EEE161 Transmission Lines and Waves

September 17, 2008

Instructor: Dr. Milica Marković
Office: Riverside Hall 3028
Email: milica@csus.edu
Web: http://gaia.ecs.csus.edu/~milica

Abstract

The lecture notes are not a substitute to lecture attendance.

1 Section 2 Electrostatics

2 Introduction to Electric Field

- a bunch of positives (negatives) would repel away from each other.
- the opposite pieces would attract each other.
- the net result is a balance! Balance is formed by tight fine mixtures of positives and negatives.
- there is no attraction/repulsion between them

What we described is exactly electrical force. All matter is a mixture of positive protons and negative electrons in a perfect balance. What is the expression for the strength and direction of this force? Coulombs law.

$$\vec{F_e} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{R_{12}} \tag{1}$$

In the above equation, ϵ_0 is electrical permittivity, q_1, q_2 electrical charge and r is the distance between charges.

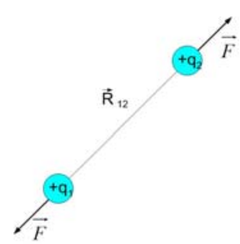


Figure 1: Vector representation of Nort-West wind of 10mph.

2.1 How perfect is this balance?

EXAMPLE

Lets calculate the repulsive force if there was a little bit of unbalance. Say that each of these two tables had 100 extra electrons. Lets try to calculate the repulsive force.

Electromagnetic force is one of four we know today. Lets discover the other forces.

2.2 Why is it that the atomic nucleus stays together when it is made out of the same kind of matter?

We just elaborated that if two charges are of the same kind, the electrostatic force will push them away from each other. It seems that there needs to be another kind of force that keeps the nucleus together. This force is called the nuclear force. This is the strongest force, but acts at a short distance. For example if we have a lot of protons in the nucleus such as in radioactive elements the nucleus can split by just lightly tapping it.

The last force is a weak-interaction force that plays role in radioactive decay.

2.3 Which four forces did we talk about today?

- Nuclear Force
- Electromagnetic Force
- Weak-Interaction Force
- Gravitational Force.

2.4 Coulombs Law

Lets review the Coulombs Law that governs the behavior of electrostatic force.

1. Like charges repel
2. Opposite charges attract
3. The force acts along the line that joins the charges
4. The strength of the force is given by the expression 2.

2.5 Whats the difference between the terms force and field?

Another bunch of questions could be:

- How do we now if we are in a gravitational field.
- What is the difference between the gravitational field and gravitational force?
- More specifically what is the meaning of the term field anyway?

Lets try to answer some of these questions.

2.6 More about the gravitational force and field

How do we know that we are in the gravitational field and not in zero gravity field? No matter how hard we try to launch ourselves in the outer space by jumping, we still come back to mother earth. If we drop a pencil, where will it go? Why is that so? The gravitational force attracts the pencil (and us). Another way to say that a gravitational force exists is to say that there is the field of force acting on an object. This is our first answer to the question: What is that term **field** anyway. Lets talk more about fields.

We know that the gravitational force acts at distance. There is no giant muscle that attracts our pencil. Earth induces a gravitational field, its influence exists at every point in space around it. This phenomenon of direct action on a distance has given rise to the concept of fields.

Lets see an example of gravitational force.

What is the source of the earths gravitational force? Earth (of course). It would be good if we can define a quantity to show what is the strength of this force at any point in space.

We can define gravitational field at any point in space through the gravitational force: If an object with mass m_m existed at the point r away from earth, it would experience the force F_g, we can say that the gravitational field at that point is equal to

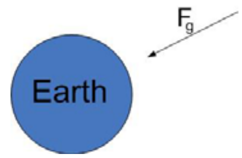


Figure 2: Gravitational Force.

$$\Psi = \frac{\vec{F_g}}{m_m} \tag{2}$$
$$\Psi = \gamma \frac{m_e}{r^2} \hat{r} \tag{3}$$

We dont need the moon in any particular spot to know what would be the gravitational field at that particular point. Note that the field does not depend on the moons mass! It depends only on the earths mass, gravitational constant and the distance to the point we want to find the field.

3 Electrostatics

3.1 More about the Electrical force and field

The same situation we have with the electrical force and field. The electric field is defined as the force that a charge would experience divided by the charge.

$$\vec{E} = \frac{\vec{F_e}}{q_2} \tag{4}$$
$$\vec{E} = \frac{q_e}{4\pi\epsilon_0 r^2} \hat{r} \tag{5}$$

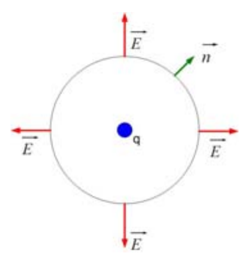


Figure 3: Electric field due to a unit charge q.

What is the source of the electrical force or field? Electric charge (of course).

3.2 Properties of Electric Charge

3.2.1 Electric charge cannot be created or destroyed.

If the total net charge of an object is q, and if that object has n_e electrons and n_p protons, then the total charge is $q = n_p e - n_e e$.

3.2.2 What is the electric field if we have more than one charge?

The total electric field at a point in space from the two charges is equal to the sum of the electric fields from the individual charges at that point.

3.2.3 What if the charge is not in air?

Lets look at Coulombs law again.

$$\vec{F_e} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{R_{12}} \tag{6}$$

Which quantity in this formula depends on the material? ϵ_0. If the charge is within a dielectric material, then we need to account for that by changing this ϵ_0 somehow. If we place the charge inside a dielectric material what do you think will happen with the atoms in the material? The atoms will get distorted and polarized. Such a polarized atom we call an electric dipole. The distortion process is called polarization. Because the material acts in such a way, the electric field around this point charge is different than if there was no material. In any dielectric medium, the electric field is defined as

$$\vec{F_e} = \frac{q_1 q_2}{4\pi\epsilon r^2}\hat{R_{12}} \tag{7}$$

$$\epsilon = \epsilon_0 \epsilon_r \tag{8}$$

We added unitless quantity ϵ_r, relative dielectric constant. ϵ_r values for different materials is shown in one of the tables in the book. Lets see its values for different materials. ϵ_r varies from 1 (air), to 2.2 (Teflon) to 80 (water).

