is expressed: This something is something *for consciousness*, and the determinate aspect of this *relating*, or of the *being* of something *for a consciousness*, is *knowing*. However, we distinguish this being-for-another from *being-in-itself*,"[29] we construct the commutative diagram above. Here, b_0 and b_1 denote being-for-another and being-in-itself respectively. In this categorical rendering, drawing heavily from Lawvere's category of categories,[30] a field of knowing is constructed as a category produced in the determinateness of the motion of conciousness to itself. Thus we can render in HoTT, $C_0 \simeq C_1$ where generic consiousncess C a homotopy equivalence and knowing K is a flow of paths between an initial object C_0 and a terminal object C_1 such that $0 \to C_0 \ \wedge \ C_1 \leftarrow 1$.

When we appeal to paths, as a formal model of Hegelian "experience" we are modeling Hegel's philosophical primacy of the motion between concepts over the concepts themselves. For us, a collection of paths is experience in the sense that Hegel claims: "it is clear both that the dialectic of sensuous-certainty is nothing but the simple history of its movement (that is, its experience) and that sensuous-certainty itself is nothing but just this history."[31] As is the case in the dialectic of sensuous-certainty, homotopical paths are the structure-preserving motion of homotopical space. Where our formal model using HoTT would use terms like "homotopy," or "structure preserving map," Hegel calls this "[its] history."

In the chapter on perception, Hegel outlines the categorical premise of identity. That is, in category theory, which in many respects HoTT is an extension of,

[29]Hegel, *The phenomenology of spirit*, p. 55

[30]F. William Lawvere. "The Category of Categories as a Foundation for Mathematics". In: *Proceedings of the Conference on Categorical Algebra*. Ed. by S. Eilenberg et al. Berlin, Heidelberg: Springer Berlin Heidelberg, 1966, pp. 1–20, p. 13

[31]Hegel, *The phenomenology of spirit*, p. 66

all objects or categories have a unique identity morphism such that an object A has an identity morphism Id_A. We also denote this as $A \to 1$. Hegel describes the identity relationship as "an indifferent unity [and] also a *One*, an *excluding unity.*"[32] He goes on to state, "the One is the *moment of negation,* as it itself relates itself to itself in a simple way and excludes others and by which *thinghood* is determined."[33] The second important property of identity in category theory, is that *all* objects have an identity morphism as such, which we denote as $\forall x \in \mathcal{U} | x \to 1$. Hegel captures the universality for our formal model uses the existential quantifier $\forall$, by claiming "to be One is to be the universal relating-itself-to-itself, and as a result of its being One, it is instead the same as all others."[34] He formulates this notion again elsewhere, writing: "*Everything has a constant difference with regard to everything else.*[35] The categorical notion of identity which Hegel captures succinctly is the foundation for concepts which are further developed and instrumentalized in HoTT.

In HoTT, the properties of identity outlined above hold only at the level of universes. That is, where category theory conceptualizes all objects as belonging to the same universe $\mathcal{U}$, HoTT's type-theoretical foundation is based on universe levels where objects belong to universes starting at $\mathrm{Type} - -2$ for contractible types, going up to $\mathrm{Type} - -1$ for propositions, $\mathrm{Type} - 0$ for sets, $\mathrm{Type} - 1$ for groupoids, and higher level types for higher inductive types (HITs). Where $\mathcal{U}$ might be in cateogrical terms what Hegel calls the "simple infinity, or the absolute concept ... the universal bloodstream ... which is itself both every difference as well as their sublatedness,"[36] the innovation of HoTT, is to show a stronger notion of universality for objects in the same universe relative to the one-universe definition of tradition category-theoretical foundations. In HoTT, all objects of the same type, or in the same universe-level are isomorphic up to homotopy. Meaning, by virtue of their identity contingent on being members of the same type, we can construct a homotopy (set of homotopical paths) between any two (or more) objects in $\mathrm{Type} - n$. Thus in the botanical example from earlier, all three objects (bud, flower, fruit) are of type plant, meaning the morphisms between each of their identity morphisms are equalities, or homotopical paths.

What Badiou calls the "count-as-One" in his set-theoretical philosophy, and Hegel calls "the *positing-into-a-one* [*Das In-eins-setzen*]"[37] is perhaps a less complex notion, than what HoTT ultimately allows us to construct with identity and homotopy. Hegel makes a gesture to what in Badiou's set-theoretical interpretation is the empty-set ($\varnothing$), writing, "the inner is *empty*, for it is only the nothingness of appearance and, positively, the simple universal."[38] Where

[32]Hegel, *The phenomenology of spirit*, p. 70
[33]Hegel, *The phenomenology of spirit*, p. 70
[34]Hegel, *The phenomenology of spirit*, p. 73
[35]Hegel, *The phenomenology of spirit*, p. 90
[36]Hegel, *The phenomenology of spirit*, p. 98
[37]Hegel, *The phenomenology of spirit*, p. 74
[38]Hegel, *The phenomenology of spirit*, p. 87

for set theory the empty set is the foundation of transfinite induction of the natural numbers, in HoTT such emptiness or pure negation, is simply equivalent to all contractible types or point-sets. In a certain reading, this is much closer what Hegel expresses about identity in the *Phenomenology.* What unifies emptiness and all discrete things is a simple universal principle. It is true that for consciousness, one is sensuously knowable and the other is not, however from a deontic position, HoTT's rendering of the concepets as fundamentally equivalent captures the core of Hegel's argument. Thus, where set theory disaggregates "*emptiness in itself*"[39] and the operation of negation, HoTT like Hegel renders the two concepts, if not wholly interchangeable only trivially distinct, or what Hegel calls a "*mere concept of law itself.*"[40]

Brandom, shows the importance of something like type-theoretical universes for understanding Hegel's thought. In his elementary example, Brandom explains how the mere act of identification for consciousness takes a different form dependent on the discrete object being identified. In his code, that action which is operationalized by pointing out a "this" can be encoded as a $/\text{this}/_i$, $/\text{this}/_j$, and $/\text{this}/_k$ for three discrete objects i, j, k.[41] By typing objects in HoTT, or any type-theoretical foundation, we are not only treating objects as discrete entities, but classifying them by type, which allows us to construct increasingly complex relationships and structures between them. Typing allows us to construct a model which distinguishes between a weak kind of "mere or indifferent" difference, and "exclusive" difference between objects. In HoTT, objects of the same type are merely different, still determinate but only trivially from one another, where objects of different types can be exclusively determinate.

Brandom creates a preliminary grammar in which discrete members of the same type are not equal ($\neq$) and members of different types are exclusive ($\#$). HoTT accounts for similar notions, but notably can create much more complex structures than mere syntactic propositions. Here we can see where a more formal approach enables richer theoretical innovations than informal prose. Though Brandom develops a preliminary syntax or propositional logic for Hegel's reasoning, his main focus—inferentialism, semantics, and pragmatics—require much more dynamic mathematical tools to be properly formalized and systematized. It is a happy accident that Brandom refers to his propositional syntax as "*types*" and "*tokenings*" given HoTT's appropriation of similar language. HoTT in place of natural language can be modeled in functional programing languages like Coq or Agda, though the essential mechanism in Brandom is preserved in this excersize using HoTT. Namely, we seek to provide more structure to the reading of Hegel's *Phenomenology.* That is not to say one is adding structure where it does not already exist in Hegel, but rather one's reading of Hegel is structural as opposed to other tendencies like historicist. Brandom's structural reading employs the logicism of Frege, Tarski[42] and Quine and the philosophy of language

[39] Hegel, *The phenomenology of spirit*, p. 88
[40] Hegel, *The phenomenology of spirit*, p. 90
[41] Brandom, *A spirit of trust*, p. 121
[42] Though taking inspiration from Tarski, Brandom makes an explicit distinction between

of Wittgenstein, where our formal approach engages more directly with synthetic language and mathematical structures rather than linguistic structures. The result is more formal and systematic then it is pragmatic, while maintaining the goal of imparting structure into the project of reading the *Phenomenology*.

For Brandom, Hegel's "metaconcepts: mediation, universality, determinateness, and negation"[43] belong to logic. Similarly for Badiou, the ontological concepts from Hegel belong to set theory. Our project in using HoTT, seeks to employ the semantic dynamism of philosophy of language in Brandom's reading and the rigid mathematical foundations of set theory in Badiou, by utilizing philosophically rich and highly structured mathematical modeling which accounts for the best qualities of both approaches.

Negarestani's reading of Brandom in *Intelligence and Spirit* emphasizes a third way that is neither purely logicist nor traditionally ontological: semantic ascent. He writes,

> *Intentional ascent*, or the complexity of thinking thoughts, demands a *semantic ascent*—that is, a hierarchical complexity of both different grades of concepts (i.e., inferentially articulated contents) and the practical know-how to use or apply concepts correctly. In ascending the hierarchy of semantic complexity, the automata attain semantic self-consciousness. They become discursively aware of how things are by thinking about thoughts through thinking about concepts that inferentially articulate those thoughts, and thereby the things thought of.[44]

The HoTT view of Hegel's dialectical motion uses type-theoretic universes to denote what Negarestani calls semantic ascent. That is the operation by which the type universe is escalated, i.e. the motion of spirit, is the dynamic process by which our formal intuitionistic system articulates the increasing complexity of thought as it transits recognition. Though HoTT denotes these transformations in terms of homotopic paths, which are properly syntactically encoded, the dynamism and intrinsic semantic inferentialism of HoTT's topological orientation avoids the pitfalls of what AA Cavia calls "zombie formalism."[45]

Hegel's computational universe

As AA Cavia argues in his book *Logiciel*,

> The homotopic version of type theory put forward by univalent foundations (Voevodsky) as a way of making explicit the logical nature

Tarski's logic and Hegel's metaphysics. One may find an objection to Tarski's use of metalinguistic structures for semantics which conflicts with the inuitionistic view of type theory and Brandom's philosophical inferentialism.

[43]Brandom, *A spirit of trust*, p. 136

[44]Reza Negarestani. *Intelligence and spirit.* Falmouth: Urbanomic, 2018, p. 334

[45]AA Cavia. *Logiciel: six seminars on computational reason.* Berlin: &&&, 2022, p. 86

> of a holistic computational worldview. This worldview, an intuitionistic view of computing attempts to offer an internal semantics for computational reason which specifies the role of mathematics and logic without simply subordinating the computational to the mathematical, or brushing external content under the carpet.[46]

Importing and employing HoTT as a formal model of Hegelian dialectics aligns the philosophical view argued by Brandom (semantic holism) and Cavia's pragmatic view (computational holism). Far from the argument that "the universe computes," Hegelianism with HoTT implies *what is universal is computational*. To further explore this distinction is to disambiguate logical positivism from dialectics. The philosophical stakes of a formal reading of Hegel are to produce a notion of the universal unburdened by the essentialism of both the logicist tendency in analytic philosophy and the poetic ontologies of continental thought. Thus, to say the universal is computational is to affirm Hegel's central notion that higher unities emerge in the total motion of contradiction. Contrast this to the speculative axiom that "the universe computes" which in a completely non-dialectical fashion treats the universe as an *a priori* object.

As Cavia notes, in HoTT "computability becomes synonymous with a continuous path within these typed spaces, casting the dual of the continuous and the discrete in a new light."[47] Thus in reading Hegel with HoTT, the computable universe is merely the fact that objects have a continuous history of dialectical transformations. From this vantage Spirit and computability are virtually indistinguishable concepts. This is not to say that Hegelian logic is pan-computationalist. Hegel maintains the Spinozist thesis that all determination is negation, such that only computational processes which are deterministic can constitute Hegel's universe. Rather, to say Hegel's universal is computable, is to say that the totality of determinations (negations), objects and their history can be captured in the proof-theoretic models which operationalize HoTT as computation; type-construction is the generative process of semantic ascent by which each discrete determinate universe continuously deforms into the next. The dialectic of digital and analogue logics univalent foundations enables maintains the consistency of Hegelian notions of universality, totality and Spirit.

In computation we see the eroding of a metaphysical *model* of the world, and in its place a metaphysics of the world proper. For Cavia, "computation is the very movement of this collapse, a rejection of the metaphysics of the 'model', for computation acts not as a frame, but as a world in its own right."[48] The realist implications of this view of computation is the inverse of our previous axiom—what is universal is computation—namely, what is computatble belongs to the universe. In this reading belongs does not have its strict set-theoretical baggage, but rather implies that what can be thought is granted ontological status.

[46]Cavia, *Logiciel*, p. 64

[47]Cavia, *Logiciel*, p. 114

[48]Cavia, *Logiciel*, p. 166; From a Marxist frame, we might reject this view of computation as idealist, but in capturing Hegel's thought, such an idealist rendering is appropriate.

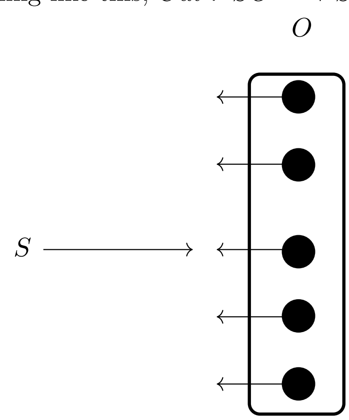
HoTT collapses the category theoretical notion that functors map a shape to a diagram, in favor of a pure immanence of the topological itself. In the notion of space offered by univalent foundations, form no longer needs such a functor to map form to the world, worlding is implicit in the structure of space itself. In HoTT, homotopies between paths can be constructed at infinitely higher levels of abstraction, up to the Type-n universe, and in functional programming languages which model HoTT like Coq, such n-Types can be verified and computed. Thus HoTT, and computational realism, allows one to operationalize the primacy of cognition in Hegel's conception of the universal.

The limits of Category-Theoretic Models

In the paradigm of *desire* and *recognition* in the self-consciousness chapter of the *Phenomenology* we can identify one area where category-theoretic schema fall short of what we can do with HoTT. Notably, to model the relation subject and object in category theory would look something like this:

$$\begin{tikzcd} S \arrow[r, bend left] & O \arrow[l, bend left] \end{tikzcd}$$

To model the relationship of subject and the field of the objective in category theory would look something like this, $Cat:\ SO^{op} \to Set$:



In HoTT, we could have something similar to the first diagram representing the rather trivial case of two (-1)–Types or mere propositions on S and O.

$$\begin{tikzcd} S \arrow[r, bend left, "f"] & O \arrow[l, bend left, "g"] \end{tikzcd}$$

Where it gets interesting is in the increasing abstraction of homotopies and path spaces we can build from this trivial case.

The Sets α and β: and The Groupoid G:

$$\begin{tikzcd}[column sep=huge] S \arrow[r, bend left=60, "f"] \arrow[r, phantom, "\alpha\Uparrow\ \Downarrow\beta"] & O \arrow[l, bend left=60, "g"] \end{tikzcd} \qquad \begin{tikzcd}[column sep=huge] S \arrow[r, bend left=60, "f"] \arrow[r, phantom, "\alpha\Uparrow\xrightarrow{G}\Downarrow\beta"] & O \arrow[l, bend left=60, "g"] \end{tikzcd}$$

And so on and so forth up to a weak ∞-groupoid or n-Type, each denoting a higher inductive level of abstraction and complexity.

Why is this model preferable to the category-theoretic or point-topological approach? Notably, the HoTT form is only superficially diagrammatic. Meaning, the diagrams included above are meant only to be illustrative of a type-theoretic or potentially computational inductive form. By reconceptualizing the relations between subject and object in terms of generic spaces, paths and homotopic path spaces, we see that each higher level of abstraction:

(1) Finds its genesis in the previous level
and,
(2) Is built on the identity relations of the subjective and objective, respectively, and the equality-relation between them.

Following the computational intuition, we see that the initial introduction of consciousness brought about by the encounter of the subject and the world, spawns an infinite induction of dialectical equality relations presupposed on the identity of the subject and object, and their intrinsic difference or otherness.

To construct the types inductively in a computational form, we can formalize using the HoTT library in Coq like such:

```
(* Import HoTT library *)
Require Import HoTT.

(* Define contractible types S and O *)
Definition S : Type := nat.
Definition O : Type := bool.

(* Show that S and O are contractible types *)
Instance contractible_S : Contr S := contr (0 : S).
Instance contractible_O : Contr O := contr (true : O).

(* Define two types alpha and beta *)
Definition α : Type := nat.
```

```
Definition β : Type := bool.

(* Define two functions f : S -> α and g : O -> β *)
Definition f : S -> α := fun s => s + 1.
Definition g : O -> β := fun o => negb o.

(* Define the type of morphisms in a groupoid as paths *)
Definition Hom (X Y : Type) : Type := X = Y.

(* Define identity morphisms *)
Definition id {X : Type} : Hom X X := eq_refl.

(* Define composition of morphisms in the groupoid *)
Definition comp {X Y Z : Type} (p : Hom X Y) (q : Hom Y Z) : Hom
↪  X Z :=
  p @ q.

(* Define inverse of morphisms *)
Definition inv {X Y : Type} (p : Hom X Y) : Hom Y X :=
  p^.

(* Define Groupoid laws (as lemmas in HoTT) *)

(* Associativity of composition *)
Lemma comp_assoc {X Y Z W : Type} (p : Hom X Y) (q : Hom Y Z) (r
↪  : Hom Z W) :
  comp p (comp q r) = comp (comp p q) r.
Proof.
  reflexivity.
Qed.

(* Left identity law *)
Lemma left_id {X Y : Type} (p : Hom X Y) :
  comp id p = p.
Proof.
  reflexivity.
Qed.

(* Right identity law *)
Lemma right_id {X Y : Type} (p : Hom X Y) :
  comp p id = p.
Proof.
  reflexivity.
Qed.

(* Left inverse law *)
```

```
Lemma inv_left {X Y : Type} (p : Hom X Y) :
  comp (inv p) p = id.
Proof.
  apply concat_Vp.
Qed.

(* Right inverse law *)
Lemma inv_right {X Y : Type} (p : Hom X Y) :
  comp p (inv p) = id.
Proof.
  apply concat_pV.
Qed.

(* Groupoid structure *)

(* A Groupoid is simply a type where every morphism has an
↪  inverse. *)
Record Groupoid := {
  Obj : Type;
  Hom : Obj -> Obj -> Type := fun X Y => X = Y; (* Hom-sets are
  ↪  path types *)
  id : forall X : Obj, Hom X X := fun X => eq_refl;
  comp : forall X Y Z : Obj, Hom X Y -> Hom Y Z -> Hom X Z :=
    fun X Y Z p q => p @ q;
  inv : forall X Y : Obj, Hom X Y -> Hom Y X :=
    fun X Y p => p^;

  (* Groupoid laws *)
  assoc : forall X Y Z W : Obj (p : Hom X Y) (q : Hom Y Z) (r :
  ↪  Hom Z W),
          comp X Z W (comp X Y Z p q) r = comp X Y W p (comp Y
          ↪  Z W q r);
  id_left : forall X Y : Obj (p : Hom X Y), comp X X Y (id X) p =
  ↪  p;
  id_right : forall X Y : Obj (p : Hom X Y), comp X Y Y p (id Y)
  ↪  = p;
  inv_left : forall X Y : Obj (p : Hom X Y), comp X Y X (inv X Y
  ↪  p) p = id X;
  inv_right : forall X Y : Obj (p : Hom X Y), comp Y X Y p (inv X
  ↪  Y p) = id Y
}.

(* Example groupoid G with objects alpha and beta *)
Definition G : Groupoid := {
  Obj := nat + bool; (* Objects are the disjoint union of nat and
  ↪  bool *)
```

```
  assoc := fun _ _ _ _ _ _ _ _ => eq_refl;
  id_left := fun _ _ _ => eq_refl;
  id_right := fun _ _ _ => eq_refl;
  inv_left := fun _ _ _ => eq_refl;
  inv_right := fun _ _ _ => eq_refl
}.
```

