

# Review of Complex Numbers

*This section aims to introduce two different ways we represent complex numbers: Cartesian coordinates, and Polar coordinates. We often use complex numbers in Cartesian coordinates when we discuss impedance or admittance. We often use complex numbers in polar coordinates to discuss magnitude and phase of voltages, currents, transfer functions, and Bode Plots. We can also represent sinusoidal signals with complex numbers with phasors. It is critically important that we understand this chapter.*

**Definition 1.** *A complex number  $z$  can be represented in the Cartesian coordinate system, as shown in Equation 1, and Polar coordinate system, as shown in Equation 2.*

$$z = x + jy \quad (1)$$

$$z = |z|e^{j\Theta} \quad (2)$$

In Equation 1, a complex number  $z$  is represented in rectangular coordinate system, where  $x$  is the real part,  $y$  is the imaginary part, and  $j = \sqrt{-1}$ .

In Equation 2, we see a complex number  $z$  in the polar coordinate system, where  $|z|$  is the magnitude, and  $\Theta$  is the angle (aka phase) of the complex number.

The geometric interpretation of these two equations is shown in Figure 1. The magnitude is the length of the triangle's hypotenuse, and the angle is the angle that the hypotenuse makes with the x-axis.

In Figure 1, we represent a complex number with a "position vector." Position vectors are vectors that start at the center of the coordinate system and end at any point in the coordinate system. We see that phasor is a vector that represents a complex number in a polar coordinate system.

You may be wondering why we represent the phase of a complex number in the polar coordinate system as  $e^{j\Theta}$  because in the circuits class, you used  $\angle\theta$ . Great question. That brings us to Euler's formula that we will discuss in section Euler's Formula.

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Learning outcomes: Sketch a complex number in rectangular and polar coordinates, and label magnitude, phase, real and imaginary parts. Derive the magnitude and phase from the real and imaginary parts of a complex number. Derive the real and imaginary parts of a complex number from the magnitude and phase.

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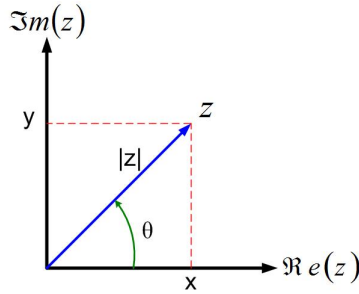


Figure 1: Visual representation of a complex number  $z$  in rectangular  $z = x + jy$  and polar coordinates  $z = |z|e^{j\theta}$ .

## Conversion between Cartesian and Polar coordinate systems

To find magnitude and angle when we know real and imaginary parts of a complex number, we use the Pythagorean Theorem to find the magnitude of the complex number as in Equation 3, and use the definition of the tangent to find the angle as in Equation 4.

$$|z| = \sqrt{x^2 + y^2} \quad (3)$$

$$\Theta = \arctan \frac{y}{x} \quad (4)$$

To find the real and imaginary part of a complex number when we know magnitude and phase, we use trigonometry. To find the real part of the complex number as in Equation 5, use the definition of cosine and sine to find the imaginary part of the complex number as in Equation 6.

$$\Re\{z\} = x = r \cos(\Theta) \quad (5)$$

$$\Im\{z\} = y = r \sin(\Theta) \quad (6)$$

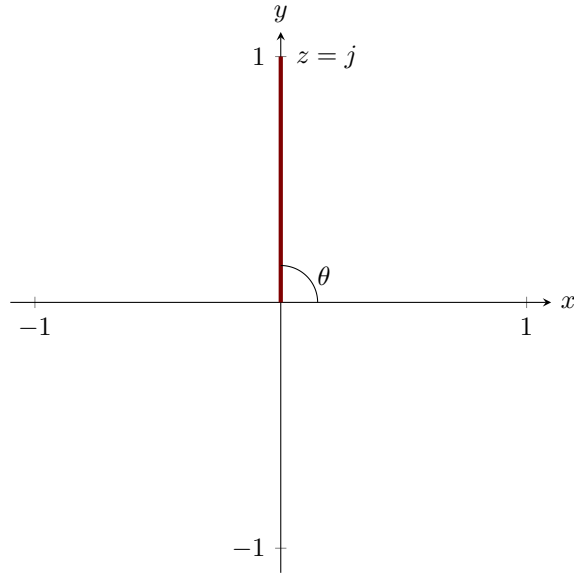
**Question 1** Explore the conversion of complex numbers between cartesian and polar coordinates.

Geogebra link: <https://tube.geogebra.org/m/b8hu8ztx>

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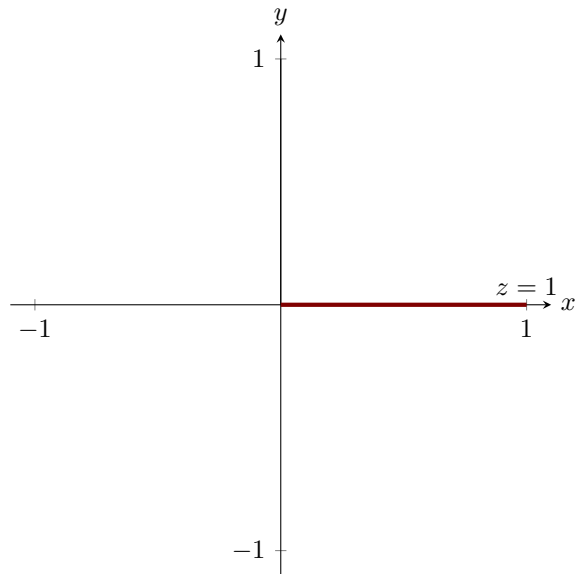
**Example 1.** Find the magnitude and phase of complex numbers  $z_1 = j$  and  $z_2 = 1$ .

**Explanation.** Complex number  $z_1 = j$  is on the  $y$ -axis where  $y=1$ . By inspection, the magnitude of  $z_1$  is  $|z| = 1$ , and the angle is  $\theta = 90^\circ$ .



The complex plane is sketched below. Complex number  $z_1 = 1$  is on the  $x$ -axis where  $x=1$ . By inspection, the magnitude of  $z_1$  is  $|z| = 1$ , and the angle is  $\theta = 0^\circ$ .

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**Question 2** Calculate magnitude and phase of complex number  $z = -j$

**Multiple Choice:**

- (a)  $|z| = 1, \theta = 180$
- (b)  $|z| = 1, \theta = -90$  ✓
- (c)  $|z| = -1, \theta = 180$
- (d)  $|z| = -1, \theta = -90$

Write magnitude and phase of complex number  $z = -1$

**Question 3**  $-1 = \boxed{e^{j180}}$