Electric field density is the quantity that we introduce here:

$$\vec{D} = \epsilon\vec{E} \tag{9}$$

3.3 Principle of Superposition

If we have two charges, the total field due to both charges is equal to the vector sum of the fields due to individual charges, see Figure 4. The field at

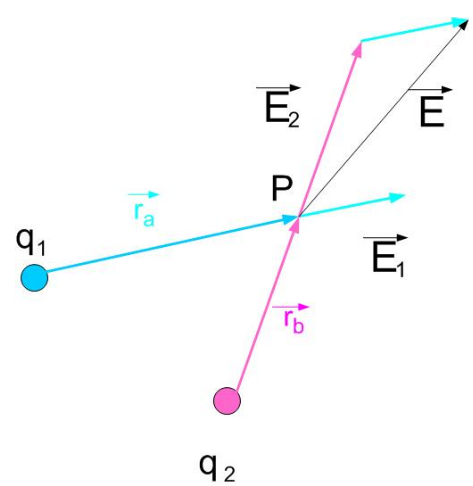


Figure 4: Electric Field due to two charges.

The fields or charges q_1 and q_2 are:

$$\vec{E_1} = \frac{q_1}{4\pi\epsilon_0 {r_a}^2}\hat{r_a} \tag{10}$$

$$\vec{E_2} = \frac{q_1}{4\pi\epsilon_0 {r_b}^2}\hat{r_b} \tag{11}$$

Where $\hat{r_a}$ and $\hat{r_b}$ are unit vectors in the direction of r_a and r_b. The total field due to both charges is

$$\vec{E} = \vec{E_1} + \vec{E_2} \tag{12}$$

3.4 Electric Field in Rectangular Coordinates

In general equation for the electric field is given as

$$\vec{E} = \frac{q_1}{4\pi\epsilon_0 {r_a}^2}\hat{r_a} \tag{13}$$

The electric field at a point $P(x, y, z)$ due to a charge q_1 positioned at a point $P_{q_1}(x_1, y_1, z_1)$ in the rectangular coordinate system is shown in Figure 5. The position vector of the point P_{q_1} is

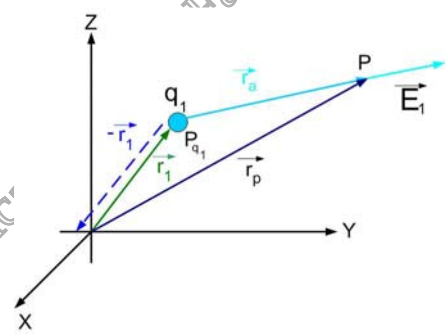


Figure 5: Electric Field due to a unit charge in Rectangular coordinate system.

$$\vec{r_1} = x_1\vec{x} + y_1\vec{y} + z_1\vec{z} \tag{14}$$

The position vector of point P is equal to

$$\vec{r_p} = x\vec{x} + y\vec{y} + z\vec{z} \tag{15}$$

The two vectors mark the beginning and the end of the distance vector $\vec{r_a}$ between points P_{q_1} and P. The vector $\vec{r_a}$ is the sum of vectors $-\vec{r_p}$ and $\vec{r_1}$

$$\vec{r_a} = \vec{r_p} + (-\vec{r_1}) \tag{16}$$

When we substitute position vectors r_1 and r_p:

$$\vec{r_a} = (x - x_1)\vec{x} + (y - y_1)\vec{y} + (z - z_1)\vec{z} \tag{17}$$

Vector $\vec{r_a}$ has the magnitude of:

$$|\vec{r_a}| = \sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2} \tag{18}$$

Unit vector in the direction of vector $\vec{r_a}$ is:

$$\hat{r_a} = \frac{\vec{r_a}}{|\vec{r_a}|} \tag{19}$$

$$\hat{r_a} = \frac{\vec{r_a}}{\sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2}} \tag{20}$$

$$\vec{E_1} = \frac{q_1}{4\pi\epsilon_0 {r_a}^2}\hat{r_a} \tag{21}$$

Where r_a is the distance between the charge q_1 and the point P. Substituting expressions for $\hat{r_a}$, and $|\vec{r_a}|$ in equation 13 we get

$$\vec{E_1} = \frac{q_1}{4\pi\epsilon_0 \sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2}^3}\vec{r_a} \tag{22}$$

Substituting

For two charges, as shown in Figure 6 equation 22 becomes

$$\vec{E} = \frac{q_1}{4\pi\epsilon_0 \sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2}^3}\vec{r_a} + \frac{q_2}{4\pi\epsilon_0 \sqrt{(x-x_2)^2 + (y-y_2)^2 + (z-z_2)^2}^3}\vec{r_b} \tag{23}$$

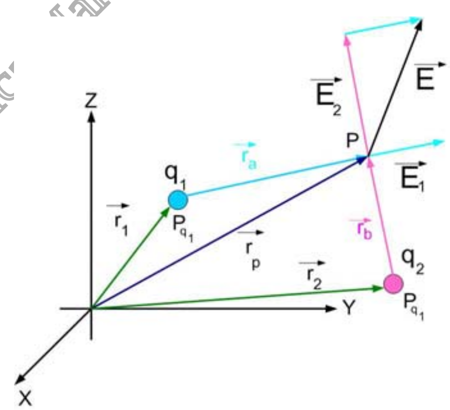


Figure 6: Electric field due to two charges in Rectangular coordinate system.

3.5 Electric Field Distributions

Example of Line Charge Distribution

Find the field at the z-axis of a loop of charge. Charge is uniformly distributed along the loop with line charge density of ρ_l.

Line charge distribution

Surface charge distribution

DISK of charge

Infinite plane

Volume charge distribution

EXAMPLE Line charge distribution Loop of wire

EXAMPLE Surface charge distribution Disk

EXAMPLE Volume charge distribution diode

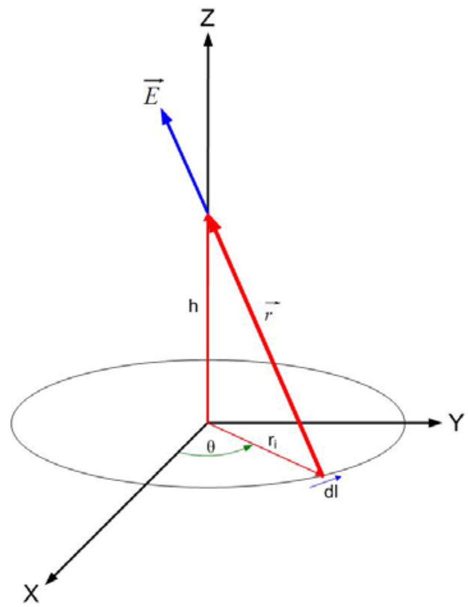


Figure 7: Loop of wire uniformly charged with line charge density ρ_l. Electric field is shown due to a very small section of the loop.

