Applied Quantitative Finance for Equity Derivatives

Second Edition

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Contents

Preface

This book presents the most significant equity derivatives models used these days. It is not a book around esoteric or cutting-edge models, but rather a book on relatively simple and standard models, viewed from the angle of a practitioner. Most books present models in an abstract manner, often disconnected from how to apply them in the real world. This book intends to fill that gap, with the ambitious goal of transforming a reader unfamiliar with equity derivatives models into a specialist of such models.

There is no introductory mathematical chapter. If the reader is interested in stochastic calculus, the very concise book of Mikosch [248] is highly recommended. Shreve's book [295] is a nice complement with a more detailed, and very accessible mathematical presentation of theorems relevant to finance. John Hull offers a good even if slightly austere introduction to financial derivatives and various rate conventions in his book [155].

The first chapter of this book introduces the specificities of the equity derivatives market in terms of modeling, with a close look at the dividend curves and the forward price. We then move on to the vanilla options, with the famous Black-Scholes model, paying attention to the various adjustments used in practice. After giving the most standard practices for European vanilla options, we follow with the issues raised by discrete cash dividends on the option price and study recent analytical approximations. Regarding American vanilla options, we detail fast and stable finite difference schemes, and proceed to analyze the inclusion of cash or proportional dividends, paying particular attention to the effect of the dividend model on the exercise boundary.

Chapter four introduces the Monte-Carlo method to price financial derivatives on a basket of equities. The parallelization of random numbers generation, the randomization of quasi-random numbers and the various ways of generating of correlated normal variates as well as the use of control variates and their caveats are carefully explained. We then present adjoint algorithmic differentiation techniques to compute sensitivities and finish the chapter with various techniques to include the American or more precisely, Bermudan exercise, in particular non-parametric regressions.

In chapter five, six and seven, we look at how to imply volatilities in practice, and common volatility representations, be it parametric, Dupire local volatility, or stochastic volatility. We describe precisely how to accurately simulate the different models with the Monte-Carlo method or through finite difference methods. In doing so, we expose the many issues that arise with the classical approach to the Dupire local volatility along with solutions and explain how to handle discrete cash dividends in the

Dupire framework. We conclude the chapter with an analysis of the particle method and its close relatives for Heston and Schobel-Zhu stochastic-local volatility models, detailing the use of quasi-random sequences with the method.

Progressively we consider other commonly traded options: forward start, digital, barrier, Asian, quanto, compo, etc. We will however not present the rarely traded options such as compound or chooser, even when they have apparently nice analytical formulas. On each subject, pricing techniques are presented in great detail, be it through the simplest analytical formula, a Monte-Carlo simulation, or the finite difference method.

In chapter ten, we have a look at common volatility derivatives, that is, variance swaps, volatility swaps and options. Discrete, continuous or model-free replication of variance swaps is analyzed. Newer listed derivatives such as VIX options and dividend derivatives are subsequently covered.

We finally present common exotics and how to evaluate those in a forward Monte-Carlo manner or in a backward PDE manner. Even if those tend to be less traded nowadays, they are still in many traders books, and remain popular in Asia, especially the autocallables.

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In this second edition, various typos have been corrected, and the text has been slightly updated. New arbitrage-free implied volatility interpolations were added to chapter five, and different types of warrants, including callable bull/bear contracts (CBBCs) are covered in chapter eight. The book layout has also been significantly updated to allow for a hardback book publication.

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Chapter One

The Forward

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The equity forward, the expected price of a stock at a future date, is key to understanding and explaining the valuation of many equity derivatives. It is directly linked to one of the simplest equity derivatives contract, the forward contract, where an exchange of cash against stock occurs at a specific future date, the maturity date.

Compared to other asset classes, for example, foreign exchange (FX) derivatives or interest rate derivatives, there are two specificities of equities that are going to play a major role in the valuation of the forward price and in the pricing of equity derivatives: the dividend and the borrow cost.

1.1 THE BORROW COST

Through a repurchase agreement contract (in common language, a repo), one can borrow money secured by a stock at a specific rate, usually lower than the rate that would be obtained by borrowing money unsecured. Therefore the stock price will grow at its repo rate r_R, while the option price will grow either at the so called risk-free rate (really the unsecured funding rate) r_F, or at the collateral rate r_C in the case of a collateralized trade in the risk-neutral measure [268].

In order to look at the evolution of repo costs, it can be more meaningful to represent the repo in terms of annualized spread s_R against the risk-free rate: $s_R = r_F - r_R$. This spread corresponds to the rate charged over the risk-free rate to go short. The spread is positive when there is demand for a security and negative when there is demand for cash. The repo spread for equity indices was traditionally very close to zero. Since the 2008 financial crisis, it is not uncommon for it to be negative, for example the 7 years repo spread on Eurostoxx offered by BNP was around -0.3% in December 2012.