Review of Complex Numbers

Invest time in studying complex numbers, because phasors are represented with complex numbers!

Definition 1. *A complex number z can be represented in the Cartesian coordinate system, as shown in Equation 1, and Polar coordinate system, as shown in Equation 2.*

$$z = x + jy \tag{1}$$
$$z = |z|e^{j\Theta} \tag{2}$$

In Equation 1, a complex number z is represented in rectangular coordinate system, where x is the real part, y is the imaginary part, and $j = \sqrt{-1}$.

In Equation 2, a complex number z is represented in the polar coordinate system, where $|z|$ is the magnitude, and Θ is the angle (aka phase) of the complex number.

Geometric interpretation of these two equations is given in Figure 1. The magnitude is the length of the hypothenuse of the triangle shown, and the angle is the angle that the hypothenuse makes with x-axis.

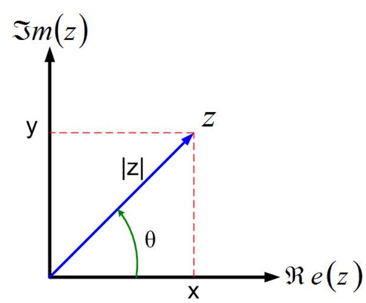

Figure 1: Visual representation of a complex number z in rectangular $z = x + jy$ and polar coordinates $z = |z|e^{j\theta}$.

You may be wondering why we represent the phase of a complex number in the polar coordinate system as $e^{j\Theta}$ because in the circuits class, you used $\angle\theta$. Great question. That brings us to Euler's formula.

Learning outcomes:
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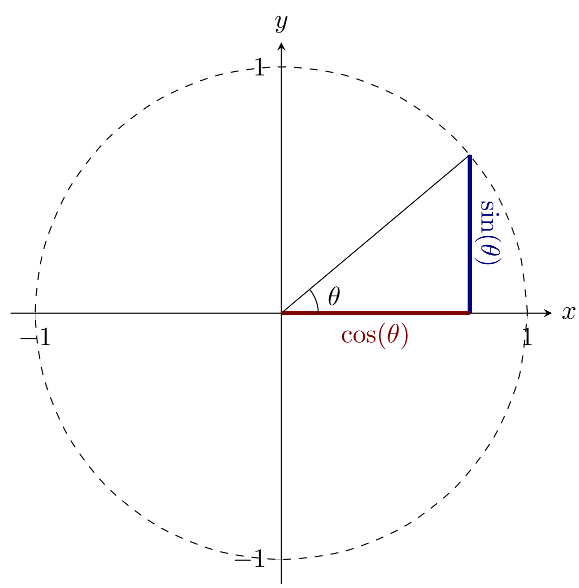
Euler's formula

Euler's formula relates to the Cartesian and Polar coordinates for complex numbers.

$$e^{j\Theta} = cos\Theta + jsin\Theta \tag{3}$$

Geometric interpretation of the Euler's formula is shown below. $z = r(\cos\theta + j\sin\theta)$, where $r\cos\theta = x$ and $r\sin\theta = y$. Euler's formula shows that number z given in Cartesian coordinates as $x+jy$ can be represented in Polar Coordinates as $e^{j\Theta}$. You have likely seen this proof in your Calculus class. **TIP: Your calculator may not know what $e^{j\theta}$ is. Check how to convert between polar and cartesian coordinates on your calculatorbefore the test.**



To find magnitude and angle when you know real and imaginary part of a complex number,

$$|z| = \sqrt{x^2 + y^2} \tag{4}$$

$$\Theta = arctg\frac{y}{x} \tag{5}$$

Operations with complex numbers

Complex Conjugate is often seen when finding the conditions for maximum power transfer.

$$z^* = (x + jy)^* = x - jy = |z|e^{-j\Theta} \tag{6}$$

Complex number addition and subtraction are often seen when complex impedances are placed in series, and the equivalent complex impedance has to be found. The easiest way to add two complex numbers is to find the Cartesian representation of both and then add the real parts separately and the imaginary part separately.