We can also represent a generic groupoid like G in another dependently typed language like cubical Agda. The Agda model has some strengths, for instance it is more intuitively readable, and because the types take the shape of mathematical objects like the empty type ($\varnothing$), the unit type ($\{\varnothing\}$), the natural numbers ($\mathbb{N}$) and the Boolean (Bool), we can create a more one-to-one correspondence between philosophical concepts and mathematical objects. Here is an example of what that would look like where we use the generic contractible types $\varnothing$ (the empty type) and $\{\varnothing\}$ (the unit type) as place holders for Subject (S) and Object (O) respectively:

```
{-# OPTIONS --cubical #-}
module ContractibleEmptyUnit where

open import Agda.Primitive
open import Cubical.Core.Everything
open import Data.Empty  -- Import for the empty type
open import Data.Unit   -- Import for the unit type
open import Data.Bool   -- Import for Bool
open import Data.Nat    -- Import for ℕ (natural numbers)

-- Define S as the empty type (∅)
S : Set
S = ⊥  -- ⊥ is the empty type in Agda

-- Define O as the unit type ({∅})
O : Set
O = ⊤  -- ⊤ is the unit type in Agda

-- Show that S () is contractible
-- The empty type is trivially contractible since it has no
↪  elements.
contractible_S : Contr S
contractible_S = contr (λ x → x , λ _ → ⊥-elim x)

-- Show that O ({∅}) is contractible
-- The unit type is contractible because there is only one
↪  element (tt).
contractible_O : Contr O
contractible_O = contr tt λ _ → refl
```

```
-- Define two types alpha (α) and beta (β)
α : Set
α = ℕ  -- alpha is the type of natural numbers

β : Set
β = Bool  -- beta is the type of booleans

-- Define a function f : S → α (from ∅ to ℕ)
f : S → α
f x = ⊥-elim x  -- No function can map from an empty type

-- Define a function g : 0 → β (from {∅} to Bool)
g : 0 → β
g tt = true  -- Since 0 is unit type, we just map the single
↪  element to true

-- Define the type of morphisms (Hom) in a groupoid as paths
Hom : (X Y : Set) → Set
Hom X Y = X ≡ Y  -- Hom-sets are path types in HoTT

-- Define identity morphism (id) for any type X
id : {X : Set} → Hom X X
id = refl  -- The identity morphism is reflexivity

-- Define composition of morphisms in the groupoid
comp : {X Y Z : Set} → Hom X Y → Hom Y Z → Hom X Z
comp p q = p • q  -- Composition is path concatenation (•)

-- Define inverse of morphisms
inv : {X Y : Set} → Hom X Y → Hom Y X
inv p = sym p  -- Inverse is just path symmetry

-- Groupoid laws

-- Associativity of composition
comp_assoc : {X Y Z W : Set} (p : Hom X Y) (q : Hom Y Z) (r : Hom
↪  Z W) →
  comp p (comp q r) ≡ comp (comp p q) r
comp_assoc p q r = refl  -- Composition is associative

-- Left identity law: composing with the identity on the left
left_id : {X Y : Set} (p : Hom X Y) → comp id p ≡ p
left_id p = refl  -- Composing with identity on the left does
↪  nothing

-- Right identity law: composing with the identity on the right
```

```
right_id : {X Y : Set} (p : Hom X Y) → comp p id ≡ p
right_id p = refl  -- Composing with identity on the right does
↪  nothing

-- Left inverse law: composing a morphism with its inverse on the
↪  left gives the identity
inv_left : {X Y : Set} (p : Hom X Y) → comp (inv p) p ≡ id
inv_left p = refl  -- Composing a morphism with its inverse on
↪  the left gives identity

-- Right inverse law: composing a morphism with its inverse on
↪  the right gives the identity
inv_right : {X Y : Set} (p : Hom X Y) → comp p (inv p) ≡ id
inv_right p = refl  -- Composing a morphism with its inverse on
↪  the right gives identity

-- Define the Groupoid structure as a record in Agda
record Groupoid : Set where
  field
    Obj : Set  -- The objects in the groupoid
    Hom : Obj → Obj → Set  -- The morphisms (Hom-sets)
    id : (X : Obj) → Hom X X  -- Identity morphisms
    comp : {X Y Z : Obj} → Hom X Y → Hom Y Z → Hom X Z  --
    ↪  Composition of morphisms
    inv : {X Y : Obj} → Hom X Y → Hom Y X  -- Inverse of
    ↪  morphisms

    -- Groupoid laws
    assoc : {X Y Z W : Obj} (p : Hom X Y) (q : Hom Y Z) (r : Hom
    ↪  Z W) →
      comp p (comp q r) ≡ comp (comp p q) r  -- Associativity
    id_left : {X Y : Obj} (p : Hom X Y) → comp (id X) p ≡ p  --
    ↪  Left identity
    id_right : {X Y : Obj} (p : Hom X Y) → comp p (id Y) ≡ p  --
    ↪  Right identity
    inv_left : {X Y : Obj} (p : Hom X Y) → comp (inv p) p ≡ id X
    ↪  -- Left inverse
    inv_right : {X Y : Obj} (p : Hom X Y) → comp p (inv p) ≡ id Y
    ↪  -- Right inverse

-- Example groupoid G with objects α and β
G : Groupoid
G = record {
  Obj = α ⊎ β  -- Objects are the disjoint union of ℕ and Bool
  ; Hom = λ X Y → X ≡ Y  -- Hom-sets are path types
  ; id = λ X → refl  -- Identity morphism is reflexivity
```

```
  ; comp = λ {X Y Z} p q → p • q  -- Composition is path
  ↪  concatenation
  ; inv = λ {X Y} p → sym p  -- Inverse is path symmetry
  ; assoc = λ _ _ _ _ _ _ _ → refl  -- Associativity holds
  ↪  trivially
  ; id_left = λ _ _ _ → refl  -- Left identity holds trivially
  ; id_right = λ _ _ _ → refl  -- Right identity holds trivially
  ; inv_left = λ _ _ _ → refl  -- Left inverse holds trivially
  ; inv_right = λ _ _ _ → refl  -- Right inverse holds trivially
}
```

While the computational-syntactic version outlined in Coq or Agda is much more tedious than the diagramatic representation, the goal is to show a general structure which Hegel sketches throughout the *Phenomenology*—one that Hegel uses to express the relationship of the subject to the objective world, but could be used to formalize a behavior between any two dialectically engaged categories and their corresponding system. The purpose of this formal expression is two-fold. In the diagramatic case, we can visualize dialectical motion and totality in its most generic form. In the example using Coq sample code, we can show how, perhaps for the first time something like Lawvere's categorical logic, brought about by reading Hegel can be formalized and operationalized as mechanical computation, bolstering in an empirical sense our earlier claim that "what is *universal* is *computable*."

The distinction between "the universe computes" and "what is *universal* is *computable*," lies in relation to the Real. In the mathematical case of real numbers, $\mathbb{R}$,[49] or in the metaphysical and psychoanalytic understanding, the real cannot be abducted[50] by the mechanical operations of computers. That is computation of the Real is indeterminate; the program does not halt. This is perhaps the greatest refutation of the pan-computationalist hypothesis: "the universe computes." For Hegel, the real, or the "infinite One,"

> is devoid of content; it gives itself its content in the [organic] shape, and in that shape it appears as that shape's process. In this extreme as simple negativity, or as *pure singularity*, the organic is in possession of its absolute freedom through which it is both safeguarded and indifferent vis-à-vis being for others and vis-à-vis the determinateness of the moments of the shape.[51]

That is to say that the Real is not a computable category in itself, but a category which must be abducted as a quality formed in relation to an organic entity.

[49] Most real numbers are computable, when referring to the Real as uncomputable we refer to the concept of the continuum without Dedekind cuts, or families of uncomputable numbers like Chaitin's constant.

[50] Here and consequently, the use of the term "abduction" is in the Piercian sense, to mean a procedure of logic which prefigures both deduction and induction. In the case of computationally abducting the Real, we are referring to a process by which the machine attempts to subsume uncomputable numbers or strings as legible in its logical framework of consistency.

[51] Hegel, *The phenomenology of spirit*, p. 166

In the HoTT-theoretical view, the Real is still not computable in itself, but a quality which can be abducted in the form of natural transformations between empirical entities we call spaces. Thus, in HoTT and in the Hegelian view what *is computable* remains the reflection of Real qualities onto organic entities (spaces) which *is universal.* That is "[t]he universal *content* of the actual way of the world has already emerged,"[52] such that "[t]he way of the world inverts the unchangeable, but in fact inverts it from the *nothingness of abstraction into the being of reality.*"[53] In Hegel's view, the ineffable Real or the uncomputable, is a nothingness of abstraction which only through engagement with the world gains its universal content, i.e. becomes computable. Thus, what is universal is computable even if a non-trivial *outside* remains uncomputable.

As Hegel claims in the Spirit chapter of the *Phenomenology*, "the category is determined for consciousness as it is in its universal truth, as essence existing *in-and-for itself.*"[54] That is to say if we are operating in the realm of consciousness, which is evident in the case of mechanical computation, conceptual categories like the Real can be abducted as universal properties which are consistent with a computational proof, what Hegel calls "its universal truth." Far from the pan-computationalist thesis, our inversion posits that the qualities or content which can be computed mechanically is the universal truth produced through the abduction of relating noncomputational forms through homotopy equivalencies.

Problems with Recursion

The issue of infinitely recursive enumeration is central to computation, and computational philosophy. Rather than rehearse the existing discourse in computational theory regarding recursion, I want to briefly state where HoTT and to a greater extent, Hegel, impose new problems for recursive enumeration. First, as can be extrapolated from the Agda sample code in the previous section, universe levels in dependently-typed languages with support for HoTT (Coq HoTT library, Cubical Agda, etc.) require explicit definition to compute. Thus we need an infinite tape, the likes of a theoretical Turing machine to compute some of the most important structures in HoTT, weak ∞-groupoids. Structures which can be represented as infinite recursion arithmetically can only be computed on an infinite time scale, or with an infinitely-long explicitly enumerated string of code.

This mirrors what Hegel terms "*absolute mediation.*" That is an infinitely mediated substance which takes the form of a stable "*immediate existence*".

> It is *absolute mediation*, like the culturally forming consciousness and the faithful consciousness, for the movement of the self is essentially that of sublating the abstraction of *immediate existence*

[52] Hegel, *The phenomenology of spirit*, p. 220
[53] Hegel, *The phenomenology of spirit*, p. 224
[54] Hegel, *The phenomenology of spirit*, p. 253

> and becoming, to itself, universal—but it does this neither through purely alienating and disrupting itself and actuality, nor does it do this by running away. Rather, it is, to itself, *immediately current* in its substance, for this substance is the intuited pure certainty of itself.[55]

We can say that *for us* weak ∞-groupoids are immediately current, as we have a way of elaborating the concept symbolically, but for machines, that is computationally, there is no general stable language for computing what can only be written as an infinite string of code. Thus, there is a gap between what is for us conceptual information, and what is for the machine aconceptual mechanization. It appears evident that there is something happening with computational recursive enumeration that is different from conceptual cognition. Something which Hegel distinguished dialectically as *becoming* and "pure being," and which Lawvere equated to homotopies and terminal objects.[56] That is, becoming is modeled by continuous deformation, and pure being is modeled by the singular and final target object within a category.

In the religion chapter of the *Phenomenology* Hegel distinguishes between the universal character of cognition which we have called "computable" and something which is *beyond* or "uncomputable," which he gives several names, chief among them is "spirit knowing itself as spirit." Hegel claims, "*Consciousness,* to the extent that it is *the understanding*, already becomes consciousness of the *supersenisble*, or consciousness of the *inner* of objective existence. However, the supersensible, the eternal, or whatever else one may call it, is *devoid of self*. Initially, it is only the *universal* which is still some distance removed from spirit knowing itself as spirit."[57] The issue of infinite recursive enumeration emphasizes the contrast between these two conceptual categories: the computable "universal" and the uncomputable "eternal," which, as Hegel claims are some distance apart. Unlike $\infty-$groupoids which cannot be recursively enumerated, topological objects of infinite dimension can be induced in typed languages with support for higher inductive types like cubical Agda.

Take the topological sphere with n dimensions (n-Sphere) as an example of such a space, where a 0-Sphere is a shape with two discrete points, the 1-sphere is a circle, and a 2-sphere is the traditionally conceived ball in 3-dimensional space. Using construction, we can computationally define a sphere of n dimensions where n is indexed by the natural numbers $\mathbb{N}$ and recursively enumerated. In Agda we can compute the n-Sphere like this:

```
module NSphere where
  open import Cubical.Foundations
  open import Cubical.Path
```

[55] Hegel, *The phenomenology of spirit*, p. 347

[56] F. William Lawvere. "Categories of Space and of Quantity". In: *The Space of Mathematics*. Ed. by Javier Echeverria, Andoni Ibarra, and Thomas Mormann. DE GRUYTER, Dec. 31, 1992, pp. 14–30, p. 29

[57] Hegel, *The phenomenology of spirit*, p. 390

```
open import Cubical.Data.Nat
open import Cubical.HITs.Suspension

-- Base case: 0-sphere (S⁰)
S⁰ : Set
S⁰ = Bool  -- S⁰ is just two points: true and false

-- Inductive definition of n-sphere (Sⁿ)
Sⁿ : ℕ → Set
Sⁿ 0       = S⁰
-- Base case: 0-sphere
S⁰ (suc n) = Suspension (Sⁿ n)
-- Suspension of the (n-1)-sphere

-- Example: Explicitly defining S¹ and S²
S¹ : Set
S¹ = Sⁿ 1
-- S¹ = Suspension of S⁰

S² : Set
S² = Sⁿ 2
-- S² = Suspension of S¹

-- Function to illustrate working with spheres
-- A constant map from Sⁿ n to Unit
constMap : ∀ {n} → Sⁿ n → Unit
constMap _ = tt
-- Every point maps to the unit element

-- A dependent function on S¹
exampleFuncOnS¹ : S¹ → Set
exampleFuncOnS¹ north = ℕ
-- North pole maps to ℕ
exampleFuncOnS¹ south = Bool
-- South pole maps to Bool
exampleFuncOnS¹ (merid true i) = ℕ
-- Path: Meridians map to ℕ
exampleFuncOnS¹ (merid false i) = Bool
-- Path: Meridians map to Bool
```

As we can see from the sample code above, a shape of infinite dimension can easily be defined computationally in a dependently-typed language like Agda, but there are some crucial differences between this kind of topological object and the homotopy type ∞-groupoid. Namely, the n-Sphere can be induced with a successor function and suspension because its infinite dimensionality is indexed by a countable set, where the ∞-groupoid has potentially infinite levels of morphisms.

This *indexing* reveals something about conceptual content in dependently-typed computation: programs can induce algebraic structures of higher complexity, even in infinite dimensions, if they have an indexical relationship to simpler algebraic structures, namely sets, however programs cannot induce high-level conceptual content because computation has no experience from which to draw indexical relationships to conceptual content. In dependently-typed programs we can explicitly define a concept given a set of logical rules, but there is no way to computationally index conceptual content, or in other words, conceptual content cannot be made computationally implicit. In Hegelian terms, we can compute knowledge—that is we can make explicit conceptual content computationally so that it is *known* to the program—but programs do not have *experience*—that is, the conceptual content remains *for us* and it is not known to the machine as conceptual content, just as Hegel says "spirit does not know itself as spirit."[58]

Computation teaches us the division between the "actual" and the "pure," where what is computable is merely linguistic structure devoid of conceptual content, and the uncomputable is a discursive surplus. Programs compute the explicit articulation of linguistic statements which give a skeletal structure for concepts, but the implicit pragmatic conceptual content remains exclusively *for us*. As the technological product of modernity computation maintains Hegel's thesis regarding the central role of alienation in modernity. Computation bifurcates linguistic structure[59] (the computable) and discursive meaning (the uncomputable) so that a fundamental alienation between mere structure and the pragmatic field of discursive meaning is intrinsic to the computational form.[60]

This interpretation, Hegel through Lawvere, posits universality in computational ontology in a new and critical light. Where universality is maintained it is essentially contingent, however it remains robust enough that the claim "all that is computable is universal" holds. Where universality is (partially) compromised is in the rejection of the computability of "absolute mediation." As noted previously there is no way to computationally enumerate infinite universe levels. This wrench in computing infinite recursive enumeration, leaves us with an infinite set of universe levels, which are each *uniformly* contingent. There is a price to pay for circumventing Russel's paradox. The core problem of traditional ZFC set theory is that the system is not complete, it must always reach outside itself for an additional axiom to find consistency. Category theory only partially resolved this problem by distinguishing between "small" categories and "large" categories. Type-theoretic foundations provide the most successful resolution of Russell's paradox with the innovation of universe levels, however, at the level of computation the contingency of universe levels means there is no generalizable form for infinite universe-level higher inductive types. Again there is no complete set of axioms that gives consistency to the system

[58]Hegel, *The phenomenology of spirit*, p. 390

[59]Here "linguistic structure" refers to syntax and a limited notion of semantics operative in machine languages.

[60]Brandom, *A spirit of trust*, pp. 500–503

irrespective of universe-level. What Hegel teaches us is that this contingency does not negate universality entirely. Instead HoTT with Hegel advances a new universality that reads univalence as a dialectical motion up the ladder of universes with no enumerable limit. That there is an abstract concept of absolute mediation which can be cognized but cannot be mechanically (i.e. computationally) manifested, poses new challenges for how we conceptualize the relationship between cognition and computation.

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Reading Capital as an Autopoietic System: A Categorical Approach

> If the process is complex, as in autopoiesis, then some time steps will leave the same for the observer, and some time steps will have a difference associated with them. Indeed, some time steps can embody the disappearance of the form and a disappearance of the eigenform.
>
> Louis H. Kaufmann, "Autopoiesis and Eigenform"

> (1) *Immediate identity*: Production is consumption, consumption is production. Consumptive production. Productive consumption. The political economists call both productive consumption. But then make a further' distinction. The first figures as reproduction, the second as productive consumption. All investigations into the first concern productive or unproductive labour; investigations into the second concern productive or non-productive consumption. (2) [In the sense] that one appears as a means for the other, is mediated by the other : this is expressed as their mutual dependence; a movement which relates them to one another, makes them appear indispensable to one another, but still leaves them external to each other. Production creates the material, as external object, for consumption; consumption creates the need, as internal object, as aim, for production. Without production no consumption; without consumption no production. [This identity] figures in economics in many different forms.
>
> Karl Marx, *Grundrisse: foundations of the critique of political economy*

Autopoiesis and Capital

This chapter uses insights from category theory (CT) and autopoietic systems theory to read capitalist accumulation as a coherent self-organizing dynamical system. This formal approach to reading Capital seeks to utilize mathematical structuralism as an analytic framework for studying the structures of capitalism. In close engagement with the "So-called primitive accumulation" chapter of Capital Vol I with occasional support from Spinoza and Hegel, this chapter uses the concepts of autopoiesis and eigenform from mathematics of computation to argue capitalist accumulation requires the perpetual inclusion of pre-capitalist "primitive accumulation" to initiate new raw materials and new laboring subjects into the capitalist commodity form and wage relation. In this sense, primitive accumulation is an eigenbehavior, or initial behavior of capitalism, which must constantly elaborate itself to sustain capital. If primitive accumulation is the eigenbehavior of capital, its structured reiteration or eigenform is imperialism. The implication of this reading is to say there is a pre-capitalist or non-capitalist germ internal to capitalism's reproduction. In category-theoretical terms, we can identify the relationship between the non-capitalist mode of "primitive accumulation" and capitalist accumulation as a universal morphism called imperialism. That is, in the universe inhabited by capitalism, imperialism preserves itself as a universal structure of the system. My intention with this talk is to give an overview of what modeling capitalism as an autopoietic system might look like with a theoretical exposition, followed by a more practical discussion of how these dynamics, that the formal system is trying to model, elaborate themselves. Namely, I would like to utilize the case study of the Philippines, where the violent seizure of land and resources (non-capitalist accumulation) exists contemporaneously with the movement of global capitalism. Using Walter Rodney's theory of underdevelopment in conjunction with the analyses of the Philippine New Democracy Movement, we can begin to see how a formal model of capitalism as an autopoietic system captures the sustained reproduction of capitalism at the site of non-capitalist superexploitation. To recognize imperialism as a universal characteristic of the capitalist form, is to implicate all subjects under capitalism in the violence of said imperialism, as constitutive of a global process. At the core of revitalizing a universalist analysis is a call to action for internationalism, and a critique of the national-chauvinism of Western social-democracy. That is, the universal is a frame of analysis necessitating that any action taken against capitalism in its local form must be coupled with a global struggle against the non-capitalist plunder which serves as an anchor for the entire system.

Interest-Bearing Capital

Objects:	**Morphisms**:
Natural Value: V_N; Necessary Value: V_n; Surplus Value: V_s	Initial Investment: i_0
Money: M, $M^{'}$, $M^{''}$, ΔM	Production: $prod_0$, $prod_1$, ..., $prod_n$
Commodity: C, $C^{'}$, C_N	Exchange: ex; ex_0, ex_1, ..., ex_n
Wages: W; Labor Power: LP; Labor Time: LT	Wage Relation: wr
Productivity: p; Savings: S; Time: t; Necessary Time: t_n	Equality: $=$

Pictured above is a very preliminary blueprint for the organization of the M-C-M' circuit outlined in *Captial Vol. I.*[1] For the purposes of our analysis, and at the risk of reductivism, I suggest the reader focus their attention to the top element ($\mathcal{N}$) for nature as it enters into the wage relation, and the relation of the M-C-M' circuit. Pictured below in Figure 2 is a simplification of this fundamental process whereby the only non-trivial morphisms are primitive accumulation (*prim.acc.*)

[1]Karl Marx. *Capital: A Critique of Political Economy, Volume 1.* Trans. by Ben Fowkes. Reprint edition. London ; New York, N.Y: Penguin Classics, May 5, 1992

$$
\begin{tikzcd}
\mathcal{N} \arrow[d] & & & & & & (t-t_n)p \arrow[d, equal] \\
V_N + W \cdot t \arrow[dd, "i_0"] & LP \cdot t \arrow[d, equal] & & C_N \arrow[d, "ex"] & & & LT_s \cdot p \arrow[d, equal] \\
& (LT & \times \arrow[d] & V_N) \arrow[r, "wr"] & (V_n & + \arrow[d] & V_s) \\
M \arrow[rr, "prod_0"] & & C \arrow[rrr, "ex_0"] & & & M' \arrow[r, equal] \arrow[d, equal] & M + \Delta M \\
& & & & (S & + & (M' - S)) \arrow[d, "prod_1"] \\
& & & & & & C' \arrow[d, "ex_1"] \\
& & & & & & M''
\end{tikzcd}
$$

Figure 1: Circulation

and the cyclical morphism (K) composed of production (*prod*) and exchange (*ex*).

Objects:	**Morphisms:**
Nature: $\mathcal{N}$	Primitive Accumulation: *prim.acc.*
Commodity: C_0, C, $C^{'}$, ..., C^n	Production: $prod_0$, $prod_1$, ..., $prod_n$
Money: M, $M^{'}$, ..., M^n	Exchange: ex_0, ex_1, ..., ex_n
	Capitalist Accumulation: $\mathcal{K}$
	$prod \circ ex = \mathcal{K}$

We can set our general model for capital, which is a discrete dynamical sys-

$$
\begin{tikzcd}
\mathcal{N} \arrow[d, "prim.acc."] \\
C_0 \arrow[d, "\mathcal{K}_0"] \\
M \arrow[rr, "\mathcal{K}"] \arrow[dr, "prod_0"] & & M^{'} \arrow[rr, "\mathcal{K}"] \arrow[dr, "prod_1"] & & M^{''} \arrow[rr, "\mathcal{K}"] \arrow[dr, "prod_2"] & & M^{'''} \arrow[rr, dashed, "\mathcal{K}"] \arrow[dr, dashed, "prod_n"] & & M^n \\
& C \arrow[ur, "ex_0"] & & C^{'} \arrow[ur, "ex_1"] & & C^{''} \arrow[ur, "ex_2"] & & C^{\,n-1} \arrow[ur, "ex_n"] &
\end{tikzcd}
$$

Figure 2: General Model for Capital

tem with an initial element ($\mathcal{N}$) or nature equal to the presheaf $\mathcal{K}^{op} \xrightarrow{F} Set$ which maps to the object capital $\rightarrow \mathcal{K}$. From a Categorical perspective we can also view the functors $\mathbb{N} \rightrightarrows M$ mapping each iteration of the money form indexed by the natural numbers to the category of money as the comeet of two partial morphisms ∂_0 and ∂_1 in M such that $\mathcal{K}$ is the constant functor mapping each instantiation of the circuit.[2] Such that, excluding the initial introduction of nature into the system via primitive accumulation, the circuit can be modeled by the tree of meets or pullbacks. Such that for $M \xrightarrow{\mathcal{K}} M'$:

$$
\begin{tikzcd}
 & & (prod, ex) \arrow[dl] \arrow[dr] & & \\
 & (prod, M) \arrow[dl] \arrow[dr] & & (M, ex) \arrow[dl] \arrow[dr] & \\
C \arrow[dr, "prod"] & & M^2 \arrow[dl, "M^{\overline{\partial}_0}"'] \arrow[dr, "M^{\overline{\partial}_1}"] & & C' \arrow[dl, "ex"] \\
 & M & & M & \\
 & M \arrow[u, "\mathcal{K}"] & & M \arrow[u, "\mathcal{K}"] & \\
 & & \mathbb{N} \arrow[ul, "\partial_0"] \arrow[ur, "\partial_1"'] & &
\end{tikzcd}
$$

Figure 3: Tree Diagram for $M \xrightarrow{\mathcal{K}} M'$ indexed by $\mathbb{N}$

Here, the morphisms *prod* and *ex* are in there generic form and should be considered the functors *prod* and *ex* representing the domain and codomain of the functor $\mathcal{K}$. Similarly M is made generic to represent the category of money where M is indexed by the natural numbers up to the limit ordinal ω such that $\omega = (0, \mathbb{N})$. This commutative diagram shows in a left-right model a closed circuit for capitalist accumulation. Thus, unlike figure 2, which generally describes primitive accumulation *and* the circulation of commodities, figure 3 highlights the motion of commodity circulation in categorical form.

