Assignment 2.

Due Friday, Jan. 19, at 2:30pm PST.

Reading: Sec. 2.2.1-2.2.4 in the Gautschi book.

1. Muller's method for finding a root of a function $f(x)$ fits a quadratic through three given points, $(x_k, f(x_k))$, $(x_{k-1}, f(x_{k-1}))$, and $(x_{k-2}, f(x_{k-2}))$, and takes the root of this quadratic that is closest to x_k as the next approximation x_{k+1}. Write down a formula for this quadratic. Suppose $f(x) = x^3 - 2$, $x_0 = 0$, $x_1 = 1$, and $x_2 = 2$. Find x_3.

2. The Chebyshev interpolation points are defined for the interval $[-1, 1]$ as $x_j = \cos\left(\frac{j\pi}{n}\right)$, $j = 0, 1, \ldots, n$. Suppose we wish to approximate a function on the interval $[a, b]$. Write down the linear transformation ℓ that maps the interval $[-1, 1]$ to $[a, b]$, with $\ell(-1) = a$ and $\ell(1) = b$. What interpolation points should we use on the interval $[a, b]$ to correspond to the Chebyshev points on $[-1, 1]$?

3. Coding problem: Consider the Runge function defined on $[-5, 5]$:

$$f(x) = \frac{1}{1 + x^2}, \quad x \in [-5, 5].$$

   (a) Plot the interpolating polynomials using equidistant nodes $x_i = -5 + \frac{10i}{n}$, $i = 0, \ldots, n$. Try $n = 5,\ 10,\ 15,$ and 20. Plot each of the interpolating polynomials, along with the function f on the same graph.

   (b) Plot the interpolating polynomials using Chebyshev nodes $x_i = 5\cos\left(\frac{i\pi}{n}\right)$, $i = 0, \ldots, n$. Again try $n = 5,\ 10,\ 15,$ and 20 and plot each of the interpolating polynomials, along with the function f on the same graph. On a separate graph, plot the difference $f(x) - p_{20}(x)$ between f and the degree 20 interpolating polynomial.

   Turn in the three plots produced in (a) and (b); you do not have to turn in your code (although you may if you wish). If you want to use `chebfun` for this work, you can download it from `www.chebfun.org`. Remember, however, that you must define the domain of your function to be $[-5, 5]$: `f = chebfun(' 1 / (1 + x^2) ',[-5,5])`. You may also do this problem without chebfun by simply using the Lagrange interpolation formula (see (2.49) and (2.52), pp. 74-75, in the text).

4. p. 125, problem 27 [Gautschi].

5. p. 127, problem 38 [Gautschi].