

# Review of Complex Numbers

*Invest time in studying complex numbers, because phasors are represented with complex numbers!*

**Definition 1.** A complex number  $z$  can be represented in Cartesian coordinate system as shown in Equation 1, and Polar coordinate system as shown in Equation 2.

$$z = x + jy \quad (1)$$

$$z = |z|e^{j\Theta} \quad (2)$$

In Equation 1, a complex number  $z$  is represented in rectangular coordinate system, where  $x$  is the real part,  $y$  is the imaginary part, and  $j = \sqrt{-1}$ .

In Equation 2, a complex number  $z$  is represented in polar coordinate system, where  $|z|$  is the magnitude and  $\Theta$  is the angle (aka phase) of the complex number.

Geometric interpretation of these two equations is given in Figure 1. The magnitude is the length of the hypotenuse of the triangle shown, and the angle is the angle that the hypotenuse makes with x-axis.

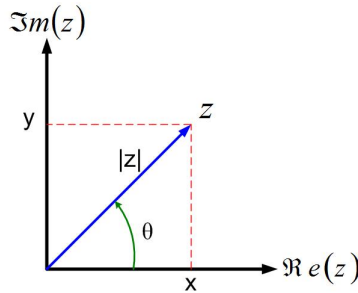


Figure 1: Visual representation of a complex number  $z$  in rectangular  $z = x + jy$  and polar coordinates  $z = |z|e^{j\theta}$ .

You may be wondering why are we representing the phase of a complex number in polar coordinate system as  $e^{j\Theta}$ , because in the circuits class you used  $\angle\theta$ . Great question. That brings us to Euler's formula.

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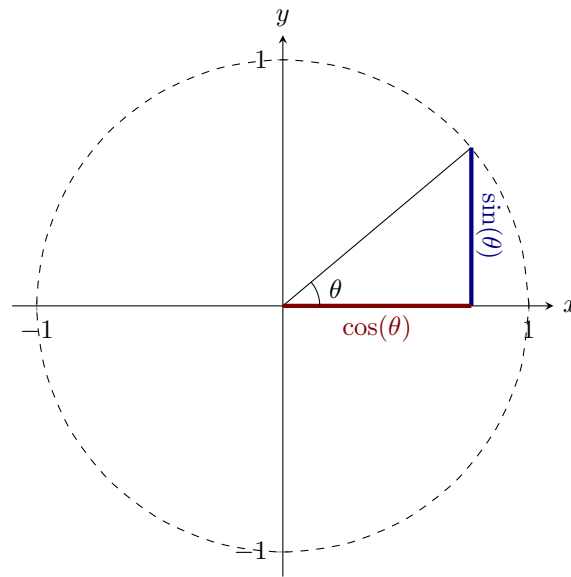
Learning outcomes:  
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## Euler's formula

Euler's formula relates the Cartesian and Polar coordinates for complex numbers.

$$e^{j\Theta} = \cos\Theta + j\sin\Theta \quad (3)$$

Geometric interpretation of the Euler's formula is shown below.  $z = r(\cos\theta + j\sin\theta)$ , where  $r\cos\theta = x$  and  $r\sin\theta = y$ . Euler's formula shows that number  $z$  given in Cartesian coordinates as  $x + jy$  can be represented in Polar Coordinates as  $e^{j\theta}$ . You have likely seen this proof in your Calculus class. **TIP: Your calculator may not know what  $e^{j\theta}$  is. Check how to convert between polar and cartesian coordinates on your calculator before the test.**



To find magnitude and angle when you know real and imaginary part of a complex number,

$$|z| = \sqrt{x^2 + y^2} \quad (4)$$

$$\Theta = \arctan \frac{y}{x} \quad (5)$$

## Operations with complex numbers

Complex Conjugate is often seen when finding the conditions for maximum power transfer.

$$z^* = (x + jy)^* = x - jy = |z|e^{-j\Theta} \quad (6)$$

Complex number addition and subtraction is often seen when dealing with complex impedances are placed in series and the equivalent complex impedance has to be found. The easiest way to add two complex numbers is to find Cartesian representation of both and then add the real parts separately and the imaginary part separately.