Figures 1-3 all give quite technical accounts of the structure of commodity circulation, where Figure 1 is mechanical, Figure two is primitively categorical, and Figure 3 has a rich categorical structure. The aim of the image below depicting a monoid, or single-object category $\mathcal{K}$ with one morphism σ is to effectively reduce the M-C-M' circuit to its most base autopoietic form. Like the ouroboros swallowing its own tail, capital continuously moves and develops while maintaining its own essential identity and structure characterizing autopoiesis.

[2] F William Lawvere. "Quantifiers and Sheaves". In: *Actes Congrès intern.* Math.1 (1970), pp. 329–334

Cat: $\mathcal{K}$
Objects: $\mathcal{K}$
Morphisms:
$Id_{\mathcal{K}},\ \sigma,\ \sigma_1,\ \sigma_2,\ \ldots,\ \sigma_n$

$$\begin{tikzcd} \mathcal{K} \arrow[loop above, "\sigma"] \end{tikzcd}$$

$\mathcal{K}^{op} \xrightarrow{F} Set$
$\mathcal{K} \mapsto Set = F(\mathcal{K})$
$\sigma \mapsto \text{function} = F(\mathcal{K}) \xrightarrow{F(\sigma)} F(\mathcal{K})$

Gangle[3] makes the key point that the top element of the presheaf of an autopoietic system is not a fixed-point as such, but rather it is idempotent. Meaning, this element has, as a property of operations *themselves* rather than a fixed point. Autopoiesis, “involves an *iterated structural equivalence* (not necessarily an identity) between a reflexive operation and the result of that operation”[4] Such that:

$$\begin{gathered} \forall \mathcal{N} \in F,\ \ F(F(\mathcal{N})) = F(\mathcal{N}) \\ F(F) = F \end{gathered} \tag{1}$$

The implication of this for the formal general model for capital relates to primitive accumulation. If the top element in the presheaf $\mathcal{K}^{op}$ which we call Nature ($\mathcal{N}$) could only once be initiated into the system of capital as a commodity (C_0) than $\mathcal{N}$ would be a fixed point for the discrete dynamical system of capital. However, we know that the seizure of land, labor and resources through non-capitalist/pre-capitalist extraction, enclosure and colonization has and does happen continuously throughout the development of global capitalism, not only in the so-called transition from feudalism, there are many such *first instances* of the commodity form. This multiplicity of *first instances* where primitive accumulation initiates nature into the system of commodities and exchange, (capitalism) is best modeled as idempotent, as Gangle et al. model. We can consider the changing restrictions V_i on the space $\mathcal{U}$ as a map of imperialism and colonization, as it demarcates where nature is being newly initiated into capitalism and what of nature remains outside of the system.

[3] Rocco Gangle. “Modeling Autopoietic Systems with Presheaves”. Logic, Methodology of Science and its Applications. New York, Aug. 2024

[4] Louis H. Kauffman. “Autopoiesis and Eigenform”. In: *Computation* 11.12 (Dec. 5, 2023), p. 247

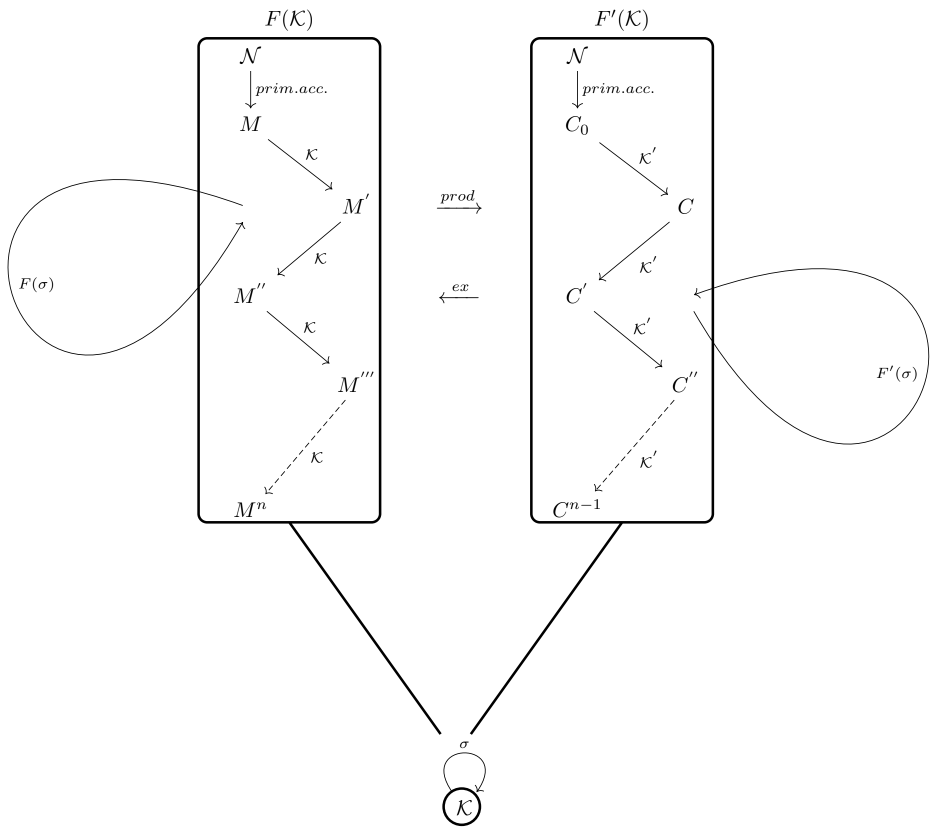


Figure 4: Spiromorphic Capitalism

Category:
$Set^{\mathcal{K}op}$

Objects:
Presheaves: $F(\mathcal{K})$, $F'(\mathcal{K})$

Elements:
N
M, $M^{'}$, . . ., M^n
C_0, C, $C^{'}$, ..., C^{n-1}

Morphisms:
Natural transformations: *prod*, *ex*
or,
$F(\mathcal{K}) \to F'(\mathcal{K})$, $F(\mathcal{K}) \to F''(\mathcal{K})$, . . .

Figure 4 shows the beginnings of a model of the full structure of the M-C-M' circuit whereby the movement of money and the circulation of commodities form presheaves, and the correspondence between them constitute the monoidal model of autopoietic capital from before.

Of course, capital is not, in and of itself a closed system. That is, in line with traditional notions of autopoiesis[5] the autopoeitic system is not only the self-iterating automaton, but the automaton in relation to an environment which codetermines it. An internal model for capital, forces of production, or sub-

[5]See Francisco J. Varela and Humberto Maturana Romesín. *Autopoiesis and cognition: the realisation of the living*. Boston studies in the philosophy of science 42. Dordrecht Boston Lancaster: D. Reidel, 1980

jectivity can be created using one object, (capital, commodity, and subject, respectively), but in order to model what Marx aimed to produce, a total social theory, one must take the step of elevating one step higher to a general topos with two objects one for capital, and one for the corresponding social system. For Marx this dialectic might be base and superstructure, and for other post-Marxists economy and ideology or materialism and discourse/culture. For our model, it is not necessary to assign a rigid term, as the site of analysis is the operations, morphisms, and relations between objects more so than the arbitrary labels applied to the objects themselves. For this system relating the objects capital ($\mathcal{K}$) and the social universe (S) we can use the notation Category: $S\mathcal{K}$.

Following Gangle's[6] topos model of autopoiesis we can construct a model for a social totality or system relating capitalism to the surrounding social environment thus:

Category: $S\mathcal{K}$
Objects: S, $\mathcal{K}$
Morphisms: $Id_S, Id_{\mathcal{K}}, \tau, \sigma, tel, det$

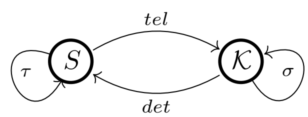
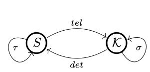
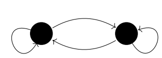


Figure 5: Social Totality

Consider the presheaf ($S\mathcal{K}^{op} \xrightarrow{F} Set$) as behavig like a sheaf over the universal topological space $\mathcal{U}$ where $\forall x \in \mathcal{U}$, F can be understood as a set F_x which changes for a given point $x \in \mathcal{U}$. (See Mac Lane and Moerdijk) From this view, the elements of F map to a chain of relations or determinations in the autopoietic model for capital ($\mathcal{K}$). Thus we can see nature ($\mathcal{N}$) in the first instance as the stable state for the generic topos $\mathcal{U}$ which through a function *prim.acc.* defined on an open set $U$of $\mathcal{U}$ restricted to *prim.acc.*$|v$ on open subsets $V \subset U$. The sheaf (F) can be constructed or "recovered" by collating the restrictions V_i over the universal space $\mathcal{U}$.[7]

To translate this technical jargon is to say that capitalism as a dynamic system is composed of a kind of cybernetic feedback through which the social totality includes a set of determinations and a teleology of correspondence between the economic mode of commodity circulation (production and exchange) and the superstructural environment that governs it (culturally, legally, etc.). This feedback loop constitutes something of a universal character and aligns with the strong Hegelian notion of totality.

A diagram:  $\in S\mathcal{K}$ Is a functor:  $\to S\mathcal{K}$

The behavior of presheaves described above is just a behavioral extension of something topologists have identified about categories and their diagrams generally. Using the example in figure 4, the diagram on the right is a functor

[6]Gangle, "Modeling Autopoietic Systems with Presheaves"

[7]Saunders Mac Lane and Ieke Moerdijk. *Sheaves in geometry and logic: a first introduction to topos theory.* Universitext. New York: Springer-Verlag, 1992

which maps a diagram of shape D to the category $S\mathcal{K}$. Meaning if we treat diagrams as a functor, we can abstract specific diagrams with more information to generic shapes containing less information where the diagram is the mapping of the shape to the category. Our analysis of the presheaf ($S\mathcal{K}^{op} \xrightarrow{F} Set$) follows from the same logic, that the diagram is a functor which maps the generic shape of capitalist accumulation (its motion) to a topos.

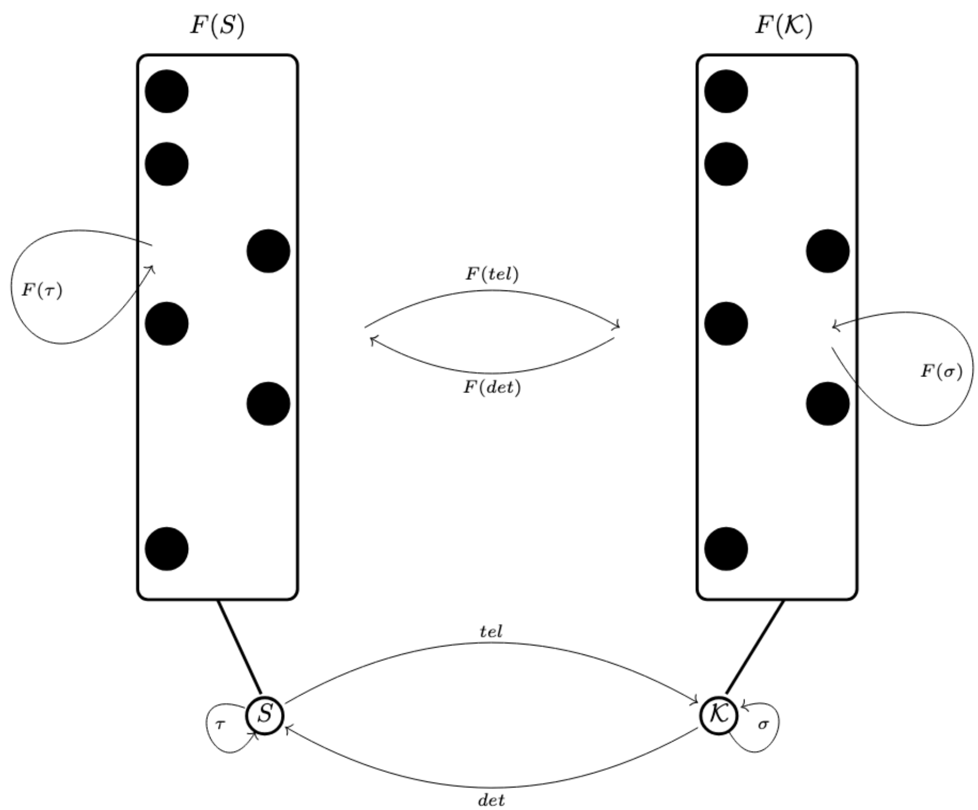


Figure 6: Social Totality Presheaf Model

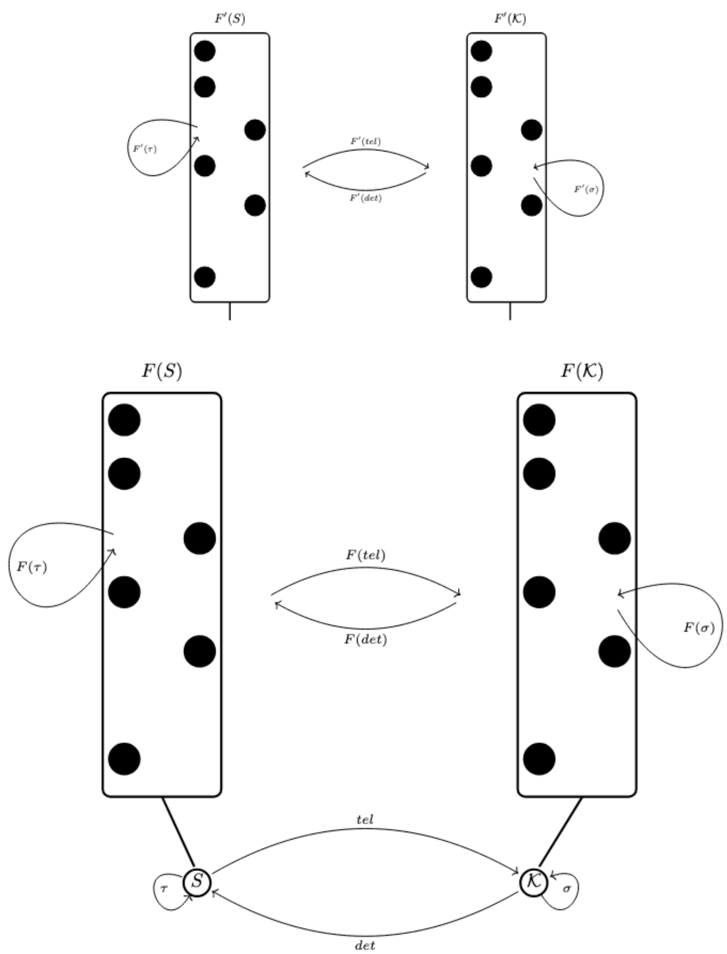
Presheaf: $S\mathcal{K}^{op} \xrightarrow{F} Set$

$S \mapsto set = F(S)$
$\mathcal{K} \mapsto set = F(\mathcal{K})$

$\tau \mapsto \text{function} = F(S) \mapsto F(S)$
$\sigma \mapsto \text{function} = F(\mathcal{K}) \mapsto F(\mathcal{K})$
$tel \mapsto \text{function} = F(S) \mapsto F(\mathcal{K})$
$det \mapsto \text{function} = F(\mathcal{K}) \mapsto F(S)$

Figure 6 is a preliminary diagram of the aforementioned presheaf showing the motion of autopoiesis in a first-order model which precedes and identically models the motion in the category $S\mathcal{K}$. This model is meant to give some structure to $S\mathcal{K}$ beyond a rudimentary motion including the update morphisms τ and σ

and the morphisms *tel* and *det* for teleology and determination. Figure 6, shows the recursive structure of the total topos, $Set^{\mathcal{SK}^{op}}$, which is why we note that Figure 5 is a preliminary diagram. Continuing to view diagrams as functors mapping a shape to a category, the topos pictured in Figure 6 contains multiple diagrams and the morphisms between them are natural transformations from one shape to another within the topos.



Topos: $Set^{\mathcal{SK}^{op}}$

Objects:
Presheaves: F, F', F'', ..., F^n

Morphisms:
Natural Transformations:
$F \to F'$, $F' \to F''$,
$F \to F''$, ..., $F^{n-1} \to F^n$,
$F \to F^n$

Figure 7: Topos of Autopoietic Social Totality

In this total model, capitalism ($\mathcal{K}$) is not only continuously updating itself, via the *det* morphism it is determining the superstructural environment (S), and via the *tel* function, the superstructural environment is altering the form of update undergone in capitalism. To Marxists this base observation is elementary, but this model allows us to visualize discrete parts of the social superstructure like for example "the state" as an element in the presheaf $F(S)$ and conceptualize it as a set with enumerable discrete elements, which through the morphism $F(\tau)$ updates itself, $F(tel)$ determines elements a crucial element in $F(\mathcal{K})$, and through $F(\sigma)$ is updated in $F(\mathcal{K})$ and through $F(det)$ is reintroduced as a determinate to the state. The dynamism of this model is that these morphisms are combinatorial and do not have one fixed teleological pattern. That is, as the history of capitalism has shown, the state does not always determine capital markets, and vice versa. An infinite multitude of determinations can happen simultaneously within these dynamical systems to represent the real dynamism of capitalism and sociality.

This topos $Set^{S\mathcal{K}^{op}}$ is one level higher than the presheaf in Figure 6 ($S\mathcal{K}^{op} \xrightarrow{F} Set$) and differs by showing that not only are the determinates of the category $S\mathcal{K}$ updating, the determinates determinates are updating in an infinite series of recursive determinations. Of course, the top element or fixed point-set of the presheaves $F^n(\mathcal{K})$ and $F^n(S)$, which serves as our stable-state or to use an Althusserian term "the last instance" is the set ($\mathcal{N}$) for nature. This is the most concrete representation of the line of theory developed in *Reading Capital*[8] and *Hegel or Spinoza*[9] which situates the origin of Marx's materialism in Spinozist terms.

To elaborate this point further, we can subsitute the complex topos-theoretic model of Figure 7 to zero in on the core function of nature ($\mathcal{N}$) in the total unfolding of capitalism and the social totality. Such is the approach of the Althusser group's materialist dialectic which looks to a Spinozist interpretation of the oft-overlooked ontological dimension of Marx's *Capital Vol. I*.[10]

For a set-theoretical definition, we can consider nature ($\mathcal{N}$) a countable infinite set, much like the set of natural numbers $\mathbb{N}$, where each instance of $\mathcal{N}$ as the top element in a presheaf $F^x(\mathcal{K})\ \mathcal{N}$ is a singleton set of a natural number. This set is well ordered with the smallest element $\{\varnothing\}$ as $\mathcal{N}_0$ belonging to the presheaf $F^n(\mathcal{K})$ and the largest element $\{\omega\}$ as $\mathcal{N}_\omega$ belonging to the presheaf $F(\mathcal{K})$. Thus this set is discrete and countably infinite with a cardinality of $\aleph_0$. We can then think of the transformation of Nature in each iteration of presheafs from $\{\varnothing\} \to \{\omega\}$ or, $\{\mathcal{N}_0, \ldots, \mathcal{N}_\omega\}$ as a symbolic chain underwriting the high-level system of $Set^{S\mathcal{K}^{op}}$. At the level of category theory all of the iterations of nature

[8] Louis Althusser and Étienne Balibar. *Reading capital: the complete edition.* Trans. by David Fernbach. London ; New York: Verso, 2015

[9] Pierre Macherey. *Hegel or Spinoza.* Trans. by Susan Ruddick. Minneapolis: University of Minnesota Press, 2011

[10] Nick Nesbitt. *Reading Capital's Materialist Dialectic: Marx, Spinoza, and the Althusserians.* BRILL, May 9, 2024

have the same value ($*$) which holds with the metaphysical argument that, under capitalism or the category $S\mathcal{K}$, at the level of commodities and circulation, all nature is alienated from its essential form. However, category theory also provides a demystifying metaphysical claim, notably that these point-sets ($*$) have an identity morphism which maps to itself $*_{Id}$.

