Exercise 1 Suppose that $\overrightarrow{\mathbf{u}} = \langle -1, 2, 2 \rangle$ and $\overrightarrow{\mathbf{v}} = \langle -2, 0, 1 \rangle$. Find a vector $\overrightarrow{\mathbf{w}}$ of magnitude 4 that is parallel to $2\overrightarrow{\mathbf{u}} - 3\overrightarrow{\mathbf{v}}$.

$$\overrightarrow{\mathbf{w}} = \left\langle \left\lceil \frac{16}{\sqrt{33}} \right\rceil, \left\lceil \frac{16}{\sqrt{33}} \right\rceil, \left\lceil \frac{4}{\sqrt{33}} \right\rceil \right\rangle$$

Hint: To begin, let's find a vector in the same direction as $2\vec{u} - 3\vec{v}$. Using the rules of addition and scalar multiplication, we find:

$$2\overrightarrow{\mathbf{u}} - 3\overrightarrow{\mathbf{v}} = \langle \boxed{4}, \boxed{4}, \boxed{1} \rangle$$

How should we proceed?

Multiple Choice:

- (a) Multiply this result by 4; that is, $\overrightarrow{\mathbf{w}} = \langle 16, 16, 4 \rangle$.
- (b) Find the magnitude of $\langle 4, 4, 1 \rangle$ and scale it appropriately if necessary. \checkmark

We compute:

$$|2\overrightarrow{\mathbf{u}} - 3\overrightarrow{\mathbf{v}}| = \sqrt{\left(\boxed{4}\right)^2 + \left(\boxed{4}\right)^2 + \left(\boxed{1}\right)^2} = \sqrt{\boxed{33}}$$

(type the components in the order of $2\overrightarrow{\mathbf{u}}-3\overrightarrow{\mathbf{v}})$

A unit vector in the direction of $\overrightarrow{\mathbf{w}}$ is thus $\frac{2\overrightarrow{\mathbf{u}} - 3\overrightarrow{\mathbf{v}}}{\left|2\overrightarrow{\mathbf{u}} - 3\overrightarrow{\mathbf{v}}\right|}$, so:

$$\hat{\mathbf{w}} = \left\langle \left\lceil \frac{4}{\sqrt{33}} \right\rceil, \left\lceil \frac{4}{\sqrt{33}} \right\rceil, \left\lceil \frac{1}{\sqrt{33}} \right\rceil \right\rangle$$

and $\overrightarrow{\mathbf{w}} = \boxed{4} \hat{\mathbf{w}}$.