Assignment 5.

Due Thurs., Nov. 9, at 11:59 pm.

Please do not share these problems or the solutions associated with them outside of our class, and do not post them publicly online. Doing so is a violation of academic conduct and can result in unfortunate consequences.

Reading: Lectures 11, 12-13

1. **Even more least squares algorithms.** Let $A \in \mathbb{C}^{m \times n}$, $m > n$, be a full rank matrix. Let $\hat{x}$ be the minimizer of $\|Ax - b\|_2$ over all choices of $x \in \mathbb{C}^n$. Consider the $(m+n) \times (m+n)$ system
$$\begin{bmatrix} I_{mm} & A \\ A^* & 0_{nn} \end{bmatrix} y = \begin{bmatrix} b \\ 0_n \end{bmatrix},$$
where 0_n is a vector of n zeros, 0_{nn} is an $n \times n$ matrix of all zeros, and I_{mm} is the $m \times m$ identity matrix. Show that the solution $\hat{x}$ to the least squares problem will be a submatrix (sub-vector?) of the vector y.

2. p. 96 Exercise 12.1

3. p. 101 Exercise 13.3 (For part (a), use a Vandermonde matrix to do the evaluation; see Example 1.1; include your plots in your write-up, you do not need to submit code for this).

4. p. 85 Exercise 11.3 (you can skip items (b) and (c), meaning you do not need to include these methods in your comparison). Please record and comment on the condition numbers $k_2(R)$, where $QR = A$ is the QR decomposition of A, and $k_2(A^*A)$, for A used in your experiments. Using key theorems from Lecture 12, what do these numbers imply about the condition number of the linear systems you must solve in this problem via (i) the QR decomposition and (ii) the normal equations? How many digits you should expect to compute accurately using these methods?