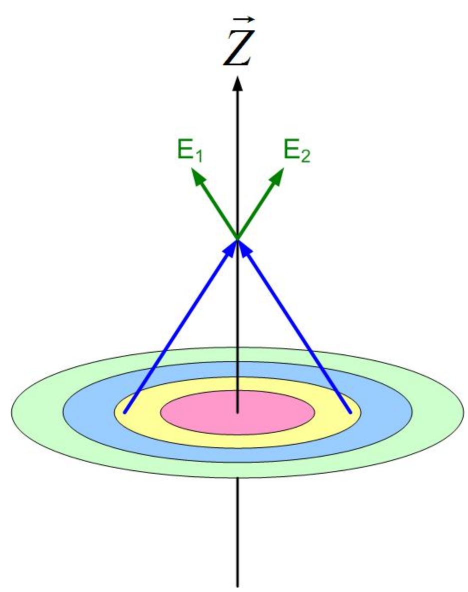


Figure 8: Disk of charge can be regarded as an infinite number of concentric rings of charge.

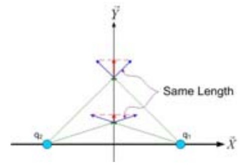


Figure 9: Electric field from two rings located on the infinite plane.

Figure 10: Application of Gauss Law to find Electric Field of wire.

3.6 Gauss Law

EXAMPLE Wire
EXAMPLE Infinite Plane
EXAMPLE Two Infinite Planes

4 Definition of Potential and Voltage

EXAMPLE Potential due to unit charge

4.1 Capacitance

What is capacitance, how does it affect circuits.

4.2 Electric Field inside Metals

4.3 Boundary Conditions

4.4 Image Theory

5 Static Magnetic Field

We talked about electric field so far. Lets change a topic to the magnetic field.

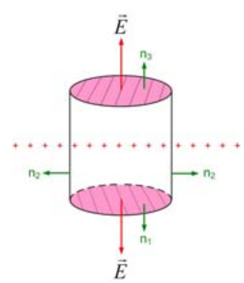


Figure 11: Infinite plane charged with positive surface charge density ρ_S.

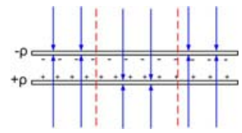


Figure 12: Vector representation of Nort-West wind of 10mph.

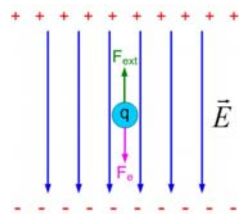


Figure 13: Forces on a unit charge in an external field $\vec{E}$.

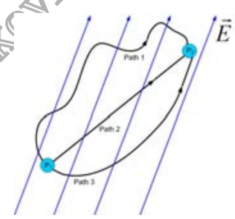


Figure 14: Vector representation of Nort-West wind of 10mph.

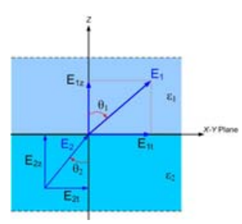


Figure 15: Boundary Conditions for Electric Field.

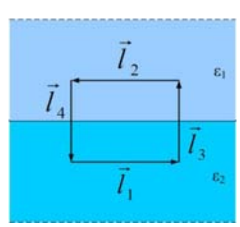


Figure 16: Integration path to find tangential fields at the boundary.

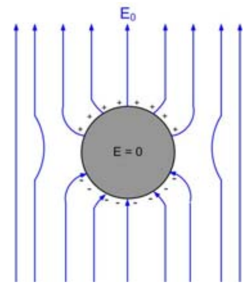


Figure 17: Metal sphere in an external electric field.

5.1 What is the source of the magnetic field?

Magnet. This is the first source of the magnetic field that people have discovered in Greece long time ago. [1] Danish scientist Oersted discovered later that the current passing through wire will deflect a compass needle. Later French scientists Biot and Savart quantified this statement:

$$B = \mu_0 \frac{I}{2\pi r} \tag{24}$$

Where B is magnetic flux density, μ_0 is magnetic permeability, I is the electric current, and r is the distance to the point where the magnetic field is measured.

HERE PICTURE WITH A WIRE AND THE magnetic field.

In a magnetic material instead of μ_0 in the above formula we have $\mu = \mu_0 \mu_r$. μ_r is relative magnetic permeability.

5.2 Constitutive parameters of a material

We have introduced two constitutive parameters so far, electric permittivity ϵ and magnetic permeability μ. The third parameter is conductivity σ. Conductivity is zero for perfect insulator and infinite for perfect conductor.

The speed of light in air is equal to

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \tag{25}$$

5.3 Charged Particle in a Static Magnetic Field

5.4 Force on a conductor carrying current

5.5 Wire frame carrying current in a static magnetic field

5.6 Biot-Savarts Law

How to find the magnetic field due to a current distribution.

5.7 Amperes Law

5.8 Inductance

Types, internal external. Ways to find inductance through energy and directly. What is inductance, how does it affect circuits.

EXAMPLE Two wire line EXAMPLE Coax EXAMPLE Internal inductance of wire, block etc.

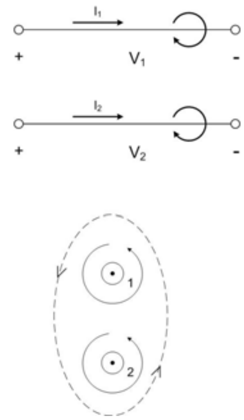


Figure 18: Mutual Inductance: Increasing the magnetic field and therefore current in one wire due to another wire in vicinity.

5.9 Mutual Inductance

5.10 Inductance in circuit theory

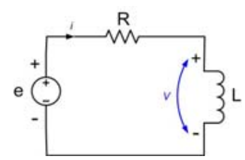


Figure 19: Simple electronic circuit with an inductance and resistance.

6 Electromagnetics

Electromagnetics is usually studied by observing static electric fields, static magnetic fields and dynamic electromagnetic fields. Static electric fields are independent from static magnetic fields. In dynamic electromagnetic fields changing electric field induces changing electric field and so on.

6.1 Induced EMF

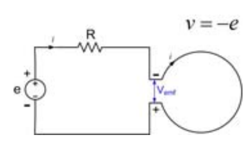


Figure 20: Induced voltage due to a loop of wire with AC current. Voltage induced is due to inductance of the loop of wire.

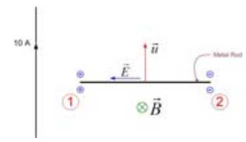


Figure 21: Example of induced motional electromotive force.

6.2 Motional EMF

7 Waves

7.1 Waves

We have seen waves in oceans, rivers and ponds. If you throw a stone into a pond it will make ripples on the surface of the pond. We know that waves have velocity, they propagate from the center of the disturbance out. There are two types of waves, transient and continuous harmonic waves. We can also observe waves in 1-D (string attached on one end), 2-D ripples in a pond or 3-D. In 1-D the disturbance varies in one variable, etc.