Spinoza	**Mathematical Theory**	**Model**
natura naturata	Set Theory	$\{\varnothing\} \to \{\omega\}$ or, $\{\mathcal{N}_0, \ldots, \mathcal{N}_\omega\}$
natura naturans	Category Theory	$*_{Id}$ or, $* \to 1$

Under the category theoretic model, we can understand the infinite set $\mathbb{N}$ as an identity function mapping all of the natural numbers to themselves under there identical quality. The notion of nature as both concrete and discrete elements as in the set-theoretic definition $\{\mathcal{N}_0, \ldots, \mathcal{N}_\omega\}$ and as a unity of substance as in the category-theoretic definition $*_{Id}$ is an elegant way of modeling the Spinozist conception of God, or Nature, as the set-theoretic definition gives us a model of *natura naturata* and the category-theoretic definition gives us a model of *natura naturans*.[11]

The two ideas of nature are not only compatible theoretically, they are compatible mathematically. Namely, if we employ the mathematical concept of "eigenform," we find that we can define the recursion of our transfinite set $\mathcal{N}_0$, $\ldots$, $\mathcal{N}_\omega\}$ in terms of generic point-sets ($*$). For this operation, let us represent $\mathcal{N}$ as a series of point-sets $*$.

$$\begin{aligned} \mathcal{N}_0 &= * \\ \mathcal{N}_1 &= ** \\ \mathcal{N}_2 &= *** \\ \mathcal{N}_3 &= **** \end{aligned} \tag{2}$$

Letting $\mathcal{N}_\omega = ^{****} + \ldots$ where ω is a limit-ordinal . Here, $\mathcal{N}_\omega$ is an eigenform, or fixed-point insofar as by definition $\mathcal{N}_\omega = *\mathcal{N}_\omega = **\mathcal{N}_\omega = ***\mathcal{N}_\omega = \ldots$

If we recall that all members of our set for nature $\{\varnothing\} \to \{\mathcal{N}_\omega\}$ are point-sets ($*$) including $\{\mathcal{N}_\omega\}$, we can recursively define all point-sets as an eigenform for point-sets such that $* = ** = *** = * \ldots$. From here, the category-theoretic definition of $\mathcal{N}$ as $* \to 1$ is a natural extension of the set-theoretic definition conditioned that we start with the "infinite place" (see Kaufmann). Starting with the infinite place as an eigenform and performing recursion, our new map

[11] Benedict de Spinoza. *Ethics.* Trans. by R. H. M. Elwes. Simon & Brown, Feb. 21, 2013

looks something like this:

$$
\begin{aligned}
\mathcal{N}_0 &= * \\
\mathcal{N}_1 &= * \\
\mathcal{N}_2 &= * \\
\mathcal{N}_3 &= * \\
&\cdots \\
\mathcal{N}_\omega &= *
\end{aligned} \tag{3}
$$

For those familiar with Cantor, this may seem like a simple operation, something along the lines of $x \to card\{x\}$. However the operation of abstracting the particular attributes of discrete nature, *natura naturata* ($\mathcal{N}_x$), to the operation of identity as explicated through infinite recursion *natura naturans* ($*$) is both the fundamental shift from set-theoretic foundations of mathematics to univalent foundations, and the essential building block for constructing a category-theoretic model of autopoiesis. Additionally, this mathematical representation, or non-representation of the abstraction of finite nature to infinite nature in the form of $\{\varnothing\} \to \{\mathcal{N}_\omega\} \to * \to 1$, provides the framework for understanding the operation whereby the Spinozist outlook can simultaneously conceive of nature as a finite mode and infinite substance.

$$
\begin{gathered}
\mathcal{N} : \{\varnothing\} \to \{\mathcal{N}_\omega\} \\
\mathcal{N}_{\mathrm{Id}} : * \to 1 \\
\mathcal{N} \to \mathcal{N}_{\mathrm{Id}} \\
\{\varnothing\} \to * \\
\{\mathcal{N}_\omega\} \to * \\
* \to 1
\end{gathered} \tag{4}
$$

Imperialism and Primitive Accumulation

Kaufman using Gödelian logic shows another side to the autopoietic form. Namely that autopoietic systems need to incorporate an exterior and are in themselves incomplete. Kaufman proves this argument in a process he calls "exiting the box" in which the box is an iterated argument of an infinitely recursive autopoietic system. Kaufman outlines a version of Russell's paradox in which he produces a list of algorithms and logically proves that such a list is necessarily incomplete and "each list [of algorithms] can be used to produce an algorithm that is not on the list."[12] This proof holds for an autopoietic system like $Set^{\mathcal{K}op}$ which you might recall is a model for capitalist accumulation excluding its superstructural environment (See figure 3). Not only does the incompleteness problem imply the model for social totality must use the category $S\mathcal{K}$, it also implies something about the continuous productions of new modes of

[12]Kauffman, "Autopoiesis and Eigenform", p. 17

accumulation "exiting the box". In our model the "box" is the category $\mathcal{K}$ or capitalist accumulation, and the morphism which is produced outside of the "box" is *prim.acc.* the morphism which introduces new raw materials into capitalism through non-capitalist accumulation. This process of always exiting the box (capitalist accumulation) by producing modes of accumulation outside the box (primitive accumulation) aligns closely with what in the *Grundrisse* Marx calls "productive consumption."[13] That is, there is a pre-capitalist or non-capitalist germ internal to capitalism's reproduction.

In an unexpected way "exiting the box" affirms Lenin's thesis on the development of monopoly capitalism in *Imperialism the Highest Stage of Capitalism.*[14] That is not to claim that imperialism is a fixed-point for capitalism, as we have established that is the element nature ($\mathcal{N}$). Rather, imperialism is the operation by which the growing self-updating mass of capital ($\mathcal{K}$) produces and reproduces a morphism outside of itself (primitive accumulation) which initiates new nature into the capitalist system. Primitive accumulation is recursively infinitely reproduced through the dynamism of Capitalism, as modeled in the infinitely recursive topos, $Set^{S\mathcal{K}^{op}}$ which produces the infinite set $\mathcal{N}$ where $\mathcal{N}$: $\{\mathcal{N}_0, \mathcal{N}_1, \ldots, \mathcal{N}_\omega\}$. From here, not only can we say that $\mathcal{N}$ is an eigenform for capitalism, but it serves as an observable token for the eigenbehavior, primitive accumulation, which when iterated and updated transfinitely constitutes imperialism.

$$\begin{gathered}
\in F^n(\mathcal{K}) : \mathcal{N}_0 \xrightarrow{prim.acc.} C_0 \xrightarrow{\mathcal{K}_0} M \\
\in F^{n-1}(\mathcal{K}) : \mathcal{N}_1 \xrightarrow{prim.acc.} C_0 \xrightarrow{\mathcal{K}_0} M \\
\in F^{n-2}(\mathcal{K}) : \mathcal{N}_2 \xrightarrow{prim.acc.} C_0 \xrightarrow{\mathcal{K}_0} M \\
\cdots \\
\in F(\mathcal{K}) : \mathcal{N}_\omega \xrightarrow{prim.acc.} C_0 \xrightarrow{\mathcal{K}_0} M
\end{gathered} \tag{5}$$

If the set $\mathcal{N}$ and its point-set iterations $\{\varnothing\} \to \{\mathcal{N}_\omega\}$ are, under the operation (*prim.acc.*), an Eigenform for capitalism, the Eigenbehavior can be modeled thus:

$$\begin{gathered}
C_0 = prim.acc.(prim.acc.(prim.acc.(\ldots | n\text{times} \\
C_0 = prim.acc.^{(n)}(\mathcal{N})
\end{gathered} \tag{6}$$

Where C_0 a commodity newly initiated into the system capital ($\mathcal{K}$) is indefinitely recursive for the function *prim.acc.*, meaning if $prim.acc.(\mathcal{N}_\omega) = C_0$,

[13]Karl Marx. *Grundrisse: foundations of the critique of political economy.* Penguin Classics. London: Penguin books, 1993

[14]Vladimir Ilich Lenin. *Imperialism the Highest Stage of Capitalism.* Martino Fine Books, 1934

then $prim.acc.(C_0) = C_0$ and $prim.acc.(prim.acc.(C_0) = C_0, \ldots$. This recursive function iterated infinitely shows that the process of primitive accumulation has one determined form, the commodity form. However, once nature has entered capital accumulation a new indefinitely recursive model arises in which the set C is acted on by the morphism $\mathcal{K}$ and becomes the base for the Eigenform M. This translation from one Eigenform to another is for political-economy the transition from non-capitalist/pre-capitalist accumulation to capitalist accumulation, and for value the shift from essence ($\mathcal{N}$) to appearance (M) via the set (C_0).

$$\begin{gathered} \mathcal{N} \xrightarrow{prim.acc.} C_0 \xrightarrow{\mathcal{K}_0} M \\ C_0 \xrightarrow{\mathcal{K}_0} M \xrightarrow{\mathcal{K}} M^{‘} \end{gathered} \tag{7}$$

Once in capitalist accumulation (Cat: $\mathcal{K}$), the Eigenform of appearance looks like such:

$$\begin{gathered} M^n = \mathcal{K}(\mathcal{K}(\ldots|n\text{times} \\ M^n = \mathcal{K}^{(n)}(M) \end{gathered} \tag{8}$$

If we normalize the three Eigenforms above as one fixed-point ($\mathcal{N}$) which we can call natural-form and two generic Eigenbehaviors (one which we can call the enclosure-form and the other the commodity-form) we have two discoveries. First, there is a stage of pre-capitalist accumulation characterized by primitive accumulation which precedes capitalism, but both systems are indefinitely recursive in tandem. Second, there is a process whereby the self-organizing system develops new recursive operations (ie. pre-capitalism to capitalism). Thus, if primitive accumulation is the germ for the self-sustaining system of capitalism, capitalism contains within it the germ of a post-capitalist economy.

By grounding our complex tops $Set^{S\mathcal{K}^{op}}$ in Eigenforms at each level, where (*) is a fixed-point for internal presheaves $F(S)$, F'(S), F''(S), ..., $\mathcal{N}$ is a fixed point for internal presheaves $F(\mathcal{K})$, F'$(\mathcal{K})$, F''$(\mathcal{K})$, ..., $prim.acc. \to C_0$ is an Eigenbehavior for M and $\xrightarrow{\mathcal{K}} \to C^{n-1}$ is an eigenbehavior for M^n, we can model complementary processes at each end of the system capitalist accumulation, one of primitive accumulation in the first instance and imperialism in the last instance.

At risk of eliding the intricate relations of the autopoietic system, capitalist accumulation (Cat: $\mathcal{K}$) and its environment (S) we can make a very generic model describing the development of concurrent movement between forms of political-economy like such.

$$\boxed{\text{Natural-form}} \longrightarrow \boxed{\text{Enclosure-form}} \longrightarrow \boxed{\text{Commodity-form}} \longrightarrow \boxed{\cdots}$$

Figure 8: Movement between Eigenforms

This very elementary map at once tells us something important about the general motion of nature, its enclosure from public to private property, and its initiation into capitalism, and obscures the details of perhaps the most important political questions our model seeks to address. Notably, what does the mechanics of the motion into capitalism tell us about the motion through and overcoming capitalism. So far, our model has taught us that systems of political-economy are both self-organizing, and dependent on a predecessor form tracing back to a set of fixed-points (*), nature ($\mathcal{N}$). The next section looks to second-order cybernetics to further theorize the balance of forces which sustain self-organizing systems, what in our system might be called class-conflict.

Conceptualizing diagrams as functors which map a shape to a category, and morphisms mapping diagrams to diagrams as natural transformations of shapes, we can view the fixed point $\mathcal{N}$ as a special space, notably a point ($*$). From this viewpoint, the morphism *prim.acc.* between a point ($\mathcal{N}$) and the diagram (generic presheaf $F(\mathcal{K})$) is a (co)limit of the diagram. In algebraic topology we would describe *prim.acc.* as a special cone over the diagram. Since, a generic morphism *prim.acc.* is iterated in every presheaf of the topos, we can say that primitive accumulation always forms a special cone over capitalist accumulation. Conveniently, these terms from algebraic topology are a nice metaphor for the influence of imperialism over the development of capitalism, that the violence of capitalism extends from a a space where it is highly concentrated (primitive accumulation) downward towards a more globally pervasive and diffuse form.

Perhaps the most simple categorical model for this universal kernel of non-capitalist or pre-capitalist activity concurrent with capitalism is one that models the idempotent and autopoietic functions of this kernel's reproduction in terms of isomorphism. This means a structure-preserving mapping of the universal kernel which constitutes the eigenbehavior and eigenform of imperialism and primitive accumulation as a state-machine in motion at the origin of capitalism's reproduction. Such a state-machine can be represented categorically, and computationally in Haskell like such:

$$\begin{tikzcd}
\mathcal{N} \arrow[dr, "\text{prim. acc.}"'] & & \\
& U \arrow[r, "\text{idemp.}(\cong)"] \arrow[dr, "\text{auto}(\cong)"'] & U \times U \arrow[d, "\text{curry}(\cong)"] \\
& & U^U
\end{tikzcd}$$

Closed Cartesian Category: Imperialism	**Objects:** $\mathcal{N}$: Initial Object (Nature) U: Universal Capitalist Kernel	**Morphisms:** $\mathcal{N} \to U$: primitive accumulation $U \cong U \times U$: idempotence $U \cong U^U$: autopoiesis $U \times U \cong U^U$: curry correspondence

Figure 9: Structure of Imperialism

```haskell
module Imperialism where

-- Objects
data N = N -- Nature
data U = U -- Universal Kernel

-- Product of U with itself (U × U)
type UxU = (U,U)

-- Exponential Object (U^U)
type UtoU = U -> U

-- Morphisms

-- Primitive Accumulation: N → U
primAcc :: N -> U
primAcc _ = U

-- Idempotence: U ≅ (U × U)
idem :: U -> (U,U)
idem u = (u,u)

-- Autopoiesis: U ≅ U^U
auto :: U -> UtoU
auto u = \_ -> u

-- Curry Correspondence: (U × U) ≅ U^U
curry :: UxU -> UtoU
curry (u1, u2) = \_ -> u1

-- Closed Cartesian Category Rules
class CCC cat where
    -- Terminal object
    terminal :: cat a ()

    -- Products
    pair :: cat a b -> cat a c -> cat a (b, c)
    fst :: cat (a, b) a
    snd :: cat (a, b) b
```

```
-- Exponentials
cur :: cat (a, b) c -> cat a (b -> c)
uncur :: cat a (b -> c) -> cat (a, b) c
```

$$
\begin{tikzcd}
\mathtt{N} \arrow[dr, "\mathtt{primAcc}"] & & \\
& \mathtt{U} \arrow[r, "\mathtt{idem}"] \arrow[dr, "\mathtt{auto}"'] & \mathtt{(U,U)} \arrow[d, "\mathtt{curry}"] \\
& & \mathtt{U\text{->}U}
\end{tikzcd}
$$

To understand imperialism this way and to verify that it computes is in itself some preliminary affirmation of our thesis that imperialism forms a universal characteristic of capitalism's reproduction. That being said, this point has been made more concretely, as what Hegel would call a "concrete universal" by the analyses of those most affected by imperialism's wrath and the consequences of underdevelopment.

Case Study: Neocolonialism and Semi-Feudalism in the Philippines

One might be primed to criticize the thesis developed thus far—that primitive accumulation is a necessary concurrence with the development of capitalism—on the grounds that while, capitalism was once constrained to small corners of Europe, in the 21st century capitalism has grown to devour every corner of the globe. While yes it is true that global capitalism draws on labor and commodities from every nation and locale, it is also true that capitalism takes diverse form of appearance in its localized setting. Take the example of the firm, whereby under global capitalism there exists concurrently everything from the standard wage-relation (the capitalist firm), to firms dependent on slave labor (pre-capitalist firms), to worker co-operatives (post-capitalist firms). At a larger scale, regions of entire nations in the global periphery of imperialism are characterized by modes of accumulation and production that do not resemble capitalism at the local level, even if they are constitutive of a global capitalist process. One such example is the Philippines.

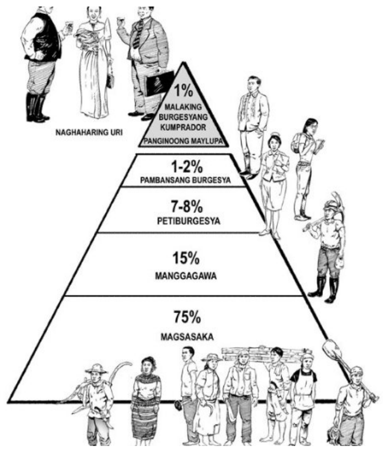


Figure 10: Class Society in the Philippines

The predominant Marxist movement in the Philippines—taking forms in the political party (Communist Party of the Philippines (CPP)), military wing (New People's Army (NPA)), diasporic youth movement (Anakbayan), and broad ideological formation (National Democracy)—assesses the current challenges facing the nation and its working class as "semi-colonialism and semi-feudalism" characterized by three fundamental problems: U.S. imperialism, feudalism, and bureaucrat capitalism.[15] The tripartite structure developed by the National Democracy Movement and elaborated by founder and leader of the CPP, Jose Mariá Sison, in his book, *Philippine Society and Revolution* under the pseudonym Amado Guerrero, captures in a political analysis and polemic what I hope to argue about the necessity of primitive accumulation's reproduction throughout the historical development of capitalism. Sison claims:

> The semifeudal character of Philippine society is principally determined by the impingement of U.S. monopoly capitalism on the old feudal mode of production and the subordination of the latter to the former. The concrete result of the intertwining of foreign monopoly capitalism and domestic feudalism is the erosion and dissolution of a natural economy of self-sufficiency in favor of a commodity economy. Being dictated by foreign monopoly capitalism, this commodity economy is used to restrict the growth of a national capitalism and force owner-cultivators and handicraftsmen into bankruptcy. It is used to keep large masses of people in feudal bondage and at the same time create a relative surplus of population, a huge reserve army of labor, that keeps the local labor market cheap. In Philippine agriculture, the old feudal mode of production persists side by side with capitalist farming chiefly for the production of a few export crops needed by the United States and other capitalist countries. As

[15]Amado Guerrero. *Philippine Society and Revolution.* Philippines: Pulang Tala Publications, 1971

> a matter of fact, the old feudal mode of production still covers more extensive areas than capitalist farms. Feudalism has been encouraged and retained by U.S. imperialism to perpetuate the poverty of the broad masses of the people, subjugate the most numerous class which is the peasantry and manipulate local backwardness for the purpose of having cheap labor and cheap raw materials from the country. It is in this sense that domestic feudalism is the social base of U.S. imperialism. The persistence of landlord exploitation is in turn under the counterrevolutionary protection of U.S. imperialism. An agrarian revolution is needed to destroy the links between U.S. imperialism and feudalism and deprive the former of its social base.[16]

Here, Sison describes the same fundamental premise of our autopoietic theory, beginning with the elaboration of imperialism and working backward toward the causes, as opposed to our analysis that begins with the universal kernel. However, Sison's analysis is itself a universalizing gesture, one that implicates the concrete situation of Philipine feudalism and neocolonialism into a vast and totalizing web of global social relations. Notably, Sison shows how the progress of industrial development in the imperial core, necessitates a dialectical progressive underdevelopment of its neocolony, such that the telos of the history of capitalism in the Philippines elaborates itself as an uneven process whereby beureaucrat capitalism progresses in the cities and feudal stagnation proliferates in the countryside.

The "semi-feudal" character of the Philippines is endemic of the continued need for pre-capitalist accumulative modes at the origin of capitalism's reproduction. As is always the case, primitive accumulation in the Philippines in its U.S. Imperial form came as an extension of previous Spanish colonialism, whereby a war of attrition granted U.S. capitalists access to Philippine resource riches and cheap labor in the first instance, later formalizing the imperial relation through legal treaties. In this historical moment the sovereign resources of the Philippines entered into the relation of U.S. finance capitalism. Though this process may look different than the enclosure of the European commons described by Marx, a fundamental principle of primitive accumulation is operative in both. Namely, acquisition of new raw materials and labor into the M-C-M' circuit from a previously uncommodified outside. Only once these material resources and labor have been subsumed into global capitalist relations under the banner of imperial conquest, does the superstructural mechanism of legal elaborate itself and the social relations of (semi)feudalism fully develop.

The continued reproduction of capitalism through primitive accumulation in the periphery is maintained through strategic underdevelopment. Walter Rodney outlines why, for Marxism, "underdeveloped" is the operative phrase compared to "developing," for "on the economic level, it is best to remain with the word 'underdeveloped' rather than 'developing,' because the latter creates the im-

[16] Guerrero, *Philippine Society and Revolution*, pp. 39–40

pression that all the countries of Africa, Asia, and Latin America are escaping from a state of economic backwardness relative to the industrial nations of the world, and that they are emancipating themselves from the relationship of exploitation. That is certainly not true, and many underdeveloped countries in Africa and elsewhere are becoming more underdeveloped in comparison with the world's great powers, because their exploitation by the metropoles is being intensified in new ways."[17] For this superexploitation to be intensified in new ways is to allude to the way in which the repetition of primitive accumulation that characterizes imperialism does not necessarily entail the development of capitalist social relations but rather the continued development of new modes of exploitative slavery, feudalism, and colonialism.

To read capitalism as an autopoietic system, with a fixed-point called an eigenform, and an initial process called an eigenbehavior, is simply to say that capitalism contains at its root a localization of pre-capitalist relations that, in itself is not characterized by self-motivated mechanical processes, but a real communication between capitalism proper and its exterior. This correspondence between the self-organizing system (Capitalism) and its external environment forms the basis of a kind of cybernetic understanding of political economy.

Capitalism's Demonic Enterprise

von Foerster in his work on self-organizing systems, what we might call a theoretical precursor to autopoietic systems, theoretically grounds the conditions by which the system relates to its environment in the mathematical concept of entropy.[18] Specifically he models the motion of the self-organizing system in relation to the environment in terms of relative order and disorder. Using Shannon's formula for redundancy ($R = 1 - \frac{H}{H_m}$) "whereby $\frac{H}{H_m}$ is the ratio of the entropy H of an information source to the maximum value, H_m, it could have while still restricted to the same symbols. Shannon calls this ratio the "relative entropy," von Foerster begins to model a spectrum ($0 \leq R \leq 1$) where $R = 0$ is total disorder and $R = 1$ is total unity with the environment.[19] This model allows for a way of theorizing the relative (in)dependence of the self-organizing system from its environment. More accurately, it models the frequency of autonomous updates in the system to novel teleological updates.

von Foerster's postulation that the self-organizing system's entropy (H) will necessarily decrease, in which case if the environment does not gain complexity (H_m stays constant), R will progressively approach 1 or unity should offer some hope to the orthodox Marxist axiom that the contradictions internal to capitalism will necessarily generate its abolition. Beyond this base observation, the

[17] Walter Rodney. *How Europe underdeveloped Africa.* Rev. pbk. ed. Washington, D.C: Howard University Press, 1981, p. 14

[18] Heinz von Foerster. *Understanding understanding: essays on cybernetics and cognition.* New York Berlin Heidelberg Hong Kong London Milan Paris Tokyo: Springer, 2003

[19] Foerster, *Understanding understanding*, p. 18

other consequence of this discovery, is that self-organizing systems tend towards their annihilation through the increase of order and structure, as it approximates the quantitative order of the ambient environment. However, this is a special case for von Foerster who suggests it is much more common for H to decrease while H_m increases. In this scenario, the entropy of the system progressively decreases while the complexity of the environment increases.

In developing this theory, von Foerster invents two figures to describe these complimentary dynamics: an internal demon (D_i) who works to decrease the entropy of the system, and an external demon (D_e) who works to increase the complexity of the environment. He argues these demons collaborate to maintain a dynamic balance.

$$H \quad \times \quad \frac{\delta H_m}{\delta t} \quad > \quad H_m \quad \times \quad \frac{\delta H}{\delta t}$$

Internal demon's results	External demon's efforts	External demon's results	Internal demon's efforts

Figure 9: von Foerster's demons

In our scheme of capitalist accumulation and its corresponding social environment, as has been born out by the history of capitalism, some balance between the work of an internal demon (say the constant decline of the rate of profit) and the work of an external demon (we increased complexity of social and ideological forms, the development of new technology, advanced nation-states, and a global financial system dependent on central banking) has perpetuated capitalism's self-organization. From this vantage point, the movement towards the horizon of the end of capitalism, what Marx and Engels called "communism:" to quote Marx precisely from *The German Ideology*: "We call communism the real movement which abolishes the present state of things. The conditions of this movement result from the premises now in existence"[20] is the negative dynamism of these relations. That is, not the simple decline of profit to zero, nor the halting of social complexity, but the process by which the ratio between them approaches redundancy.

von Foerster nearly concludes his paper with two primary theses, that self-organizing systems:

(1) generate order from order;
(2) generate order from noise.[21]

The implications of these theses for thermodynamics, communication theory,

[20] Karl Marx and Friedrich Engels. *The German Ideology.* Prometheus, 1845
[21] Foerster, *Understanding understanding*, p. 10

and cybernetics was the entry-point for second-order cybernetics. For our purposes it models two general claims about the system of capitalist accumulation which are useful; that in order to reproduce itself, capitalism must subsume structure from its environment—we see this in the way that an increasing set of cultural, ideological and natural substances are reduced to the commodity form—second, capitalism must regulate the natural complexity of the world and of life under its own semiotic rules and structure such that cultural formations which do not suit its reproduction are mutilated into legible forms which service its reproduction.