$$z_1 = x_1 + jy_1 \quad (7)$$

$$z_2 = x_2 + jy_2 \quad (8)$$

$$z_1 + z_2 = x_1 + x_2 + j(y_1 + y_2) \quad (9)$$

Multiplication and Division are often seen in the calculation of the transfer function of a circuit. The easiest way to divide two complex numbers is to find the polar representation of both and then divide the amplitudes and subtract the phases.

$$z_1 = |z_1|e^{j\Theta_1} \quad (10)$$

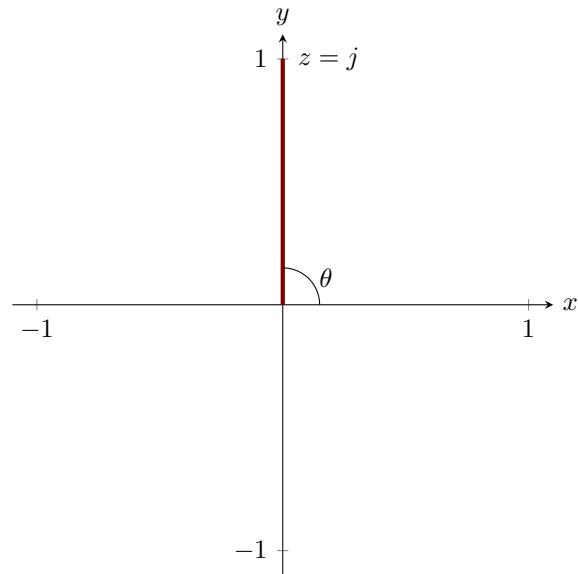
$$z_2 = |z_2|e^{j\Theta_2} \quad (11)$$

$$\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|}e^{j\Theta_1 - \Theta_2} \quad (12)$$

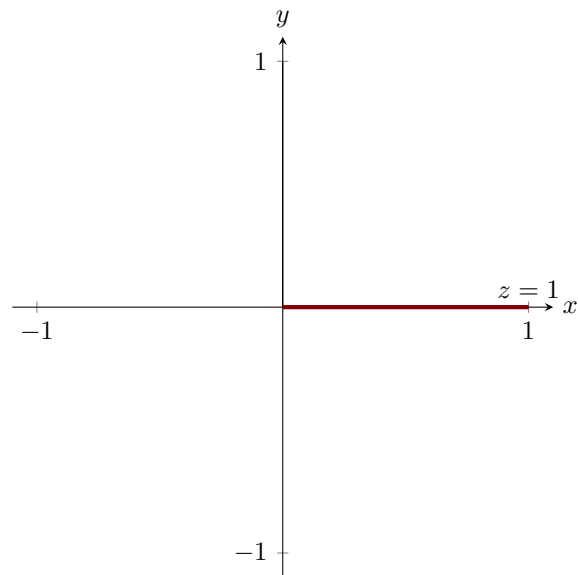
**Example 1.** Find the magnitude and phase of a complex numbers  $z_1 = j$  and  $z_2 = 1$ .

**Explanation.** Complex number  $z_1 = j$  is on the  $y$ -axis where  $y=1$ . By inspection, the magnitude of  $z_1$  is  $|z_1| = 1$ , and the angle is  $\theta = 90^\circ$ .

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Complex plane is sketched below. Complex number  $z_1 = 1$  is on the x-axis where  $x=1$ . By inspection, the magnitude of  $z_1$  is  $|z| = 1$ , and the angle is  $\theta = 0^\circ$ .



**Question 1** Calculate magnitude and phase of complex number  $z = -j$

**Multiple Choice:**

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(a)  $|z| = 1, \theta = 180$

(b)  $|z| = 1, \theta = -90$  ✓

(c)  $|z| = -1, \theta = 180$

(d)  $|z| = -1, \theta = -90$

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Write magnitude and phase of complex number  $z = -1$

**Question 2**  $-1 = \boxed{e^{j180}}$

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