SHOW 3-D WAVES FROM THE ULABY CD

Lets review sinusoidal signals first.

$$y(t) = Acos(\omega t + \theta) \tag{26}$$

PLOT OF THE SINUSOIDAL WAVE VS TIME

PLOT OF THE SINUSOIDAL WAVE VS SPACE

Well talk in Lecture 3 about what is the difference and similarities between the two plots shown above

For additional reading see Ulaby and Feynman Lectures on Physics Vol. II 1-1. [2]

[1]to be exact

[2]May the force be with you.

8 Lecture 2 Review of basic concepts

8.1 Reading

Ulaby Chapter 1, The beginning of the lecture is adapted from the book Feynmann lectures on Physics, Vol. II 1-1

8.2 Purpose of the lecture and the main point

Review of basic concepts.

8.3 What does it mean when we say a medium is lossy or lossless?

Lossless medium: Electromagnetic wave power is not turning into heat. Lossy medium: Electromagnetic wave is heating up the medium, therefore its power is decreasing as $e^{-}\alpha x$.

medium	attenuation constant α [dBm/km]
coax	60
waveguide	2
fiber-optic	0.5

In guided wave systems such as transmission lines and waveguides the attenuation of power with distance follows approximately $e^{-2\alpha x}$. The power radiated by an antenna falls off as $1/r^2$.

8.4 Decibels?

What is a dB? dB is a ratio of two power or two voltages. For example if we want to say that the output power is twice the input power we say that the power gain is 3dB.

$$G = 10log\frac{P_{out}}{P_{in}} \tag{27}$$

EXAMPLE Find the output power if the input power is 1W and the power gain is 6dB.

8.5 Review of complex numbers

1. A complex number z can be represented in Cartesian eqn. or Polar eqn. 29 form.

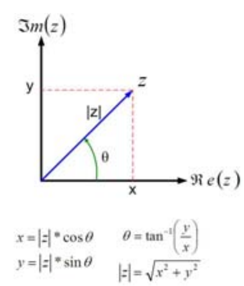


Figure 22: Complex number z in rectangular and polar coordinates.

$$z = x + jy \tag{28}$$
$$z = |z|e^{j\Theta} \tag{29}$$

x is the real part, y is the imaginary part, $|z|$ is the magnitude and Θ is the angle of the complex number.

2. Euler's Identity

$$e^{j\Theta} = cos\Theta + jsin\Theta \tag{30}$$

3. Cartesian and polar form representation

$$|z| = \sqrt{x^2 + y^2} \tag{31}$$
$$\Theta = arctg\frac{y}{x} \tag{32}$$

4. Complex Conjugate

$$z^* = (x + jy)^* = x - jy = |z|e^{-j\Theta} \tag{33}$$

5. Complex number addition

$$z_1 = x_1 + jy_1 \tag{34}$$
$$z_2 = x_2 + jy_2 \tag{35}$$
$$z_1 + z_2 = x_1 + x_2 + j(y_1 + y_2) \tag{36}$$

6. Multiplication

$$z_1 = x_1 + jy_1 \tag{37}$$
$$z_2 = x_2 + jy_2 \tag{38}$$
$$z_1 z_2 = (x_1 x_2 y_1 y_2) + j(x_2 y_1 + x_1 y_2) \tag{39}$$

   Prove it!

7. Division

$$z_1 = |z_1|e^{j\Theta_1} \tag{40}$$
$$z_2 = |z_2|e^{j\Theta_2} \tag{41}$$
$$\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|}e^{j\Theta_1 - \Theta_2} \tag{42}$$

   What happens in Cartesian coordinates?

8. Power and Square root

$$z_1 = |z_1|e^{j\Theta_1} \tag{43}$$
$$z_1^n = |z_1|^n e^{jn\Theta} \tag{44}$$
$${}^n\sqrt{z_1} ={}^n \sqrt{|z_1|}e^{j\frac{\Theta}{n}} \tag{45}$$

9. Some examples. Find the magnitude and phase of a complex number

$$-1 \tag{46}$$
$$j \tag{47}$$
$$\sqrt{j} \tag{48}$$

8.6 Phasors

Let's assume we don't know how to solve the circuit in the frequency domain. The next question is how do we solve a circuit in the time domain?

Example HERE PICTURE OF A SIMPLE RC CIRCUIT WITH A SINUSOIDAL SOURCE AND THE OUTPUT OVER THE CAPACITOR.

the final equation is

$$v_s(t) = Ri + \frac{1}{C}\int idt \tag{49}$$

Where

$$v_s(t) = Acos(\omega t + \Theta) \tag{50}$$

We see that even for the simplest possible circuit we get a differential equation in the time domain.

Use of phasors simplify the equations to solve a circuit significantly. Instead of differential equations we get a set of linear equations.

Let's look at the forcing function 50 first. What is the important information here? Amplitude and phase. All currents and voltages in a circuit will have the same cos expression in them, but amplitude and phase information will change depending on the circuit topology. It would be good if we could loose this $cos(\omega t)$ from the picture and replace it with some function that is simpler to deal with. Let's try and add a piece to this cos.

$$Acos(\omega t + \Theta) + jAsin(\omega t + \Theta) \tag{51}$$

It seems that we made the expression more complicated. However, if we remember Euler's identity, the expression becomes

$$Ae^{j(\omega t+\Theta)} = Ae^{(}j\Theta)e^{(}j\omega t) \tag{52}$$

We see that now we have extracted the phase and amplitude information and separated it from the exponential expression. The piece that contains the amplitude and phase information we call phasor $V_S(j\omega)$.

$$Ae^{(}j\Theta)e^{(}j\omega t) = V_S(j\omega)e^{(}j\omega t) \tag{53}$$

Why is this expression better then the one with a cos? We'll let's express all time-dependent variables as well as derivatives and integrals in this fashion.

$i(t)$	$I(j\omega)e^{j\omega t}$
$v(t)$	$V(j\omega)e^{j\omega t}$
$\frac{v(t)}{dt}$	$j\omega V(j\omega)e^{j\omega t}$
$\int i(t)dt$	$\frac{I(j\omega)}{j\omega}e^{j\omega t}$

We now replace the time-domain quantities in equation 49 with these newly developed expressions.

$$V_S(j\omega)e^{j\omega t} = RI(j\omega)e^{j\omega t} + \frac{I(j\omega)}{j\omega C}e^{j\omega t} \tag{54}$$

A common term in the previous equation is $e^{j\omega t}$. We can now write the equation as

$$V_S(j\omega) = RI(j\omega) + \frac{I(j\omega)}{j\omega C} \tag{55}$$

Example Since this is a linear equation, we can easily solve it!