The result of this process, when iterated over and over, is an ideological reduction of the world's natural complexity as the universe of essences is mapped to a muted world of commodified appearances, and a growth in the total complexity of social structure, to which, in turn, capitalism must adapt itself. In what ways can we talk about universality from this point of view? There is of course the universal dimension of negative determinations that I've outlined, whereby the precapitalist kernel of accumulation undergirds the expansion and maintenance of capitalism proper. It is a universality which grounds the historicity of capitalism as an ever evolving, but nonetheless stable form through which the social totality elaborates itself. Then again there is an affirmative valence through which we can view universality. That is the political situation by which imperialism and neocolonialism in the Global South is inextricably linked to exploitation in the Global North. That the autopoietic structure, which at its heights looks something like global capitalism in the present day spirals out of a concrete form of universal mystification in the commodity. This is Marx's project in *Capital*, and why the first chapter is the commodity, and the twenty-third chapter of volume one is so-called primitive accumulation, all of which precedes the critique of wealth of nations in volumes 2 and 3. To speak of universality in the affirmative, at least as a Marxist, is to never lose sight that the fight for free time and better working conditions within the firm is at a fundamentally metaphysical level bound up in the fight against non-capitalist forms of accumulation, the feudal agricultural plantations, and mining sites predicated on slave labor which constitute the global economy. And so the point here is somewhat polemical, that a global solidarity which seeks to undermine the essential pre-capitalist kernel at the origin of capitalism's reproduction is the only means by which a truly emancipatory project can emerge.

If the mechanical processes of capitalism adapt themselves in the mode we call autopoiesis, what is the task of the anti-capitalist? One can, and I suggest must, argue for the uprooting of the relation of imperialism as the genesis for capitalism's reproduction. That is to say the struggle of the wage slave seeking emancipation in the Global North is only an anti-capitalist struggle if it is sutured solidaristically with a commitment to the revolutionary anti-imperialist struggle in the Global South. The universalism of imperialism can only be defeated by a more radically universal solidarity engaged in the Hegelian life-and-death struggle for recognition.

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One-Universe: Categorical Universalism, and a Non-Representational Turn in Psychoanalytic Theory

> I leave the question open–what is the nature of the principle which governs what is at issue, namely the subject? Is it assimilable, reducible, symbolizable? Is it something? Or can it neither be named, nor grasped, but only structured?
>
> Jacques Lacan, *Le Séminaire – Livre II*

> It turns out that a flexible and effective formulation of the present notions of category theory can be given with a more modest addition to the standard axiomatic set theory: The assumption that there is one universe.
>
> Saunders Mac Lane, "One Universe as a Foundation for Category Theory"

Why Category Theory?

This paper aims to extend the existing and provide a new formal language for expressing key concepts in Lacanian psychoanalysis, namely by moving formal Lacanianism forward from its use of set theory towards category theory and homotopy type theory. The imperative to formalize Lacan seeks to take Lacan at his word when he argues "I posited that this matheme is the pivotal point of all teaching. In other words, mathematical teaching is the only teaching. The rest is banter."[1] Just as great contributions have been made toward furthering Lacan's work in theory and practice, the mathematical tools at Lacan's disposal have been largely updated and surpassed. My project is to reintroduce these innovations in mathematics (namely category theory and its successors) to the corpus of Lacanianism. Specifically, this paper examines the issue of universality at the injunction of Lacan and the traditions of Hegel and Marx. Theorists such as Žižek, McGowan, Badiou,[2]and Zupančič[3] have all sought to address issues of a renewed universalism at the intersection of psychoanalysis and German idealism. This paper contributes to this contemporary debate by reviving and reintroducing the formal impulse in Lacanianism with special attention to the

[1] Jacques Lacan. *or worse: the seminar of Jacques Lacan, book XIX*. ed. by Jacques-Alain Miller. Trans. by A. R. Price. English edition. Cambridge ; Medford, MA: Polity, 2018, p. 17

[2] Alain Badiou. *Being and event*. London ; New York: Continuum, 2005

[3] Alenka Zupančič. *What is sex?* Short circuits. Cambridge, MA: MIT Press, 2017

category theoretical hypothesis positing “one-universe.”[4] This paper will include preliminary category-theoretical graphs of Lacan’s “formulas of sexuation,” the phallic function (Φx), a model for language (metonymy and metaphor) and a model for general subjectivity.

Jacqueline Rose suggested to me that “when one uses the term ‘subject position’ they are no longer thinking psychoanalytically.” This statement struck me as evidently true, but nonetheless troubling as I had used this precise phrase in psychoanalytic discourse when referring to the elements of the phallic function (Φx) in Lacanian set-theory. The resolution to this problem came from an unfamiliar place, namely Lawvere’s category theory. By thinking and theorizing sets as a category one arrives at a more psychoanalytic understanding of subjectivity than the limitations of set-theoretic definitions allow. In category theorist William Lawvere’s formulation, “Philosophically, it may be said that these developments partially support the thesis that even in set theory and elementary mathematics it is also true as has long been felt in advanced algebra and topology, namely that the substance of mathematics resides not in Substance (as it is made to seem when $\in$ is the irreducible predicate, with the accompanying necessity of defining all concepts in terms of a rigid elementhood relation) but in Form (as is clear when the guiding notion is isomorphism-invariant structure, as defined, for example, by universal mapping properties).”[5] The philosophical transition from the subject as substance, which lends itself to theorizing the subject in terms of “subject position(s),” to the subject as abstract form accounts for a more psychoanalytically attune formalization of subjectivity. In category theory elements of the universe are no longer elements defined by membership ($\in$) as in set theory, but as *mappings*. Mappings are a much more accurate analog to the kind of conception of subjectivity in psychoanalysis we are trying to formalize than elements. Subjects are defined as structures which cohere to the formal operation of the phallic function (the category of subjectivity), rather than the substance of a set called humanity.

Category theory is, from this perspective, a method to extend the psychoanalytic teachings of Lacan to a more general philisophical structuralism. As category theorist and philosopher Steve Awodey claims, “the language and methods of category theory can be used to determine a notion of ‘structure’ that is both precise and yet flexible enough to do some philosophical work.”[6][7] From this same position, we choose category theory from set theory in order to build a

[4]Saunders Mac Lane. “One universe as a foundation for category theory”. In: M. Barr et al. *Reports of the Midwest Category Seminar III*. vol. 106. Series Title: Lecture Notes in Mathematics. Berlin, Heidelberg: Springer Berlin Heidelberg, 1969, pp. 192–200

[5]F William Lawvere. “An elementary theory of the category of sets (long version) with commentary”. In: *Reprints in Theories and Applications of Categories* (No. 11 2005), pp. 1–35, p. 7

[6]Steve Awodey. “An Answer to Hellman’s Question: ‘Does Category Theory Provide a Framework for Mathematical Structuralism?’†”. In: *Philosophia Mathematica* 12.1 (Feb. 1, 2004), pp. 54–64, p. 54

[7]Steve Awodey. “Structure in Mathematics and Logic: A Categorical Perspective”. In: *Philosophia Mathematica* 4.3 (Sept. 1, 1996), pp. 209–237

more complete philosophical model than set theory's limitations allow.

Alain Badiou in his master treatise on ontology and set theory claimed, "But in ontology there is no procedure, only structure. There is not a-truth, but construction of the concept of the being multiple of any truth."[8] In the move from the undecidability paradigm of set theory towards category theory, we find ourselves in a situation which at once strengthens and invalidates Badiou's claim. It strengthens the claim insofar as the abstraction of sets into the category Set only furthers the generic quality of the subject in a mathematical ontology, perhaps to the point of no longer being ontology and rather a general metaphysics. Category theory invalidates Badiou's claim, because the formula "no procedure, only structure" no longer holds in category theory where "procedure," what we call operations, are objects and morphisms, they *are* structure.

A Preliminary Model of Sexuation

$$
\begin{tikzcd}
& A(\exists ! x \; \overline{\Phi x}) \arrow[loop above, "1_A"] \arrow[dl, "auth."'] \arrow[d, "auth."] \arrow[dr, "auth."] & \\
\Phi x_\alpha \arrow[dr, "lack"'] & \Phi x_\beta \arrow[d, "lack"] & \Phi x_\gamma \arrow[dl, "lack"] \\
& a \arrow[loop below, "1_a"] &
\end{tikzcd}
$$

Figure 1: Sexuation of Four Subjects

Objects:	**Morphisms:**	**Id Morphisms:**
Big Other: $A(\exists ! x\overline{\Phi x})$	$A \xrightarrow{authority} \Phi_x$	Identity on A: 1_A
Subjects: $\Phi x_{\alpha,\beta,\gamma}$	$\Phi x \xrightarrow{lack} a$	Identity on a: 1_a
Objet a: a		

a is a terminal object of sets Φx_α, Φx_β, and Φx_γ As a single-point set, or singleton, its only mapping is the identity morphism 1_a or $a \to a$.

In traditional Category theory notation, a singleton set like a takes the notation 1, for our purposes, to maintain the theoretical richness of our set-theoretic

[8]Badiou, *Being and event*, pp. 356–7

definition of a we will continue to use the notation a or $\{\varnothing\}$.[9] [10] Point-set definition is something traditional ZFC set theory is philosophically ambiguous on. In this framework it is known that all objects in set theory including individual elements of sets, are themselves sets. These sets containing one element, also known as singletons or point-sets are given little to no special metaphysical status in the organization of set theory. Conversely in elementary category theory, what Lawvere called the "elementary theory of the category of sets" (ETCS) these special sets and indeed all objects are given unique demarcated status from other mathematical objects like posets, categories, maps, morphisms and functors. By providing a vocabulary which designates the concepts as unique from one another, CT distinguishes its philosophical stakes from set theory. The introduction of this vocabulary introduces with it the concept of type, which becomes core feature of type theory and later homotopy type theory.

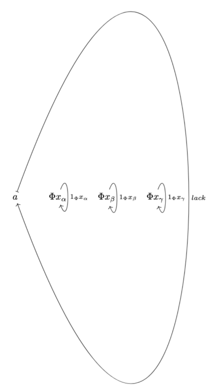


Figure 2: Three Subjects[11]

In HoTT, contractible types, sometimes called "mere propositions" or in a set theoretical frame singletons are equal to 1.[12] In HoTT type levels, contractible types are at the level type -2, which for historical reasons in the lowest level type. For mathematicians, these types contain the least information and thus the least structure of all mathematical objects and types. Contractible types have two basic properties (Identity and homotopy equivalence). That is, there is an identity morphism mapping contractible types to themselves, and all point-sets are homotopy equivalent. There is no more information or structure to

[9]Saunders Mac Lane. *Categories for the Working Mathematician*. Vol. 5. Graduate Texts in Mathematics. New York, NY: Springer New York, 1978

[10]F. William Lawvere and Stephen H. Schanuel. "The category of sets". In: *Conceptual Mathematics: A First Introduction to Categories*. Ed. by F. William Lawvere and Stephen H. Schanuel. 2nd ed. Cambridge: Cambridge University Press, 2009, pp. 11–36

[11]Figure 2 is a creative rendition of Figure 1

[12]The Univalent Foundations Program. *Homotopy Type Theory: Univalent Foundations of Mathematics*. Institute for Advanced Study: `https://homotopytypetheory.org/book`, 2013, p. 124

these types other than this "mere" truth-value. It is also true that all higher types—which is all other types because contractible types are the lowest—are built on these elementary structures.

$$\text{Identity: } * \to 1 :\equiv id_x$$

$$\text{Homotopy} \equiv \begin{cases} f : * \to 1 \text{ and } g : 1 \to * \\ f \circ g \sim 1_* \text{ and } g \circ f \sim 1_1 \\ \text{or,} \\ H : * \times [0,1] \to 1 \end{cases}$$

In our model of Lacanian psychoanalysis, the *petit objet a* is one such singleton or point-set. In mathematic and philosophical consistency with HoTT, *petit objet a* plays the same role, it is an object with the least structure and information in the Lacanian system, its only structure is its identity and homotopy to itself, and it is the theoretical foundation upon which lack, in the first instance, and many other psychoanalytic structures in later instances are constructed.

In the crisis of continental philosophy and critical theory in which we find ourselves, where truth is a concept we only approach tentatively, these mere propositions, which are the lowest order of information in the thinking of mathematicians may be the philosophically rich foundation for a new approach to truth in analysis.

As Badiou has explicated in detail the object ∅ or the empty set is a rich topic for philosophical inquiry.[13] Cantor, Zermelo, Gödel, and von Neumann have shown that all of the natural numbers can be inductively constructed from the empty set.[14] [15] [16]

$$\begin{array}{ccccc} \varnothing, & \{\varnothing\}, & \{\varnothing,\{\varnothing\}\}, & \{\varnothing,\{\varnothing\},\{\varnothing,\{\varnothing\}\}, & \{\varnothing,\{\varnothing\},\{\varnothing,\{\varnothing\},\{\{\varnothing,\{\varnothing\},\{\varnothing,\{\varnothing\}\}\}\},\ldots \\ 0, & 1, & 2, & 3, & 4,\ldots \end{array}$$

For our purposes the object $\{\varnothing\}$, is philosophically multivalent. It is at once, a) the Badiouan "count-as-one"; b) the Žižekian "void"; c) the Lacanian *objet petit a*; and perhaps most importantly, d) the object which in our category-theoretic model allows for the construction of the morphism "lack" $(-\Phi)$.

[13] Badiou, *Being and event*

[14] Transfinite induction of sets from empty sets is not unique to the natural numbers ($\mathbb{N}$) but through the power set axiom and axiom of foundation in ZFC can construct all sets including $\mathbb{R}$ and functions mapping a set X to a set Y ($f : X \to Y$).

[15] Georg Cantor. *Contributions to the founding of the theory of transfinite numbers.* Repr. of the ed. LaSalle, Ill., Open Court Publ. Co., 1915. Dover books on intermediate and advanced mathematics. New York: Dover Publ, 1955

[16] John Von Neumann. "An Axiomatization of Set Theory". In: G. Frege and K. Gödel. *From Frege to Gödel.* Ed. by Jean van Heijenoort. 4. pr. Cambridge, Mass: Harvard Univ. Pr, 1981, pp. 393–413

$$\forall \Phi x \in \Phi x \ldots \Phi x_n \to a$$

That each subject (Φx) maps to the terminal object a, is an instantiation of the central Lacanian proposition that "all subjects are castrated" $\forall x.\Phi x$ or to give the feminine "there is no non-castrated subject" $\overline{\exists x}.\overline{\Phi x}$.[17] Category-theoretic graphs allow us to visualize and schematize what the mathemes and Lacan's lectures allude to, that there is a universal function or morphism which maps subjects to the *objet petit a* and it is lack.[18]

In a Hegelian frame, we can make the same argument as presented above, namely that the constitutive law of subjectivity relies on a determinative operation of negation. As Hegel states in the Self-Consciousness chapter of the *Phenomenology*, "It is in confronting the other that the I is itself. At the same time, it reaches out over and beyond this other, which, for the I, is likewise only itself."[19] Hegel's preliminary argument on subjectivity is the basis of the two morphisms for each (castrated subject): a morphism relating the the other (lack) and an identity morphism mapping "the I to itself," or put another way, "*self-consciousness* is the only motionless tautology of 'I am I.'"[20] As for what lies beyond the scope of the diagram, the ineffable Real, Hegel claims, "the *essence* is infinity as the *sublation* of all differences, the pure movement rotating on its own axis, its own motionless being as absolutely restless infinity."[21] No doubt, this restless infinity refers to the continuum, a topic which constitutes a central concern for Badiou's set-theoretical work in *Being and Event*, and need not be reiterated here.

We should note a distinction between the term "terminal object" and the common understanding of the term "determinate" in continental philosophy. When we say that the *objet petit a* is the terminal object of subjects that is to say that their is a relationship called lack which is constitutively determinative of subjectivity. That is not to say however, that the *objet petit a* is the zero-point of subjectivity. Far from it, the function or morphism "lack" has a determinative role in structuring subjectivity, but this does not necessitate that the *objet a* is a real object from which the phenomenon "lack" emanates. Rather in the category-theoretic constructable universe, which I believe is very close to the Hegelian system, objects are related through mappings called morphisms, which are themselves operations.[22][23] That is to say that category theory operates at

[17] Lacan, *or worse*

[18] We of course have our two subjects which contradict the mathemes listed above, "the subject who speaks negation" $\exists ! x \ \overline{\Phi x}$, and the hysteric $\overline{\forall x} \ \Phi x$. These are accounted for in our category-theoretic graph, but could be further theorized in a sequel to this paper which formally maps the four discourses.

[19] Georg Wilhelm Friedrich Hegel. *The phenomenology of spirit.* Ed. and trans. by Terry P. Pinkard. First paperback edition. Cambridge Hegel translations. Cambridge New York Port Melbourne New Delhi Singapore: Cambridge University Press, 2018, p. 102

[20] Hegel, *The phenomenology of spirit*, p. 103

[21] Hegel, *The phenomenology of spirit*, p. 104

[22] Solomon Feferman and G. Kreisel. "Set-Theoretical foundations of category theory". In: M. Barr et al. *Reports of the Midwest Category Seminar III.* vol. 106. Series Title: Lecture Notes in Mathematics. Berlin, Heidelberg: Springer Berlin Heidelberg, 1969, pp. 201–247

[23] Mac Lane, "One universe as a foundation for category theory"

a level of abstraction which allows us to manipulate and study phenomena as if they were objects with the understanding that they are representative a processes. In doing so, we avoid essentializing these operations, like negation by reducing them to things.

If we take a generic subject like the set Φx_α to be an infinite countable set like the natural numbers $\mathbb{N}$, two things are immediately apparent: a) Any subject has infinitely many attributes (elements); and b) these elements are organized as discrete elements, meaning each element is bounded on both sides by a cut, or negative void ($\varnothing$). This digitality in the internal model of the set Φx_α is the same operation, namely negative determination modeled by the morphism "lack" in the category-theoretic model for general subjectivity. Lacan already detailed this digital logic in his description of the "chain of signification" or metonymy in his *Ecrits*.[24]

In *Ecrits* Lacan models metonymy as such:

$$f(S...S')$$

A later Lacan using set theory may have modeled it like this:

$$\Phi x_\alpha : \{S_{\alpha 0}, S_{\alpha 1}, S_{\alpha 2}, \ldots, S_{\alpha n}\}$$

At the core of both models, though admittedly the later is a more accurate model, is a digital logic. That is to say that just as signifiers are discrete in Lacan's theory of language, via Saussurean structural linguistics, subjects shift from one element to the next, each negatively determined by a cut. This set theoretic definition of the phallic function tends towards a philisophical isolationism. To systematize Lacan's thought, perhaps with an intervention from German idealism, we need the category-theoretic extension to model the relationship between concepts. Set theory, as a project like *Being and Event* proves, provides a beautiful framework for modeling ontology, but stops far short of modeling a general metaphysics. With category theory we are no longer bound to ontology, or a theory of the subject, but our model of the subject, is constructable into a model of subjectivity, which in-turn is constructable to a total category-theoretic universe. In homotopy type theory, set-theoretic foundations like the ones deployed by Badiou are referred to as "strict categories" alluding to the metaphysical limitations of their reach. [25]

The tradition of thought from Spinoza, to Marx, to Lacan when modeled in category theory is not simply the metaphoric unicity of the three theorists' systematic thought, but a mathematical proof of the formal isomorphism of a theory of the subject, a theory of the social, and a theory of the universe.[26] In a

[24]Jacques Lacan. *Ecrits: The first complete edition in English.* Trans. by Bruce Fink. New York, NY: Norton, 2006

[25]Univalent Foundations Program, *Homotopy Type Theory: Univalent Foundations of Mathematics*, p. 307

[26]Mac Lane, "One universe as a foundation for category theory"

sequel to this paper, using homotopy type theory, one can model the structure of this isomorphism in the category-theoretic universe.

The Universal

I began to outline some of the reasons category theory is a fruitful analytic for Lacanian psychoanalysis and its application in metaphysics. One use-case which I will spend the remainder of my time on is that of the universal. Theorists such as Badiou, Žižek, McGowan, Zupančič, and others have all sought in recent years to devleop a theory revitalizing universality. As I am sure many would agree, finding a new universality has become one of the central problematics of critical theory and an issue of great political importance. A major stumbling block for the formalization of Lacanianism especially in its set theoretic application, came from Paul Cohen's discovery and proof that many of the most important questions in set theory namely large cardinals and the continuum hypothesis are undecidable.[27] Badiou continues to hold out hope that the system of set theory will get new axioms where these undecidable problems will be resolved. Despite drawing major influence and inspiration from Badiou, from a Lacanian perspective this is totally wrong and forecloses on the Real entirely.

Kiarina Kordela, citing Žižek, makes an important critique of "neo-Spinozism," the likes and Deleuze, Hardt and Negri, and in turn levels a similar critique against Alain Badiou.

