Visualization of waves on lossless transmission lines

$$\tilde{V}(z) = \tilde{V}_0^+ e^{-\gamma z} + \tilde{V}_0^- e^{\gamma z}$$
$$\tilde{I}(z) = \tilde{I}_0^+ e^{-\gamma z} + \tilde{I}_0^- e^{\gamma z}$$

In this equation $\tilde{V}(z)$ is the total voltage anywhere on the line (at any point z), $\tilde{I}(z)$ is the total current anywhere on the line (at any point z), $\tilde{V_0}^+$ and $\tilde{V_0}^-$ are the **phasors** of forward and reflected voltage waves at the load (where z=0), and $\tilde{I_0}^+$ and $\tilde{I_0}^-$ are the phasors of forward and reflected current wave at the load (where z=0).These voltages and currents are also phasors and have a constant magnitude and phase in a specific circuit, for example $\tilde{V_0}^+ = |\tilde{V_0}^+|e^{\Phi} = 4e^{25^0}$, and $\tilde{I_0}^+ = |\tilde{I_0}^+|e^{\Phi} = 5e^{-40^0}$. We can get the time-domain expression for the current and voltage on the transmission line by multiplying the phasor of the voltage and current with $e^{j\omega t}$ and taking the real part of it.

$$v(t) = Re\{(\tilde{V}_0^+ e^{(-\alpha - j\beta)z} + \tilde{V}_0^- e^{(\alpha + j\beta)z})e^{j\omega t}\}$$
$$v(t) = |\tilde{V}_0^+|e^{-\alpha z}\cos(\omega t - \beta z + \angle\tilde{V}_0^+) + |\tilde{V}_0^-|e^{\alpha z}\cos(\omega t + \beta z + \angle\tilde{V}_0^-) \tag{1}$$

If the signs of the ωt and βz terms are oposite the wave moves in the forward $+z$ direction. If the signs of ωt and βz are the same, the wave moves in the $-z$ direction.

In the next several sections, we will look at how to find the constants β, $\tilde{V}_0^+$, $\tilde{V}_0^-$, $\tilde{I}_0^+$, $\tilde{I}_0^-$. In order to find the constants, we will introduce the concepts of transmission line impedance Z_0, reflection coefficient $\Gamma(z)$, input impedance Z_{in}.

Example 1. *We will show next that if the signs of the ωt and βz have the opposite sign, as in Equation 2, the wave moves in the forward $+z$ direction. If the signs of ωt and βz are the same, as in Equation 3, the wave moves in the $-z$ direction. In order to see this, we will visualize Equations 2 and 3 using Matlab code below.*

Learning outcomes: dentify whether the wave travels in the positive or negative direction from the equation of a wave. Describe how signal flows on a transmission line. Describe forward and reflected wave on a transmission line. Sketch forward and reflected wave as a function of distance, and explain how the graph changes as time passes.

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$$v_f(t) = |\tilde{V}_0^+|e^{-\alpha z}\cos(\omega t - \beta z + \angle\tilde{V}_0^+) \tag{2}$$

$$v_r(t) = |\tilde{V}_0^-|e^{\alpha z}\cos(\omega t + \beta z + \angle\tilde{V}_0^-) \tag{3}$$

Explanation. *Figure 1 shows forward and reflected waves on a transmission line. z represents the length of the line, and on the y-axis is the magnitude of the voltage. The red line on both graphs is the voltage signal at a time .1 ns. We would obtain Figure 1 if we had a camera that can take a picture of the voltage, and we took the first picture at t_1 = .1 ns on the entire transmission line. The blue dotted line on both graphs is the same signal .1 ns later, at time t_2 =.2 ns. We see that the signal has moved to the right in 1 ns, from the generator to the load. On the bottom graph, we see that at a time .1 ns, the red line represents the reflected signal. The dashed blue line shows the signal at a time .2 ns. We see that the signal has moved to the left, from the load to the generator.*

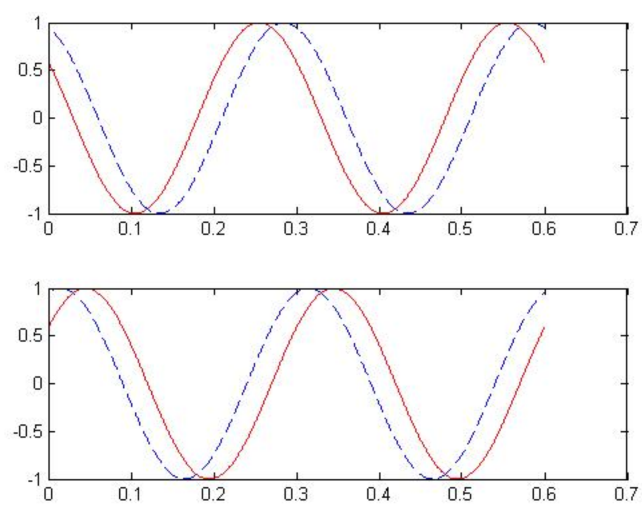

Figure 1: Forward (top) and reflected (bottom) waves on a transmission line.

```
clear all
clc
f = 10^9;
w = 2*pi*f
c=3*10^8;
beta=2*pi*f/c;
lambda=c/f;
t1=0.1*10^(-9)
```

```
t2=0.2*10^(-9)
x=0:lambda/20:2*lambda;

y1=sin(w*t1 - beta.*x);
y2=sin(w*t2 - beta.*x);
y3=sin(w*t1 + beta.*x);
y4=sin(w*t2 + beta.*x);

subplot (2,1,1),

    plot(x,y1,'r'),...
            hold on
    plot(x,y2,'--b'),...
    hold off
subplot (2,1,2),

    plot(x,y3,'r')
        hold on
    plot(x,y4,'--b')
    hold off
```

Using Matlab code above, repeat the visualization of signals in the previous section for a lossy transmission line. Assume that $\alpha = 0.1\,Np$, and all other variables are the same as in the previous section. How do the voltages compare in the lossy and lossless cases?

Question 1 *In the following simulation, we have three waves as a function of distance z. One is fixed $\cos(\beta z + 0^0)$ with a constant phase of 0^0, and for the other two signals the phase can be changed manually by changing the slider t that represents time. In the simulation, $\beta = 1$ and $\omega = 1$. This simulation is realistic only if time moves forward from 0 to 5. Observe how phase change ωt as the time increases from 0 to 5, then answer the question below.*

Geogebra link: `https://tube.geogebra.org/m/x5q7p7jx`

The sign in front of βz and ωt is opposite for the forward going wave.

Multiple Choice:

(a) *True* ✓

(b) *False*