Once we solve it, how do we find the time-domain expression again? First we need to multiply the phasor with $e^{j\omega t}$ then find the real part of the expression.

$$I(j\omega)e^{j\omega t} \tag{56}$$

$$i(t) = Re\{I(j\omega)e^{j\omega t}\} \tag{57}$$

How do we now solve a circuit using phasors? We replace all impedances with their phasor expressions, find the phasor expression for the current and then find the time-domain expression for the current. Using this techinique we can find only the steady-state expression for the current/voltage.

Example Find the voltage accross the resistor in the circuit below.

HERE GOES A PICTURE OF THE CIRCUIT WITH AN INDUCTOR AND RESISTOR

The phasor expression for impedance shows us what happens with certain impedances at different frequencies.

cicruit element	impedance	low frequencies $f \rightarrow 0$	high frequencies $f \rightarrow \inf$
capacitor	$\frac{1}{j\omega C}$	∞	0
inductor	$j\omega L$	0	∞

9 Lecture 3 Introduction to Transmission Lines

9.1 Reading

Ulaby Chapter 1, The beginning of the lecture is adapted from the book Feynmann lectures on Physics, Vol. II 1-1

9.2 Purpose of the lecture and the main point

Introduction to transmission lines

9.3 What is a transmission line?

Any wire, cable or line that guides energy from one point to another is a transmission line. Whenever you make a circuit on a breadboard, every wire you attach is a transmission line. Wheather we see the propagation effects on a line depends on the line length. So, at lower frequencies we do not see that the signal (wave) actually propagates from one end of the wire to the other.

9.4 What is wavelength?

9.5 Wave types

Types of waves include acoustic waves, mecahnical pressure waves, electromagnetic (EM) waves. Here we will focus on EM waves and transmission lines for EM waves.

HERE GOES A PICTURE OF TRANSMISSION LINE WITH THE GENERATOR ON ONE END AND LOAD ON THE OTHER with AA sending end and BB receiving end and length l

How much time it takes for this signal to go from AA end to BB end? $t = \frac{l}{c}$, where $c = 310^8$. If the signal at end AA is

$$v_{AA'}(t) = Acos(\omega t) \tag{58}$$

Then at the other end the signal is

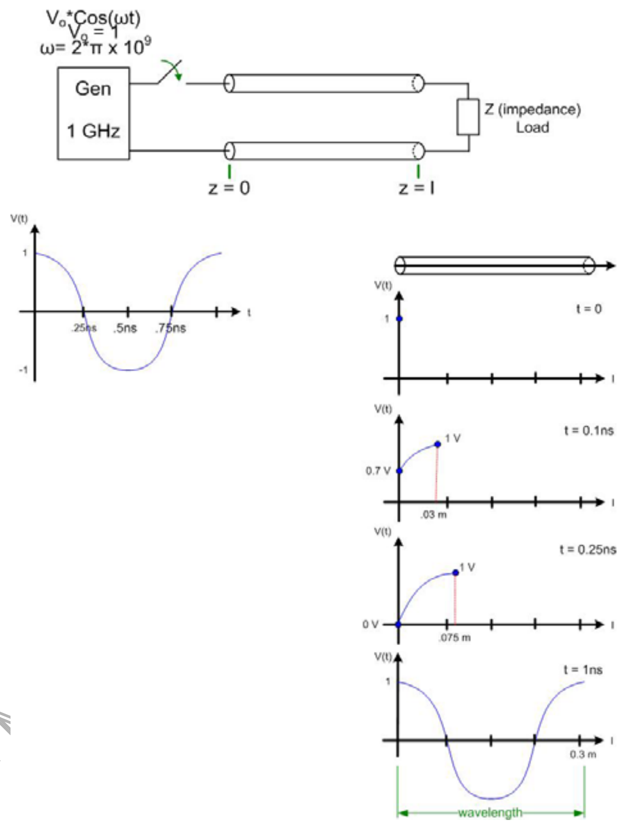


Figure 23: Section of a coaxial cable.

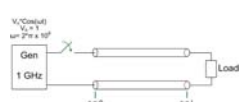


Figure 24: Electronic Circuit with an emphasis on cables that connect the generator and the load.

$$v_{BB'}(t) = v_{AA'}(t - \frac{l}{c}) \tag{59}$$

$$v_{BB'}(t) = Acos(\omega(t - \frac{l}{c})) \tag{60}$$

$$v_{BB'}(t) = Acos(\omega t - \omega\frac{l}{c})) \tag{61}$$

$$v_{BB'}(t) = Acos(\omega t - \frac{\omega}{c}l)) \tag{62}$$

Since we know that $\omega = 2\pi f$

$$v_{BB'}(t) = Acos(\omega t - \frac{2\pi f}{c}l)) \tag{63}$$

The quantity $\frac{c}{f}$ is the wavelength λ

$$v_{BB'}(t) = Acos(\omega t - \frac{2\pi}{\lambda}l)) \tag{64}$$

The quantity $\frac{2\pi}{\lambda}$ is the propagation constant β
Finally the expression for the voltage at BB end is

$$v_{BB'}(t) = Acos(\omega t - \beta l) \tag{65}$$

$$v_{BB'}(t) == Acos(\omega t - \Psi) \tag{66}$$

We see that at BB the signal will experience a phase shift.
We will derive this equation later again from the Telegrapher's equations 9.8.
Now let's see how the length of the line l affects the voltage at the end BB or a wire. Look at Equation 67.

$$v_{BB'}(t) = Acos(\omega t - 2\pi\frac{l}{\lambda})) \tag{67}$$

1. If $\frac{l}{\lambda} < 0.01$ The angle $2\pi\frac{l}{\lambda}$ is of the order of 0.0314 rad or about 2^0. This phase is obviosly something that we don't have to worry about. When the length of the transmission line is muchn smaller than λ the wave propagation on the line can be ingnored.
2. If $\frac{l}{\lambda} > 0.01$, say $\frac{l}{\lambda} = 0.1$, then the phase is 20^0, which is a significant phase shift. In this case it may be necessary to account for reflected signals, power loss and dispersion on the transmissio line.

Dispersion is an effect where different frequencies travel with different speeds on the transmission line.
Example Find what is the length of the cable at which we need to take into account transmission line effects if the frequency of operation is 10 GHz.

9.6 Types of transmission lines

Coax, two wire line, microstrip etc
HERE PICTURES OF DIFFERENT LINES

9.7 Propagation modes on a transmission line

Coax, two wire line, microstrip etc can be approximated as TEM up to the 30-40 GHz (unshielded), up to 140 GHz shielded.

1. TEM E, M field is entirely transverse to the direction of propagation
2. TE, TM E or M field is in the direction of propagation

9.8 Derivation of the wave equation on a transmission line

In this section we will derive what is the expression for the signal along a wire as a function of space z. So far in curriculum we have only been talking about what is the expression for the signal as a function of time.