> Žižek's criticism of "Neo-Spinozism" equally applies to Alain Badiou, whose attempt to rescue truth, his self-differentiation from Deleuze notwithstanding, falls within the paradigm of "NeoSpinozism." Badiou's path toward truth endeavors to bypass the ideological nature of representation by means of a recourse to logical formalism and "mathematics," as the direct revelation of the "ontological situation," that is, as a presumably pre- or suprarepresentational field ("objective" or "Absolute Knowledge") in which universal truths and Being reveal themselves transparently, without any ideological distortion (Badiou, 2003, p. 170).[28]

In the iteration of this critique against Badiou, one is really concerned with the Badiou of *Being and Event* and *Conditions*, as a younger Badiou, say in his works *The Concept of the Model* or "On Mark and Lack," had much more deference for the subtleties of non-representational ontology and anti-positivism than his more mature works present.[29] Notably, category theory, especially in

[27]Paul J. Cohen. *Set theory and the continuum hypothesis.* Dover ed. Dover books on mathematics. Mineola, N.Y: Dover Publications, 2008

[28]Aglaia Kiarina Kordela. *$urplus: Spinoza, Lacan.* SUNY series, insinuations. Albany: State University of New York Press, 2007, p. 109

[29]Alain Badiou. *The concept of model: an introduction to the materialist epistemology of mathematics.* Trans. by Luke Zachary Fraser and Tzuchien Tho. Transmission. Melbourne: re.press, 2007

its attempt to leave the undecidability problems of set theory at the level of their, to use a Lacanian term, constitutional "inconsistency," attempts to evade the positivist implications of Badiou's mantra that "mathematics is ontology." Rather, category theory seeks only to construct a universe where its own almost non-axiomatic rules hold. In contrast to set theory category theory does not rely on a system of axioms to give it structure; its structure is defined practically not in its truth-value, as is the case with set theory and Badiouan ontology extending from it.

The import of category theory into continental philosophy and critical theory is similar in orientation to Badiou's project with set theory, however the form and structure of set theory, its stakes and goals, necessitate some key differences.

Category theory, especially in its philosophical instantiation has different epistemological stakes from the purely set theoretic. Much like the debates pertaining to new axioms in set theory, category theory roots much of its empirical value not in *truth qua truth* but in pragmatics, or what mathematicians often call fruitfulness. CT models are in essence a descriptive tool for abstract processes which are otherwise difficult to conceptualize. The fruitfulness of CT resides in varied branches of mathematics like algebraic geometry and topology, or in our case the formalization of continental philosophy, because it enables us to see what is formally challenging to express without making direct claims to truth in itself. In that sense, despite being what one might consider a rationalist method, CT maintains what many of us find most compelling about the Continental (anti-positivist) approach to science. Constructability as opposed to logical positivism is a framework that is compatible with the kinds of contradictory logics in Hegel, Marx and Lacan. Contrary to the radical empiricism of the Deleuzian tradition, CT especially in our deployment of it makes very modest epistemological claims and rejects outright an appeal to "absolute knowledge" present in vitalism and positivism alike. In this sense CT can represent processes and operations as objects, morphisms, and mappings, but does not engage in the kind of reasoning that reifies these mathematical concepts as "things."

For Badiou, the axiom "mathematics is ontology" holds, but the statement, "the one is…" is unthinkably vulgar and reductive. From a CT perspective, the opposite is true; we have various uses for the one (terminal objects, singleton sets, identity mappings, one-universe, etc.) where these concepts have practical and philosophical value in our models, but to claim that mathematics has a one-to-one mapping with a branch of philosophy is a positivist essentialism.

Category theory, which emerged out of the crisis of set theoretic foundations, posits a different constructable universe. In the early years of category theory, mathematicians debated whether axiomatically the system necessitated the construction of unique and incompatible universes in which to work, but quickly Saunders Mac Lane posited the one-universe hypothesis.[30][31] A similar trajec-

[30] Feferman and Kreisel, "Set-Theoretical foundations of category theory"
[31] Mac Lane, "One universe as a foundation for category theory"

tory has unfolded in critical theory and the political Left more broadly over the past half century. The explosion of new social movements and their respective identity politics created a terrain where theorizing the social did not hold as one theory sought to traverse from say the universe of feminism to the universe of decolonial Marxism, to pick two examples at random. My hope is that just as the formalization of category theory and Mac Lane's singular constructable universe created a framework from which mathematicians can theorize complex abstract structures. Category theory can be a tool for theorizing the social in universal terms.

To posit an example, in Seminar XI, Lacan refers to the field of the Other and its relationship to subjectivity, namely language by claiming " The signifier, producing itself in the field of the Other, makes manifest the subject of its signification."[32] Using category theory we can theorize the field of the other as a mathematical object called a vector space.[33] In category theory a vector space, lets say $\boldsymbol{Vct_A}$ over a fixed field such as the field of the other $\vec{A}(\vec{x})$ has a universal functor U which maps the vector space to the category of sets, which in our case is the general phallic function or subjectivity Φx such that $U : \boldsymbol{Vct_A} \to \Phi x$ This universal map behaves such that each element in the field of the Other, in other words every signifier, can be mapped to subjects (subsets of subjectivity) universally. An arrow j such that $j : S \to U(V_A)$ is a universal morphism of what Lacan would call language. That is to say j, language, is a universal arrow mapping subjects S to the universal U.[34]

$$\begin{tikzcd}
\Phi x \arrow[r, "j"] \arrow[d, equal] & A \\
\Phi x \arrow[r, "f"] & \Phi x' \arrow[u, "A_{f'}"']
\end{tikzcd}$$

Figure 3: Language

[32]Jacques Lacan. *The four fundamental concepts of psycho-analysis.* Ed. by Jacques-Alain Miller. Norton paperback psychology. New York: Norton, 1981, p. 207

[33]A note to mathematicians, I am aware that fields and vector spaces are different mathematical objects which behave differently, especially with regard to things like cartesian products in fields versus only scalar multiplication in vector spaces. I choose to translate Lacan's field of the Other as a vector space, primarily because it resembles more closely what Lacan conceptually means, and because vector spcaes behave much better than fields in category theory.

[34]Mac Lane, *Categories for the Working Mathematician*, p. 56

Objects:	**Morphisms:**
Subject: Φx	Language: $\Phi x \xrightarrow{j} A$
Subject': $\Phi' x$	Metonymy: $\Phi x \xrightarrow{f} \Phi' x$
Field of the Other: A	Metaphor: $\Phi' x \xrightarrow{A_{f'}} A$

Law of Language: $\forall \Phi x_n \in \Phi x, \Phi x_n \mapsto \exists A_x$
Law of Metonymy: $\forall \Phi x_n \in \Phi x \; \exists f : \Phi x \mapsto \Phi x'$
Law of Metaphor: $\forall \Phi x'_n \in \Phi x' \; \exists A_{f'} : \Phi x'_n \mapsto A_x$

In the above diagram first we have a mapping of the category of subjects, subjectivity (Φx) mapped to the Universal (U) through a universal morphism (j) or language. Next we see the universal mapping of an element of the subject α mapped through language to the vector space covering the field of the Other (A) of the field of the Other $\vec{A}(\vec{x})$ or a set of signifiers. Below is the mapping of metonymy (f) which maps the same element of the subject α to the next element of the subject α in the set of the subject α, ($\Phi x'_\alpha$). Last the arrow $A_{f'}$ is the mapping metaphor which maps the set of signifiers to the (now) speaking subject, uniquely from metonymy (f).

We theorize metaphor as the mapping of a signifier to the subject-element prime, in line with Lacan's formula for metaphor in his *Ecrits*.[35]

$$\text{Lacan} : f(\frac{S'}{S})S \cong S(+)s \tag{1}$$

$$\text{CT} : A_{f'} \circ f = j \tag{2}$$

We theorize metonymy as the mapping of the subject in the first instance to the subject as the next element in the chain.

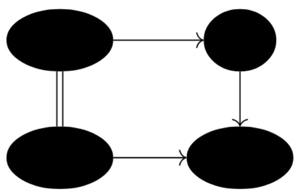
$$\text{Lacan} : f(S \ldots S')S \cong S(-)s \tag{3}$$

$$\text{CT} : S \xrightarrow{f} S' \tag{4}$$

For Lacan, the bar (—) and the crossed bar (+) were the formal symbols best approximating the relationship between synchrony and diachrony in his structural linguistic system. The category-theoretic definitions seek to replace this formal symbol and its ambiguity with morphisms and mappings which signify operations and relations between conceptual objects like subjects and signifiers. The mapping above seeks to show, much like Lacan in "The Instance of the Letter in the Unconscious" the relation or non-relation between the linguistic operations of metaphor and metonymy. The advantage of the category-theoretic model is we can construct a general model for language that incorporates concepts like metaphor and metonymy, rather than operating with a long list of disparately-related mathemes and formulae.

[35]See, Lacan, *Ecrits*, pp. 428–9

Similarly, we can see at work in this model of language the circuit Lacan outlined in *Seminar XI*, the losange ($\Diamond$). In his theory of alienation, Lacan describes his discrete logic in another form, that is the *vel* requires that the production of the signifier occurs in the field of the Other, and that between the subject and the field of the Other is a gap of non-meaning.[36] The diamond operation, which Lacan conviniently calls a "topology," is mapped appropriately in category theory as opposed to concrete algebra, $S\Diamond a$, precisely because operations can be rendered as observable objects in category theory. Hence we can represent the diamond operation as our universal morphism j, such that $j : S \to U$.



$$\xrightarrow{j_0}$$

$$\begin{tikzcd} \Phi x_\alpha \arrow[r, "j"] \arrow[d, equal] & A \arrow[d, "A_{f'}"] \\ \Phi x_\alpha \arrow[r, "f"] & \Phi x'_\alpha \end{tikzcd}$$

Figure 4: Universality of Language

Objects: Topology in the Shape of Language (Left),
Diagram of Language (Right)

Universal Functor: Topology $\xrightarrow{j_0}$ Diagram

The above diagram shows from a category theoretical, but also topological perspective, what we mean when we claim language maps the subject to the universal. Here a topological space of relations between four objects makes a category which maps to our diagram for language. The space shows the relational qualities, or bear structure of language, where the diagram enters those relations into a system of signification which constitutes language. Our universal functor "j_0" maps those relations to the diagram which constitutes language.

This short exposition is meant to demonstrate the potential of bridging psychoanalysis and metaphysics using category theory and category-theoretic modeling as a tool, and in no way is meant to be exhaustive. Aspirationally, this project of formalization is intended to pick up on and further a tradition of mathematical structuralism within Lacanianism, which Lacan only scraped the surface of and has been neglected in the subsequent years.

From CT to Type Theory

Employing what Thorsten Alterkirch calls "naïve Type Theory" we can begin to reconceptualize concepts which we previously defined using both set-theoretic and category-theoretic foundations under what mathematicians call univalent foundations. This new way of thinking through the foundations of mathematics

[36] Lacan, *The four fundamental concepts of psycho-analysis*, p. 211

is meant to be intuitionistic like the others mentioned, but because of its restrictions allows us to work at higher levels of abstraction. The work of Martin-Löf in Type Theory has lead to the development of Homotopy Type Theory (HoTT) which is becoming increasingly useful to philosophers of mathematics. For our purposes, naïve Type Theory enables us to make generic, to borrow a term from Badiou and Laruelle, the objects of our analysis. For instance, *objet petit a* which we defined as the point-set $\{\varnothing\}$ can be made generic as the type for point sets $(*)$ defined by its quality $(* : \mathbf{Type})$and identity $(*_{\mathrm{Id}})$. From this vantage the function "lack" which under category-theoretic definition is a mapping of $a \to \Phi x_i$ can be rethought and genericized as $\lambda x.* : \mathbb{N} \to \mathbb{N}$. Here lack is a universal morphism defined as statically mapping the point-set (*objet petit a*) to the map of the natural numbers into itself, which, as stated previously is the generic form of subjectivity (Φx). The most important innovation of our model coming from the type-theoretic definition is that the sets of subjects $(\Phi x_\alpha, \Phi x_\beta, \Phi x_\gamma, \ldots)$ are no longer expressed through the proposition "in subjectivity" $(\in \Phi x)$ but as elements of the type of subjects. Thus, we can make a similar argument to Mac Lane and Lawvere regarding one-universe in the elementary category of sets (ECTS) in univalent foundations simply stating $\Phi x : \mathbf{Type}$ where $\mathbf{Type}$ is the metavariable for all universes $\mathbf{Type}_i$.

Another way of type-theorizing the subject is to think of subjectivity as a class (Φx) with the type of elements:

$$0_{\Phi x}, 1_{\Phi x}, 2_{\Phi x}, \ldots, (\Phi x - 1)_{\Phi x} : \overline{\Phi x} \tag{5}$$

Such that:

$$\Phi x_\alpha = \Phi x_\beta = \Phi x_\gamma \tag{6}$$

where = indicates has the same number of elements.

Equation (5) shows that the type subjects has infinitely many (or at least an arbitrarily large number of) elements. Equation (6) shows the equality of elements of the type subjects to one another emphasizing their generic character under the univalence principle. This principle allows us to state other properties of the type elegantly like the function type. Taking lack as an example we can type subjectivity as:

$$\overline{\Phi x^*} = \overline{*} \to \overline{\Phi x} \tag{7}$$

To generalize further we can express the generic quality of function types acting on subject types using a formulation like this:

$$\prod x : \Phi x_\alpha.* = \Phi x_\beta.* \tag{8}$$

$$\forall\ \Phi x_x : \Phi x\ \ \&\ \ \forall *$$

$$\Phi x_x \times * :\equiv \prod x : \mathrm{Bool.if}\ x\ \mathrm{then}\ \Phi x_x\ \mathrm{else}\ * \tag{9}$$

We can write equation (8) in English as the function type lack is always equal to itself for all subjects to *objet a*. Here lack is not indexed by any one element of the type subjects (any one subject) but the generic type, subjects, to the point-set. Thus lack is a type which acts upon a domain (subjects) and a codomain (*objet a*). Equation (9) shows that relationships like the cartesian product of a subject to *objet a* can be derived from adding a Boolean proposition to a generic product type.

These changing definitions may seem trivial as, in general terms, our model is making similar claims in all three foundations (set-theoretic, category-theoretic and type-theoretic). Indeed it is probably easier to see the change represented in set-theory to category-theory than category-theory to type-theory. However, regarding the question of universality, type theory allows us to enter the generic where the freedom of ST and CT prevent us. Following Badiou's exposition on the importance of the generic for an emancipatory theory of the subject, and Laruelle's insistance on the generic as a new basis for a non-philosophy of science, type theory's move to reframe subjects as belonging to the type "subjects" rather than as subsets of the set "subjectivity" proves radically more emancipatory. In the category theoretical model, which itself is highly indebted to the set-theoretical model of Lacan, subjects are alone in an empty universe, left to indulge in their own unique individuality. As members of the subject type, subjects are still discrete, but are part of a generic famility where their unique qualities are rendered trivial, and their relationship to *objet petit a* is identical in every case as generic lack.

On Cybernetics and Models

Further yet, and markedly a superior model to the previous preliminary models, pictured below is a diagram recuperating the brief cybernetic intervention in Lacan's thinking, which is most present in *SII: The Ego in Freud's Theory and in the Technique of Psychoanalysis*. One might ask, "if this model is superior, why include the others?" The central motivating question regarding the mathematical work in Lacan, is how to reconcile the varied forms of Lacan's thinking into one coherent body of work. Short of achieving this full ambition, category theory serves as a mechanism, or to borrow a term from Lacanian parlance, a *supplement* to the cybernetic teaching in *SII* and the set theoretical and topological teaching in *SXIX* onward. Pictured below is a joint cybernetic and category theoretical diagram depicting a single system for both language and subjectivity—because for Lacan, language and subjectivity are coconstitutive concepts.

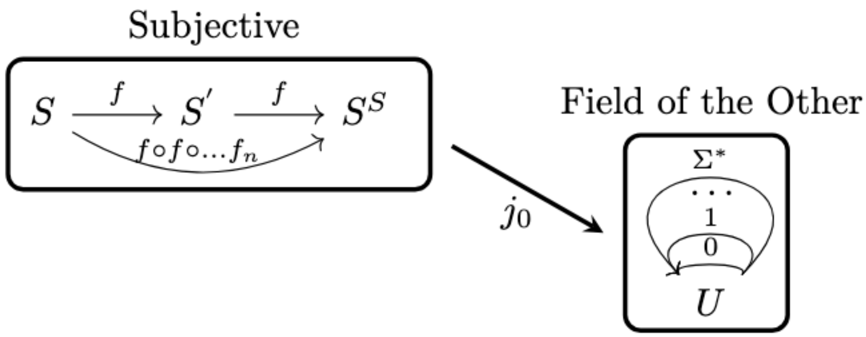


Figure 5: A Cybernetic Machine Model of Language

Categories:	**Objects:**	**Morphisms:**
Subjective	Subject: S	Subject Transformations: $S \xrightarrow{f} S^S$
Field of the Other	Monoid U	Morphisms $U \to U$ indexed by countably infinite alphabet Σ
Functors:		
Language Functor: j_0		All Speech Acts énoncé $\forall$é $\in j_0$

At first glance, the diagram appears to depict two systems or categories, one called "Subjective" and another called "Field of the Other." However, as those with even a cursory exposure to cybernetics know, as Lacan said, "the first machine [is] dependent on the image of the second, hanging on its unitary functioning, and consequently captivated by its procedures. Hence a circle, which can be vast, but whose essential set-up is given by this imaginary dual relation."[37] This is to say the two machines, or categories describe a single relation, which to maintain the continuity from the previous diagrams is a functor called j_0 or language. The machines depict a feedback loop through which the subject sends a signal to the Field of the Other, which is rendered as a concatenation of morphisms on the monoid U, in other terms a string of symbols in the Field of the Other. The function I just described in directly analogous to the speech-act or énoncé in Lacan's thought. At a fundamental level, what defines this system, the cybernetic interaction of these two automata, is the relation of speech by which the subject becomes a speaking subject, and some string of symbols is imbued with significance. A generalized collection of all the instances of these relations, or speech morphisms, is the functor j_0, or language in general.

It is important to remind the reader that no one part or component of this model can be extricated from the model as a whole. For Lacan, "when one speaks of

[37] Jacques Lacan. *The ego in Freud's theory and in the technique of psychoanalysis, 1954-1955*. Ed. by Jacques-Alain Miller. Trans. by Sylvana Tomaselli. Norton paperback bk. 2. New York, N.Y: W.W. Norton, 1991. 343 pp., p. 54

subjectivity, the problem is not to turn the subject into an entity"[38] Furthermore, "the subject is no one. It is decomposed, in pieces. And it is jammed, sucked in by the image, the deceiving and realised image, of the other…that is where it finds its unity."[39] All of this is to say that the model only has coherence as a unity by which the subject must be the subject of language, and symbols must be brought into the speech relation when mediated by subjectivity. There is an operative dialectic whereby the ontological status of the speaking subject is entirely dependent on a world of symbols in the Field of the Other and a circular or circuit-like relation between the automata. Conversely the symbols are merely, to use an example from the Lacanian cannon, hieroglyphs in the desert, without a speaking subject to enter them into a linguistic relation.

Now that we have established the total contingency of the system on each of its component parts, we can begin to unpack the categorical and computational implications of such a model. Bringing into frame the philosophical contingency of models themselves is necessary at this step. What is it we want this model to do? Conceptuatlizing the model as an analytical tool, my goal is for the model to do more than the traditional Lacanian diagram, whose role is primarily demonstrative. Additionally, we want our model to take up the task of the Lacanian matheme, that is, within certain schematic parameters, to make a truth claim axiomatically. Finally, as a cybernetic or computational model, we want it to be verifiable. That is, it can be instrumentalized mechanically as a computation.[40] If our model is successful, it does all these things at once. In order for it to be successful, one must not think of the system as a static or ossified being, but as infinitely churning machinic circuits. These dynamic flows will seem familiar to those who have studied the libidinal economy of psychoanalysis, and has a natural isomorphism to the dynamics of movement in Hegel's dialectic and Marx's circuits of commodity circulation.

Though the aim of using category theory is to provide more structure to the model, the precise internal structure of the objects S (the subject) remains unknowable. Rather we generalize the structure in terms of its function in the system and its relation to other sub-systemic objects. For instance, the Subjec-

[38] Lacan, *Le Séminaire - Livre II*, p. 53

[39] Lacan, *Le Séminaire - Livre II*, p. 54

[40] One of the great strengths of critical theory and of Continental philosophy more broadly has been its ability to give the necessary attention to contingency that the empiricists and rationalists of formal science eschew. That is the true value of a critical practice. However when too mired down in the omniprescence of contingency, which contemporary theory takes as the *a priori* background for philosophy, our claims lack power beyond the trust and intuition of our readers. That is to say, theory often implores readers to trust its claims on the merits that they have some intuitive or provocative staying power. The past decades have taught us that this allergy to truth claims leaves our practice vulnerable to what I can only call "theoretical chauvinists": those who are willing to make claims boldly and authoritatively on the basis of status, privilege and charisma. Read these essays as an experiment in a truly radical practice of philosophy. By taking up the tools of formalization offered by mathematics, and verification offered by computation, we have a way of insulating the critical tradition from self-defeat and the conservative abuse of appeals to reason, without surrendering the centrality of contingency which is critical theory's virtue.

tive is comprised of mappings (f) from one instance of subjectivity to another. When all instances of f are concatenated, forming the signifying chain, they map to a terminal exponential object S^S representing the space of transformations between finite-states of subjectivity. Thus the subjective takes the form in computational theory of a deterministic finite-state-automaton (FSA). The language functor j_0 maps elements in the Subjective (finite states of subjectivity) to the symbolic order in the Field of the Other. The structure of the field of the Other is also an automaton. The Field of the Other is an non-deterministic infinite state automaton, closely resembling a Universal Turing Machine (TM). This machine representing by the monoid U for universal computes strings from an infinite language generated by the infinite alphabet Σ, generating the infinite language of finite strings Σ^*. For those familiar with automata theory and computational linguistics this should all sound very familiar. However, for those with less exposure to formal linguistic and computational theory, let me formulate this in terms familiar to a Lacanian: the subjective is a machine that represents in the most generic terms the position of subjectivity vis-a-vis the symbolic order, each instance of the subject is linked by a computational or mechanical process which model metaphor and metonymy; the sum total of these computations is referred to as the space of transformations of the subject, and can be considered to model the general condition of subjectivity. The Field of the Other is another machine which takes signals from the Subjective (or speech-acts) and translates them into strings of symbols in the symbolic order. The machine itself is the total collection of signifiers available to the speaking subject. When a signal or map comes from the subjective into the Field of the Other any information which cannot be expressed symbolically is information loss (lack). The total collection of maps (énoncé) from the subjective to the Field of the Other is the language relation functor j_0.

