Electrostatic Field

More about the Electrical force and field

We talked about the Electric Force in the previous section. To find the force on one charge, we had to know the position of all charges and what is their charge. To introduce Electric Field, we'll call charge q_1 the source of the force and charge q_2 the charge that feels the force. Charge one produces the force, and charge two feels the force.

By introducing the concept of the field, we can separate the cause of the field and the effect that the field has on other charges. We can find a field due to a charge or charge distribution, and once we know this field, we don't have to keep track of the source charge. We can just find the field's effect on other charges or charge distributions.

In the Equation 1 q_1 and q_2 are charges, r is the distance between the two charges, and $\hat{r}$ is the unit vector directed from charge one to charge two. The electric field of a source charge q_1 is defined as the force that a charge q_1 would impress on a positive charge q_2, divided by the amount of charge q_2, as shown in Equation 2. One thing to remember is that in the definition of the electric field, we always assume that the charge q_2 is positive! This way, we remove the ambiguity of the field direction. The direction of the electric field at a point P from charge q_1 is always in the direction of the force that would act on a positive charge q_2 placed at that point, as shown in Figure 1.

$$\overrightarrow{\mathbf{F_2}} = \frac{q_1 q_2}{4\pi\varepsilon r^2}\hat{r} \tag{1}$$

$$\overrightarrow{\mathbf{E}} = \frac{\overrightarrow{\mathbf{F_2}}}{q_2} \tag{2}$$

$$\overrightarrow{\mathbf{E}} = \frac{q_1}{4\pi\varepsilon_0 r^2}\hat{r} \tag{3}$$

If you carefully look at the Equation 3, you see that the electric field depends only on the source charge q_1.

If the electric charge is in a medium other than air, the electric field becomes

Learning outcomes: Explain definition of electric field. Derive and calculate electric field due to several point charges using vector algebra.

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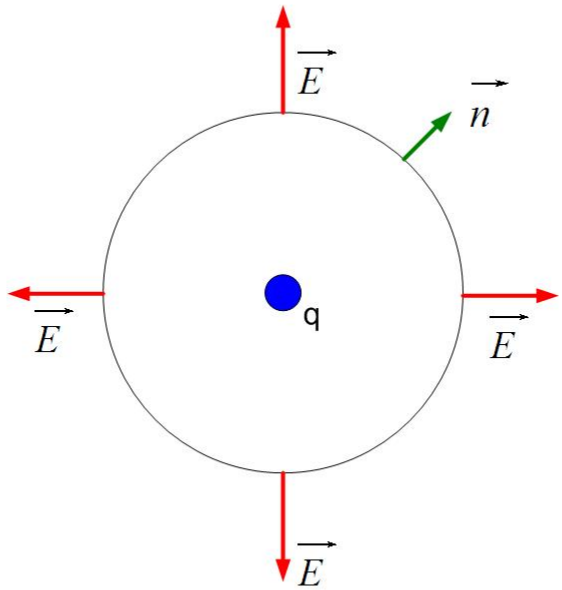

Figure 1: Electric field due to a unit charge q.

$$\overrightarrow{\mathbf{E}} = \frac{q_1}{4\pi\varepsilon_0\varepsilon_r r^2}\hat{R_{12}} \tag{4}$$

$$\varepsilon = \varepsilon_0\varepsilon_r \tag{5}$$

We added a unitless quantity, ε_r, called the relative dielectric constant, or relative permittivity, of a material. ε_r values for different materials can be found online. For example, you can see its values for different materials here `https://en.wikipedia.org/wiki/Relative_permittivity`.

Problem 1 *Find the electric field at a center of the Cartesian Coordinate system if positive charges q are placed at points (0,1) and (-1,0).*

Multiple Choice:

(a) *impossible to say*

(b) *zero* ✓

(c) *one*

(d) *infinity*

positive charge that is in an electric field experiences a force that is

Problem 2 *A positive charge that is in an electric field E experiences a force that is*

Multiple Choice:

(a) *Impossible to say*

(b) *In the same direction as E* ✓

(c) *Perpendicular to E*

(d) *In the opposite direction from E*

Principle of Superposition

What is the electric field if we have more than one charge?

The total electric field at a point in space from the two charges is equal to the sum of the electric fields from the individual charges at that point.