We want to derive the equations for the case when the transmission line is longer then the fraction of a wavelength. To make sure that we don't encounter the transmission line effects to start with, we can look at the piece of a transmission line that is much smaller then the fraction of a wavelenth. In other words we chop the transmission line into small pieces to make sure there are no transmission line effects, as the pieces are shorter then the fraction of a wavelength.

Plan:

- Look at an infinitensimal length of a transmission line Δz.
- Represent that piece with an equivalent circuit.
- Write KCL, KVL for the piece in the time domain (we get differential equations)
- Apply phasors (equations become linear)
- Solve the linear system of equations to get the expression for the voltage and current on the transmission line as a function of z.

Let's follow the plan now.

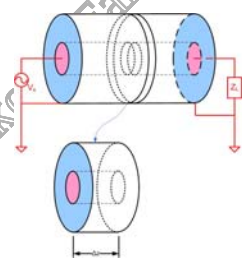


Figure 25: Section of a coaxial cable.

HERE PICTURE OF THE TRANSMISSION LINE CHOPPED INTO PIECES FIG1
FIG 2 ONE PIECE OF A TRANSMISSION LINE EQUIVALENT CIRCUIT
KVL

$$-v(z,t) + R\Delta z i(z,t) + L\Delta z \frac{\partial i(z,t)}{\partial t} + v(z+\Delta z, t) = 0$$

KCL

$$i(z,t) = i(z+\Delta z) + i_{CG}(z+\Delta z, t)$$
$$i(z,t) = i(z+\Delta z) + G\Delta z v(z+\Delta z, t) + C\Delta z \frac{\partial v(z+\Delta z, t)}{\partial t}$$

Rearrange the KCL and KVL Equations 68, 71 divide them with Δz Equations 69, 72 let $\Delta z \to 0$ and recognize the expression for the derivative Equations, 70, 73.

KVL

$$-(v(z+\Delta z,t) - v(z,t)) = R\Delta z i(z,t) + L\Delta z \frac{\partial i(z,t)}{\partial t} \tag{68}$$
$$-\frac{v(z+\Delta z,t) - v(z,t)}{\Delta z} = Ri(z,t) + L\frac{\partial i(z,t)}{\partial t} \tag{69}$$
$$-\frac{v(z,t)}{\partial z} = Ri(z,t) + L\frac{\partial i(z,t)}{\partial t} \tag{70}$$

KCL

$$-(i(z+\Delta z,t)-i(z,t)) = G\Delta z v(z+\Delta z,t) + C\Delta z\frac{\partial v(z+\Delta z,t)}{\partial t} \quad (71)$$
$$-fraci(z+\Delta z,t)-i(z,t)\Delta z = Gv(z+\Delta z,t) + C\frac{\partial v(z+\Delta z,t)}{\partial t} \quad (72)$$
$$-\frac{i(z,t)}{\partial z} = Gv(z+\Delta z,t) + C\frac{\partial v(z+\Delta z,t)}{\partial t} \quad (73)$$

We just derived Telegrapher's equations in time-domain:

$$-\frac{v(z,t)}{\partial z} = Ri(z,t) + L\frac{\partial i(z,t)}{\partial t}$$
$$-\frac{i(z,t)}{\partial z} = Gv(z+\Delta z,t) + C\frac{\partial v(z+\Delta z,t)}{\partial t}$$

These are two differential equations with two unknowns. It is not impossible to solve them, however we would prefer to have linear differential equations. So what do we do now?

Express time-domain variables as phasors!

$$v(z,t) = Re\{V(z)e^{j\omega t}\}$$
$$i(z,t) = Re\{I(z)e^{j\omega t}\}$$

And we get the Telegrapher's equations in phasor form

$$-\frac{\partial V(z)}{\partial z} = (R+j\omega L)I(z) \quad (74)$$
$$-\frac{\partial I(z)}{\partial z} = (G+j\omega C)V(z) \quad (75)$$

Two equations, two unknowns. To solve these equations, we first integrate both equations over z,

$$-\frac{\partial^2 V(z)}{\partial z^2} = (R+j\omega L)\frac{\partial I(z)}{\partial z}$$
$$-\frac{\partial^2 I(z)}{\partial z^2} = (G+j\omega C)\frac{\partial V(z)}{\partial z}$$

Now rearange the previous equations

$$-\frac{1}{(R+j\omega L)}\frac{\partial I(z)}{\partial z} = \frac{\partial^2 V(z)}{\partial z^2} \quad (76)$$
$$-\frac{1}{(G+j\omega C)}\frac{\partial V(z)}{\partial z} = \frac{\partial^2 I(z)}{\partial z^2} \quad (77)$$

now substitute Eq.76 into Eq.74 and Eq.77 into Eq.75 and we get

$$-\frac{\partial^2 V(z)}{\partial z^2} = (G+j\omega C)(R+j\omega L)V(z) -$$
$$-\frac{\partial^2 I(z)}{\partial z^2} = (G+j\omega C)(R+j\omega L) - I(z)$$

Or if we rearrange

$$-\frac{\partial^2 V(z)}{\partial z^2} - (G+j\omega C)(R+j\omega L)V(z) = 0 \quad (78)$$
$$-\frac{\partial^2 I(z)}{\partial z^2} - (G+j\omega C)(R+j\omega L)I(z) = 0 \quad (79)$$

The above Eq.78 and Eq.79 are the equations of the current and voltage wave on a transmission line. $\gamma = (G + j\omega C)(R + j\omega L)$ is the complex propagation constant. This constant has a real and an imaginary part.

$$\gamma = \alpha + j\beta$$

where α is the attenuation constant and β is the phase constant.

$$\alpha = Re\{\sqrt{(G + j\omega C)(R + j\omega L)}\}$$
$$\beta = Im\{\sqrt{(G + j\omega C)(R + j\omega L)}\}$$

What is the general solution of the differential equation of the type 78 or 79?

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$
$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

In this equation V_0^+ and V_0^- are the **phasors**[3] of forward and backward going voltage waves, and I_0^+ and I_0^- are the phasors of forward and backward going current waves

The time domain expression for the current and voltage on the transmission line we get

$$v(t) = Re\{(V_0^+ e^{(-\alpha + j\beta)z} + V_0^- e^{(\alpha + j\beta)z})e^{j\omega t}\}$$
$$v(t) = |V_0^+|e^{-\alpha z}cos(\omega t + \beta z + \angle V_0^+) + |V_0^-|e^{-\alpha z}cos(\omega t - \beta z + \angle V_0^-)$$

We'll look at the Matlab program the next class to see that if the signs of the ωt and βz are the same the wave moves in the forward $+z$ direction. If the signs of ωt and βz are opposite, the wave moves in the $-z$ direction.