To render this model computationally, and to verify its logical consistency, I have written the following Haskell program:

```haskell
module FuncLang where

import Data.Hashable (hash)

-- Subjective

-- Objects
data State
    = Initial          -- Represents S
    | Successor State  -- Represents S′
    | Final            -- Represents S^S (final state)
    deriving (Show, Eq)

-- Define the successor function
f :: State -> State
```

```haskell
f Initial = Successor Initial  -- Transition from S to S'
f (Successor s) =
    case s of
        Final -> Final          -- If already at S^S, stay there
        _     -> Successor (f s) -- Otherwise, recurse deeper
f Final = Final

-- Field of the Other

-- Objects

data U = U deriving (Show) -- Monoid of the Symbolic

-- Morphims

-- Define the morphisms as natural numbers
newtype Morphism = Morphism Int deriving (Show, Eq)

-- Identity morphism (corresponds to 0)
identityMorphism :: Morphism
identityMorphism = Morphism 0

-- Composition of morphisms
composeMorphisms :: Morphism -> Morphism -> Morphism
composeMorphisms (Morphism m) (Morphism n) = Morphism (m + n)

-- Action of a morphism on U (trivial in this case)
applyMorphism :: Morphism -> U -> U
applyMorphism _ U = U

-- Language Relation (Functor j_0)

-- On Objects
jOb :: State -> U
jOb _ = U

-- On Morphisms
jMorph :: (State -> State) -> [Morphism]
jMorph stateTransition =                -- stateTransition is a
↪  place-holder for the parameter 'f'
    let initialState = Initial
        states = iterate f initialState  -- Generate infinite
        ↪  sequence of states
        indices = takeWhile (/= Final) states  -- Stop at Final
        ↪  state
```

```
    hashedIndices = map (\s -> Morphism (hash (show s) `mod`
    ↪  100)) indices
in hashedIndices
```

The module FuncLang, expressed in the above Haskell program is an implementation of the commutative diagram between the categories "Subjective" and "Field of the Other" with universal functor "j_0" representing the language relation. The model is a correspondence between a recursive infinite deterministic automaton (Subjective) and an infinite nondeterminatistic automaton (The Field of the Other). The subjective is a machine with potentially infinite states with successive unary logic gates determinatively concatenating to some final state S^S representing the space of transformations on the Subject (S). At each iteration of S the language functor (j_0) maps the Subject to some concatenation of morphisms in the monoid U in the Field of the Other. This concatenation of morphisms is expressed as a string in the language Σ^* with alphabet Σ of countably infinite symbols (the symbolic order). For this reason the countably infinite set of strings generated by the Field of the Other is the final state or stable state of the automaton Field of the Other. From this perspective, each element of the HomSet of the Language functor (j_0) is an instance of the énoncé or speaking subject, meaning the subject at some point in time (some S) maps to some string of symbols (concatenation of morphisms) in the symbolic order. Thus the total HomSet(Subjective, Field of the Other) is the total (countably infinite) set of possible enunciations of language by a speaking subject. Even further, the Language functor is the very structure that maps the subject to the structured relationship of all symbols, which Lacan called the symbolic order.

As for the Field of the Other, this infinite state machine is discrete from the system "the Subjective" insofar as in Lacanian teaching, the set of symbols (here the alphabet Σ) and the relations between them are their own structure independent of Subjects. Take this brief dialogue from Seminar II:

> M. Rigeut: You have a kind of basic language of communication, a kind of universal language, and the symbols you speak of are always coded in function of this basic language.
>
> M. Lacan: What strikes me in what you just said […]—is this—when one illustrates the phenomenon of language with something as formally purified as mathematical symbols—and that is one of the reasons for putting cybernetics on the agenda—when one gives a mathematical notation of the *verbum*, one demonstrates in the simplest possible way that language exists completely independent of us. Numbers have properties which are absolute. […] All this can circulate in all manner of ways in the universal machine, which is more universal than anything you could imagine.[41]

The use of the term language here is tricky, but what should be distilled is that

[41] Lacan, *Le Séminaire - Livre II*, p. 284

the machinic system of signifiers which we are calling "Field of the Other" has its own self-sustaining logic that is universal and independent of the subject. However, when brought into contact with the subject, it forms a cybernetic system of subjectivity which gives rise to a language relation (j_0) between the subjective and the universal state machine of signifiers. Taken as a total system, subjectivity traces the dynamical interaction (speech-acts) between the subject and the universal regime of signifiers.

What is the structure of this seemingly random mapping of states in the Subjective to strings of speech, or concatenations of morphisms in the Field of the Other? In the Haskell program we use something called a hash function to give this speech-act structure. This hash function is a cryptographic tool used to take any input of information from the Subjective and render it as some series of digits in the Field of the Other. This tool is used to store information in cryptography because it can reliably map inputs to outputs without being able to easily determine the function of those mappings. This models nicely the speech-act where-by concepts are efficiently rendered symbolically as speech without a clearly identifiable logic to the translatability of such ethereal concepts in the mind to concrete linguistic strings. Similarly, Lacan invokes Freud's explanation of a similar function in the unconscious. When we think of a hash function, we should "think of that very strange game Freud mentions at the end of *Psychopathology of Everyday Life,* which consists in inviting the subject to say numbers at random."[42] The psychoanalytic point, which maps effectively onto the hash function, is that the series of numbers which appear random to the conscious belong to a highly structured and deterministic set that is unknowable at the level of the énoncé. As Lacan claims, "from the point of view of probabilities, what he chose goes well beyond anything we might expect from pure chance."[43] Already in Freud we have a primitive model of a computational process.

$$\text{law)}\ \ \Gamma_s \forall \text{é} \in j_0 \mapsto \exists x \in \Sigma^* \times \mathbb{N}^{\mathbb{N}} \tag{10}$$

The logical statement above denotes the general structure of the functor j_0 which sends a signal from the Subjective the the Field of the Other. In the Haskell program this is captured by the hash function. The statement reads: "in the context of subjectivity (Γ_s), for all instances of speech (énoncé) in the language relation (j_0), there exists some string (x) composed of concatenations of symbols in the alphabet (Σ) in the Field of the Other." This logical form denotes the operation of a speech-act or énoncé (é) under the conditions specified by the machine we are calling "Subjectivity." To see this machine in action, take the canonical example of the speech-act producing the string "hello world!"

$$\text{ex)}\ \ \Gamma_s \exists ! \text{é} \in j_0 \mapsto \text{hello\_world!} \in \Sigma^* \times \mathbb{N}^{12} \tag{11}$$

[42]Lacan, *Le Séminaire - Livre II*, p. 56
[43]Lacan, *Le Séminaire - Livre II*, p. 56

The equation reads: "in the context of subjectivity (Γ_s), there exists exactly one speech-act which enunciates the twelve character string 'hello world!'" To illustrate this speech act categorically would look something like this

$$S \xrightarrow{é} U \xrightarrow{!} U \xrightarrow{d} U \xrightarrow{l} U \xrightarrow{r} U \xrightarrow{o} U \xrightarrow{w} U \Longrightarrow U \xrightarrow{o} U \xrightarrow{l} U \xrightarrow{l} U \xrightarrow{e} U \xrightarrow{h} U$$

where each morphism on the monoid U is a character in the alphabet (Σ) that is concatenated to compose the string, and é an instance of the language relation (j_0), also called an element of the homset between the Subjective and the Field of the Other.

The same kind of logical statement can be used to denote the most simple universal Turing Machine (TM)

$$\text{Turing Machine: } \Gamma_{TM} \forall x \mapsto \exists y \in \{0,1\} \times 2^{\mathbb{N}} \tag{12}$$

The above reads: "in the context of a universal Turing Machine (Γ_{TM}) for all inputs x, there exists an output y of binary symbols {0,1} concatenated by some countable length before terminating ($2^{\mathbb{N}}$)."[44] It should be noted, following Turing's thesis, the model subjectivity can be replicated in the model Turing Machine such that $\Gamma_s \in \Gamma_{TM}$; in fact $\forall\ \Gamma \in \Gamma_{TM}$ where Gamma is a computational machine, which reads: "following that all computational machines can be modeled by a universal Turing Machine, the machine Subjectivity can as well."

Incorporating category theory, cybernetics and Haskell, serves as a way to properly reel in the verbosity of psychoanalytic discourse with the full expressiveness of formal systems.

Computational Sexuation: Toward a Turing-Lacan Correspondence

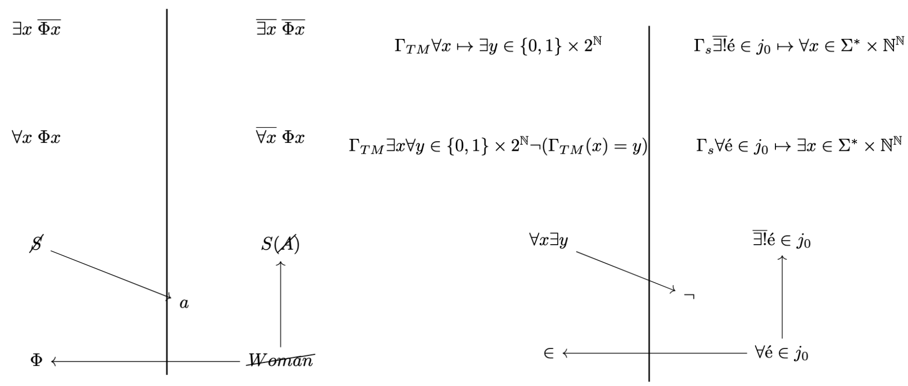
The lesson from Hegel to Marx to Lacan is that computation is in the discourse of the hysteric. That is to say each example shows that the limitations of computation are reflective of a base desire for the human to apprehend what he cannot, to abduct the real. In computational semantics, we can say that there is no program (é) in the set of programs (j_0) that universally applies to all finite strings and infinite sequences of natural numbers. Such that there is no universal decision procedure for a given domain. In psychoanalytic discourse we can say, "there is no speech act that says all that can be said at once; there is no speech-act that universally satisfies speech." The correspondence from computation to Lacanian discourse would look something like this:

[44] I've chosen to use the set {0,1}, but theoretically it can be any two-element set which in category theory is canonically called 2 and in logic is canonically called Bool.

$$\Gamma_s \overline{\exists !} \text{é} \in j_0 \mapsto \forall x \in \Sigma^* \times \mathbb{N}^{\mathbb{N}} \tag{13}$$

$$\overline{\exists x}\ \overline{\Phi x} \tag{14}$$

The following corresponds to one position in the feminine side of Lacan's formula's of sexuation. A full extension of the model is provided below:



Pictured above on the left is a reproduction of Lacan's formulas of sexuation diagram from SXIX. To the right is an interpolation of the same diagrams and concepts using the computational framework developed by Turing. I will aim to show here how fundamental concepts from the philosophy of computing are analogous to the Lacanian subject. Beginning with the right side of each diagram, referred to as the feminine side,

Compare the machine:

$$\Gamma_s \forall \text{é} \in j_0 \mapsto \exists x \in \Sigma^* \times \mathbb{N}^{\mathbb{N}} \tag{15}$$

to the statement:

$$\Gamma_s \overline{\exists !} \text{é} \in j_0 \mapsto \forall x \in \Sigma^* \times \mathbb{N}^{\mathbb{N}} \tag{16}$$

This is my computational gloss on what Badiou calls "On Mark and Lack," That all énoncés are programs which constitute a discrete speech-act, and that there is no single speech-act (program) that successfully expresses all we call language. That there is a core constituitive lack of uncomputables (computation) and undecidables (mathematics) that we can successfully gesture at, or point to, but cannot cognize (Hegel), calculate (Marx), enunciate (Lacan); that we cannot compute. This corresponds to the feminine situation in Lacan. On the bottom right, is a universal condition of subjectivity, that the subject can speak; on the top right, is the law or exception to this statement which constitutes it, what Badiou calls a "truth-condition." The truth condition for feminine subjectivity is "there is no Woman." Computationally, this is rendered as "there is no decidability program which models speech."

On the left, or masculine sides of the diagrams are the universal conditions of computing associated with a universal Turing machine. The top left position: "there exists a subject outside of the phallic function" ($\exists x\ \overline{\Phi x}$) corresponds to the image of a universal Turing machine which can compute all finite strings of the binary alphabet without halting. In both Lacan and theory of computation, this is the impossible position of the uncastrated subject and a machine where all numbers are computable. On the bottom left is its "truth-condition" or constitutive limit, rendered in Lacan that all subjects are in the phallic function ($\forall x\ \Phi x$) and in computational terms:

$$\Gamma_{TM} \exists x \forall y \in \{0,1\} \times 2^{\mathbb{N}} \neg (\Gamma_{TM}(x) = y) \tag{17}$$

which reads, "in the context of a universal Turing machine, there exist numbers such that for all finite strings in the language generated by a binary alphabet the machine does not produce an output." This is what we call halting, and it constitutes a central problem called the *Entscheidungsproblem* in the theory of computation, that Turing and Church proved unsolvable.[45][46] In psychoanalytic terms, this proof of the impossibility of the decision problem is merely affirmation of the discovery of the unconscious. In a more positivist frame, Turinng himself makes a similar claim, that "the justification [for his hypothesis regarding the decidability of the halting problem] lies in the fact that human memory is necessarily limited."[47] Needless to say, the correspondence between psychoanalysis and classical computing continues to the formal level.

As noted earlier, all computational machines, including the machine "Subjectivity" (Γ_s) can be modeled in the paradigm of a universal Turing machine (Γ_{TM}). This maps onto the Lacanian proposition that there is no woman, or rather that the feminine situation of subjectivity is itself, at the level of elaboration, operative in the masculine. Following Lacan, the elaboration of computational models, which themselves represent something impossible or non-computational, maps to his statement that such models are "markers. It's a marker that doesn't hold up even for an instant, that is neither instructive nor teachable, if we do not conjoin it to another quantifier that is set down within the four terms, namely the quantifier that is said to be universal."[48] That is to say that we must enunciate these systems at the level of signification for illustrative purposes in order to make a more profound or universal argument. Such is the focus of the core

[45] A. M. Turing. "On Computable Numbers, with an Application to the Entscheidungsproblem". In: *Proceedings of the London Mathematical Society* 2-42.1 (1937), pp. 230–265

[46] Alonso Church. "An unsolvable problem, of elementary number theory". In: *American Journal of Math* 58 (1936), pp. 345–363

[47] Turing, "On Computable Numbers", p. 231

[48] Lacan, *or worse*, p. 178

Freudian point ($\forall x\ \Phi x$) “that the only libido is masculine.”[49]

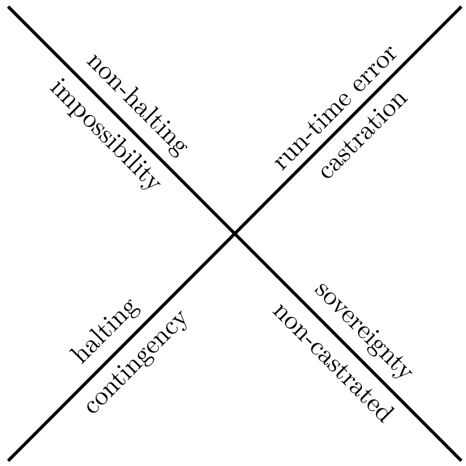
$$\Gamma_s \in \Gamma_{TM}, \forall \Gamma \in \Gamma_{TM}$$
$$\ldots$$
$$\text{there is no woman, all libido is male}$$

Another way of interpreting the matheme, is that all subjects (who desire) are castrated or constituted by a lack. In the computational system, this is merely the statement that all computational machines are bound to the halting problem.

$\Gamma_{TM} \forall x \mapsto \exists y \in \{0,1\} \times 2^{\mathbb{N}}$ Pan-Computationalism $\exists x\ \Phi x$ Uncastrated Father	$\Gamma_s \overline{\exists !}é \in j_0 \mapsto \forall x \in \Sigma^* \times \mathbb{N}^{\mathbb{N}}$ Undecidability of Speech $\overline{\exists x}\ \overline{\Phi x}$ Discovery of the Unconscious
$\Gamma_{TM} \exists x \forall y \in \{0,1\} \times 2^{\mathbb{N}} \neg(x = (y))$ Entscheidungsproblem $\forall x\ \Phi x$ Universal Castration	$\Gamma_s \forall é \in j_0 \mapsto \exists ! x \in \Sigma^* \times \mathbb{N}^{\mathbb{N}}$ Pan-Linguistic Subject $\overline{\forall x} \Phi x$ “Nay-saying” Subject

We can imagine a diagonal axis connecting the top left quadrant to the bottom right quadrant signifying computational non-halting and subjective non-castration. This is the axis of authority and sovereignty and constitutes an impossible position. The opposing axis connecting the bottom left and top right quadrants signifies halting in computation and castration in the subject; this is the axis of contingency, and constitutes the constitutive exception of the real situation. Of course, sexuation and the discourse accompanying subjectivity and computation is born out of the dialectical engagement of all four positions which themselves constitute a whole.

[49]Lacan, *or worse*, p. 178



The composite of the four mathemes comprising the two sexes correspond directly to the two machines, one universal (Γ_{TM}) and one modeling human speech (Γ_s) and their respective generalities and contingencies. Following the goal of this project more broadly, the computational rendering is not meant to merely metaphorically represent some abstract concept, but rather to exist as a functional model with its own explanatory power. To show a correspondence between computation and psychoanalysis is to take quite literally Freud's assertion that the psyche is an "apparatus."[50] The reverse point which is equally important is to highlight what we might call a subjective dimension to computational machines. The implication of this discovery is to radically reimagine computational contingency, to use logical positivist tools to undermine the project of political and philosophical positivism.

Computing the Subject: LLMs and the Future of Computation

It is crucial that this machine (Subjectivity) composed of the two automata (the Subjective and the Field of the Other) produces subjective thought (pre-symbolic), symbols (symbolic) and the relation between them (j_0) but does not in itself produce *a posteriori* signified meaning. In other words, we have a cybernetic model of the speaking subject but we do not have an interpellation machine. As Lacan notes, "One can imagine an indefinite number of levels, where all this turns around and circulates. The world of signs and functions, and it has no signification whatsoever."[51] A Lacanian cybernetic machine forecloses on the idea that meaning is produced in this subjective frame. Lacan suggests that signification happens at another scene: "when we stop the machine."[52]

[50]Or a complex system of several interactive circuits.
[51]Lacan, *Le Séminaire - Livre II*, p. 284
[52]Lacan, *Le Séminaire - Livre II*, p. 284

That is to say it is in temporal ruptures in the exchange of signals between the Subjective and the Universal system of signifiers that something like meaning can emerge.

What then might we ask of machines that deal in natural language, that simulate speech and reasoning computationally? In recent years this kind of machine, namely Large Language Models (LLMs), have come to dominate the discourse of contemporary computation. These models primarily through the operations of multiplication on matrices seek to simulate linguistic and multi-modal reasoning and generate speech.

The paradigm at the intersection of classical computing and subjectivity outlined in this chapter does not map neatly onto learning models. All of the interesting questions regarding learning models' limitations have little to do with classical computing's questions of (un)computability. Similarly, though we all use language normatively everyday, we rarely acknowledge the formal underpinnings of mark and lack, though undoubtedly the laws of language and subjectivity undergird our usage. Thus, the motivation for philosophical inquiry into mark and lack maps to complexity theory in computation, as (some) analytic philosophy's pragmatist occupation with the normative, alethic qualities of language map to the more pragmatic machines like learning models. However, these questions of computability vis-a-vis Hegel, Marx, and Lacan are of course operative in learning models, just as metaphysics, political-economy, and the psyche are operative in everyday life. From this perspective, the question "why study classical computing?" is analogous to the question "why theory?" The proof of that question is left as an exercise to the reader!

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Appendix

Chapter 1

Primitive Botanical Coq Program

```
(* Assume plant is a type *)
Variable plant : Type.

(* Assume P is a type *)
Variable P : Type.

(* Declare three objects of type P *)
Variables bud flower fruit : P.

(* Declare paths (equalities) between bud and flower, and flower
↪  and
fruit using Coq's equality *)
Variable p : bud = flower.
Variable q : flower = fruit.

(* We can model a path between bud and fruit using transitivity
↪  of equality *)
Definition r : bud = fruit := eq_trans p q.

(* Inductive definition of path, generalized over a type A *)
Inductive path {A : Type} (x : A) : A -> Type :=
| idpath : path x x.

(* Declare inductive paths between bud and flower, and flower and
↪  fruit *)
Variables p' : path bud flower.
Variables q' : path flower fruit.

(* Define path composition for the inductive path type *)
Definition compose_paths {A : Type} {x y z : A} (p : path x y) (q
↪  : path y z) : path x z :=
```

```
  match p with
  | idpath => q
  end.

(* Compose paths homotopically *)
Definition homotopical_path := compose_paths p' q'.

(* Declare two paths between bud and flower *)
Variables p1 p2 : path bud flower.

(* Define a higher path (path between paths) *)
Definition higher_path : path p1 p2 := idpath.

(* Define the composed path between bud and fruit *)
Definition composed_path : path bud fruit := compose_paths p' q'.
```

Universe Coq Program

```
(* Declare two types 'a' and 'b', each in their respective
↪  universes 'A' and 'B' *)
Universe A B.

(* Declare 'a' of type Type A and 'b' of type Type B *)
Variable a : Type@{A}.
Variable b : Type@{B}.

(* The function type 'a -> b' lives in a universe that is one
↪  level higher than
the maximum of the universes A and B *)
Check (a -> b : Type@{1 + max(A, B)}).
```

Advanced Botanical Coq Program

```
(* First define the identity function of the type plant *)
Definition id {plant : Type} (x : plant) := x.
Definition identity_plant (x : plant) := x.

(* s.t. *)
identity_plant : plant -> plant

(* Then show that bud, flower, and fruit are all members of
↪  type plant *)
Inductive plant : Type :=
 bud : plant
 flower : plant
| fruit : plant.
```

```
(* Show the generic path p between the three elements *)
Definition p : bud = flower -> flower = fruit -> bud = fruit
 ↪  :=
fun Hbudflower Hflowerfruit => eq_trans Hbudflower
 ↪  Hflowerfruit.

(* Following the operation for assigning function types,
it follows that the mapping (bud to flower to fruit) should
↪  have the type plant -> plant *)

Definition plant_mapper (x : plant) : plant :=
match x with
 bud => flower
 flower => fruit
| fruit => suc(fruit)
end.
(* Here suc, is a function which returns the next (successor)
 ↪  element in the type plant *)

(* The identity map is of type plant -> plant *)
Check identity_plant.
(* identity_plant : plant -> plant *)

(* The mapping plant_mapper is also of type plant -> plant,
 ↪  demonstrating that functions
on the same type are at the same type level. *)
Check plant_mapper.
(* plant_mapper : plant -> plant *)
```

