Applied Quantitative Finance for Equity Derivatives

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Preface

This book presents the most significant equity derivatives models used these days. It is not a book around esoteric or cutting-edge models, but rather a book on relatively simple and standard models, viewed from the angle of a practitioner. Most books present models in an abstract manner, often disconnected from how to apply them in the real world. This book intends to fill that gap, with the ambitious goal of transforming a reader unfamiliar with equity derivatives models into a specialist of such models.

There is no introductory mathematical chapter. If the reader is interested in stochastic calculus, the very concise book of Mikosch [Mik98] is highly recommended. Shreve's book [Shr04] is a nice complement with a more detailed, and very accessible mathematical presentation of theorems relevant to finance. John Hull offers a good even if slightly austere introduction to financial derivatives and various rate conventions in his book [Hul99].

The first chapter of this book introduces the specificities of the equity derivatives market in terms of modeling, with a close look at the dividend curves and the forward price. We then move on to the vanilla options, with the famous Black-Scholes model, paying attention to the various adjustments used in practice. After giving the most standard practices for European vanilla options, we follow with the issues raised by discrete cash dividends on the option price and study recent analytical approximations. Regarding American vanilla options, we detail fast and stable finite difference schemes, and proceed to analyze the inclusion of cash or proportional dividends, paying particular attention to the effect of the dividend model on the exercise boundary.

Chapter four introduces the Monte-Carlo method to price financial derivatives on a basket of equities. The parallelization of random numbers generation, the randomization of quasi-random numbers and the various ways of generating of correlated normal variates as well as the use of control variates and their caveats are carefully explained. We then present adjoint algorithmic differentiation techniques to compute sensitivities and finish the chapter with various techniques to include the American or more precisely, Bermudan exercise, in particular non-parametric regressions.

In chapter five, we look at how to imply volatilities in practice, and common volatility representations, be it parametric, Dupire local volatility, or stochastic volatility. We describe precisely how to accurately simulate the different models with the Monte-Carlo method or through finite difference methods. In doing so, we expose the many issues that arise with the classical approach to the Dupire local volatility along with solutions and explain how to handle discrete cash dividends in the Dupire framework. We conclude the chapter with an analysis of the particle method and its close relatives for Heston and Schobel-Zhu stochastic-local volatility models, detailing the use of quasi-random sequences with the method.

Progressively we consider other commonly traded options: forward start, digital,

barrier, Asian, quanto, compo, etc. We will however not present the rarely traded options such as compound or chooser, even when they have apparently nice analytical formulas. On each subject, pricing techniques are presented in great detail, be it through the simplest analytical formula, a Monte-Carlo simulation, or the finite difference method.

In chapter eight, we have a look at common volatility derivatives, that is, variance swaps, volatility swaps and options. Discrete, continuous or model-free replication of variance swaps is analyzed. Newer listed derivatives such as VIX options and dividend derivatives are subsequently covered.

We finally present common exotics and how to evaluate those in a forward Monte-Carlo manner or in a backward PDE manner. Even if those tend to be less traded nowadays, they are still in many traders books, and remain popular in Asia, especially the autocallables.