[3] complex numbers having an amplitude and the phase

10 Lecture 4

Introduction to MATLAB

11 Lecture 5

11.1 Reading

11.2 Purpose of the lecture and the main point

11.3 Relating forward and backward current and voltage waves on the transmission line

$$V(z) = V_0^+ exp^{-\gamma z} + V_0^- exp^{\gamma z} \tag{80}$$
$$I(z) = I_0^+ exp^{-\gamma z} + I_0^- exp^{\gamma z} \tag{81}$$

In this equation V_0^+ and V_0^- are the phasors of forward and backward going voltage waves, and I_0^+ and I_0^- are the phasors of forward and backward going current waves

We will relate the phasors of forward and backward going voltage and current waves.

When substitute the voltage wave equation into Telegrapher's Eq. 74. The equation is repeated here Eq.82.

$$-\frac{\partial V(z)}{\partial z} = (R + j\omega L)I(z) \tag{82}$$
$$\gamma V_0^+ e^{-\gamma z} - \gamma V_0^- e^{\gamma z} = (R + j\omega L)I(z) \tag{83}$$

We now rearrange Eq.83

$$I(z) = \frac{\gamma}{R + j\omega L}(V_0^+ exp^{-\gamma z} + V_0^- exp^{\gamma z})$$
$$I(z) = \frac{\gamma V_0^+}{R + j\omega L} e^{-\gamma z} - \frac{\gamma V_0^-}{R + j\omega L} e^{\gamma z} \tag{84}$$

Now we compare Eq.84 with the Eq.81.

$$I_0^+ = \frac{\gamma V_0^+}{R + j\omega L}$$
$$I_0^- = -\frac{\gamma V_0^-}{R + j\omega L}$$

We can define the characteristic impedance of a transmission line as the ratio of the voltage to current amplitude of the forward going wave.

$$Z_0 = \frac{V_0^+}{I_0^+}$$
$$Z_0 = \frac{R + j\omega L}{\gamma}$$
$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

11.4 Lossless transmission line

In many practical applications $R \to 0$ [4]and $G \to 0$[5]. This is a lossless transmission line.

In this case the transmission line parameters are

[4]metal resistance is low
[5]dielectric conductance is low

- Propagation constant

$$\gamma = sqrt(R + j\omega L)(G + j\omega C)$$
$$\gamma = \sqrt{j\omega L j\omega C}$$
$$\gamma = j\omega\sqrt{LC} = j\beta$$

- Transmission line impedance

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$
$$Z_0 = \sqrt{\frac{j\omega L}{j\omega C}}$$
$$Z_0 = \sqrt{\frac{L}{C}}$$

- Wave velocity

$$v = \frac{\omega}{\beta}$$
$$v = \frac{\omega}{\omega\sqrt{LC}}$$
$$v = \frac{1}{\sqrt{LC}}$$

- Wavelength

$$\lambda = \frac{2\pi}{\beta}$$
$$\lambda = \frac{2\pi}{\omega\sqrt{LC}}$$
$$\lambda = \frac{2\pi}{\sqrt{\epsilon_0\mu_0\epsilon_r}}$$
$$\lambda = \frac{c}{f\sqrt{\epsilon_r}}$$
$$\lambda = \frac{\lambda_0}{\sqrt{\epsilon_r}}$$

11.5 Voltage Reflection Coefficient, Lossless Case

The equations for the voltage and current on the transmission line we derived so far are

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \tag{85}$$
$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z} \tag{86}$$

At $z = 0$ the impedance of the load has to be

$$Z_L = \frac{V(0)}{I0}$$

Substitute the boundary condition in Eq.85

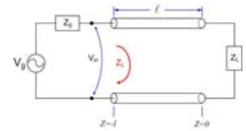


Figure 26: Transmission Line connects generator and the load.

$$Z_L = Z_0 \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \tag{87}$$

We can now solve the above equation for V_0^-

$$\begin{aligned}
\frac{Z_L}{Z_0}(V_0^+ - V_0^-) &= V_0^+ + V_0^- \\
(\frac{Z_L}{Z_0} - 1)V_0^+ &= (\frac{Z_L}{Z_0} + 1)V_0^- \\
\frac{V_0^-}{V_0^+} &= \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1} \\
\frac{V_0^-}{V_0^+} &= \frac{Z_L - Z_0}{Z_L + Z_0} \qquad (88)
\end{aligned}$$

The quantity $\frac{V_0^-}{V_0^+}$ is called voltage reflection coefficient Γ. It relates the reflected to incident voltage phasor. Voltage reflection coefficient isi n general a complex number, it has a magnitude and a phase.

Examples

1. $100\,\Omega$ transmission line is terminated in a series connection of a $50\,\Omega$ resistor and 10 pF capacitor. The frequency of operation is 100 MHz. Find the voltage reflection coefficient.
2. For purely reacive load $Z_L = jX_L$ find the reflection coefficient.

The end of this lecture is spent in the lab making a Matlab program to make a movie of a wave moving left and right.

12 Lecture 6

12.1 Reading

12.2 Purpose of the lecture and the main point

12.3 Standing Waves

In the previous section we introduced the voltage reflection coefficient that relates the forward to reflected voltage phasor.

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{89}$$

If we substitute this expression to Eq.85[6] we get for the voltage wave

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \tag{90}$$

$$V(z) = V_0^+ e^{-j\beta z} + \Gamma V_0^+ e^{j\beta z}$$

$$V(z) = V_0^+ (e^{-j\beta z} - \Gamma e^{j\beta z}) \tag{91}$$

since $\Gamma = |\Gamma| e^{j\Theta_r}$ Eq.92 becomes

$$V(z) = V_0^+ (e^{-j\beta z} - |\Gamma| e^{j\beta z + \Theta_r}) \tag{92}$$

and for the current wave

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z} \tag{93}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} + \Gamma \frac{V_0^+}{Z_0} e^{j\beta z}$$

$$I(z) = \frac{V_0^+}{Z_0} (e^{-j\beta z} - \Gamma e^{j\beta z}) \tag{94}$$

The voltage and the current waveform on a transmission line are therefore given by Eqns.92, 94. Now we have two equations and one unknown V_0^+! We will solve these two equations in Lecture 7. Now let's look at the physical meaning of these equations.

In Eq.92, Γ is the voltage reflection coefficient, V_0^+ is the phasor of the forward going wave, z is the axis in the direction of wave propagation, β is the phase constant[7], Z_0 is the impedance of the transmission line[8]. $V(z)$ is a complex number, phasor. We will find the magnitude and phase of the voltage on the transmission line.