Groupoid Coq Program

```
(* Import HoTT library *)
Require Import HoTT.

(* Define contractible types S and O *)
Definition S : Type := nat.
Definition O : Type := bool.

(* Show that S and O are contractible types *)
Instance contractible_S : Contr S := contr (0 : S).
Instance contractible_O : Contr O := contr (true : O).

(* Define two types alpha and beta *)
Definition α : Type := nat.
Definition β : Type := bool.
```

```
(* Define two functions f : S -> α and g : O -> β *)
Definition f : S -> α := fun s => s + 1.
Definition g : O -> β := fun o => negb o.

(* Define the type of morphisms in a groupoid as paths *)
Definition Hom (X Y : Type) : Type := X = Y.

(* Define identity morphisms *)
Definition id {X : Type} : Hom X X := eq_refl.

(* Define composition of morphisms in the groupoid *)
Definition comp {X Y Z : Type} (p : Hom X Y) (q : Hom Y Z) : Hom
↪  X Z :=
  p @ q.

(* Define inverse of morphisms *)
Definition inv {X Y : Type} (p : Hom X Y) : Hom Y X :=
  p^.

(* Define Groupoid laws (as lemmas in HoTT) *)

(* Associativity of composition *)
Lemma comp_assoc {X Y Z W : Type} (p : Hom X Y) (q : Hom Y Z) (r
↪  : Hom Z W) :
  comp p (comp q r) = comp (comp p q) r.
Proof.
  reflexivity.
Qed.

(* Left identity law *)
Lemma left_id {X Y : Type} (p : Hom X Y) :
  comp id p = p.
Proof.
  reflexivity.
Qed.

(* Right identity law *)
Lemma right_id {X Y : Type} (p : Hom X Y) :
  comp p id = p.
Proof.
  reflexivity.
Qed.

(* Left inverse law *)
Lemma inv_left {X Y : Type} (p : Hom X Y) :
```

```
  comp (inv p) p = id.
Proof.
  apply concat_Vp.
Qed.

(* Right inverse law *)
Lemma inv_right {X Y : Type} (p : Hom X Y) :
  comp p (inv p) = id.
Proof.
  apply concat_pV.
Qed.

(* Groupoid structure *)

(* A Groupoid is simply a type where every morphism has an
↪  inverse. *)
Record Groupoid := {
  Obj : Type;
  Hom : Obj -> Obj -> Type := fun X Y => X = Y; (* Hom-sets are
  ↪  path types *)
  id : forall X : Obj, Hom X X := fun X => eq_refl;
  comp : forall X Y Z : Obj, Hom X Y -> Hom Y Z -> Hom X Z :=
    fun X Y Z p q => p @ q;
  inv : forall X Y : Obj, Hom X Y -> Hom Y X :=
    fun X Y p => p^;

  (* Groupoid laws *)
  assoc : forall X Y Z W : Obj (p : Hom X Y) (q : Hom Y Z) (r :
  ↪  Hom Z W),
          comp X Z W (comp X Y Z p q) r = comp X Y W p (comp Y
          ↪  Z W q r);
  id_left : forall X Y : Obj (p : Hom X Y), comp X X Y (id X) p =
  ↪  p;
  id_right : forall X Y : Obj (p : Hom X Y), comp X Y Y p (id Y)
  ↪  = p;
  inv_left : forall X Y : Obj (p : Hom X Y), comp X Y X (inv X Y
  ↪  p) p = id X;
  inv_right : forall X Y : Obj (p : Hom X Y), comp Y X Y p (inv X
  ↪  Y p) = id Y
}.

(* Example groupoid G with objects alpha and beta *)
Definition G : Groupoid := {
  Obj := nat + bool; (* Objects are the disjoint union of nat and
  ↪  bool *)
  assoc := fun _ _ _ _ _ _ _ => eq_refl;
```

```
  id_left := fun _ _ _ => eq_refl;
  id_right := fun _ _ _ => eq_refl;
  inv_left := fun _ _ _ => eq_refl;
  inv_right := fun _ _ _ => eq_refl
}.
```

Groupoid Agda Program

```
{-# OPTIONS --cubical #-}
module ContractibleEmptyUnit where

open import Agda.Primitive
open import Cubical.Core.Everything
open import Data.Empty  -- Import for the empty type
open import Data.Unit   -- Import for the unit type
open import Data.Bool   -- Import for Bool
open import Data.Nat    -- Import for ℕ (natural numbers)

-- Define S as the empty type (∅)
S : Set
S = ⊥  -- ⊥ is the empty type in Agda

-- Define O as the unit type ({∅})
O : Set
O = ⊤  -- ⊤ is the unit type in Agda

-- Show that S () is contractible
-- The empty type is trivially contractible since it has no
↪  elements.
contractible_S : Contr S
contractible_S = contr (λ x → x , λ _ → ⊥-elim x)

-- Show that O ({∅}) is contractible
-- The unit type is contractible because there is only one
↪  element (tt).
contractible_O : Contr O
contractible_O = contr tt λ _ → refl

-- Define two types alpha (α) and beta (β)
α : Set
α = ℕ  -- alpha is the type of natural numbers

β : Set
β = Bool  -- beta is the type of booleans

-- Define a function f : S → α (from ∅ to ℕ)
```

```
f : S → α
f x = ⊥-elim x  -- No function can map from an empty type

-- Define a function g : 0 → β (from {∅} to Bool)
g : 0 → β
g tt = true  -- Since 0 is unit type, we just map the single
↪  element to true

-- Define the type of morphisms (Hom) in a groupoid as paths
Hom : (X Y : Set) → Set
Hom X Y = X ≡ Y  -- Hom-sets are path types in HoTT

-- Define identity morphism (id) for any type X
id : {X : Set} → Hom X X
id = refl  -- The identity morphism is reflexivity

-- Define composition of morphisms in the groupoid
comp : {X Y Z : Set} → Hom X Y → Hom Y Z → Hom X Z
comp p q = p • q  -- Composition is path concatenation (•)

-- Define inverse of morphisms
inv : {X Y : Set} → Hom X Y → Hom Y X
inv p = sym p  -- Inverse is just path symmetry

-- Groupoid laws

-- Associativity of composition
comp_assoc : {X Y Z W : Set} (p : Hom X Y) (q : Hom Y Z) (r : Hom
↪  Z W) →
  comp p (comp q r) ≡ comp (comp p q) r
comp_assoc p q r = refl  -- Composition is associative

-- Left identity law: composing with the identity on the left
left_id : {X Y : Set} (p : Hom X Y) → comp id p ≡ p
left_id p = refl  -- Composing with identity on the left does
↪  nothing

-- Right identity law: composing with the identity on the right
right_id : {X Y : Set} (p : Hom X Y) → comp p id ≡ p
right_id p = refl  -- Composing with identity on the right does
↪  nothing

-- Left inverse law: composing a morphism with its inverse on the
↪  left gives the identity
inv_left : {X Y : Set} (p : Hom X Y) → comp (inv p) p ≡ id
```

```
inv_left p = refl  -- Composing a morphism with its inverse on
↪  the left gives identity

-- Right inverse law: composing a morphism with its inverse on
↪  the right gives the identity
inv_right : {X Y : Set} (p : Hom X Y) → comp p (inv p) ≡ id
inv_right p = refl  -- Composing a morphism with its inverse on
↪  the right gives identity

-- Define the Groupoid structure as a record in Agda
record Groupoid : Set where
  field
    Obj : Set  -- The objects in the groupoid
    Hom : Obj → Obj → Set  -- The morphisms (Hom-sets)
    id : (X : Obj) → Hom X X  -- Identity morphisms
    comp : {X Y Z : Obj} → Hom X Y → Hom Y Z → Hom X Z  --
    ↪  Composition of morphisms
    inv : {X Y : Obj} → Hom X Y → Hom Y X  -- Inverse of
    ↪  morphisms

    -- Groupoid laws
    assoc : {X Y Z W : Obj} (p : Hom X Y) (q : Hom Y Z) (r : Hom
    ↪  Z W) →
      comp p (comp q r) ≡ comp (comp p q) r  -- Associativity
    id_left : {X Y : Obj} (p : Hom X Y) → comp (id X) p ≡ p  --
    ↪  Left identity
    id_right : {X Y : Obj} (p : Hom X Y) → comp p (id Y) ≡ p  --
    ↪  Right identity
    inv_left : {X Y : Obj} (p : Hom X Y) → comp (inv p) p ≡ id X
    ↪  -- Left inverse
    inv_right : {X Y : Obj} (p : Hom X Y) → comp p (inv p) ≡ id Y
    ↪   -- Right inverse

-- Example groupoid G with objects α and β
G : Groupoid
G = record {
  Obj = α ⊎ β  -- Objects are the disjoint union of ℕ and Bool
  ; Hom = λ X Y → X ≡ Y  -- Hom-sets are path types
  ; id = λ X → refl  -- Identity morphism is reflexivity
  ; comp = λ {X Y Z} p q → p • q  -- Composition is path
  ↪  concatenation
  ; inv = λ {X Y} p → sym p  -- Inverse is path symmetry
  ; assoc = λ _ _ _ _ _ _ _ → refl  -- Associativity holds
  ↪  trivially
  ; id_left = λ _ _ _ → refl  -- Left identity holds trivially
  ; id_right = λ _ _ _ → refl  -- Right identity holds trivially
```

```
  ; inv_left = λ _ _ _ → refl  -- Left inverse holds trivially
  ; inv_right = λ _ _ _ → refl  -- Right inverse holds trivially
}
```

NSphere Agda Program

```
module NSphere where
  open import Cubical.Foundations
  open import Cubical.Path
  open import Cubical.Data.Nat
  open import Cubical.HITs.Suspension

  -- Base case: 0-sphere (S⁰)
  S⁰ : Set
  S⁰ = Bool  -- S⁰ is just two points: true and false

  -- Inductive definition of n-sphere (Sⁿ)
  Sⁿ : ℕ → Set
  Sⁿ 0       = S⁰
  -- Base case: 0-sphere
  S⁰ (suc n) = Suspension (Sⁿ n)
  -- Suspension of the (n-1)-sphere

  -- Example: Explicitly defining S¹ and S²
  S¹ : Set
  S¹ = Sⁿ 1
  -- S¹ = Suspension of S⁰

  S² : Set
  S² = Sⁿ 2
  -- S² = Suspension of S¹

  -- Function to illustrate working with spheres
  -- A constant map from Sⁿ n to Unit
  constMap : ∀ {n} → Sⁿ n → Unit
  constMap _ = tt
  -- Every point maps to the unit element

  -- A dependent function on S¹
  exampleFuncOnS¹ : S¹ → Set
  exampleFuncOnS¹ north = ℕ
  -- North pole maps to ℕ
  exampleFuncOnS¹ south = Bool
  -- South pole maps to Bool
  exampleFuncOnS¹ (merid true i) = ℕ
  -- Path: Meridians map to ℕ
```

```
exampleFuncOnS¹ (merid false i) = Bool
-- Path: Meridians map to Bool
```

Infinite Sphere Haskell Program

```haskell
module Sphere where

data Empty -- Empty Type

data Suspension a = North  South  Merid a -- Constructs
↪  Suspension

top :: Suspension () -- Pole Definition
top = North

merid :: a -> Suspension a -- Meridian Definition
merid x = Merid x

type Sphere0 = Suspension Empty -- Suspension on the Empty Type
↪  (S^-1 or Empty Set)

ex1 :: Suspension () -- Suspension on the Unit Type (S^0 or Point
↪  Set)
ex1 = North

ex2 :: Suspension ()
ex2 = Merid ()

ex3 :: Suspension Bool -- Suspension on the Boolean Type (S^1 or
↪  Loop)
ex3 = North

ex4 :: Suspension Bool
ex4 = Merid True

ex5 :: Suspension (Bool, Bool) -- Suspension on the Loop (S^2 or
↪  3D Sphere)
ex5 = North

ex6 :: Suspension (Bool, Bool)
ex6 = Merid (True, False)

newtype Mu f = Roll { unRoll :: f (Mu f)} -- Recursive type
↪  definition
type InfSuspension = Mu Suspension -- Infinite Suspension
↪  definition
```

```haskell
exInf :: Suspension (Mu Suspension) -- Suspension on Infinite recursive type (S^:lo∞)
exInf = North

exInfty :: Suspension (Mu Suspension)
exInfty = Merid (Roll exInfty)
```

Chapter 2

Composition Model of Circulation

$$
\begin{tikzcd}
M \arrow[r, "prod"] \arrow[rr, bend left, "\mathcal{K}"] \arrow[dr, equal] \arrow[ddr, bend right, "prod"'] & C \arrow[r, "ex"] \arrow[d, "i_0"] & M' \\
& M \arrow[ur, "\mathcal{K}"'] \arrow[d, "prod"] & \\
& C \arrow[uur, bend right, "ex"'] &
\end{tikzcd}
$$

Morphism Composition: $\mathcal{K} = ex \circ prod$
$ex = \mathcal{K} \circ i_0$
$i_0 = prod^{op}$

Imperialism Haskell Program

```haskell
module Imperialism where

-- Objects
data N = N -- Nature
data U = U -- Universal Kernel

-- Product of U with itself (U × U)
type UxU = (U,U)

-- Exponential Object (U^U)
type UtoU = U -> U

-- Morphisms

-- Primitive Accumulation: N → U
primAcc :: N -> U
primAcc _ = U
```

```haskell
-- Idempotence: U ≅ (U × U)
idem :: U -> (U,U)
idem u = (u,u)

-- Autopoiesis: U ≅ U^U
auto :: U -> UtoU
auto u = \_ -> u

-- Curry Correspondence: (U × U) ≅ U^U
curry :: UxU -> UtoU
curry (u1, u2) = \_ -> u1

-- Closed Cartesian Category Rules
class CCC cat where
    -- Terminal object
    terminal :: cat a ()

    -- Products
    pair :: cat a b -> cat a c -> cat a (b, c)
    fst :: cat (a, b) a
    snd :: cat (a, b) b

    -- Exponentials
    cur :: cat (a, b) c -> cat a (b -> c)
    uncur :: cat a (b -> c) -> cat (a, b) c
```

$$\begin{tikzcd}
\mathtt{N} \arrow[dr, "\mathtt{primAcc}"] & & \\
& \mathtt{U} \arrow[r, "\mathtt{idem}"] \arrow[dr, "\mathtt{auto}"'] & \mathtt{(U,U)} \arrow[d, "\mathtt{curry}"] \\
& & \mathtt{U\text{->}U}
\end{tikzcd}$$

Chapter 3

Trivial Categorical Structure of Sets (Factorization)

$$x = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \tag{1}$$

$$\begin{tikzcd}
 & & 8 & 12 & \\
 & & 4 \arrow[u] \arrow[ur] & 6 \arrow[u] & \\
10 & & 2 \arrow[ll] \arrow[u] \arrow[ur] & & 9 \\
 & 7 & & 3 \arrow[uu] \arrow[ur] & \\
 & 5 \arrow[uul] & 1 \arrow[l] \arrow[r] \arrow[ul] \arrow[ur] \arrow[uu] & 11 & \\
 & & \varnothing \arrow[u] & &
\end{tikzcd}$$

FuncLang Haskell Program

```haskell
module FuncLang where

import Data.Hashable (hash)

-- Subjective

-- Objects
data State
    = Initial          -- Represents S
    | Successor State   -- Represents S'
    | Final            -- Represents S^S (final state)
    deriving (Show, Eq)

-- Define the successor function
f :: State -> State
f Initial = Successor Initial   -- Transition from S to S'
f (Successor s) =
    case s of
        Final -> Final          -- If already at S^S, stay there
        _     -> Successor (f s) -- Otherwise, recurse deeper
f Final = Final

-- Field of the Other
```

```haskell
-- Objects

data U = U deriving (Show) -- Monoid of the Symbolic

-- Morphims

-- Define the morphisms as natural numbers
newtype Morphism = Morphism Int deriving (Show, Eq)

-- Identity morphism (corresponds to 0)
identityMorphism :: Morphism
identityMorphism = Morphism 0

-- Composition of morphisms
composeMorphisms :: Morphism -> Morphism -> Morphism
composeMorphisms (Morphism m) (Morphism n) = Morphism (m + n)

-- Action of a morphism on U (trivial in this case)
applyMorphism :: Morphism -> U -> U
applyMorphism _ U = U

-- Language Relation (Functor j_0)

-- On Objects
jOb :: State -> U
jOb _ = U

-- On Morphisms
jMorph :: (State -> State) -> [Morphism]
jMorph stateTransition =                  -- stateTransition is a
↪  place-holder for the parameter 'f'
    let initialState = Initial
        states = iterate f initialState  -- Generate infinite
        ↪  sequence of states
        indices = takeWhile (/= Final) states  -- Stop at Final
        ↪  state
        hashedIndices = map (\s -> Morphism (hash (show s) `mod`
        ↪  100)) indices
    in hashedIndices
```

Lacan Haskell Program

```haskell
module Lacan where

import Control.Category
```

```haskell
import Prelude hiding (id, (.))

-- Objects in the category
data LacanObj
    = BigOther          -- A
    | PhiX String       -- Φx with a parameter (e.g., α, β, γ)
    | Petite_a          -- a
    deriving (Show, Eq)

-- Morphisms in the category
data LacanMorph
    = IdM LacanObj               -- Identity morphism for each
    ↪  object
    | Auth LacanObj LacanObj    -- auth: A → Φx
    | Lack LacanObj LacanObj    -- lack: Φx → a
    deriving (Show, Eq)

-- The Lacan category with objects and morphisms
data Lacan a b = Morph LacanMorph

instance Category Lacan where
    -- Identity morphism for each object
    id = Morph (IdM BigOther)  -- Default, but should be
    ↪  object-dependent

    -- Composition of morphisms
    Morph (IdM a) . f = f   -- Identity acts as a neutral element
    f . Morph (IdM a) = f

    Morph (Auth a b) . Morph (Auth _ c)
        | b == c     = Morph (Auth a c)  -- Compose auth morphisms
        | otherwise = error "Invalid composition"

    Morph (Lack a b) . Morph (Auth _ c)
        | b == c     = Morph (Lack a c)  -- Compose auth and lack
        ↪  morphisms
        | otherwise = error "Invalid composition"

    Morph (Lack a b) . Morph (Lack _ c)
        | b == c     = Morph (Lack a c)  -- Compose lack morphisms
        | otherwise = error "Invalid composition"

    _ . _ = error "Invalid composition"

-- Functor instance: Mapping Lacan to Haskell functions
instance Functor Lacan where
```

```haskell
    fmap f (Morph (IdM x)) = Morph (IdM (f x))    -- Identity
    ↪  morphism mapping
    fmap f (Morph (Auth a b)) = Morph (Auth (f a) (f b))
    fmap f (Morph (Lack a b)) = Morph (Lack (f a) (f b))

-- auth. morphisms
authAlpha :: Lacan LacanObj LacanObj
authAlpha = Morph (Auth BigOther (PhiX "α"))

authBeta :: Lacan LacanObj LacanObj
authBeta = Morph (Auth BigOther (PhiX "β"))

authGamma :: Lacan LacanObj LacanObj
authGamma = Morph (Auth BigOther (PhiX "γ"))

-- lack morphisms
lackAlpha :: Lacan LacanObj LacanObj
lackAlpha = Morph (Lack (PhiX "α") Petite_a)

lackBeta :: Lacan LacanObj LacanObj
lackBeta = Morph (Lack (PhiX "β") Petite_a)

lackGamma :: Lacan LacanObj LacanObj
lackGamma = Morph (Lack (PhiX "γ") Petite_a)

-- Composition examples
authToLackAlpha :: Lacan LacanObj LacanObj
authToLackAlpha = lackAlpha . authAlpha -- BigOther → Φx_a → a

authToLackBeta :: Lacan LacanObj LacanObj
authToLackBeta = lackBeta . authBeta    -- BigOther → Φx_β → a

authToLackGamma :: Lacan LacanObj LacanObj
authToLackGamma = lackGamma . authGamma -- BigOther → Φx_γ → a
```

$$
\begin{tikzcd}
 & \texttt{BigOther} \arrow[dl, "\texttt{Auth}"'] \arrow[d, "\texttt{Auth}"] \arrow[dr, "\texttt{Auth}"] & \\
\texttt{PhiX String} \arrow[dr, "\texttt{Lack}"'] & \texttt{PhiX String} \arrow[d, "\texttt{Lack}"] & \texttt{PhiX String} \arrow[dl, "\texttt{Lack}"] \\
 & \texttt{Petite\_a} &
\end{tikzcd}
$$

Language Haskell Program

```haskell
module Language where

-- Objects
data LanguageObj
    = Subject -- Φx
    | Subject' -- Φx′
    | FieldOfOther -- A
    deriving(Show,Eq)

-- Morphisms
data LanguageMorph
    = IdM LanguageObj -- Identity Morphisms
    | Eq LanguageObj LanguageObj -- Equality Relation
    | Language LanguageObj LanguageObj -- Φx → A
    | Metonymy LanguageObj LanguageObj -- Φx → Φx′
    | Metaphor LanguageObj LanguageObj -- A → Φx′
    deriving (Show, Eq)
-- Composition of morphisms
compose :: LanguageMorph -> LanguageMorph -> Maybe LanguageMorph
compose (Language a b) (Metaphor b' c)
     b == b'   = Just (Metonymy a c)
     otherwise = Nothing
compose _ _ = Nothing

verifyCommutativity :: Bool
verifyCommutativity =
    let j = Language Subject FieldOfOther
        f = Metonymy Subject Subject'
        af' = Metaphor FieldOfOther Subject'
        idX = IdM Subject
    in compose j af' == Just f
```

Lacan-Badiou-CT Correspondence

Lacan Badiou CT

$$
\begin{tikzcd}
 & A \arrow[dl, "auth."'] \arrow[d, "auth."] \arrow[dr, "auth."] & & & & & \mathbb{R} \arrow[dl] \arrow[d] \arrow[dr] & & & & & \mathbf{Cat} \arrow[dl] \arrow[d] \arrow[dr] & \\
\Phi x_\alpha \arrow[dr, "lack"'] & \Phi x_\beta \arrow[d, "lack"] & \Phi x_\gamma \arrow[dl, "lack"] & {} \arrow[r, "F"] & {} & \mathbb{N} \arrow[dr] & \mathbb{N} \arrow[d] & \mathbb{N} \arrow[dl] & {} \arrow[r, "G"] & {} & \mathbf{Set} \arrow[dr] & \mathbf{Set} \arrow[d] & \mathbf{Set} \arrow[dl] \\
 & a & & & & & \{\varnothing\} & & & & & 1 &
\end{tikzcd}
$$

Categories:	**Functors:**	**Maps:**
Lacan	F: Lacan $\to$ Badiou	$A \mapsto \mathbb{R} \mapsto \mathbf{Cat}$
Badiou ($\in \mathbf{Set}$)	G: Badiou $\to$ CT	$\Phi x \mapsto \mathbb{N} \mapsto \mathbf{Set}$
CT (Topos $\in \mathbf{2} - \mathbf{Cat}$)		$a \mapsto \{\varnothing\} \mapsto 1$