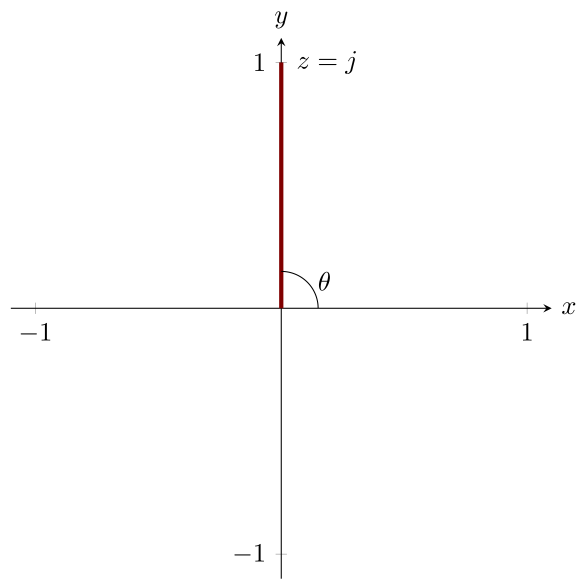
$$z_1 = x_1 + jy_1 \tag{7}$$
$$z_2 = x_2 + jy_2 \tag{8}$$
$$z_1 + z_2 = x_1 + x_2 + j(y_1 + y_2) \tag{9}$$

Multiplication and Division are often seen in the calculation of the transfer function of a circuit. The easiest way to divide two complex numbers is to find the polar representation of both and then divide the amplitudes and subtract the phases.

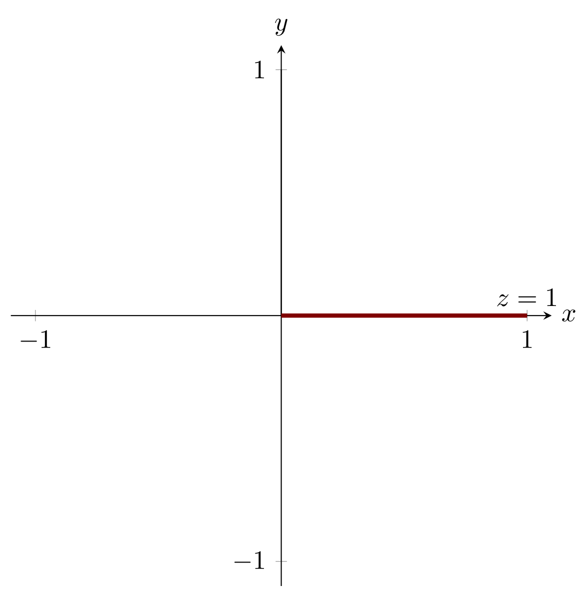
$$z_1 = |z_1|e^{j\Theta_1} \tag{10}$$
$$z_2 = |z_2|e^{j\Theta_2} \tag{11}$$
$$\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|}e^{j\Theta_1 - \Theta_2} \tag{12}$$

Example 1. *Find the magnitude and phase of complex numbers $z_1 = j$ and $z_2 = 1$.*

Explanation. *Complex number $z_1 = j$ is on the y-axis where y=1. By inspection, the magnitude of z_1 is $|z| = 1$, and the angle is $\theta = 90^o$.*



The complex plane is sketched below. Complex number $z_1 = 1$ is on the x-axis where x=1. By inspection, the magnitude of z_1 is $|z| = 1$, and the angle is $\theta = 0^o$.



Question 1 *Calculate magnitude and phase of complex number $z = -j$*

Multiple Choice:

(a) $|z| = 1, \theta = 180$

(b) $|z| = 1, \theta = -90$ ✓

(c) $|z| = -1, \theta = 180$

(d) $|z| = -1, \theta = -90$

Write magnitude and phase of complex number $z = -1$

Question 2 $-1 = \boxed{e^{j180}}$