The magnitude of a complex number can be found as $|z| = \sqrt{zz^*}$[9].

$$|V(z)| = \sqrt{V(z)V(z)^*}$$

$$|V(z)| = \sqrt{V_0^+(e^{-j\beta z} - |\Gamma| e^{j\beta z + \Theta_r}) V_0^+ (e^{j\beta z} - |\Gamma| e^{-(j\beta z + \Theta_r)})}$$

$$|V(z)| = V_0^+ \sqrt{(e^{-j\beta z} - |\Gamma| e^{j\beta z + |\Theta_r})(e^{j\beta z} - |\Gamma| e^{-(j\beta z + \Theta_r)})}$$

$$|V(z)| = V_0^+ \sqrt{1 + |\Gamma| e^{-(2j\beta z + |\Theta_r)} + |\Gamma| e^{j2\beta z + \Theta_r} + \Gamma^2)}$$

$$|V(z)| = V_0^+ \sqrt{1 + |\Gamma|^2 + |\Gamma|(e^{-(2j\beta z + \Theta_r)} + e^{(j2\beta z + \Theta_r)})}$$

$$|V(z)| = V_0^+ \sqrt{1 + |\Gamma|^2 + 2|\Gamma| cos(2\beta z + \Theta_r)} \tag{95}$$

The magnitude of the total voltage on the transmission line is given by Eq.95. It seems like a complicated function.

[6] repeated here as 90
[7] imaginary part of the complex propagation constant
[8] defined as the ratio of forward going voltage and current
[9] Prove this in Carthesian and Polar coordinate system

- Let's start from a simple case when the voltage reflection coefficient on the tranmission line is $\Gamma = 0$ and draw the magnitude of the total voltage.
  HERE PICTURE OF THE FLAT LINE
- Let's look at another case, $\Gamma = 0.5$ and $\Theta_r = 0$. The equation for the magnitude

  $$|V(z)| = V_0^+ \sqrt{\frac{5}{4} + cos 2\beta z} \tag{96}$$

  PICTURE OF STANDING WAVE PATERN FOR THIS
  The function 96 is at it's maximum when $cos(2\beta z) = 1$ or $z = \frac{k}{2}\lambda$, and the function value is $V(z) = 1.5V_0^+$. It is at it's minimum when $cos(2\beta z) = -1$ or $z = \frac{2k+1}{4}\lambda$ and the function value is $V(z) = 0.5V_0^+$
  It is important to mention here that the function that we see looks like a cosine with an average value of V_0^+, but **it is not**. The minimums of the function are sharper then the maximums, so when the reflection coefficient is at it's maximum of $\Gamma = 1$ the function looks like this:
  PICTURE OF STANDING WAVE PATTERN WITH SHORT
- General Case.
  In general the voltage maximums will occur when $cos(2\beta z) = 1$

  $$\begin{aligned} |V(z)_{max}| &= V_0^+ \sqrt{1 + \|Gamma|^2 + 2|\Gamma|} \\ |V(z)_{max}| &= V_0^+ \sqrt{(1 + \|Gamma|)^2} \\ |V(z)_{max}| &= V_0^+ (1 + |\Gamma|) \end{aligned} \tag{97}$$

  In general the voltage minimums will occur when $cos(2\beta z) = -1$,

  $$\begin{aligned} |V(z)_{min}| &= V_0^+ \sqrt{1 + \|Gamma|^2 - 2|\Gamma|} \\ |V(z)_{min}| &= V_0^+ \sqrt{(1 - \|Gamma|)^2} \\ |V(z)_{min}| &= V_0^+ (1 - |\Gamma|) \end{aligned} \tag{98}$$

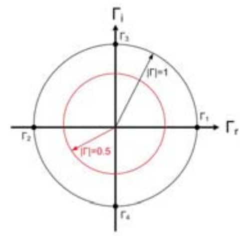
  The ratio of voltage minimum on the line over the voltage maximum is called the Voltage Standing Wave Ratio (VSWR) or just Standing Wave Ratio (SWR).

  $$\begin{aligned} SWR &= \frac{V(z)_{max}}{V(z)_{min}} \\ SWR &= \frac{1 + |\Gamma|}{1 - |\Gamma|} \end{aligned} \tag{99}$$

  The voltage maximum position on the line is where

  $$\begin{aligned} cos(2\beta z) &= 1 \\ 2\beta z + \Theta_r &= 2n\pi \\ z &= \frac{2n\pi - \Theta_r}{2\beta} \\ z &= \frac{2n\pi - \Theta_r}{4\pi} \end{aligned} \tag{100}$$

12.4 Smith Chart



$$\Gamma_1 = 1$$
$$\Gamma_2 = -1$$
$$\Gamma_3 = j = e^{j*90}$$
$$\Gamma_4 = -j = e^{-j*90}$$

Figure 27: All points on the circle have the constant magnitude of the reflection coefficient.

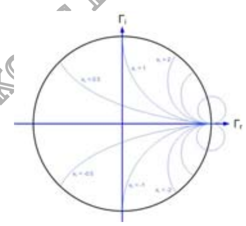


Figure 28: All points on the circle have the constant imaginary part of the impedance.

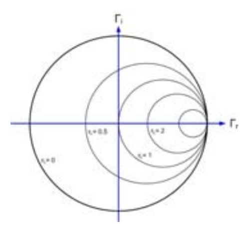


Figure 29: All points on the circle represent the constant real part of the impedance.

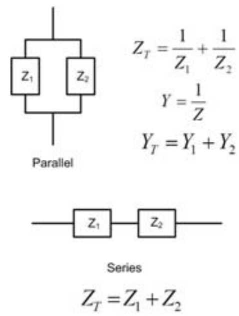


Figure 30: It is easier to use admittance when the circuit elements are in parallel and impedance when the circuit elements are in series.

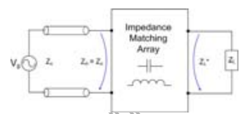


Figure 31: The result of impedance matching.

12.5 Brief review of impedance and admittance

12.6 Transmission Line Matching

13 Lecture 7

13.1 Reading

13.2 Purpose of the lecture and the main point

14 Lecture 8

14.1 Reading

14.2 Purpose of the lecture and the main point

15 Lecture 9

15.1 Reading

15.2 Purpose of the lecture and the main point

16 Lecture 10

16.1 Reading

16.2 Purpose of the lecture and the main point

17 Lecture 11

17.1 Reading

17.2 Purpose of the lecture and the main point

18 Lecture 12

18.1 Reading

18.2 Purpose of the lecture and the main point

19 Lecture 13

19.1 Reading

35 Lecture 29

PRESENTATION OF TERM PROJECTS

36 Lecture 30

PRESENTATION OF TERM PROJECTS