If we have two charges, the total field due to both charges is equal to the vector sum of the fields due to individual charges, see Figure 2. The field at

The fields or charges q_1 and q_2 are:

$$\overrightarrow{\mathbf{E_1}} = \frac{q_1}{4\pi\varepsilon_0 {r_a}^2}\hat{r_a} \tag{6}$$

$$\overrightarrow{\mathbf{E_2}} = \frac{q_1}{4\pi\varepsilon_0 {r_b}^2}\hat{r_b} \tag{7}$$

Where $\hat{r_a}$ and $\hat{r_b}$ are unit vectors in the direction of r_a and r_b. The total field due to both charges is

$$\overrightarrow{\mathbf{E}} = \overrightarrow{\mathbf{E_1}} + \overrightarrow{\mathbf{E_2}} \tag{8}$$

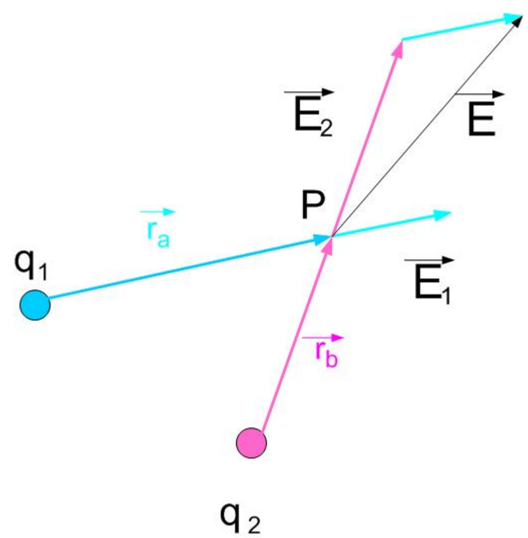

Figure 2: Electric Field due to two charges.

Electric Field in Rectangular Coordinates

The general equation for the electric field is given as

$$\overrightarrow{\mathbf{E}} = \frac{q_1}{4\pi\varepsilon_0 {r_a}^2}\hat{r_a} \tag{9}$$

The electric field at a point $P(x, y, z)$ due to a charge q_1 positioned at a point $P_{q_1}(x_1, y_1, z_1)$ in the rectangular coordinate system is shown in Figure 3. The position vector of the point P_{q_1} is

$$\overrightarrow{\mathbf{r_1}} = x_1\overrightarrow{\mathbf{x}} + y_1\overrightarrow{\mathbf{y}} + z_1\overrightarrow{\mathbf{z}} \tag{10}$$

The position vector of point P is equal to

$$\overrightarrow{\mathbf{r_p}} = x\overrightarrow{\mathbf{x}} + y\overrightarrow{\mathbf{y}} + z\overrightarrow{\mathbf{z}} \tag{11}$$

The two vectors mark the beginning and the end of the distance vector $\overrightarrow{\mathbf{r_a}}$ between points P_{q_1} and P. The vector $\overrightarrow{\mathbf{r_a}}$ is the sum of vectors $-\overrightarrow{\mathbf{r_p}}$ and $\overrightarrow{\mathbf{r_1}}$

$$\overrightarrow{\mathbf{r_a}} = \overrightarrow{\mathbf{r_p}} + (-\overrightarrow{\mathbf{r_1}}) \tag{12}$$

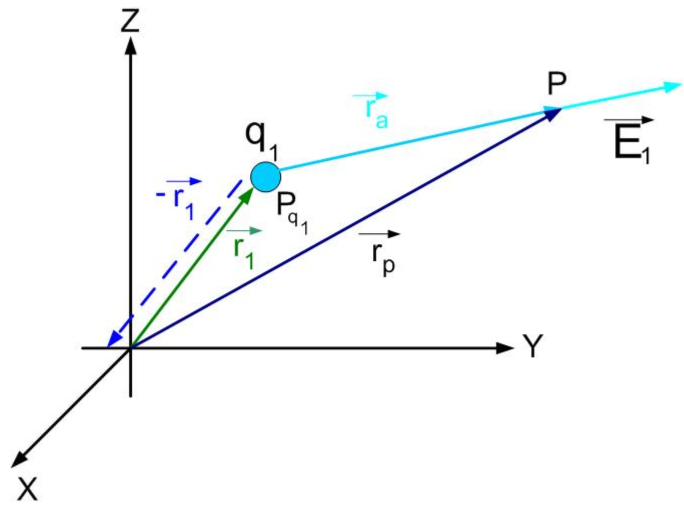

Figure 3: Electric Field due to a unit charge in Rectangular coordinate system.

