# Propagation constant and loss

## Lossless transmission line

In many practical applications, conductor loss is low $R \rightarrow 0$, and dielectric leakage is low $G \rightarrow 0$. These two conditions describe a lossless transmission line.

In this case, the transmission line parameters are

- Propagation constant

$$\begin{aligned}\gamma &= \sqrt{(R+j\omega L)(G+j\omega C)}\\ \gamma &= \sqrt{j\omega L j\omega C}\\ \gamma &= j\omega\sqrt{LC} = j\beta\end{aligned}$$

- Transmission line impedance will be defined in the next section, but it is also here for completeness.

$$\begin{aligned}Z_0 &= \sqrt{\frac{R+j\omega L}{G+j\omega C}}\\ Z_0 &= \sqrt{\frac{j\omega L}{j\omega C}}\\ Z_0 &= \sqrt{\frac{L}{C}}\end{aligned}$$

- Wave velocity

$$\begin{aligned}v &= \frac{\omega}{\beta}\\ v &= \frac{\omega}{\omega\sqrt{LC}}\\ v &= \frac{1}{\sqrt{LC}}\end{aligned}$$

---

Learning outcomes: Explain parts of propagation constant and what they represent. Calculate the phase and attenuation constant for specific transmission lines.

Author(s): Milica Markovic

- Wavelength

$$\lambda = \frac{2\pi}{\beta}$$
$$\lambda = \frac{2\pi}{\omega\sqrt{LC}}$$
$$\lambda = \frac{2\pi}{\sqrt{\varepsilon_0\mu_0\varepsilon_r}}$$
$$\lambda = \frac{c}{f\sqrt{\varepsilon_r}}$$
$$\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_r}}$$

## Voltage and current on lossless transmission line

On a lossless transmission line, where $\gamma = j\beta$ current and voltage simplify to

$$\tilde{V}(z) = \tilde{V}_0^+ e^{-j\beta z} + \tilde{V}_0^- e^{j\beta z}$$
$$\tilde{I}(z) = \tilde{I}_0^+ e^{-j\beta z} + \tilde{I}_0^- e^{j\beta z}$$

## What does it mean when we say a medium is lossy or lossless?

In a lossless medium, electromagnetic energy is not turning into heat; there is no amplitude loss. An electromagnetic wave is heating a lossy material; therefore, the wave's amplitude decreases as $e^{-\alpha x}$.

| medium | attenuation constant $\alpha$ [dB/km] |
|---|---|
| coax | 60 |
| waveguide | 2 |
| fiber-optic | 0.5 |

In guided wave systems such as transmission lines and waveguides, the attenuation of power with distance follows approximately $e^{-2\alpha x}$. The power radiated by an antenna falls off as $1/r^2$. As the distance between the source and load increases, there is a specific distance at which the cable transmission is lossier than antenna transmission.

## Low-Loss Transmission Line

This section is optional.

In some practical applications, losses are small, but not negligible. $R << \omega L$ and $G << \omega C$.

In this case, the transmission line parameters are

- Propagation constant

  We can re-write the propagation constant as shown below. In somel applications, losses are small, but not negligible. $R << \omega L$ and $G << \omega C$, then in Equation 2, $RG << \omega^2 LC$.

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \tag{1}$$

$$\gamma = j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right) - \frac{RG}{\omega^2 LC}} \tag{2}$$

$$\gamma \approx j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)} \tag{3}$$

  Taylor's series for function $\sqrt{1+x} = \sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)}$ in Equation 3 is shown in Equations 4-5.

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - ...\, for\, |x| < 1 \tag{4}$$

$$\gamma \approx j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)} = j\omega\sqrt{LC}\left(1 - \frac{j}{2}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)\right) \tag{5}$$

  The real and imaginary part of the propagation constant $\gamma$ are:

$$\alpha = \frac{\omega\sqrt{LC}}{2}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right) \tag{6}$$

$$\beta = \omega\sqrt{LC} \tag{7}$$

  We see that the phase constant $\beta$ is the same as in the lossless case, and the attenuation constant $\alpha$ is frequency independent. All frequencies

---

metal resistance is lower than the inductive impedance

dielectric conductance is lower than the capacitive impedance of a modulated signal are attenuated the same amount, and there is no dispersion on the line. When the phase constant is a linear function of frequency, $\beta = const\,\omega$, then the phase velocity is a constant $v_p = \frac{\omega}{\beta} = \frac{1}{const}$, and the group velocity is also a constant, and equal to the phase velocity. In this case, all frequencies of the modulated signal propagate at the same speed, and there is no distortion of the signal.

## Transmission-line parameters R, G, C, and L

To find the complex propagation constant $\gamma$, we need the transmission-line parameters R, G, C, and L. Equations for R, G, C, and L for a coaxial cable are given in the table below.

| Transmission-line | R | G | C | L |
|---|---|---|---|---|
| Coaxial Cable | $\frac{R_{sd}}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$ | $\frac{2\pi\sigma}{\ln b/a}$ | $\frac{2\pi\varepsilon}{\ln b/a}$ | $\frac{\mu}{2\pi}\ln b/a$ |

Where $R_{sd} = \sqrt{\pi f \mu_m/\sigma_m}$ is the resistance associated with skin-depth. $f$ is the frequency of the signal, $\mu_m$ is the magentic permeability of conductors, $\sigma_c$ is the conductivity of conductors.

**Example 1.** *Calculate capacitance per unit length of a coaxial cable if the inner radius is 0.02 m, the outer radius is 0.06 m, the dielectric constant is* $\varepsilon_r = 2$. *Use the applet below, Matlab, Matematica, or other software that you use.*

*Geogebra link:* `https://tube.geogebra.org/m/whkrg2pu`