Assignment 7.

Due Thurs., Nov. 30, at 11:59 pm.

Please do not share these problems or the solutions associated with them outside of our class, and do not post them publicly online. Doing so is a violation of academic conduct and can result in unfortunate consequences.

Reading: Lectures 24-29

1. pg. 188, Exercise 24.1
2. pg. 188, Exercise 24.2 parts (c) and (d)
3. pg. 218, Exercise 28.1
4. (*from A. Greenbaum*) For $A \in \mathbb{R}^{m \times m}$, define the matrix exponential:

$$e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \ldots = \sum_{k=0}^{\infty} \frac{A^k}{k!}.$$

Suppose A is diagonalizable: $A = X \Lambda X^{-1}$. Show that $e^{tA} = X e^{t\Lambda} X^{-1}$, where

$$e^{t\Lambda} = \begin{bmatrix} e^{t\lambda_1} & & \\ & \ddots & \\ & & e^{t\lambda_m} \end{bmatrix}.$$

5. (*From A. Greenbaum*) Consider the matrix

$$A = \begin{bmatrix} 2 & -1 & & \\ -1 & \ddots & \ddots & \\ & \ddots & \ddots & -1 \\ & & -1 & 2 \end{bmatrix}.$$

Taking A to be a 10 by 10 matrix, try the following:

(a) What information does Gerschgorin's theorem give you about this matrix?

(b) Implement the power method to compute an approximation to the eigenvalue of largest absolute value and its corresponding eigenvector. Include in your writeup a printout your of code together with the eigenvalue/eigenvector pair that you computed. Once you have a good approximate eigenvalue, look at the error in previous approximations. Create a plot showing the error at each iteration, and comment on the rate of convergence. Does it match the theory?

(c) Using $s = 1$ as a shift in inverse iteration, find the eigenvalue that is closest to 1 and its corresponding eigenvector. Include in your writeup a printout of your code together with the eigenvalue/eigenvector pair that you computed. Comment on the rate of convergence of inverse iteration with $s = 1$ as a shift.