When we substitute position vectors r_1 and r_p:

$$\overrightarrow{\mathbf{r_a}} = (x - x_1)\overrightarrow{\mathbf{x}} + (y - y_1)\overrightarrow{\mathbf{y}} + (z - z_1)\overrightarrow{\mathbf{z}} \tag{13}$$

Vector $\overrightarrow{\mathbf{r_a}}$ has the magnitude of:

$$|\overrightarrow{\mathbf{r_a}}| = \sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2} \tag{14}$$

Unit vector in the direction of vector $\overrightarrow{\mathbf{r_a}}$ is:

$$\hat{r_a} = \frac{\overrightarrow{\mathbf{r_a}}}{|\overrightarrow{\mathbf{r_a}}|} \tag{15}$$

$$\hat{r_a} = \frac{\overrightarrow{\mathbf{r_a}}}{\sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2}} \tag{16}$$

$$\overrightarrow{\mathbf{E_1}} = \frac{q_1}{4\pi\varepsilon_0 {r_a}^2}\hat{r_a} \tag{17}$$

Where r_a is the distance between the charge q_1 and the point P. Substituting expressions for $\hat{r_a}$, and $|\overrightarrow{\mathbf{r_a}}|$ in equation 9 we get

$$\overrightarrow{\mathbf{E_1}} = \frac{q_1}{4\pi\varepsilon_0\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}^3}\overrightarrow{\mathbf{r_a}} \tag{18}$$

Substituting

For two charges, as shown in Figure 4 equation 18 becomes

$$\begin{aligned}\overrightarrow{\mathbf{E}} = &\frac{q_1}{4\pi\varepsilon_0\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}^3}\overrightarrow{\mathbf{r_a}} + \\ &\frac{q_2}{4\pi\varepsilon_0\sqrt{(x-x_2)^2+(y-y_2)^2+(z-z_2)^2}^3}\overrightarrow{\mathbf{r_b}}\end{aligned} \tag{19}$$

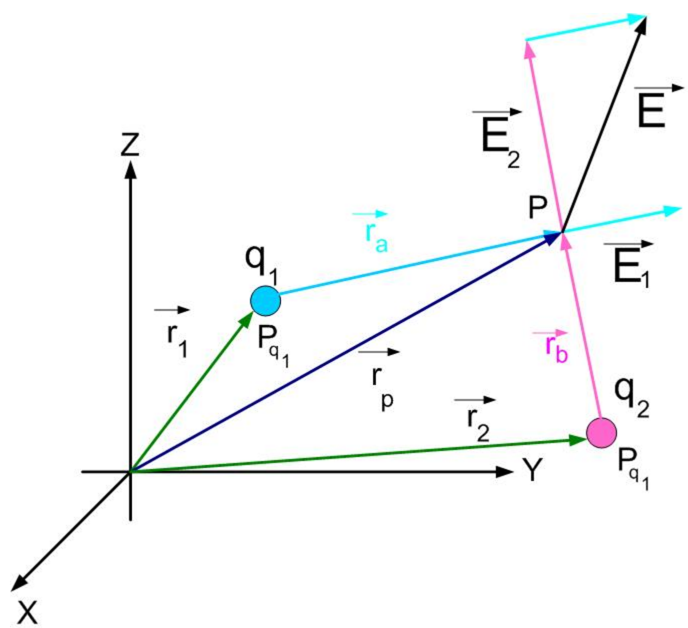

Figure 4: Electric field due to two charges in Rectangular coordinate system.

Example 1. *Find the point P where total electric field is zero inside of an equilateral triangle, if the three charges of magnitude 3 nC, 3 nC, and 3 nC are placed in the corners of equilateral triangle of side 2 m. Use the app below to confirm your result.*

Geogebra link: `https://tube.geogebra.org/m/kupge9gc`